Interfering doorway states and giant resonances. II. Transition strengths

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The mixing of the doorway components of a giant resonance (GR) due to the interaction via common decay channels influences significantly the distribution of the multipole strength and the energy spectrum of the decay products of the GR. The concept of the partial widths of a GR becomes ambiguous when the mixing is strong. In this case, the partial widths determined in terms of the K and S matrices must be distinguished. The photoemission turns out to be most sensitive to the overlapping of the doorway states. At high excitation energies, the interference between the doorway states leads to a restructuring towards lower energies and apparent quenching of the dipole strength. [S0556-2813(97)01807-4]

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I. INTRODUCTION

In [1] we investigated analytically as well as numerically the dipole giant resonance (GR) as a collective excitation in an open quantum system. In the energy domain of the GR, both internal (due to the Hermitian residual interaction) and external (due to the interaction via common decay channels) mixings are equally important. At the first stage, $k+1$ doorway states are formed, with k being the number of decay channels. These states inherit two different types of collectivity which are called, according to their origin, internal and external collectivity, respectively. The doorway resonances formed in such a manner still interfere with one another due to the external residual interaction. Finally a few resonance states with appreciable dipole strengths are formed. The interference gives, generally, rise to an essential redistribution of the dipole strength and shifts it towards lower energies.

The investigations show further that two of the resonance states share the main part of the total dipole strength and are therefore most responsible for the manifestations of the GR. The properties of these two doorway components of the GR crucially depend on the degree of their overlapping. In the case of weak overlapping they have comparable escape widths but the dipole strength of the lower lying state is small. Quite opposite, a large degree of overlapping leads to the appearence of two states with similar dipole strengths whereas the escape width of the lower lying state is dynamically reduced.

In the present paper, we study the cross section pattern in order to elucidate the role of the external interaction and the interplay of both types of collectivity in the experimentally measurable values. Of special interest are the transition strengths into specific channels when the interaction via the energy continuum is strong.

In Sec. II, we describe the overlapping of doorway resonances in the context of the general resonance scattering theory. The concept of the partial escape widths in the case of overlapping resonances is reexamined from both inside (K-matrix) and outside (T-matrix) viewpoints. The transition strengths in the particle channels are analytically analyzed in Sec. III. The photoemission, which turns out to be especially sensitive to the degree of overlapping of the doorway states, is studied in Sec. IV. In Sec. V, we discuss the interaction of the doorway states described in [1] with the background of complicated compound states which leads to an internal damping of the collective exitation. We show in Sec. VI some numerical results obtained in the same model (without internal damping), but with the restrictions being removed which were introduced into the analytical investigation. The numerical calculations confirm the main features of the interference between the different types of doorway states as they follow from the analytical study. Finally, we summarize the results in Sec. VII and draw some conclusions. Of interest is, above all, the apparent loss of the collective dipole strength at high excitation energy.

All symbols used in this paper are the same as in [1]. We cite an equation in [1] by writing its number in brackets with the upper index [1], e.g., $(2.1)^{[1]}$ means Eq. (2.1) in paper [1].

II. CROSS SECTIONS AND PARTIAL WIDTHS

In the vicinity of an isolated resonance state dw the Hermitian K matrix is represented in the form

$$\hat{K}(E)=\frac{\hat{\mathbf{A}}_{\rm dw}^{T}\hat{\mathbf{A}}_{\rm dw}}{E-E_{\rm dw}}, \qquad (2.1)$$

where the row vector $\hat{\mathbf{A}}_{\rm dw}$ is composed of the k real decay amplitudes $A^{c}_{\rm dw}$ of the doorway state into the individual channels $c=1,2,\ldots,k$ and the superscript T means transposition. The pole of this matrix lies on the real energy axis at the energy $E_{\rm dw}$ of the doorway state. Equation (2.1) leads to the standard single-resonance Breit-Wigner formula

$$\hat{T}_{\rm dw}(E)=\frac{\hat{K}(E)}{1+(i/2)\hat{K}(E)}=\frac{\hat{\mathbf{A}}_{\rm dw}^{T}\hat{\mathbf{A}}_{\rm dw}}{E-E_{\rm dw}+(i/2)\Gamma_{\rm dw}} \qquad (2.2)$$

for the transition matrix. Though the pole of the transition matrix is shifted to the point $E=\mathcal{E}_{\rm dw}=E_{\rm dw}-(i/2)\Gamma_{\rm dw}$ in the complex energy plane, both matrices have the same residues. In particular, the residues $\Gamma^c_{\rm dw}=(A^c_{\rm dw})^2$ of the diagonal elements of these matrices give the partial escape widths of the state dw relative to the channels c. The Hermiticity of the K matrix automatically provides the unitarity of the scattering matrix $\hat{S}(E)=I-i\hat{T}(E)$ implying the well-known connection

$$\Gamma_{\rm dw}=\hat{\mathbf{A}}^2_{\rm dw}=\sum_c \Gamma^c_{\rm dw} \tag{2.3}$$

between the total width, $\Gamma_{\rm dw}$, and the partial widths of the resonance dw. In what follows we omit all nonresonant effects. They can, if necessary, be easily taken into account by standard methods.

Using the parametrization (2.2), the partial widths of the resonance state can be extracted from the experimental data. Averaging the cross section of the reaction $c'\to c$ over all initial channels c', one obtains, with the help of the unitarity condition, the strength

$$\sigma^c(E)=-\frac{\sigma_0}{\pi}\mathrm{Im}T^{cc}_{\rm dw}(E)=\sigma_0\frac{1}{2\pi}\frac{\Gamma_{\rm dw}}{(E-E_{\rm dw})^2+(1/4)\Gamma^2_{\rm dw}}\Gamma^c_{\rm dw}$$

$$=\sigma_0\frac{2}{\pi}\frac{\Gamma^c_{\rm dw}}{\Gamma_{\rm dw}}\sin^2\delta(E) \tag{2.4}$$

of the transition into the channel c. Here, $\delta_{\rm dw}(E)$ defined by

$$\tan\delta_{\rm dw}(E)=-\frac{1}{2}\frac{\Gamma_{\rm dw}}{E-E_{\rm dw}} \tag{2.5}$$

is the resonance scattering phase. The factor

$$\sigma_0\frac{1}{2\pi}\frac{\Gamma_{\rm dw}}{(E-E_{\rm dw})^2+(1/4)\Gamma^2_{\rm dw}}$$

describes the total cross section of the doorway state excitation. Below we set the factor σ_0 to unity measuring all cross sections in units of this quantity. The maximal value

$$\sigma^c(E_{\rm dw})=\frac{2}{\pi}\frac{\Gamma^c_{\rm dw}}{\Gamma_{\rm dw}}\equiv\frac{2}{\pi}\mathcal{B}^c_{\rm dw} \tag{2.6}$$

of the transition strength (2.4) is proportional to the branching ratio of the decay into the channel c. The integration over the whole resonance region gives the partial width itself,

$$\int_{-\infty}^{\infty}dE\,\sigma^c(E)=\Gamma^c_{\rm dw}. \tag{2.7}$$

Due to Eq. (2.3) it follows from Eqs. (2.6) and (2.7) that

$$\frac{\pi}{2}\sum_c \sigma^c(E_{\rm dw})=1,\quad \sum_c\int_{-\infty}^{\infty}dE\,\sigma^c(E)=\Gamma_{\rm dw}. \tag{2.8}$$

The above discussion implies a good separation of the different resonance states dw so that any interference between them can be neglected. A more careful analysis is however needed when the widths of the relevant doorway states become comparable with their spacings. In this case one has to use the formulas of the general theory of resonance reactions [2–5]. Here, the transition matrix

$$\hat{T}(E)=A^T\frac{1}{E-\mathcal{H}}A \tag{2.9}$$

is composed of the three matrix factors which describe the formation of the intermediate unstable system, its propagation, and subsequent disintegration. If there are $N_{\rm dw}$ doorway resonance states near the excitation energy E coupled to k decay channels, the matrix A consists of k $N_{\rm dw}$-dimensional column vectors $\mathbf{A}^c$ connecting all internal states with each channel c. These vectors are real because of time-reversal invariance. In the following we neglect a possible smooth energy dependence of the components A^c_n over the whole energy domain considered. The validity of such an assumption is not always obvious and deserves a special consideration. It may lead to further complications.

The evolution of the intermediate open system is described by the Green's matrix

$$\mathcal{G}(E)=\frac{1}{E-\mathcal{H}} \tag{2.10}$$

corresponding to the non-Hermitian effective Hamiltonian

$$\mathcal{H}=H-\frac{i}{2}AA^T=H-\frac{i}{2}W \tag{2.11}$$

which has been investigated in detail in part I of this paper [1]. The factorized form of the interaction W via the continuum ensures the unitarity of the scattering matrix for arbitrarily overlapping resonances [4,5]. However, the simple Breit-Wigner parametrization (2.2) loses its validity in general.

The propagator $\mathcal{G}(E)$ of the unstable system satisfies the Dyson equation

$$\mathcal{G}(E)=G(E)-\frac{i}{2}G(E)W\mathcal{G}(E), \tag{2.12}$$

where

$$G(E)=\frac{1}{E-H} \tag{2.13}$$

is the resolvent of the Hermitian part H of the effective Hamiltonian (2.11). Subsequent iterations in the anti-Hermitian part of the effective Hamiltonian lead [6] to

$$\mathcal{G}(E)=G(E)-\frac{i}{2}G(E)A\frac{1}{1+(i/2)\hat{K}(E)}A^TG(E) \tag{2.14}$$

with

$$\hat{K}(E)=A^T\frac{1}{E-H}A=A^TG(E)A. \tag{2.15}$$

The relation (2.14) casts again [compare with the first equality in Eq. (2.2)] the transition matrix (2.9) into the explicitly unitary form

$$\hat{T}(E)=\frac{\hat{K}(E)}{1+(i/2)\hat{K}(E)}. \tag{2.16}$$

The elements of both the K- and T-channel space matrices

$$K^{cc'}(E)=\mathrm{Tr}[G(E)\mathbf{A}^c(\mathbf{A}^{c'})^T], \tag{2.17}$$

$$T^{cc'}(E)=\mathrm{Tr}[\mathcal{G}(E)\mathbf{A}^c(\mathbf{A}^{c'})^T], \tag{2.18}$$

being the traces in the Hilbert space of the internal motion, are independent of the choice of a basis in this space. In the eigenbasis of the intrinsic Hermitian part H of the effective Hamiltonian (2.11), the matrix $\hat{K}$ is presented as the sum

$$\hat{K}(E)=\sum_r \frac{\hat{\mathbf{A}}_r^T\hat{\mathbf{A}}_r}{E-\varepsilon_r} \tag{2.19}$$

of pole terms similar to the single-resonance expression (2.1). The row vectors $\hat{\mathbf{A}}_r$ consist of the real components

$$A_r^c=\mathbf{\Phi}^{(r)}\cdot\mathbf{A}^c, \tag{2.20}$$

where the eigenvector $\mathbf{\Phi}^{(r)}$ of the Hermitian matrix H belongs to the eigenenergy ε_r. The positive residues

$$\Gamma_r^c=(A_r^c)^2 \tag{2.21}$$

at the poles of the diagonal elements of the matrix (2.19), which characterize the coupling of the intrinsic state $\mathbf{\Phi}^{(r)}$ to the channels c, are the partial escape widths discussed in part I, Eq. (2.17)[1].

Analogously, the pole (resonance) parametrization of the transition matrix (2.9),

$$\hat{T}(E)=\sum_{\mathrm{dw}} \frac{\hat{\mathbf{A}}_{\mathrm{dw}}^T\hat{\mathbf{A}}_{\mathrm{dw}}}{E-\mathcal{E}_{\mathrm{dw}}}, \tag{2.22}$$

is achieved by diagonalizing the total effective Hamiltonian (2.11) (rather than only the Hermitian part H as above) with the help of a transformation Ψ which is complex since the Hamiltonian $\mathcal{H}$ is not Hermitian. Its complex eigenvalues

$$\mathcal{E}_{\mathrm{dw}}=E_{\mathrm{dw}}-\frac{i}{2}\Gamma_{\mathrm{dw}} \tag{2.23}$$

determine the energies and total widths of the overlapping resonance states. The decay amplitudes of these states are [compare with Eq. (2.20)]

$$A_{\mathrm{dw}}^c=\mathbf{\Psi}^{(\mathrm{dw})}\cdot\mathbf{A}^c \tag{2.24}$$

with $\Psi^{(\mathrm{dw})}$ being the eigenvectors of the effective Hamiltonian $\mathcal{H}$. Together with these eigenvectors, the residues at the resonance poles are also complex. Therefore, the resonances are mixed in the transition amplitudes with nonzero relative phases. In particular, the residues are equal to

$$(A_{\mathrm{dw}}^c)^2=|A_{\mathrm{dw}}^c|^2\exp(2i\phi_{\mathrm{dw}}^c) \tag{2.25}$$

in the elastic scattering amplitudes. Here, the resonance mixing phases ϕ_{dw}^c are introduced.

Unlike the case of an isolated resonance described by Eqs. (2.1) and (2.2), the residues of the K and T matrices at individual poles do not coincide if the doorway resonance states overlap. One can find the connection between the decay vectors $\hat{\mathbf{A}}_r$ and $\hat{\mathbf{A}}_{\mathrm{dw}}$ starting with the eigenvalue problem $\mathcal{H}\Psi^{(\mathrm{dw})}=\mathcal{E}_{\mathrm{dw}}\Psi^{(\mathrm{dw})}$ presented in the intrinsic eigenbasis of the Hermitian part H. Simple transformations lead then to the matrix equation

$$\left[I+\frac{i}{2}\hat{K}(\mathcal{E}_{\mathrm{dw}})\right]\hat{\mathbf{A}}_{\mathrm{dw}}=0. \tag{2.26}$$

The determinant $\det[I+(i/2)\hat{K}(\mathcal{E}_{\mathrm{dw}})]$ is equal to zero at any resonance pole $\mathcal{E}_{\mathrm{dw}}$ of the T matrix (2.16). Therefore, for each resonance dw, a nontrivial solution of the homogeneous linear system (2.26) exists. The proper solutions are finally fixed by the Bell-Steinberger relation (2.31) (see below).

The square moduli

$$\Gamma_{\mathrm{dw}}^c=|A_{\mathrm{dw}}^c|^2\equiv|\mathbf{\Psi}^{(\mathrm{dw})}\cdot\mathbf{A}^c|^2 \tag{2.27}$$

are just the quantities which are usually interpreted as the partial widths of the resonance state dw. In the case of overlapping resonances, these widths differ from the partial widths (2.21) defined in terms of the K matrix. Therefore we conclude that one has to distinguish between the T-matrix partial widths (TPW's) (2.27) extracted from the T matrix, and the K-matrix partial widths (KPW's) (2.21) drawn from the matrix $\hat{K}$.

The transformation matrix Ψ satisfies the matrix equation

$$\mathcal{H}\Psi=\Psi\mathcal{E}, \tag{2.28}$$

where $\mathcal{E}$ is the diagonal matrix of resonance energies $\mathcal{E}_{\mathrm{dw}}$. This transformation is complex orthogonal [6],

$$\Psi^T\Psi=\Psi\Psi^T=1. \tag{2.29}$$

However, for the Hermitian matrix

$$U=\Psi^{\dagger}\Psi \tag{2.30}$$

the inequality $U\neq I$ holds so that the overlapping resonance states are not orthogonal (for illustration see [7]). The matrix U appears in the well-known Bell-Steinberger relation [8] (see also a compact matrix version of this relation in [6]):

$$\hat{\mathbf{A}}_{\mathrm{dw}}^*\cdot\hat{\mathbf{A}}_{\mathrm{dw}'}=iU_{\mathrm{dw\,dw}'}(\mathcal{E}_{\mathrm{dw}'}-\mathcal{E}_{\mathrm{dw}}^*). \tag{2.31}$$

Its diagonal part gives the relation

$$\Gamma_{\mathrm{dw}}=\frac{1}{U_{\mathrm{dw}}}|\hat{\mathbf{A}}_{\mathrm{dw}}|^2=\frac{1}{U_{\mathrm{dw}}}\sum_c |A_{\mathrm{dw}}^c|^2 \tag{2.32}$$

between the total widths and TPW (2.27). Here

$$U_{\mathrm{dw}}=1+2\sum_n (\mathrm{Im}\Psi_n^{(\mathrm{dw})})^2>1 \tag{2.33}$$

is the corresponding diagonal matrix element of the matrix U. Because of Eqs. (2.32) and (2.33), the inequality condition

$$\Gamma_{\rm dw} < \sum_c \Gamma^c_{\rm dw} \tag{2.34}$$

holds in contrast to the equality (2.3) characteristic for an isolated resonance.

As it follows from Eq. (2.32), the TPW can be formally renormalized as

$$\tilde{\Gamma}^c_{\rm dw} = \frac{1}{U_{\rm dw}} \Gamma^c_{\rm dw}, \tag{2.35}$$

[9,7] leading to the equality

$$\Gamma_{\rm dw} = \sum_c \tilde{\Gamma}^c_{\rm dw} \tag{2.36}$$

also for overlapping resonances. It should be emphasized however that neither the $\Gamma^c_{\rm dw}$ nor the renormalized quantities $\tilde{\Gamma}^c_{\rm dw}$ coincide with the KPW Γ^c_r from Eq. (2.21) in the case of overlapping resonances. The only relation between them,

$$(\mathbf{A}^c)^2 = \sum_r \Gamma^c_r = \sum_{\rm dw} \Gamma^c_{\rm dw} \exp(2i\phi^c_{\rm dw}) \leqslant \sum_{\rm dw} \Gamma^c_{\rm dw}, \tag{2.37}$$

follows from the completeness of the sets of the corresponding eigenvectors. Similarly, the energies ε_r differ from the energies $E_{\rm dw}$ of the resonance eigenstates. In the second equality (2.37) additional phase factors appear in the sum over the resonance states. The imaginary part of this sum vanishes since the contributions of different resonances perfectly compensate one another.

Condition (2.37) results in the integral sum rules

$$\int_{-\infty}^{\infty} dE\,\sigma^c(E) = -\frac{1}{\pi}\int_{-\infty}^{\infty} dE\,{\rm Im}T^{cc}(E) = (\mathbf{A}^c)^2 = \sum_r \Gamma^c_r \tag{2.38}$$

and

$$\sum_c \int_{-\infty}^{\infty} dE\,\sigma^c(E) = {\rm Tr}W = \sum_{\rm dw} \Gamma_{\rm dw} \tag{2.39}$$

instead of Eqs. (2.7) and (2.8) which are valid for an isolated resonance. The integration is extended here over the whole energy region, occupied by the overlapping resonance states. Equation (2.38) leads to the sum of the KPW Γ^c_r, Eq. (2.21), rather than to the sum of the TPW $\Gamma^c_{\rm dw}$, Eq. (2.27). Therefore, one cannot learn much on the latter or even on their sum $\Sigma_{\rm dw}\Gamma^c_{\rm dw}$ from the integral (2.38) despite the expectation sometimes being expressed in the scientific literature. Still less information can be drawn from the maxima of the total cross section since their heights and positions are connected with the widths and energies of the overlapping resonances in a very complicated way. At last, Eq. (2.39) fixes only the sum of the total widths of all resonances.

A useful generalization of the sum rule (2.38) reads

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} dE\,{\rm Im}T^{cc'}(E) = \mathbf{A}^c \cdot \mathbf{A}^{c'} = X^{cc'}, \tag{2.40}$$

where the $k\times k$ matrix [10,6]

$$\hat{X} = A^T A \tag{2.41}$$

of the scalar products of the real amplitude vectors $\mathbf{A}^c$ appears.

III. TRANSITION AMPLITUDES AND PARTIAL TRANSITION STRENGTHS

Similar to Sec. III in [1] [see Eqs. (3.1)[1] and (3.2)[1]], we introduce the enlarged transition matrix

$$\hat{\mathcal{T}}(E) = \mathcal{A}^T \mathcal{G}(E) \mathcal{A}, \tag{3.1}$$

$$\mathcal{A} = (\mathbf{A}^0 \equiv \sqrt{2i}\mathbf{D}\ \ \mathbf{A}^1 \cdots \mathbf{A}^k), \tag{3.2}$$

containing along its main diagonal the function

$$\mathcal{T}^{00}(E) \equiv 2i\mathcal{P}(E) = 2i\mathbf{D}^T \mathcal{G}(E)\mathbf{D} \tag{3.3}$$

besides the $k\times k$ block $T(E)$, Eq. (2.9). The function $\mathcal{P}(E)$ together with

$$P(E) = \mathbf{D}^T G(E)\mathbf{D} \tag{3.4}$$

from (3.3)[1] is closely connected to the photoemission (see Sec. IV).

The Green's matrix $\mathcal{G}(E)$, Eq. (2.10), is therefore needed for the description of the evolution of the intermediate unstable system excited in reactions. In [1], a special doorway basis has been introduced which is adjusted to the strong coherent non-Hermitian interaction

$$\mathcal{H}^{(\rm int)} = \mathbf{D}\mathbf{D}^T - \frac{i}{2}W \tag{3.5}$$

[Eq. (4.1)[1]] by which the GR is created. In this basis, the $(k+1)\times(k+1)$ doorway block of the total Green's matrix is the only one which has to be calculated. The influence of the trapped states [1] is included in a self-energy matrix which contains the coupling between the doorway and trapped states. It manifests itself, as mentioned in [1], in the fine structure variations of the transition amplitudes in the energy region of the unperturbed parental levels. Neglecting this fine structure, one reduces the problem to the calculation of the Green's matrix $\mathcal{G}^{(\rm dw)}(E)$ of the doorway effective Hamiltonian

$$\mathcal{H}^{(\rm dw)} = \begin{pmatrix} \mathcal{H}^{(\rm coll)} & \chi^T \\ \chi & \tilde{\mathcal{H}} \end{pmatrix} \tag{3.6}$$

[Eq. (4.23)[1]].

The upper 2×2 block

$$\mathcal{H}^{(\rm coll)} = \begin{pmatrix} \varepsilon_0 + \sin^2\Theta\,\mathbf{D}^2 & \sin\Theta\cos\Theta\,\mathbf{D}^2 \\ \sin\Theta\cos\Theta\,\mathbf{D}^2 & \varepsilon_0 + \cos^2\Theta\,\mathbf{D}^2 \end{pmatrix} - \frac{i}{2}\langle\gamma\rangle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.7}$$

in Eq. (3.6) contains only two states which are strongly mixed by the competing internal and external interactions characterized by the parameters $\mathbf{D}^2$ and $\langle\gamma\rangle$, respectively. Here $\mathbf{D}$ is the N-dimensional vector of the dipole matrix elements, $\langle\gamma\rangle$ is the mean value of the nonzero eigenvalues γ^c of the external interaction matrix W (or, equivalently, of the eigenvalues of the matrix $\hat{X}$), and Θ stands for the angle between the dipole vector $\mathbf{D}$ and the k-dimensional Hilbert subspace spaned by the k decay vector $\mathbf{A}^c$. We mark this block by the subscript (coll) since only its eigenstates possess internal collectivity when the coupling χ is neglected.

The $(k-1)\times(k-1)$ block $\widetilde{\mathcal{H}}$ describes the $k-1$ doorway states with energies close to ε_0 and mean widths $\langle\gamma\rangle$. Contrary to the states of the first group, these states carry no internal collectivity.

The two groups of doorway states are coupled via the continuum by the anti-Hermitian interaction

$$\chi=-\frac{i}{2}(\mathbf{0w}), \tag{3.8}$$

which can be expected to be moderately weak. Its strength is characterized by the dispersion Δ_γ of the eigenvalues γ^c [see $(4.28)^{[1]}$].

Representing the doorway Green's matrix $\mathcal{G}^{(\mathrm{dw})}(E)$ in the block form complementary to Eq. (3.6), one obtains the following expression:

$$\mathcal{G}^{(\mathrm{coll})}(E)=\frac{1}{E-\mathcal{H}^{(\mathrm{coll})}-\mathcal{Q}(E)}$$

$$=\frac{1}{\Lambda(E)}\begin{pmatrix} E-\varepsilon_0-\cos^2\Theta\,\mathbf{D}^2+\frac{i}{2}\omega(E) & \sin\Theta\cos\Theta\,\mathbf{D}^2 \\ \sin\Theta\cos\Theta\,\mathbf{D}^2 & E-\varepsilon_0-\sin^2\Theta\,\mathbf{D}^2 \end{pmatrix} \tag{3.9}$$

for its upper collective block with the function $\Lambda(E)$ given by

$$\Lambda(E)\equiv(E-\varepsilon_0)(E-\varepsilon_{\mathrm{coll}})+\frac{i}{2}\omega(E)(E-\varepsilon_0-\sin^2\Theta\,\mathbf{D}^2)=0, \tag{3.10}$$

$$\omega(\mathcal{E})=\langle\gamma\rangle-\frac{i}{2}q(\mathcal{E}). \tag{3.11}$$

This result extends the formula for the Green's function $(3.11)^{[1]}$ of the internal collective vibration in a closed system to the consideration of decaying collective modes.

In the doorway picture just described the elements of the matrix (3.1) are presented as

$$\mathcal{P}(E)=\mathbf{D}^T\mathcal{G}^{(\mathrm{coll})}(E)\mathbf{D}=\mathbf{D}^2\,\frac{E-\varepsilon_0+(i/2)\sin^2\Theta\,\omega(E)}{\Lambda(E)}, \tag{3.12}$$

$$T^{cc'}(E)=T^{cc'}_{\mathrm{coll}}(E)+\widetilde{T}^{cc'}(E), \tag{3.13}$$

where

$$T^{cc'}_{\mathrm{coll}}(E)=\left(A_1^c-\frac{i}{2}q^c(E)\right)\left(A_1^{c'}-\frac{i}{2}q^{c'}(E)\right)\frac{E-\varepsilon_0-\sin^2\Theta\,\mathbf{D}^2}{\Lambda(E)} \tag{3.14}$$

and

$$\widetilde{T}^{cc'}(E)=\sum_\alpha\frac{A^c_\alpha A^{c'}_\alpha}{E-\widetilde{\mathcal{E}}_\alpha}. \tag{3.15}$$

The quantities A_1^c, A_α^c [Eq. $(4.10)^{[1]}$] are the components of the (real) decay vectors $\mathbf{A}^c$ in the doorway basis. It is worth noting that the collective parts of the transition amplitudes vanish at the energy

$$E_v=\varepsilon_0+\sin^2\Theta\,\mathbf{D}^2. \tag{3.16}$$

The amplitudes (3.15), being sums of independent Breit-Wigner terms, themselves contain no interference effects. Indeed, all A^c_α, which connect the states inside the lower block of the Hamiltonian (3.6) to the continuum, are real and [as one can easily check with the help of Eqs. $(4.36)^{[1]}-(4.38)^{[1]}$]

$$\sum_c (A^c_\alpha)^2=\widetilde{\gamma}^\alpha. \tag{3.17}$$

All interference effects are included in the collective part (3.14). In particular, the mixing of the two different groups of the doorway resonances in Eq. (3.6) is described by the self-energy function

$$q(E) \equiv -4\mathcal{Q}_{11}(E) = \sum_{\alpha} \frac{w^{(\alpha)2}}{E - \tilde{\mathcal{E}}_{\alpha}} \tag{3.18}$$

[Eq. (4.40)[1]], and the functions

$$q^c(E) = \sum_{\alpha} \frac{w^{(\alpha)} A_{\alpha}^c}{E - \tilde{\mathcal{E}}_{\alpha}}. \tag{3.19}$$

All these functions are complex because of the complex doorway energies $\tilde{\mathcal{E}}_{\alpha}$. Therefore, although the dependence on the channel indices c, c' in the collective part (3.14) has the desirable factorized form, the factors are generally complex and energy dependent. As a result, the locations of the maxima in the cross sections are not connected, contrary to the case of isolated resonances, with the positions and the residues of the poles of the K or T matrices in any simple way. If however the collective resonances do not overlap too strongly, all the functions $q(E)$ vary slowly within the energy region of the maximum arising from the giant resonance state and can approximately be considered as some complex constants.

The residues of the elastic reaction amplitudes are expressed in terms of the complex energies of the doorway resonances as

$$\mathrm{Res} T^{cc}(\mathcal{E}_{\mathrm{dw}}) = \left(A_1^c - \frac{i}{2} q^c(\mathcal{E}_{\mathrm{dw}}) \right)^2 \left[1 + \frac{1}{4} \sin^2 2\Theta \, \frac{\mathbf{D}^4}{(\mathcal{E}_{\mathrm{dw}} - \varepsilon_0 - \sin^2\Theta \, \mathbf{D}^2)^2} + \frac{1}{4} q'(\mathcal{E}_{\mathrm{dw}}) \right]^{-1}. \tag{3.20}$$

In contrast to the real residues (2.21) of the K matrix, they are complex and carry information, hidden in the quantities q^c, on the transition vectors $\hat{\mathbf{A}}_r$ of all the overlapping resonance states. The concept of the T-matrix partial widths of GR's, generally, becomes irrelevant when its doorway components strongly overlap. The only information on the partial widths which one can extract from the experimentally observed transition strengths $\sigma^c(E)$ is the sum rule (2.38) for the KPW.

The above formulas simplify appreciably if one neglects the coupling $\mathbf{w}$ between the two doorway blocks in Eq. (3.6). In such an approximation only the two upper collective doorway states dw=0, 1, described in detail in Sec. IV C of [1], share the total dipole strength and contribute in the GR. The energy dependence of the corresponding collective part

$$\sigma_{\mathrm{coll}}^c(E) = \frac{1}{2\pi} (A_1^c)^2 \langle \gamma \rangle \frac{(E - E_v)^2}{(E - \varepsilon_0)^2 (E - \varepsilon_{\mathrm{coll}})^2 + (1/4) \langle \gamma \rangle^2 (E - E_v)^2} \tag{3.21}$$

of the total strength

$$\sigma^c(E) = -\frac{1}{\pi} \mathrm{Im} T^{cc}(E) = \sigma_{\mathrm{coll}}^c(E) + \tilde{\sigma}^c(E) \tag{3.22}$$

of the transition into a particular decay channel c turns out to have the same universal form as in the single-channel model of [11]. In this respect, expression (3.21) is analogous to the universal Breit-Wigner formula (2.4). According to (4.13)[1], the condition

$$\sum_c (A_1^c)^2 = \langle \gamma \rangle \tag{3.23}$$

is satisfied [compare Eq. (2.3)].

The Hermitian K matrix reduces in the same approximation to [1]

$$\hat{K}(E) = \frac{\hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d}{E - \varepsilon_{\mathrm{coll}}} + \frac{\hat{X}_{\perp}}{E - \varepsilon_0} \tag{3.24}$$

with

$$A_d^c = (\mathbf{d} \cdot \mathbf{A}^c), \quad \hat{X}_{\perp} = \hat{X} - \hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d. \tag{3.25}$$

The strengths (3.21) reveal two equally high maxima

$$\sigma_{\mathrm{coll}}^c(\varepsilon_0) = \sigma_{\mathrm{coll}}^c(\varepsilon_{\mathrm{coll}}) = \frac{2}{\pi} \frac{(A_1^c)^2}{\langle \gamma \rangle} \tag{3.26}$$

just at the poles of the K matrix (3.24). Taking Eq. (3.23) into account, these relations are quite similar to Eq. (2.6). Further, in close analogy with the first equation in Eq. (2.8),

$$\frac{\pi}{2} \sum_c \sigma_{\mathrm{coll}}^c(\varepsilon_0) = \frac{\pi}{2} \sum_c \sigma_{\mathrm{coll}}^c(\varepsilon_{\mathrm{coll}}) = 1. \tag{3.27}$$

Nevertheless, at arbitrary values of the overlapping parameter λ, the quantities $(A_1^c)^2$ coincide neither with KPW nor with TPW. They are not the residues at the poles of the K or T matrices and therefore cannot be ascribed to any internal eigenstates.

It could seem that the situation is improved by writing, for example,

$$\sigma_{\mathrm{coll}}^c(\varepsilon_{\mathrm{coll}}) = \frac{2}{\pi} \frac{(A_d^c)^2}{\langle \gamma \rangle \cos^2\Theta} = \frac{2}{\pi} \frac{\Gamma_{\mathrm{coll}}^c}{\langle \gamma \rangle \cos^2\Theta}. \tag{3.28}$$

Here, $\Gamma_{\mathrm{coll}}^c \equiv (A_d^c)^2 = (A_1^c)^2 \cos^2\Theta$ are the KPW of the intrinsic collective state with the energy $\varepsilon_{\mathrm{coll}}$ while in the denominator the sum of all the widths, Eq. (3.23), stands. The same is valid for the KPW $\Gamma_0^c = (A_1^c)^2 \sin^2\Theta$ of the intrinsic eigenstate with the energy ε_0. Nevertheless, it should be stressed that the right-hand side (RHS) in Eq. (3.28) is not the standard branching ratio since the denominator $\langle\gamma\rangle\cos^2\Theta$ generally has nothing to do with the total width of the corresponding doorway state [1]. Only in the limit $\lambda\ll 1$ of a very weak overlapping is this condition fulfilled and the maxima of the collective transition strengths provide the ordinary branching ratios $\mathcal{B}^c_{\mathrm{dw}=0,1}$, Eq. (2.6), of the isolated doorway states dw=0, 1 (4.52)[1].

However, the ratios of the KPW are

$$\frac{\Gamma^c_r}{\Gamma^{c'}_r}=\frac{\sigma^c_{\mathrm{coll}}(\varepsilon_r)}{\sigma^{c'}_{\mathrm{coll}}(\varepsilon_r)},\quad r=0,\mathrm{coll} \tag{3.29}$$

independently of the value of λ. Thus, we conclude that the parameters of the K matrix can be directly extracted from the maxima of the collective part of the transition strengths σ^c.

The transition strengths (3.21) drop to zero at the point $E=E_v$, Eq. (3.16), which lies in between the two maxima. The maxima are therefore well separated and their widths on the half heights may be introduced in the two-level approximation. They can be explicitly found from Eq. (3.21) to be

$$\Gamma_{0;1/2}=\frac{1}{2}\left[1-\frac{1}{2}\left(\sqrt{1+\frac{4}{\lambda^2}+\frac{4}{\lambda}\cos 2\Theta}-\sqrt{1+\frac{4}{\lambda^2}-\frac{4}{\lambda}\cos 2\Theta}\right)\right]\langle\gamma\rangle \tag{3.30}$$

and

$$\Gamma_{1;1/2}=\frac{1}{2}\left[1+\frac{1}{2}\left(\sqrt{1+\frac{4}{\lambda^2}+\frac{4}{\lambda}\cos 2\Theta}-\sqrt{1+\frac{4}{\lambda^2}-\frac{4}{\lambda}\cos 2\Theta}\right)\right]\langle\gamma\rangle. \tag{3.31}$$

Although the sum

$$\Gamma_{0;1/2}+\Gamma_{1;1/2}=\Gamma_{\mathrm{dw}=0}+\Gamma_{\mathrm{dw}=1}=\langle\gamma\rangle \tag{3.32}$$

depends neither on λ, nor on Θ, each of the terms of the sum does depend on the degree of overlapping. Thus, the ratios

$$\frac{(A^c_d)^2}{\Gamma_{\mathrm{dw};1/2}} \tag{3.33}$$

do not characterize individual resonance states and cannot be interpreted as their branching ratios.

The same is valid for the TPW. In the two-level approximation, the residues (3.20) at the poles $\mathcal{E}_{\mathrm{dw}=0,1}$ can be presented in a very simple form:

$$\mathrm{Res}T^{cc}(\mathcal{E}_{\mathrm{dw}})=\frac{(A^c_1)^2}{\langle\gamma\rangle}\Gamma_{\mathrm{dw}}\frac{\mathcal{E}_{\mathrm{dw}}-\mathcal{E}^*_{\mathrm{dw}'}}{\mathcal{E}_{\mathrm{dw}}-\mathcal{E}_{\mathrm{dw}'}}. \tag{3.34}$$

This gives

$$\Gamma^c_{\mathrm{dw}}=\frac{(A^c_1)^2}{\langle\gamma\rangle}\Gamma_{\mathrm{dw}}\sqrt{\frac{1+[\tan\delta_{\mathrm{dw}}(E_{\mathrm{dw}'})-\tan\delta_{\mathrm{dw}'}(E_{\mathrm{dw}})]^2}{1+[\tan\delta_{\mathrm{dw}}(E_{\mathrm{dw}'})+\tan\delta_{\mathrm{dw}'}(E_{\mathrm{dw}})]^2}} \tag{3.35}$$

for the TPW of the collective states. Here, $\delta_{\mathrm{dw}}(E_{\mathrm{dw}'})$ is the scattering phase (2.5) on the resonance dw taken at the energy of the resonance dw$'$. These phases vanish only when the resonances are well isolated.

The last factor on the RHS of Eq. (3.35) is just the diagonal matrix element U_{dw}, Eq. (2.33), of the Bell-Steinberger nonorthogonality matrix (2.30). Using the results of Sec. IV C of [1], one can present the latter factor explicitly in terms of the mixing parameters Θ and λ,

$$U_{\mathrm{dw}=0,1}=\frac{1}{\sqrt{2}}\left[1+\frac{1+(1/4)\lambda^2}{\sqrt{[1-(1/4)\lambda^2]^2+\lambda^2\cos^2 2\Theta}}\right]^{1/2}. \tag{3.36}$$

In both limiting cases, $\lambda\ll 2$ and $\lambda\gg 2$, this factor goes to unity while it is maximal in the intermediate region of $\lambda\approx 2$. In particular, for $\lambda=2$

$$U_{0,1}=\begin{cases}\dfrac{1}{\sqrt{1-\tan^2\Theta}}, & 0<\Theta<\dfrac{\pi}{4},\\[2ex] \dfrac{1}{\sqrt{1-\cot^2\Theta}}, & \dfrac{\pi}{4}<\Theta<\dfrac{\pi}{2}.\end{cases} \tag{3.37}$$

The quantity (3.37) becomes infinite for $\Theta=\pi/4$ as mentioned in [1].

The factor U disappears from the ratios

$$\frac{\Gamma^c_{\mathrm{dw}}}{\Gamma^{c'}_{\mathrm{dw}}}=\frac{\sigma^c_{\mathrm{coll}}(\varepsilon_r)}{\sigma^{c'}_{\mathrm{coll}}(\varepsilon_r)}=\frac{\Gamma^c_r}{\Gamma^{c'}_r} \tag{3.38}$$

of the TPW while the sum of Γ^c_{dw}

$$\sum_c \Gamma^c_{\mathrm{dw}}=\Gamma_{\mathrm{dw}}U_{\mathrm{dw}} \tag{3.39}$$

depends, contrary to the sums of the KPW, on the degree of overlapping via the Bell-Steinberger factor U.

It has been shown in [9] that the energy spectrum of the decay products of an arbitrary two-level unstable system can generally be expressed in terms of the resonance energies $\mathcal{E}_{0,1}$, the T-matrix ''partial widths'' $\tilde{\Gamma}^c_{\mathrm{dw}}$, Eq. (2.35), which are renormalized due to overlapping, and one additional real mixing parameter which satisfies a sum rule following from the Bell-Steinberger relation (2.31). The situation is even simpler in our quasi-single-channel case [see the remark below Eq. (3.21)] where the latter parameter is easily found explicitly [9] as a function of the complex resonance energies. The resulting expression is remarkably simple,

$$\sigma^c_{\mathrm{coll}}(E)=\frac{2}{\pi}\frac{\tilde{\Gamma}^c_{\mathrm{dw}}}{\Gamma_{\mathrm{dw}}}\sin^2[\delta_0(E)+\delta_1(E)]. \tag{3.40}$$

[Note that, due to Eq. (3.35), the ratio $\tilde{\Gamma}^c_{\mathrm{dw}}/\Gamma_{\mathrm{dw}}$ is really the same for both doorway states dw=0, 1.] This yields for the transition strengths at the energy of a doorway resonance

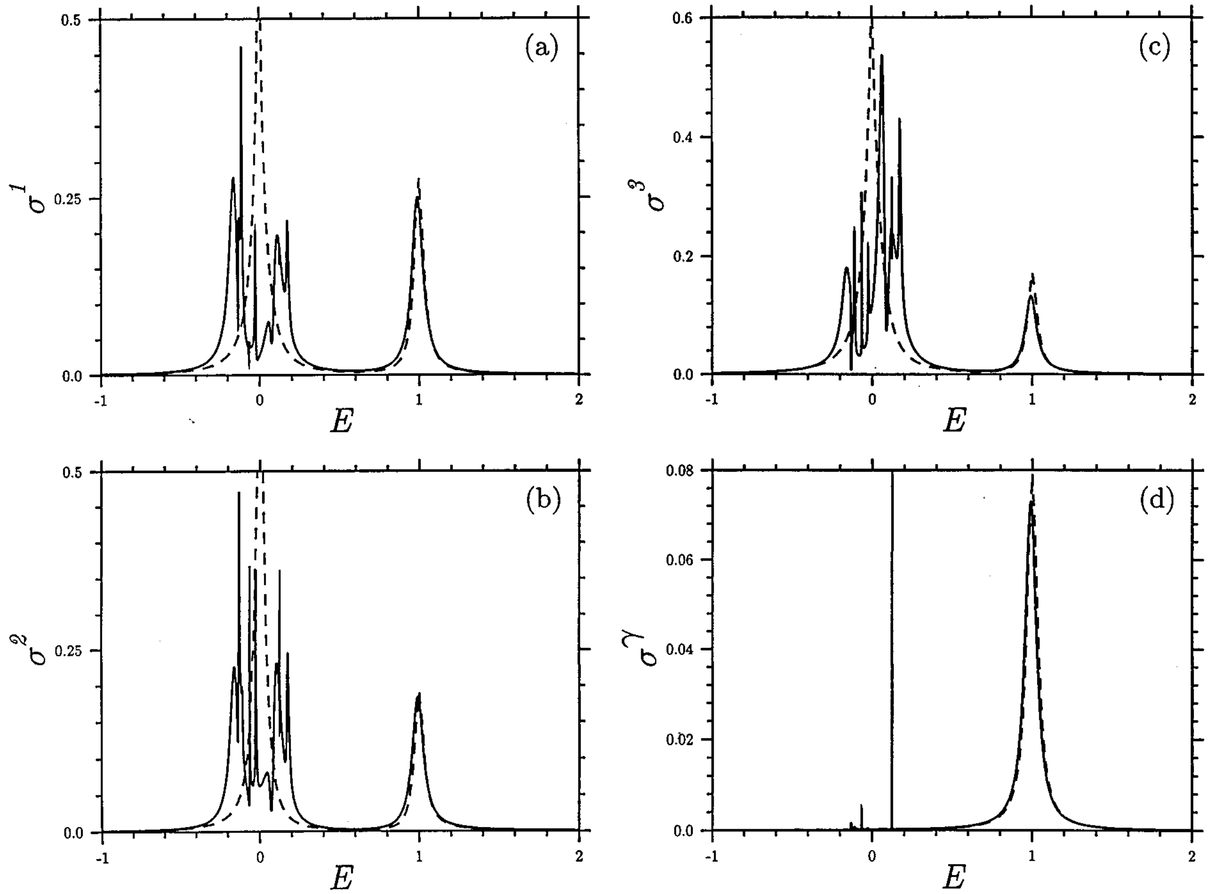

FIG. 1. The transition strengths into particle (a)–(c) and photo (d) channels for $\lambda=0.1$ and the electromagnetic interaction strength $\alpha_{\rm el}=0.01$. The resonance states are the same as in Fig. 2 in [1]. The dashed lines correspond to the case of parental levels fully degenerated ($\Delta_e=0$).

$$\sigma^c_{\rm coll}(E_{\rm dw})=\frac{2}{\pi}\frac{\tilde{\Gamma}^c_{\rm dw}}{\Gamma_{\rm dw}}\cos^2\delta_{\rm dw'}(E_{\rm dw}) \tag{3.41}$$

instead of Eq. (2.6) for an isolated resonance. The transition strengths do not attain their maximal values at the resonance energies when the resonances overlap. For this reason we have, in particular, for the first sum rule in Eq. (2.8)

$$\frac{\pi}{2}\sum_c \sigma^c(E_{\rm dw})=\cos^2\delta_{\rm dw'}(E_{\rm dw})<1. \tag{3.42}$$

One can easily convince oneself that both phases $\delta_{\rm dw'}(E_{\rm dw})$ drop to zero when $\lambda\ll 2$ and the resonances are isolated. However, in the opposite case of $\lambda\gg 2$ only the phase $\delta_0(E_1)$ of the narrow resonance is small. The other phase, $\delta_1(E_0)$, belonging to the level with the large width $\sim\langle\gamma\rangle$ is close to $\pi/2$. The cross section (3.21) has a narrow dip at the energy $E=E_v$ of the state dw=0. In the limit of very large λ the narrow state decouples and gets invisible in the particle cross sections. At the same time, this state acquires a large dipole strength due to the external interaction [1] and brightly manifests itself in the photochannel.

IV. PHOTOEMISSION

The process of photoemission by the collective states turns out to be most sensitive to their interference. To take the electromagnetic radiation into account, one has to add to the anti-Hermitian part of the effective Hamiltonian $\mathcal{H}$ the new term

$$-\frac{i}{2}W_{\rm el}=-\frac{i}{2}\alpha_{\rm el}\mathbf{D}\mathbf{D}^T \tag{4.1}$$

describing the radiation of the same multipolarity as the internal coupling vector $\mathbf{D}$. Therefore, the corresponding external coupling amplitude

$$\mathbf{A}^{(\rm rad)}=\sqrt{\alpha_{\rm el}}\mathbf{D} \tag{4.2}$$

is proportional to this vector with the constant $\alpha_{\rm el}$ characterizing the strength of the electromagnetic interaction.

The elastic matrix element of the K matrix in the photochannel is equal to

$$K^\gamma(E)=(\mathbf{A}^{(\rm rad)})^T G(E)\mathbf{A}^{(\rm rad)}=\alpha_{\rm el}P(E) \tag{4.3}$$

[see Eqs. (3.3) and (3.5)[1]]. The radiation KPW are therefore proportional to the dipole strengths $f^r=(\mathbf{d}\cdot\mathbf{\Phi}^{(r)})^2$, Eq. (3.15)[1], of the intrinsic eigenstates $\mathbf{\Phi}^{(r)}$,

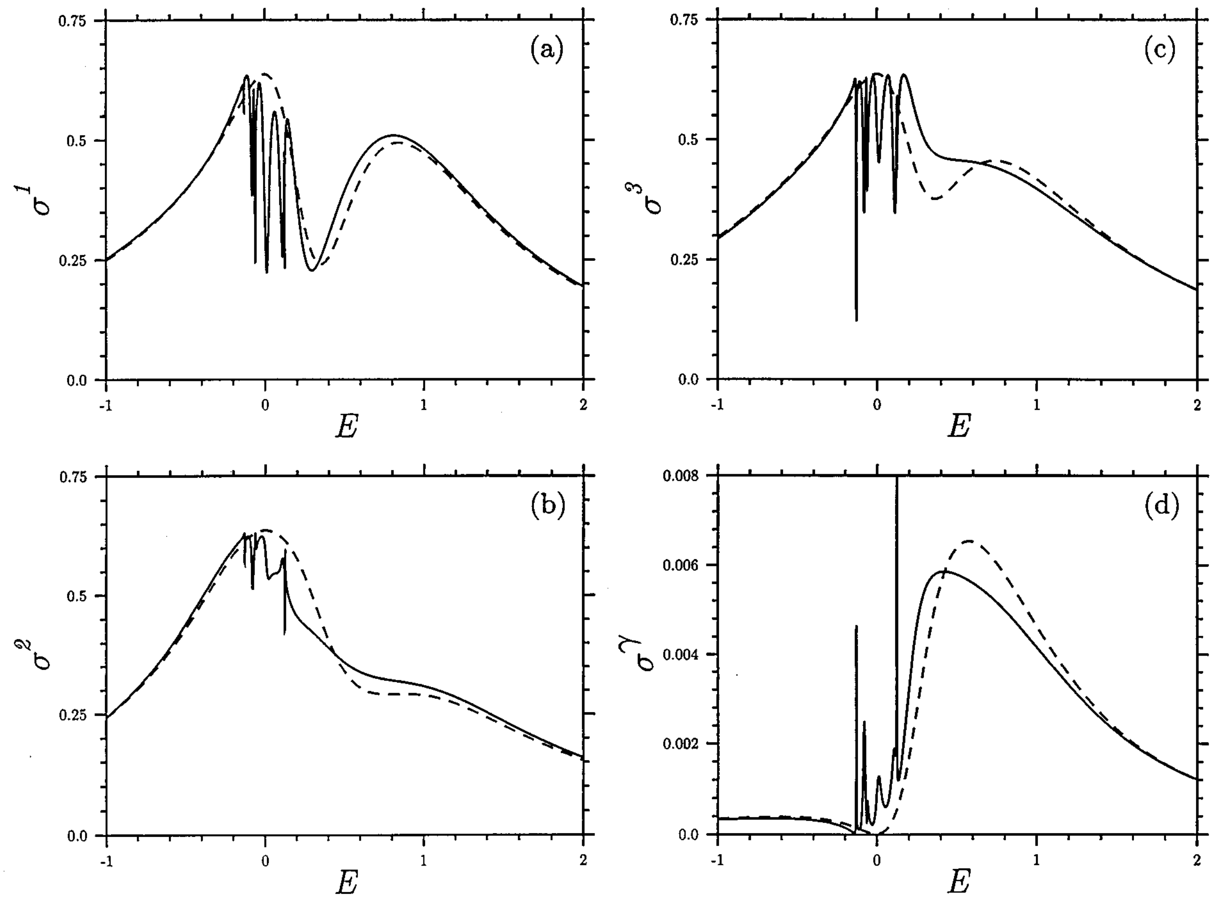

FIG. 2. The same as in Fig. 1 but for $\lambda=2$.

$$\Gamma_r^{(\mathrm{rad})}=\alpha_{\mathrm{el}}\mathbf{D}^2 f^r. \tag{4.4}$$

Since, according to Eq. (3.24)[1],

$$f^1=1-\kappa^2, \quad f^r\sim\frac{\kappa^2}{N-1} \quad (r\neq 1), \tag{4.5}$$

one can immediately see that, in the limit of small κ, the internal collective state appropriates the main part of the total radiation width $\alpha_{\mathrm{el}}\mathbf{D}^2$. When $\kappa\to 0$, only the pole at the energy $\varepsilon_{\mathrm{coll}}$ survives in the radiation K-matrix element.

The photoemission from the GR depends however upon the dipole strengths $\tilde{f}^s$ of the unstable doorway states $\mathbf{\Psi}^{(s)}$, Eqs. (4.29)[1], (4.31)[1], and (4.32)[1], rather than upon the intrinsic quantities (4.5). It is easy to see that the photoelastic scattering amplitude is obtained from the function $\mathcal{P}(E)$, [Eqs. (3.3) and (3.12)], by substituting $\mathbf{D}^2$ by $[1-(i/2)\alpha_{\mathrm{el}}]\mathbf{D}^2$ when calculating the collective Green's matrix (3.9). In the two-level approximation, this leads to the result

$$\sigma^{(\mathrm{rad})}(E)=\frac{1}{2\pi}\alpha_{\mathrm{el}}\mathbf{D}^2\langle\gamma\rangle\frac{(E-\varepsilon_0)^2(\cos^2\Theta+\alpha_{\mathrm{el}}/\lambda)+(1/4)\alpha_{\mathrm{el}}\mathbf{D}^2\langle\gamma\rangle\sin^4\Theta}{[(E-\varepsilon_0)(E-\varepsilon_{\mathrm{coll}})-(1/4)\alpha_{\mathrm{el}}\mathbf{D}^2\langle\gamma\rangle\sin^2\Theta]^2+(1/4)\langle\gamma\rangle^2[(1+\alpha_{\mathrm{el}}/\lambda)(E-\varepsilon_0)-\sin^2\Theta\,\mathbf{D}^2]^2}. \tag{4.6}$$

For small values of the parameter λ, the principal maximum of the photoemission strength lies at the energy $\varepsilon_{\mathrm{coll}}$. Near this point the expression (4.6) reduces to the standard Breit-Wigner cross section

$$\sigma^{(\mathrm{rad})}(E)=\frac{\Gamma_{\mathrm{GR}}^{(\mathrm{tot})}}{(E-\varepsilon_{\mathrm{coll}})^2+(1/4)[\Gamma_{\mathrm{GR}}^{(\mathrm{tot})}]^2}\Gamma_{\mathrm{GR}}^{(\mathrm{rad})} \tag{4.7}$$

with the radiation and total widths

$$\Gamma_{\mathrm{GR}}^{(\mathrm{rad})}=\alpha_{\mathrm{el}}\mathbf{D}^2, \quad \Gamma_{\mathrm{GR}}^{(\mathrm{tot})}=\langle\gamma\rangle\cos^2\Theta+\alpha_{\mathrm{el}}\mathbf{D}^2, \tag{4.8}$$

respectively. The giant resonance is formed in this case by the sole doorway state dw=1 with $f^1=\tilde{f}^1=1$. With growing

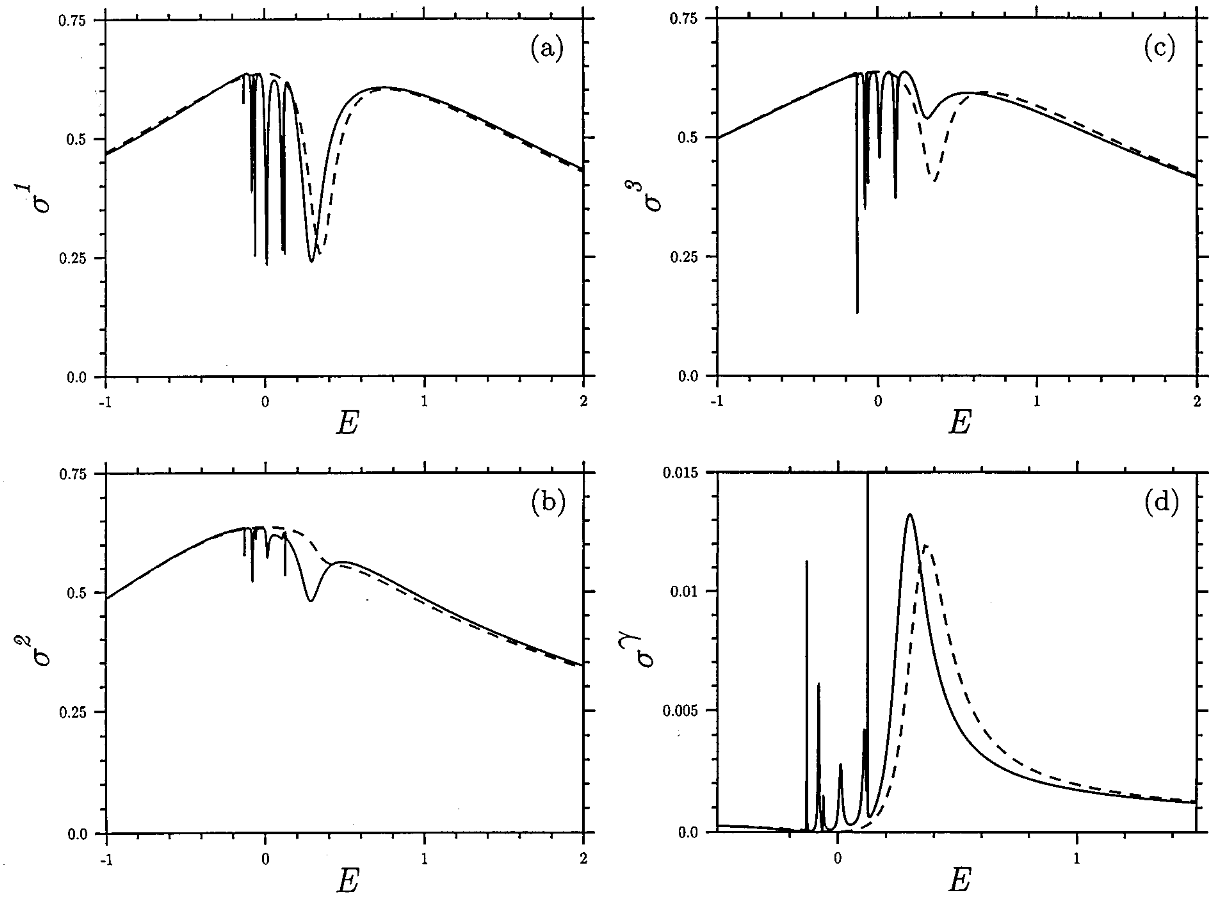

FIG. 3. The same as in Fig. 1 but for $\lambda=5$. Note the different E scale in (d).

λ, the radiation branching ratio $\mathcal{B}^{(\mathrm{rad})}=\Gamma_{\mathrm{GR}}^{(\mathrm{rad})}/\Gamma_{\mathrm{GR}}^{(\mathrm{tot})}$ decreases as long as λ does not approach the critical value 2.

The picture changes noticeably for very large values of λ ($\gg 2$). The main maximum is displaced to the point $E=E_v$, Eq. (3.16), where the transition strengths into the particle channels have an interference dip due to the narrow collective state dw=0. The energy dependence is of Breit-Wigner shape but the radiation and total widths become equal to

$$\Gamma_0^{(\mathrm{rad})}=\alpha_{\mathrm{el}}\mathbf{D}^2\sin^2\Theta=\alpha_{\mathrm{el}}\mathbf{D}^2\tilde{f}^0,$$

$$\Gamma_0^{(\mathrm{tot})}=\frac{1}{\lambda^2}\langle\gamma\rangle\sin^2 2\Theta+\alpha_{\mathrm{el}}\mathbf{D}^2\sin^2\Theta. \tag{4.9}$$

The peak contains only the part $\sin^2\Theta$ of the total radiation transition strength. It is naturally ascribed to the collective state dw=0 which acquired the dipole strength $\tilde{f}^0=\sin^2\Theta$ [see Eq. (4.55)[11]], due to the interaction via continuum. The nucleon width of this state diminishes and the radiation branching ratio $\mathcal{B}^{(\mathrm{rad})}$ increases together with λ. Therefore, the radiation appears as a narrow line near the centroid of the broad resonance dw=1 which is visible only in the particle channels. The radiation from this broad collective state is suppressed and manifests itself only as a long tail which stretches towards higher energies. The radiation from the narrow state dw=0 becomes therefore the brightest manifestation of the giant resonance in the photoemission.

In the most interesting intermediate domain of parameters $\alpha_{\mathrm{el}}\ll\lambda\ll 1/\alpha_{\mathrm{el}}$ the photoemission strength is

$$\sigma^{(\mathrm{rad})}(E)\approx-\frac{\alpha_{\mathrm{el}}}{2\pi}\mathrm{Im}\mathcal{P}(E). \tag{4.10}$$

The interference of the radiation from the two resonances is strongest when $\lambda\approx 2$. The frequency spectrum of the radiation is broad in this case, its characteristic width is $\sim\mathbf{D}^2$ and the radiation intensity remains small even in its maximum. Generally, the shape of the spectrum is not Lorenzian when $\lambda\approx 2$.

V. SPREADING WIDTH

We now discuss the interaction of the collective modes with the sea of the complicated background states. The spectrum of the background states is extremely dense at high excitations so that statistical methods are the only relevant ones to use in this case. As in [11], we suggest that the doorway states couple effectively to $N_{\mathrm{bg}}\gg N_{\mathrm{dw}}$ compound states which lie in the energy domain of the GR and have no direct access to the continuum. We also assume that the coupling matrix elements $V_{\mathrm{dw\,bg}}$ are random Gaussian variables with zero mean value. Then, after averaging over the background fluctuations, the doorway Green's function changes in the limit $N_{\mathrm{bg}}\to\infty$ as $\mathcal{G}^{(\mathrm{dw})}(E)\to\mathcal{G}^{(\mathrm{dw})}[E-\Delta+(i/2)\Gamma^{\downarrow}]$ [11] where Δ and $\Gamma^{\downarrow}$ are the energy shift and spreading width, respectively. Neglecting their possible slow energy

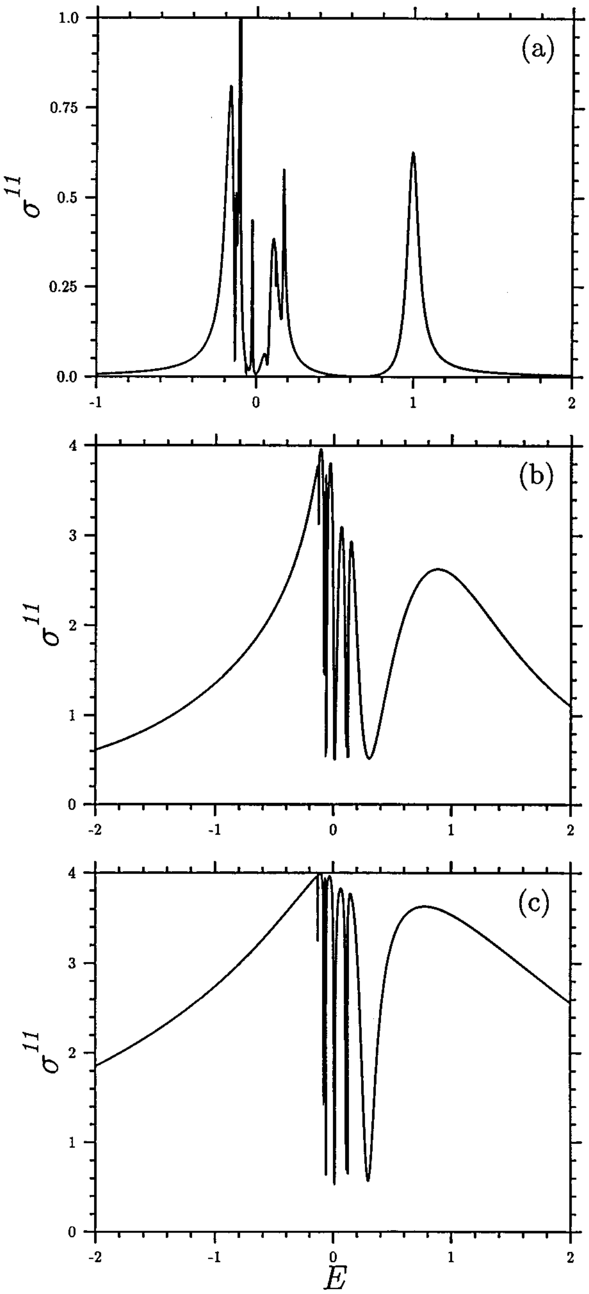

FIG. 4. The elastic cross section for $\lambda = 0.1$ (a), $\lambda = 2$ (b), and $\lambda = 5$ (c). The resonance states are the same as in Fig. 2 in [1]. Note the different E scale in (a).

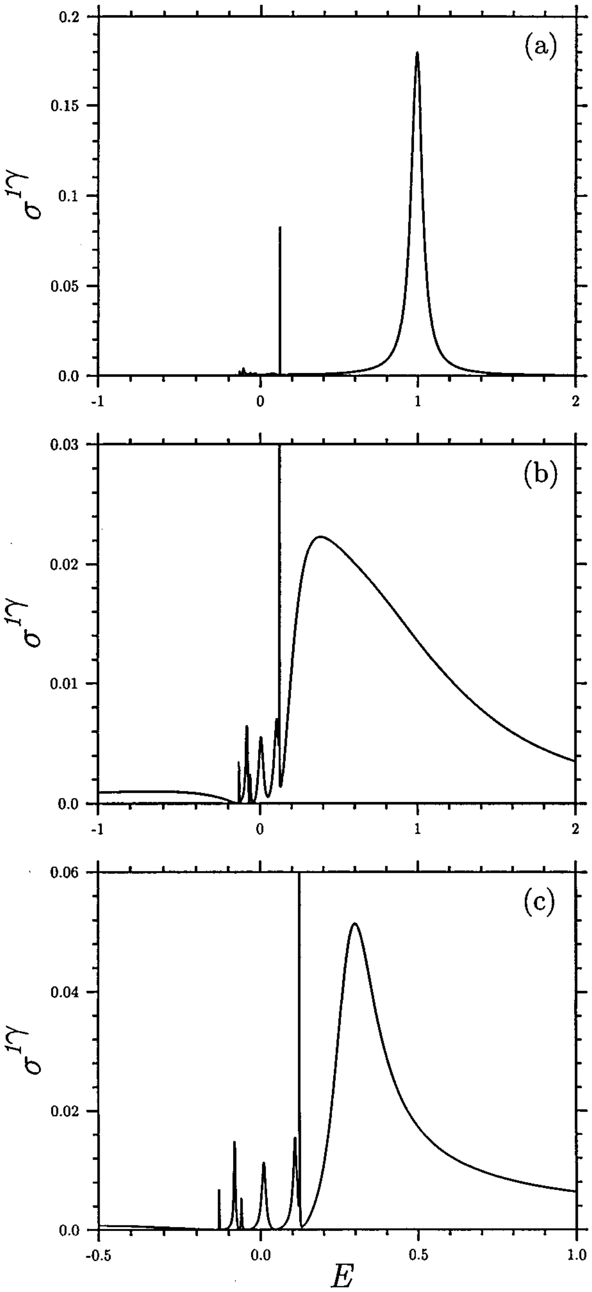

FIG. 5. The photonuclear cross section for $\lambda = 0.1$ (a), $\lambda = 2$ (b), and $\lambda = 5$ (c). The resonance states are the same as in Fig. 2 in [1]. Note the different E scale in (c).

dependence in the whole domain of the GR, we can fully incorporate the Hermitian shift Δ (which is in fact small due to statistical reasons) into the mean position ε_0. The only effect of the interaction with the background states is then the additional shift of the poles of the transition amplitudes along the imaginary direction in the complex energy plane. Note that under such conditions the integral sum rule (2.38) survives the transformations made.

We will not present here the rather cumbersome general expressions. Confining ourselves for the sake of simplicity to the two-level approximation, the shift considered does not influence the relation established in Sec. IV C of [1] between the energy shifts and dipole strengths of the collective doorway states. We suggest further that the displacement $\mathbf{D}^2$ is smaller than both the escape and spreading widths. It can then easily be shown that the transition strength corresponding to the particle emission in a channel c acquires the Breit-Wigner shape

$$\overline{\sigma^c(E)} = \frac{(A_1^c)^2}{2\pi} \frac{\Gamma_{\rm tot}}{(E - E_{\rm centr})^2 + (1/4)\Gamma_{\rm tot}^2} \tag{5.1}$$

with the centroid $E_{\rm centr} = \varepsilon_0 + \cos^2\Theta \mathbf{D}^2$ and the total width $\Gamma_{\rm tot} = \langle \gamma \rangle + \Gamma^{\downarrow}$. Let us remind the reader that condition (3.23) holds for the quantities $(A_1^c)^2$.

The evolution of the averaged γ strength $\overline{\sigma^{(\mathrm{rad})}(E)}$, when the escape width $\langle\gamma\rangle$ changes from values smaller than $\Gamma^{\downarrow}$ to larger ones, is appreciably richer. The strength transforms smoothly from

$$\overline{\sigma^{(\mathrm{rad})}(E)}=\frac{1}{2\pi}\alpha_{\mathrm{el}}\mathbf{D}^2\frac{\Gamma^{\downarrow}}{(E-\varepsilon_{\mathrm{coll}})^2+(1/4)(\Gamma^{\downarrow})^2} \quad (5.2)$$

for $\langle\gamma\rangle \ll \Gamma^{\downarrow}$ to

$$\overline{\sigma^{(\mathrm{rad})}(E)}=\frac{1}{2\pi}\alpha_{\mathrm{el}}\sin^2\Theta\,\mathbf{D}^2\frac{\Gamma^{\downarrow}}{(E-E_v)^2+(1/4)(\Gamma^{\downarrow})^2} \quad (5.3)$$

in the opposite limit $\langle\gamma\rangle \gg \Gamma^{\downarrow}$. In the intermediate region, the maximum monotonously decreases and moves towards lower energies. The shape of the radiation spectrum is not Lorentzian when both widths are of comparable value. It is worth noting that the width of the γ spectrum is always determined mainly by the spreading width. The escape width $\langle\gamma\rangle$ drops out not only from Eq. (5.2) but also from Eq. (5.3). This is due to the fact that the radiating state dw=0 becomes almost trapped.

Equation (5.3) implies the loss of an appreciable part $(=\cos^2\Theta)$ of the radiation strength if the total escape width of the GR noticeably exceeds the spreading width. The contribution of the broader collective state which is described by the right long tail in Fig. 3(d) (see next section) is invisible in Eq. (5.3). It is well known that the spreading width in fact strongly exceeds the total escape width of giant resonances at moderate excitation energies. However, in very hot nuclei the opposite condition seems to be fulfilled. According to experimental data [12,13] as well as theoretical arguments of statistical nature [14], the spreading width saturates with the excitation energy whereas the escape width continues to grow.

VI. NUMERICAL RESULTS

The behavior of the dipole strengths, energies, and widths of the interfering resonance states is reflected in the cross section pattern as shown above analytically by using mainly the two-level approximation. Below, we show the results of numerical investigations performed under less restrictive assumptions. The (purely illustrative) calculations are performed with the same 10 levels and 3 channels as in [1]. Damping is not taken into account, i.e., the results are true only for $\langle\gamma\rangle$, $\mathbf{D}^2 \gg \Gamma^{\downarrow}$ (see the discussion in Sec. V).

In Figs. 1–3 we show the energy dependence of the transition strengths into particle and photochannels for the three values of the overlap parameter $\lambda = 0.1$, 2, and 5. As in the figures in [1], the energy E is measured in units of the total energy displacement $\mathbf{D}^2$. Due to the strong interference, the pattern is noticeably different in the different final channels. One sees nicely the shift of the maximum at the higher energy towards lower energies which is predicted by the two-level approximation. Moreover, the fragmentation of the maximum at the lower energy into a number of resonances can be seen which, of course, disappears in the limit of degenerate unperturbed levels e_n. At last, the growing restructuring of the dipole strength with increasing external coupling in favor of the lower lying components is seen in Figs. 1(d)–3(d). For example, the summed strength above $E>0$ amounts to 99%, 87%, and 85% in the case of the degenerate unperturbed spectrum (dashed lines). As to the maximum value of the transition strength into the photochannel at the higher energy, it drops down by a factor of more than 10 when λ increases from 0.1 to 2, while a narrow high peak appears in agreement with the analytical consideration at lower energy when λ becomes large.

The elastic and photonuclear reaction cross sections are shown in Figs. 4 and 5. They are calculated for the same three values $\lambda=0.1$, 2, and 5 as the transition strengths in Figs. 1–3. Both the shift of the dipole resonance to lower energies and the loss of its dipole strength are seen very clearly also in these values.

Thus, the following scenario takes place. Provided that the coupling (3.8) is negligible, the two collective doorway states dw=0, 1 fully exhaust the total dipole strength so that only they can radiate γ rays. The radiation pattern determined by these doorway states turns out to be very sensitive to their degree of overlapping: as long as the energy displacement of one of them is appreciably larger than the sum of the particle escape widths (i.e., $\lambda \ll 1$) only one of them radiates. If, however, they overlap ($\lambda \sim 1$) the interference leads to a strong redistribution of the dipole strength as well as the escape width between the two states. When the degree of overlapping exceeds some critical value ~ 2 the escape width of one of the states starts to decrease (dynamical trapping effect). This effect is governed by the avoided crossing of two resonances described in detail in [15]. In the limit of strong overlapping, $\lambda \gg 1$, the nearly trapped state acquires an appreciable dipole strength and therefore would radiate, in the absence of any internal damping, a narrow electromagnetic line in the vicinity of the centroid of the broad bump which is observed only in the particle channels. The broad state, which also possesses noticeable dipole strengths, contributes mostly to a long radiation tail stretched towards larger energies.

The coupling (3.8) admixes the other doorway states and leads to an additional restructuring of the total dipole strength in favor of the low lying components.

VII. SUMMARY

On the basis of a phenomenological schematic model we investigated the interferences between the doorway components of a giant multipole resonance. The overlapping of different components influences significatly the resonance spectrum and the cross section pattern since their interaction via the energy continuum creates, at a certain critical value of the external coupling, strong redistributions of the widths and dipole strengths of the doorway states. The resulting GR pattern is formed mainly by two specific collective doorway states. Both states possess comparable dipole strengths but acquire essentially different escape widths. While the broader state determines the picture in the particle channels, the brightest feature in the photoemission would be, in the absence of any internal damping, the relatively narrow radiation line from another nearly ''trapped'' doorway component which lies at somewhat lower energies.

The internal damping due to the coupling of the doorway states to the background of complicated states smears out the effects of the interference as long as the spreading width exceeds the total escape widths of the doorway components. In very hot nuclei it is possible, however, that the escape widths become larger than the spreading width [12,13] which is expected to saturate with increasing excitation energy. If so, the interference picture is not completely spoiled. The internal damping only widens the line radiated by the narrow doorway state though it completely masks the tail from the broad one. Therefore, the visible bulk of the GR γ emission originates from a specific state with dynamically reduced particle escape width but large dipole moment (the trapped collective state) while the emission from the broader state is suppressed being spread over a wide energy range. This manifests itself as a seeming loss of a part of the dipole strength of GR and as a shift of the GR to lower energy.

Both the shift down of a part of the dipole strength and the loss of some part of the dipole strength itself are discussed at present in connection with experimental results obtained for the excitation of collective modes in hot nuclei (see, e.g., the Proceedings of the Gull Lake Nuclear Physics Conference on Giant resonances, 1993 [16]). The γ-ray multiplicity from the decay of giant dipole resonances is shown experimentally to increase with the excitation energy in agreement with the 100% sum rule strength as long as it is not too high. At higher energies, however, its saturation signals the quenching of the multiplicity and the existence of a limiting energy for the γ emission from the giant dipole resonance. The different existing theoretical approaches can only partly explain the experimental situation observed [17].

The results obtained in the present paper point to a new mechanism which could possibly shed an additional light on the problem. To our mind, the saturation of the γ multiplicity observed experimentally at about 250 MeV excitation energy in heavy nuclei [16,17] may be, at least partly, explained by the interference phenomena discussed in the present paper. Further investigations of this interesting question are necessary.

ACKNOWLEDGMENTS

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