Chiral perturbation theory and nucleon polarizabilities

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(Received 8 November 1996)

The available experimental data concerning the unpolarized cross section for the Compton scattering on the nucleon at low energy are compared with the predictions of the heavy baryon chiral perturbation theory (HBChPT) at the order q^3. [S0556-2813(97)50804-1]

PACS number(s): 13.60.Fz, 12.39.Fe, 14.20.Dh

In recent years remarkable progress has been achieved in the effort to extend the scheme adopted by the chiral perturbation theory (ChPT) [1,2] from the meson to the baryon sector (for a recent review see Ref. [3]). The ChPT predictions for the nucleon Compton effect have never been properly tested because most of the existing experimental data are spread over a wide energy region above the pion production threshold and very few are concentrated in the low energy region. Since a significant deviation from the data would have an impact at QCD level, it is worth having a closer look at the experimental results obtained over the recent years at Moscow [4], Illinois [5], Mainz [6], and Saskatoon [7,8].

For real photons, the amplitude T_{fi} of the Compton scattering on the nucleon $\gamma(k)N(p)\rightarrow\gamma'(k')N'(p')$, can be written in term of six invariant amplitudes $A_i(\nu^2,t)$ free of both kinematical singularities and constraints [9], where

$$\nu=\frac{s-u}{4m}\ ,\quad t=(k-k')^2$$
$$\text{with}\ \ s=(k+p)^2,\quad u=(k-p')^2\ .$$

Below the pion production threshold $[\omega_{th}=M_\pi(1+M_\pi/2m)\simeq 150$ MeV in the lab system] they are real functions and can be decomposed in Born and non-Born contributions:

$$A_i(\nu^2,t)=A_i^{\mathrm{B}}(\nu^2,t)+A_i^{\mathrm{NB}}(\nu^2,t)\qquad (i=1,6). \tag{1}$$

The Born contribution is associated with the pole diagrams where a single nucleon is exchanged in both s and u channels and is determined uniquely by the mass m, the electric charge Z, and the anomalous magnetic moment κ (in units of $e/2m$) of the nucleon, and its expression can be found in Ref. [9]. The non-Born parts of the invariant amplitudes depend upon the structure of the nucleon and, since in the lab system one has $(z=\cos\theta)$

$$t=-2\omega\omega'(1-z)\ ,\quad \nu^2=\omega\omega'+\frac{t^2}{16m^2}\ ,$$
$$\text{with}\ \ \omega'=\omega\left[1+\frac{\omega}{m}(1-z)\right]^{-1}\ ,$$

they can be expanded in a power series of the crossing-even parameter $\omega\omega'$

$$A_i^{\mathrm{NB}}(\nu^2,t)=a_i+\omega\omega'[a_{i,\nu}-2(1-z)a_{i,t}]+\cdots, \tag{2}$$

where

$$a_i=A_i^{\mathrm{NB}}(0,0)\ ,\quad a_{i,\nu}=\left.\frac{\partial A_i^{\mathrm{NB}}}{\partial\nu^2}\right|_{\nu=t=0},\quad a_{i,t}=\left.\frac{\partial A_i^{\mathrm{NB}}}{\partial t}\right|_{\nu=t=0}. \tag{3}$$

As a consequence of the decomposition (1) the amplitude T_{fi} can also be written as a sum of a Born and a non-Born contribution. The latter is determined by the structure constants introduced in the expansion (2) and the expression of its spin-independent part in the lab system is:

$$\frac{N}{8\pi m}\,\overline{T_{fi}^{\mathrm{NB}}}=\omega\omega'[(\vec{e}\,'^{*}\cdot\vec{e})\alpha+(\vec{s}\,'^{*}\cdot\vec{s})\beta]+O(\omega^2\omega'^2)\quad\left(N=\sqrt{1-\frac{t}{4m^2}}\right), \tag{4}$$

where ω and $\vec{e}$ are the energy and the polarization three-vector of the initial photon, $\vec{s}=\vec{e}\times\vec{k}$ (the primed quantities refer to the final photon) and the constants

$$\alpha\ =\ -\frac{1}{4\pi}\,(a_1+a_3+a_6)\ ,\quad \beta\ =\ \frac{1}{4\pi}\,(a_1-a_3-a_6)\ , \tag{5}$$

are usually interpreted as the electric and magnetic (Compton) polarizabilities of the nucleon. From Eq. (4) we see that these structure-dependent corrections start giving contributions to T_{fi} at the second order in ω. Due to the interference with the Thomson amplitude [the energy-independent term proportional to $Z(\vec{e}\,'^{*}\cdot\vec{e})$ in the Born term], their contributions to the differential cross section result in a $O(\omega^2)$ effect in the case of the proton $(Z=1)$, and in a $O(\omega^4)$ in the case of the neutron $(Z=0)$ [10].

It can be shown [11] that the spin-dependent part of $T_{fi}^{\rm NB}$ starts giving contributions to $O(\omega^3)$ and is determined by the following combinations of the four constants a_2, a_4, a_5, and a_6:

$$\alpha_1 = \frac{1}{2\pi m}(a_2 - a_4 - a_6),$$

$$\alpha_2 = -\frac{1}{2\pi m}(a_2 - a_4 + a_5),$$

$$\beta_1 = \frac{1}{2\pi m}(a_2 + a_4 + a_6),$$

$$\beta_2 = -\frac{1}{2\pi m}(a_2 + a_4 + a_5). \qquad (6)$$

Similarly to the analogous quantities γ_i introduced in Ref. [11], they will be referred to as ''spin-polarizabilities.'' Due to the interference with the spin-dependent terms of the first order in $T_{fi}^{\rm B}$, they give rise to a $O(\omega^4)$ correction to the unpolarized differential cross section [12]. In the case of a polarized nucleon and circularly polarized photons, these terms appear as an effect of $O(\omega^3)$. For the sake of completeness, the relationship between these quantities and the γ_i of Ref. [11], defined in the Breit frame, is given by

$$\alpha_1 = 4\gamma_3, \quad \alpha_2 = 2\gamma_1,$$

$$\beta_1 = -4(\gamma_2 + \gamma_4), \quad \beta_2 = 2(\gamma_2 + 2\gamma_4).$$

As an example, we consider the form assumed by the non-Born part of the amplitude in the case of forward and backward scattering

$$\frac{1}{8\pi m} T_{fi}^{\rm NB}(0) = \omega^2[(\alpha+\beta)(\vec{e}\,'^* \cdot \vec{e}) + i\omega\gamma\vec{\sigma}\cdot(\vec{e}\,'^* \times \vec{e})] + O(\omega^4),$$

$$\frac{1}{8\pi m} T_{fi}^{\rm NB}(\pi) = \omega\omega'[(\alpha-\beta)(\vec{e}\,'^* \cdot \vec{e}) - i\nu\delta\vec{\sigma}\cdot(\vec{e}\,'^* \times \vec{e})] + O(\omega^2\omega'^2),$$

where (γ is what is named ''spin-polarizability'' in Ref. [15])

$$\gamma = \frac{1}{2}(\alpha_2 - \beta_2) = \frac{1}{2\pi m} a_4,$$

$$\delta = -\frac{1}{2}(\alpha_2 + \beta_2) = \frac{1}{2\pi m}(a_2 + a_5). \qquad (7)$$

Moreover, the $O(\omega^4)$ contribution to $\overline{T}_{fi}^{\rm NB}$ in Eq. (4) is determined by the constants a_1, a_3, a_5, and, in the proton case, also by the following combinations of the derivative constants defined in Eq. (3):

$$\alpha_d = -\frac{1}{4\pi}(a_{1,d} + a_{3,d} + a_{6,d}),$$

$$\beta_d = \frac{1}{4\pi}(a_{1,d} - a_{3,d} - a_{6,d}) \quad (d = \nu, t). \qquad (8)$$

Also, these terms can be seen as an effect of $O(\omega^4)$ in the unpolarized differential cross section.

In the relativistic ChPT [13], the nucleon is treated as a fully relativistic Dirac field. Since the nucleon mass does not vanish in the chiral limit, the consistent chiral power counting scheme present in the meson sector is lost. This difficulty is overcome by considering an extreme nonrelativistic limit for baryons, i.e., treating them as heavy sources (HBChPT) [14]. In this framework, the predictions for the nucleon polarizabilities at one-loop level (order q^3) are:

$$\alpha_p = \alpha_n = 10\beta_p = 10\beta_n = \frac{5e^2 g_A^2}{384\pi^2 F_\pi^2 M_\pi} = 12.1\times 10^{-4}\ \text{fm}^3 \qquad (9)$$

($F_\pi = 93.3$ MeV, $g_A = 1.26$, $M_\pi = 139.57$ MeV) [15]. This is exactly the leading chiral singularity found in the relativistic approach [16] and it agrees quite well with the experimental values quoted in the review analysis of Ref. [17]

$$\alpha_p = (12.0 \pm 0.9)\times 10^{-4}\ \text{fm}^3,$$

$$\beta_p = (2.2 \pm 0.9)\times 10^{-4}\ \text{fm}^3,$$

$$\alpha_n = (12.5 \pm 2.5)\times 10^{-4}\ \text{fm}^3,$$

$$\beta_n = (3.3 \pm 2.7)\times 10^{-4}\ \text{fm}^3. \qquad (10)$$

At $O(q^3)$, proton and neutron appear undistinguishable also at the level of their spin-polarizabilities $\alpha_{1,2}$ and $\beta_{1,2}$ defined in Eq. (6). Contrary to α and β, these structure parameters depend on the amplitude A_2 which receives contribution also from the t-channel π^0 exchange [9]. From Ref. [3] one has

$$A_2^{\pi^0}(t) = -\frac{e^2 g_A}{4\pi^2 F_\pi^2}\frac{m}{t - M_\pi^2}, \qquad (11)$$

which, owing to the small pion mass, is a rapidly varying function of t and therefore is usually not expanded in terms of $\omega\omega'$ [12]. By extracting this contribution from the non-Born amplitude, for the remaining part of the spin polarizabilities, one has [3]:

$$\tilde{\alpha}_1 = -\tilde{\beta}_1 = \frac{1}{2}\tilde{\alpha}_2 = \frac{e^2 g_A^2}{96\pi^3 F_\pi^2 M_\pi^2} = 4.4\times 10^{-4}\ \text{fm}^4, \quad \tilde{\beta}_2 = 0. \qquad (12)$$

Let us note that of the two quantities γ and δ defined in Eq. (7), only the latter receives contributions from the π^0-exchange graph, and thus:

$$\gamma = \tilde{\alpha}_1 = -\tilde{\delta} = 4.4\times 10^{-4}\ \text{fm}^4. \qquad (13)$$

Unfortunately, no experimental data on the spin polarizabilities are yet available for a comparison.

At the higher order, the situation could change considerably since the Δ excitation is a large magnetic effect and

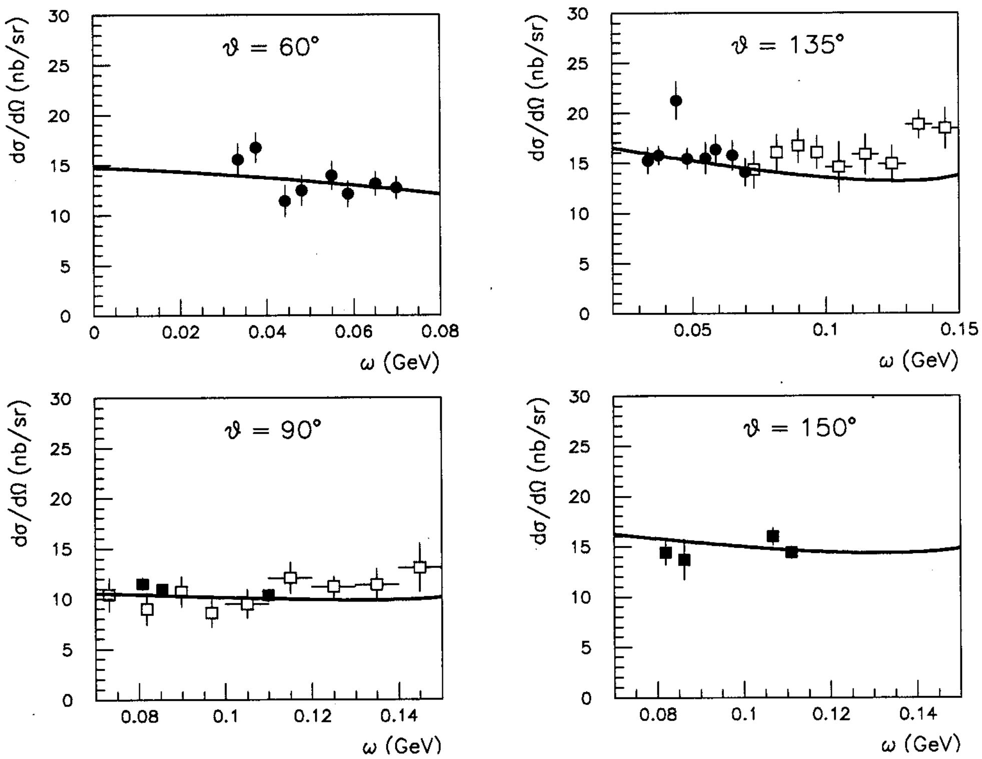

FIG. 1. The Illinois (circles), Saskatoon (1995) (open squares), and Moscow (closed squares) data for the unpolarized cross section at fixed scattering angle in the lab system as a function of the energy of the incoming photon beam. The solid line is the prediction of the HBChPT at $O(q^3)$ of Ref. [3].

starts contributing at $O(q^4)$ in α and β and at order $O(q^5)$ in γ. A full calculation at order $O(q^4)$ shows a large cancellation effect between Δ and loops in β_p, with the consequence that, aside from β_n, the predictions of Eq. (9) are not strongly modified at order $O(q^4)$ [3]

$$\alpha_p=(10.5\pm2.0)\times10^{-4}\ \mathrm{fm}^3,$$

$$\beta_p=(3.5\pm3.6)\times10^{-4}\ \mathrm{fm}^3,$$

$$\alpha_n=(13.4\pm1.5)\times10^{-4}\ \mathrm{fm}^3,$$

$$\beta_n=(7.8\pm3.6)\times10^{-4}\ \mathrm{fm}^3, \tag{14}$$

and remain consistent with the experimental data of Eq. (10). The situation for $\gamma_{p,n}$ appears more controversial. Although the $O(q^4)$ and $O(q^5)$ corrections have not been completely calculated yet, the sole addition of the Δ to the one-loop prediction in the relativistic approach strongly modifies the result of Eq. (13) [15]:

$$\gamma_p=-1.5\times10^{-4}\ \mathrm{fm}^4, \quad \gamma_n=-0.46\times10^{-4}\ \mathrm{fm}^4.$$

Though far from being conclusive, this result turns out to be nonetheless in good accordance with the dispersion calculation based on the latest pion photoproduction multipoles taken from the SAID database [18]

$$\gamma_p=-1.34\times10^{-4}\ \mathrm{fm}^4, \quad \gamma_n=-0.38\times10^{-4}\ \mathrm{fm}^4,$$

and it presents a clear element of surprise: indeed the same multipole analysis seems to be incompatible with the DHG-sum rule [18].

The comparison with the experimental values of the differential cross section obtained at Illinois [5] and Saskatoon (1995) [8] is shown in the upper part of Fig. 1 and looks very favorable only for lower energies than 100 MeV. The good agreement in this energy region is confirmed by the Moscow data [4] which, in spite of the criticism raised in Ref. [17], lie perfectly on top of the theoretical predictions of the HBChPT at $O(q^3)$ [3] up to an energy of approximately 110 MeV (see lower part of Fig. 1).

The authors of Ref. [3] claim that ''if one trusts the q^3 approximation up to the pion production threshold, one finds agreement with the few Saskatoon (1993) [7] data in this range.'' However, this is strictly true only at forward angles; as a matter of fact, in the narrow energy region spanned by these data (135–145 MeV) the agreement with the $O(q^3)$ predictions progressively deteriorates at backward angles, as is clearly noticeable in Fig. 2. This trend, already at work in the Saskatoon (1995) data (see Fig. 1), is further confirmed by the two Mainz points at 180° shown in Fig. 3(a). The effect looks to be anything but small: the Saskatoon (1993) data at $\theta=90°$ and 141° and the Mainz data exceed the $O(q^3)$ ChPT predictions by more than 25%. No further comment can be added to this statement if the comparison with theory is maintained with the full $O(q^3)$ calculation. However, more insight into the nature of this discrepancy can be gained with the recourse to the $O(\omega^4)$ expansion of the non-Born part of the differential cross section. This expansion is fully determined by the structure parameters entering Eq. (2) and its validity can be easily tested against the full $O(q^3)$ prediction by using the values of Eqs. (9) and (12) and the derivative contributions

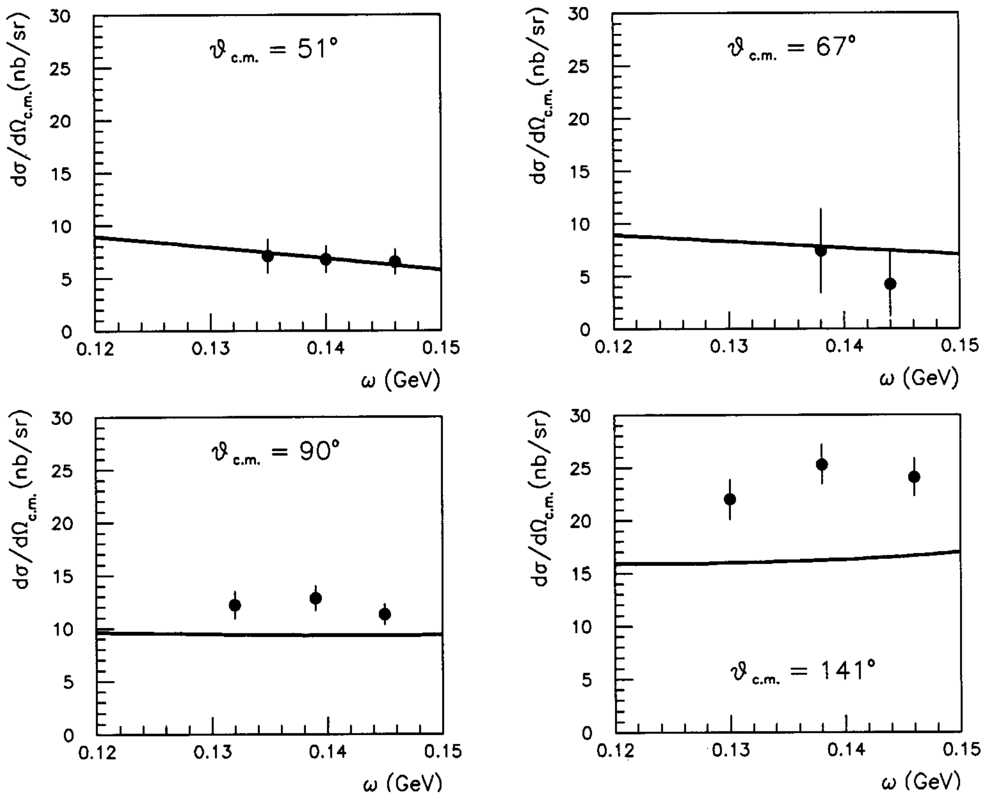

FIG. 2. The Saskatoon (1993) data for the unpolarized cross section at four different scattering angles in the c.m. system. Solid line as in Fig. 1.

$$\alpha_\nu = \frac{26}{7}\alpha_t = \frac{13}{50}\frac{\alpha}{M_\pi^2}, \quad \beta_\nu = -\frac{1}{3}\beta_t = \frac{1}{10}\frac{\beta}{M_\pi^2}$$

that follow from the low energy expansion of the $O(q^3)$ invariant amplitudes given in Ref. [3]. As shown in Fig. 3(a) the agreement between the full $O(q^3)$ calculation and its $O(\omega^4)$ approximation is particularly good in the backward direction, which is the region where most of the discrepancies are concentrated.

This energy expansion has always been used as a good phenomenological tool where the polarizabilities have been considered as free parameters to be determined with the experimental data; this is exactly the way in which the values of Eq. (10) have been obtained. At $\theta = 180°$, the non-Born contribution to the expansion of the differential cross section is ($r_0 = e^2/4\pi m \simeq 1/137m$)

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi}^{\rm NB} = -r_0\left(\frac{\omega}{\omega'}\right)^2\Bigg\{2(\alpha-\beta)\omega\omega' + \Big[4(\alpha-\beta)+2m(\tilde{\alpha}_1+\tilde{\beta}_1)-m(4\kappa+\kappa^2) \times(\tilde{\alpha}_2+\tilde{\beta}_2)+4m^2(\alpha_\nu-\beta_\nu)-16m^2(\alpha_t-\beta_t) - \frac{2m^2}{r_0}(\alpha-\beta)^2\Big]\frac{\omega^2\omega'^2}{2m^2}\Bigg\}. \tag{15}$$

In the proton case, the $O(q^4)$ prediction for $(\alpha-\beta)$ that stems from Eq. (14)

$$(\alpha-\beta)_p = (7.0 \pm 4.1)\times 10^{-4}\ \mathrm{fm}^3 \tag{16}$$

is consistent with the world average $(\alpha-\beta)_p = (9.8 \pm 1.5 \pm 0.8)\times 10^{-4}$ fm^3 quoted in Ref. [17], but, as

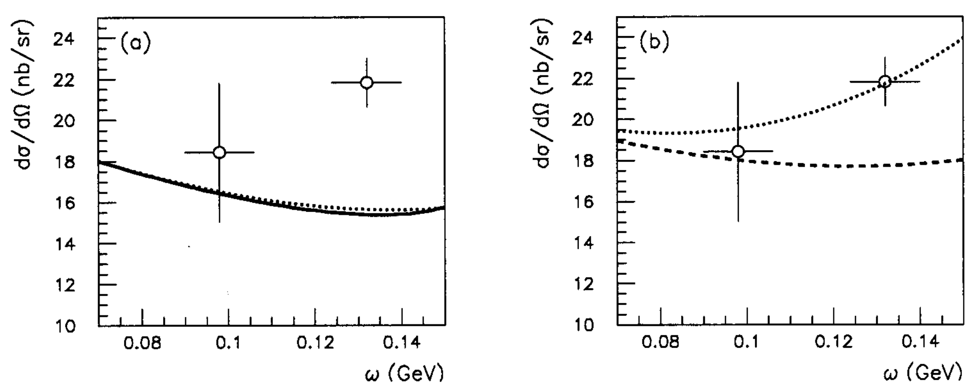

FIG. 3. The Mainz data for the unpolarized cross section at $\theta = 180°$ in the lab system. In (a) the $O(q^3)$ prediction of Ref. [3] (solid line) is compared with its expansion to $O(\omega^4)$ (dotted line). As for the curves in (b), see the text.

shown by the dashed line in Fig. 3(b), its sole insertion in Eq. (15) is not enough to reproduce the data. Indeed, one can easily verify that, in order to recover the two Mainz points, huge corrections to the remaining parameters involved are needed. The dotted line in Fig. 3(b) shows the behavior of the cross section in the backward direction when 100% corrections are applied to all of them and $(\alpha-\beta)$ is set at the central value of Eq. (16).

In conclusion, the data suggest that in the vicinity of the pion threshold and in the backward region, the higher order terms are far from being small, and thus a full calculation is seriously needed. It is clear that better experimental data in the backward direction in this energy range are also required. Furthermore, since α_i and β_i appear in the $O(\omega^3)$ spin-dependent part of $T_{fi}^{\rm NB}$ [11], the Compton scattering of circularly polarized photons on longitudinally polarized nucleons will be more sensitive to these structure constants. From this point of view the recent LSC proposal at LEGS (BNL) [19] looks very appealing.

[1] S. Weinberg, Physica A **96**, 327 (1979).
[2] J. Gasserand H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985); **B250**, 539 (1985).
[3] V. Bernard, N. Kaiser, and Ulf-G. Meiβner, Int. J. Mod. Phys. E **4**, 193 (1995).
[4] P. S. Baranov *et al.*, Sov. J. Nucl. Phys. **21**, 355 (1975).
[5] F. J. Federspiel, R. A. Eisenstein, M. A. Lucas, B. E. MacGibbon, K. Mellendorf, A. M. Nathan, A. O'Neill, and P. D. Wells, Phys. Rev. Lett. **67**, 1511 (1991).
[6] A. Zieger, R. Van de Vyer, D. Christmann, A. DeGraeve, C. Van den Abeele, and B. Ziegler, Phys. Lett. B **278**, 34 (1992).
[7] E. Hallin *et al.*, Phys. Rev. C **48**, 1497 (1993).
[8] B. E. MacGibbon, G. Garino, M. A. Lucas, A. M. Nathan, G. Feldman, and B. Dolbilkin, Phys. Rev. C **52**, 2097 (1995).
[9] V. A. Petrun'kin, Sov. J. Part. Nucl. **12**, 278 (1981).
[10] A. I. L'vov, Sov. J. Nucl. Phys. **42**, 583 (1985).
[11] S. Ragusa, Phys. Rev. D **47**, 3757 (1993).
[12] I. Guiasu and E. E. Radescu, Phys. Rev. D **18**, 651 (1978).
[13] J. Gasser, M. E. Sainio, and A. Švarc, Nucl. Phys. **B307**, 779 (1988).
[14] E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); **259**, 353 (1991).
[15] V. Bernard, N. Kaiser, J. Kambor, and Ulf-G. Meiβner, Nucl. Phys. **B388**, 315 (1992).
[16] V. Bernard, N. Kaiser, and Ulf-G. Meiβner, Phys. Rev. Lett. **67**, 1515 (1991); Nucl. Phys. **B373**, 346 (1992).
[17] A. Nathan, in *Chiral Dynamics: Theory and Experiment*, Lecture Notes in Physics LNP 452, edited by A. M. Bernstein and B. R. Holstein (Springer-Verlag, Berlin, 1995).
[18] A. M. Sandorfi, M. Khandaker, and S. Whisnant, Phys. Rev. D **50**, R6681 (1994).
[19] A. M. Sandorfi *et al.*, LEGS Progress Report BNL-63358, 1996.