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Superfluidity in asymmetric nuclear matter

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The onset of superfluidity in isospin-asymmetric nuclear matter is investigated within the BCS theory. A neutron-proton superfluid state in the channel $^3S_1$-$^3D_1$ comes about from the interplay between thermal excitations and separation $\delta\mu$ of the two Fermi surfaces. The superfluid state disappears above the threshold value of the density-asymmetry parameter $\alpha=(n_n-n_p)/n\simeq 0.35$. For large enough shift between the two Fermi surfaces $\delta\mu=\frac{1}{2}(\mu_n-\mu_p)$ the transition to the normal state becomes a first-order transition and a second gap solution develops. This solution, however, corresponds to a metastable superfluid state which is unstable with respect to the transition to the normal state. [S0556-2813(97)50402-X]

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Microscopic calculations, based on the BCS theory for the bulk nuclear matter, show that the isospin-asymmetric matter supports Cooper-type pair correlations in the $^3S_1$-$^3D_1$ partial-wave channel due to the tensor component of the nuclear force. The energy gap for this pairing has been found to be of the order of 11 MeV for the infinite nuclear matter at the saturation density within simple approaches which do not include medium-polarization effects [1–3]. Because of relatively large values of the gap, pairing in the $^3S_1$-$^3D_1$ channel could have important implications for nuclear physics and nuclear astrophysics. Models of neutron stars, which permit pion or kaon condensation such as the nucleon star model recently proposed by Brown and Bethe [4], could give a major role to the neutron-proton superfluidity in the interpretation of neutron star rotation dynamics and its thermal evolution. Furthermore, experimental evidence on neutron-proton pairing could be obtained from the disassembling phase of the compound system formed in heavy-ion collisions [5]. In this case, one has a unique chance to study the crossover from the BSC neutron-proton pairing to a Bose condensate of deuterons [5,6].

The existence of the pair correlations crucially depends upon the overlap between the neutron and proton Fermi surfaces. If the system is driven out of the isospin-symmetric state, one expects a suppression of the pairing correlations. At zero temperature, a small asymmetry is enough to prevent, at least in the BCS model, the formation of Cooper pairs of neutrons and protons with momenta $\vec{k}$ and $-\vec{k}$. The superfluidity may be restored either by thermal excitations which smear out the two Fermi surfaces or by collective motion of the pairs which results in a shift of the two Fermi spheres with respect to each other. For most applications, the question arises of how large an isospin asymmetry could a superfluid neutron-proton state sustain.

In the present Rapid Communication we study the pairing in an infinite asymmetric nuclear matter. The isospin-singlet pairing is assumed to be decoupled from the isospin-triplet one, because the *SD* coupled channels contain the dominant part of the attractive pairing force. Within this approximation, an extension of the BCS theory to asymmetric nuclear matter in the Gorkov approach is straightforward. Assuming the superfluid state to be a unitary triplet state (see also [3]) the spin dependence is explicitly worked out. Then the proton/neutron propagator can be cast in the form ($\hbar=1$)

$$G^{(p/n)}_{\sigma,\sigma'}(\vec{k},\omega_m)=-\delta_{\sigma,\sigma'}\,\frac{i\omega_m+\xi_k\mp\delta\varepsilon_{\vec{k}}}{(i\omega_m+E^+_{\vec{k}})(i\omega_m-E^-_{\vec{k}})}\,, \qquad (1)$$

while the neutron-proton anomalous propagator acquires the form

$$F^{\dagger}_{\sigma,\sigma'}(\vec{k},\omega_m)=-\frac{\Delta^{\dagger}_{\sigma,\sigma'}(\vec{k})}{(i\omega_m+E^+_{\vec{k}})(i\omega_m-E^-_{\vec{k}})}\,, \qquad (2)$$

where ω_m are the Matsubara frequencies, the upper sign corresponds to protons and the lower one to neutrons. Here the quasiparticle energy spectrum is separated in two branches

$$E^{\pm}_{\vec{k}}=\sqrt{\xi^2_{\vec{k}}+\Delta^2_{\vec{k}}}\pm\delta\varepsilon_{\vec{k}}\,, \qquad (3)$$

where

$$\xi_{\vec{k}}\equiv\frac{1}{2}(\varepsilon^{(n)}_{\vec{k}}+\varepsilon^{(p)}_{\vec{k}}), \qquad \delta\varepsilon_{\vec{k}}\equiv\frac{1}{2}(\varepsilon^{(p)}_{\vec{k}}-\varepsilon^{(n)}_{\vec{k}}),$$

and $\varepsilon^{(n,p)}_{\vec{k}}$ are the single particle energies of neutrons and protons. In the case of the free Fermi gas (to be considered below) $\varepsilon^{(n,p)}_{\vec{k}}=k^2/2m-\mu^{(n,p)}$, $\mu^{(n,p)}$ being the chemical potentials for neutrons and protons, respectively.

From Eqs. (1)–(3) we obtain the BCS gap equation for asymmetric nuclear matter using the angle-averaging procedure, which has been proved to be a quite good approximation [3]. We find

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$$\Delta_l(k) = -\sum_{k'}\sum_{l'} V_{ll'}(k,k')\frac{\Delta_{l'}(k')}{2\sqrt{\xi_{k'}^2 + D(k')^2}} \times [1 - f(E_{\vec{k}}^{+}) - f(E_{\vec{k}}^{-})], \tag{4}$$

where $D(k)^2 \equiv \Delta_0(k)^2 + \Delta_2(k)^2$ is the angle-averaged neutron-proton gap function, $f(E) = [1 + \exp(\beta E)]^{-1}$ is the Fermi distribution function, β being inverse temperature. The driving term $V_{ll'}$ is the bare interaction in the SD channel. The partial densities of neutrons and protons are obtained from Eq. (1) after summation over frequencies,

$$n^{(p/n)} = \sum_{\vec{k},\sigma}\left\{\frac{1}{2}\left(1 + \frac{\xi_{\vec{k}}}{\sqrt{\xi_{\vec{k}}^2 + \Delta_{\vec{k}}^2}}\right) f(E_{\vec{k}}^{\pm}) + \frac{1}{2}\left(1 - \frac{\xi_{\vec{k}}}{\sqrt{\xi_{\vec{k}}^2 + \Delta_{\vec{k}}^2}}\right)[1 - f(E_{\vec{k}}^{\mp})]\right\}. \tag{5}$$

The coupled system of Eqs. (4) and (5) is a generalization of the BCS equations for isospin-asymmetric nuclear matter. These should be solved self-consistently. In the present work we give the results obtained with a single particle spectrum of a free Fermi gas. This will allow us to demonstrate the new qualitative behavior of the system and separate the effects resulting from the modifications of single particle spectra. Inclusion of the full single particle spectrum is straightforward. In the symmetrical case its effect is a suppression of the pairing correlation roughly by a factor of 0.5 at the saturation density [3]. The numerical results given below are obtained using the separable version of the Paris potential (PEST 4) of Ref. [7]. Using the bare interaction in the gap equation might appear to be a very strong simplification because, as well known, the medium polarization strongly reduces the magnitude of the gap (see Ref. [8] for the influence of this effect on the pairing in the $^1S_0$ state). However, at the same time, the medium polarization tends to enhance the effective mass, which could lead to a cancellation of the previous effect.

There are two alternative ways of describing the isospin-asymmetric superfluid state. Either one can fix the total density and consider a deviation from the symmetric case in terms of the partial densities of neutrons and protons, or one may fix the total chemical potential of the system and consider deviations in the chemical potentials of neutrons and protons. The two corresponding macroscopic states could show different properties as, for instance, in the case of β equilibrium among protons, neutrons, and electrons. Let us start with the first case. Figure 1 shows the values of the pairing gap in the $^3S_1$-$^3D_1$ state as a function of the temperature for a fixed total density $n = 0.132$ fm^{-3}. The asymmetry parameter is defined as $\alpha = (n_n - n_p)/n$. The superfluid state exists in the range of asymmetry parameter $0 \leqslant \alpha \leqslant 0.35$. It can be seen that the gap is successively suppressed and the critical temperature is lowered with increasing asymmetry. The gap as a function of temperature shows a maximum which bends down to lower temperatures with increasing asymmetry. Above the value of $\alpha \sim 0.3$ the zero-temperature gap vanishes, which results in an appearance of a lower critical temperature. This two-valued behavior of the

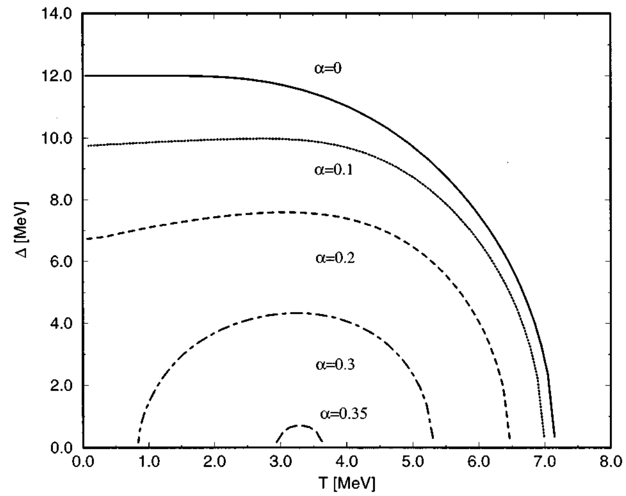

FIG. 1. Pairing gap in the $^3S_1$-$^3D_1$ pairing channel as a function of temperature at a fixed density $n = 0.132$ fm^{-3}.

critical temperature at a fixed density is consistent with the results [6,9] obtained by using the Thouless criterion [10]. We note that the lower critical temperature is, in fact, associated with the existence of a metastable state, which is signaled by a reduction of the gap with decreasing temperature. The system would undergo a phase transition to a normal state at a higher temperature, corresponding to the onset of an instability at the point where the gap starts to decrease (see the discussion below).

Let us turn to the case when the total chemical potential of the system is fixed and the deviation from the symmetric case is given by the chemical potential excess $\delta\mu = \frac{1}{2}(\mu_n - \mu_p)$. The results are displayed in Fig. 2. First of all, it should be noticed that the partial densities of the system are changing along the $\delta\mu$=const curve; therefore, the two different solutions found for the same temperature correspond to different densities. In the range $0 \leqslant \delta\mu \leqslant 7$ MeV the trend of the function $\Delta(T)$ vs T resembles that of the symmetrical case. For $T \leqslant 1$, the gap is constant and equal to its symmetric matter value Δ_{sym} regardless of the value of $\delta\mu$. In a restricted range of $\Delta_{\mathrm{sym}}/2 \leqslant \delta\mu \leqslant \Delta_{\mathrm{sym}}$ two solutions

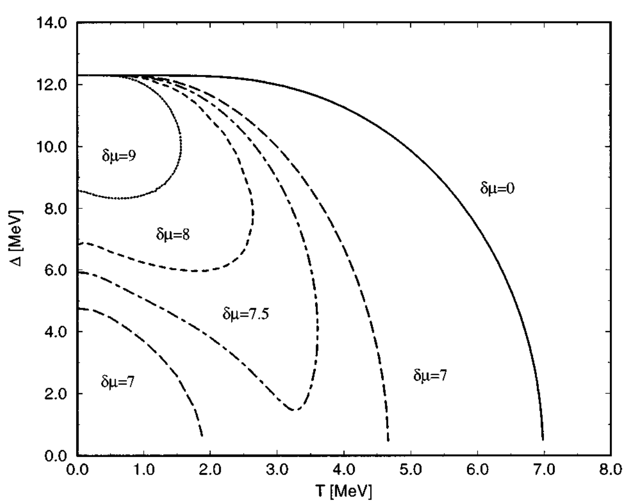

FIG. 2. The same as in Fig. 1 for fixed total chemical potential $\mu = 32$ MeV.

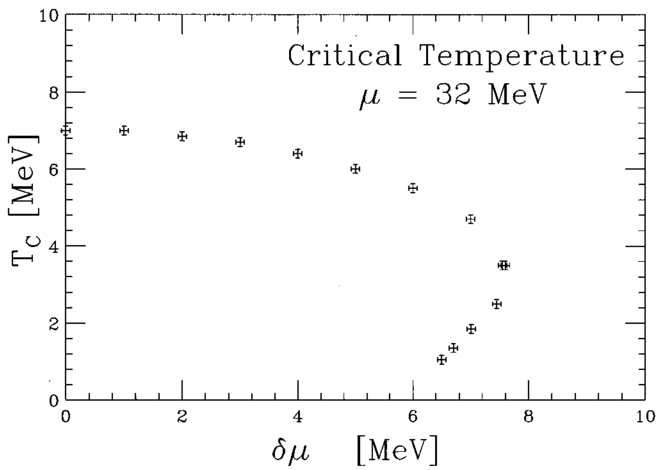

FIG. 3. Critical temperature obtained from the gap equation by taking the limit of vanishing gap.

of the gap equation are simultaneously present at the same temperature. They approach each other with increasing $\delta\mu$ and coincide at the threshold of $\delta\mu \approx \Delta_{\rm sym}$ where the superfluid state disappears. In this range of $\delta\mu$ finite gap persists in the $T \to 0$ limit, which is in contrast to the case with fixed total density. It vanishes only when the temperature is increased above T_c, where the system undergoes a first-order phase transition to the normal state. With increasing asymmetry the temperature range where the superfluid phase exists decreases and then tends to zero in the limit $\delta\mu \to \Delta_{\rm sym}$.

In Fig. 3, the critical temperature is plotted as a function of $\delta\mu$ showing that the BCS prediction is consistent with the one based on the Thouless criterion [6,10,9]. It shows the behavior of the BCS solution in the limit $\Delta(T) \to 0$, however by definition does not provide any information on the state with a finite gap.

The instability of the lower solution of the gap equation can be proved by calculating the change in thermodynamic potential $\Delta\Omega = \Omega_s - \Omega_n$, when the system goes from the normal to the superfluid state. In this note a simple estimate is given using the weak coupling approximation. At $T=0$ the higher solution is $\Delta_1 = \Delta_{\rm sym}$ in the range of asymmetry $0 \leqslant \delta\mu \leqslant \Delta_{\rm sym}$. The lower solution $\Delta_2 = \sqrt{\Delta_{\rm sym}(2\delta\mu - \Delta_{\rm sym})}$ only exists in the range $\Delta_{\rm sym}/2 \leqslant \delta\mu \leqslant \Delta_{\rm sym}$. The main features of this simple approximation survive in the exact calculation as can be seen in Fig. 2 and Fig. 3, though the BCS relation $\Delta = {\rm const} \cdot T_c$ is largely violated in our case. The change in the thermodynamic potentials in these two cases are

$$\Omega_s^{(1)} - \Omega_n = -\tfrac{1}{2}\; N(0)\Delta_1^2, \tag{6}$$

$$\Omega_s^{(2)} - \Omega_n = -N(0)\left[\tfrac{1}{2}\Delta_2^2 + \delta\mu\left(\sqrt{\delta\mu^2 - \Delta_2^2} - \delta\mu\right)\right]. \tag{7}$$

For each relevant $\delta\mu$, $\Omega_s^{(2)}$ is always larger than Ω_n and $\Omega_s^{(1)}$ therefore the lower solution is unstable. The same holds true at finite temperature according to the fact that the leading temperature-dependent correction to the thermodynamic potential arises solely from the normal state contribution and tends to reduce Ω_n as the temperature increases [11].

In summary, we studied the onset of superfluidity in asymmetric nuclear matter in the framework of the BCS approach. The separation of the Fermi surfaces of neutrons and protons prevents the formation of Cooper pairs at zero temperature. However, thermal excitations permit a superfluid state to set in on asymmetric matter at finite temperature. The interplay between thermal excitations and chemical potential shift determines the temperature and asymmetry thresholds for vanishing of superfluidity. For large separations of the two Fermi surfaces, the transition to the normal state becomes a first-order phase transition. A second (lower) solution of the gap equation has been found for fixed Fermi-energy level separation larger than $\Delta_{\rm sym}/2$. However, corresponding superfluid state is metastable, i.e., it is unstable with respect to a transition to the normal state. The stability of this state may be retrieved in the case when the Cooper pairs are moving with a non-vanishing total momentum [12].

A detailed study of the thermodynamics of the asymmetric superfluid state and of its stability is in progress.

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