First-forbidden β decay of ^{17}N and ^{17}Ne

D. J. Millener*
Brookhaven National Laboratory, Upton, New York 11973
and Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195
(Received 15 January 1997)

It is shown that differences, due to charge-dependent effects, in the ^{17}N and ^{17}Ne ground-state wave functions account for the fact that the experimentally measured branch for the β^+ decay of ^{17}Ne to the first excited state of ^{17}F is roughly a factor of two larger than expected on the basis of nuclear matrix elements which reproduce the corresponding β^- branch in the decay of ^{17}N. [S0556-2813(97)50504-8]

PACS number(s): 23.40.Hc, 21.60.Cs, 27.20.+n

By measuring positrons in coincidence with 495-keV γ rays deexciting the $1/2^+$ first-excited state of ^{17}F, Borge *et al.* [1] have obtained a branch of 1.65(16)% for the first-forbidden β^+ decay of ^{17}Ne to the $1/2^+$ state. This is a very interesting result because the measured branch is roughly a factor of two larger than expected on the basis of nuclear matrix elements which reproduce the corresponding β^- branch of 3.0(5)% [2,3] in the decay of ^{17}N. Recently, Ozawa *et al.* [4] have confirmed the magnitude of the β branch in ^{17}Ne decay, obtaining a value of 1.44(16)% by a method which utilizes a 32 MeV/nucleon radioactive beam of ^{17}Ne.

The β-decay rate is given by $ft=6170$ sec. For the $1/2^-\rightarrow 1/2^+$ transitions of interest, $f=f^{(0)}+f^{(1)}$ where the superscript refers to the spherical tensor rank of the β-decay operator. In general, $f^{(0)}$ is much larger than $f^{(1)}$ and, to a very good approximation,

$$f^{(0)}=I_0(\xi' v+\tfrac{1}{3}W_0 w+\xi w')^2, \tag{1}$$

where $\xi=\pm\alpha Z/2R$ for $\beta^{\mp}$ decay, with Z the charge of the daughter nucleus and $R=3.499$ fm for $A=17$, and

$$w=\lambda\sqrt{3}\hat{J}_f/\hat{J}_i\langle J_f T_f|||ir[C_1,\sigma]\frac{1}{\sqrt{2}}\tau|||J_i T_i\rangle C, \tag{2}$$

$$\xi' v=-\lambda\sqrt{3}\hat{J}_f/\hat{J}_i\langle J_f T_f|||\frac{i}{M}[\sigma,\nabla]\frac{1}{\sqrt{2}}\tau|||J_i T_i\rangle C\chi^2_{\rm Ce} \tag{3}$$

with C being the isospin Clebsch-Gordan coefficient and $\lambda=1.26$. Energies are expressed in units of the electron rest mass and, with I_0 the integrated phase-space factor for allowed decays divided by the square of the Compton wavelength for the electron, the nuclear matrix elements are in fm. The matrix element w' is closely related to w and takes a value $\sim 0.7w$ [5]. These expressions are based on a systematic expansion of the electron radial wave functions developed by Behrens and Bühring [6], the arcane notation for the nuclear matrix elements in first-forbidden decays being historical (see [5] for details and definitions of the rank-1 matrix elements).

Aside from the use of first-forbidden β decay as a spectroscopic tool, there has been great interest in rank-0 decays for two reasons. The first dates back to the suggestion [7] that the matrix element $\xi' v$ of the timelike piece of the axial current γ_5 should be strongly enhanced by meson-exchange currents, largely one-pion-exchange. This enhancement is now well established at $\sim 60\%$ for light nuclei [8] and even larger for heavy nuclei [9]. It is often taken into account, as is done below, by multiplying $\xi' v$ by a factor $\varepsilon_{\rm mec}$. The second reason relates to the similarity of the operators for parity-mixing and rank-0 first-forbidden β decay [10]. As a result of these fundamental interests, a large literature exists on many aspects of first-forbidden β decay and parity-mixing in light nuclei. The present treatment of the ^{17}N and ^{17}Ne decays, first studied theoretically by Towner and Hardy [11], is based on a systematic study [12] of $J^{\pm}\rightarrow J^{\mp}$ decays of ^{11}Be, ^{15}C, ^{16}C, ^{16}N, ^{17}N, ^{18}Ne, ^{19}Ne, and ^{20}F.

For the $1\hbar\omega$ basis used in [12], the $1/2^-$ initial-state wave functions have a particularly simple form in a weak-coupling representation, namely that of a $0p$-shell hole coupled to $(1s0d)^2$ eigenstates (notation $J^{\pi}_n;T$)

$$|1/2^-;3/2\rangle=0.967|1/2^-\otimes 0^+_1;1\rangle-0.224|3/2^-\otimes 2^+_1;1\rangle+0.109|1/2^-\otimes 0^+_2;1\rangle+\cdots. \tag{4}$$

In fact, the three components listed account for 99.7% of the wave function. For the dominant component, only the $1s^2_{1/2}$ component contributes to the matrix element $\boldsymbol{\sigma}\cdot\mathbf{r}$ and $\boldsymbol{\sigma}\cdot\mathbf{p}$, one $s_{1/2}$ nucleon making a transition to fill the $p_{1/2}$ hole with the other forming the single-particle final state. The same is true for the third component, which augments the first (the $0^+_2;1$ state has a dominant $1s^2_{1/2}$ component). A small $d_{3/2}\rightarrow p_{3/2}$ amplitude, arising from the second component of the $1/2^-$ wave function, is important because the single-particle matrix element is large (larger than $s\rightarrow p$ by a factor of $\sqrt{5}$ for harmonic oscillator wave functions) and interferes destructively with the dominant $1s_{1/2}\rightarrow 0p_{1/2}$ amplitude. This is a common feature of all the transitions studied in [12]. The radial single-particle matrix elements are computed with Woods-Saxon wave functions obtained by adjusting the well depth to match the separation energy from the initial or final

*Permanent address: Brookhaven National Laboratory, Upton, New York 11973. Electronic address: millener@bnl.gov

TABLE I. Parameters governing the decays of ^{17}N and ^{17}Ne to the first-excited states of ^{17}O and ^{17}F. Separation energies are given for the 0_1^-;1 core states in ^{16}N and ^{16}F; the values for the 1_1^- core state are 0.28 MeV and 0.19 MeV higher, respectively.

	W_0 (MeV)	I_0	$S_{n/p}(s_{1/2})$ (MeV)	$S_{p/n}(p_{1/2})$ (MeV)
^{17}N	8.32	0.3051	6.00 (n)	13.03 (p)
^{17}Ne	13.52	2.380	1.48 (p)	16.80 (n)

state to the appropriate physical core states of the A-1 system [12]. For the $1s_{1/2}\rightarrow 0p_{1/2}$ contribution, the only important parent states are the lowest 0^- and 1^- states of ^{16}N or ^{16}F. The separation energies are given in Table I along with the decay energies and the phase-space integrals I_0. Since the separation energies are close to the Hartree-Fock energies, the Woods-Saxon wave functions should be a good approximation to one-nucleon overlap functions [13].

For the rank-0 contribution to the β-decay rates, the calculation gives

$$f^{(0)}(\mathrm{N}) = 0.3051(10.971\varepsilon_{\mathrm{mec}} - 4.216)^2, \tag{5}$$

$$f^{(0)}(\mathrm{Ne}) = 2.380(11.585\varepsilon_{\mathrm{mec}} - 3.009)^2, \tag{6}$$

$$f^{(0)}(\mathrm{Ne}') = 2.380(15.278\varepsilon_{\mathrm{mec}} - 3.969)^2, \tag{7}$$

where the first two lines correspond to using identical nuclear structure, the small differences in matrix elements being due to the use of Woods-Saxon wave functions bound at the physical separation energies [note the energy-dependent factors in Eq. (1) for the second term]. The resulting f values are compared with experiment in Table II for two values of the enhancement due to meson-exchange currents (see Table IV of [8] for theoretical estimates of $\varepsilon_{\mathrm{mec}}$). Including the calculated $f^{(1)}$ values, it can be seen that the predicted value for the β branch in ^{17}Ne is less than $\sim$0.9% for values of $\varepsilon_{\mathrm{mec}}$ which produce agreement with the ^{17}N data (0.77% to reproduce the central value).

For the case denoted by Ne′ in Eq. (7) and the last line of Table II, the ^{17}Ne ground-state wave function has been modified to take into account charge-dependent effects which differ for $1s$ and $0d$ orbits. Now, with a 45–50 % enhancement from meson-exchange currents, the calculated β-decay rates are in agreement, within the error bars, for both nuclei.

That there should be substantial T_z-dependent effects is evident from the 376 keV difference in Coulomb energies for the $0d_{5/2}$ and $1s_{1/2}$ orbits at $A=17$. For $A=18$, the large shift

TABLE II. Comparison of theoretical and experimental β-decay rates via f values. f_{exp} for ^{17}Ne decay is derived from the average 1.55(12)% of the two measurements [1,4] for the β branch.

	$f^{(0)}$		$f^{(1)}$	f_{exp}
$\varepsilon_{\mathrm{mec}}$	1.4	1.5		
^{17}N	37.9	45.7	6.5	44.4(74)
^{17}Ne	415	491	21	873(64)
^{17}Ne′	722	854	21	873(64)

TABLE III. Excitation energies (MeV) of 0^+ $T=1$ states relative to the lowest such state. The 0_2^+ states are mainly $4p2h$ in nature. In the case of ^{18}F, it should be noted that the lowest 0^+ state obtains extra binding energy from the charge-independence breaking np interaction [16].

J_n^π	^{18}O	^{18}F	^{18}Ne
0_3^+	5.336	5.094	4.590
0_2^+	3.630	3.711	3.576

in the excitation energy of the the third 0^+ state in ^{18}Ne (Table III) led to its identification as a largely $1s_{1/2}^2$ configuration [14]. The shift in the $s_{1/2}^2$ diagonal matrix element relative to $d_{5/2}^2$ in going from ^{18}O to ^{18}Ne will also lead to more $s_{1/2}^2$ in the ^{18}Ne ground-state wave function and hence, when coupled to a $p_{1/2}$ hole, to an enhancement of the rank-0 matrix element for the β^+ decay of ^{17}Ne. This effect is amplified by the cancellation between the $s_{1/2}\rightarrow p_{1/2}$ and $d_{3/2}\rightarrow p_{3/2}$ contributions.

To make a rough estimate of this effect, the Wildenthal USD interaction [15] is used to obtain $(sd)^2$ wave functions for ^{18}O ($\epsilon_{5/2}=-3.9478$, $\epsilon_{1/2}=-3.1635$, $\epsilon_{3/2}=1.6466$, upper half diagonal of two-body matrix elements -2.8197, -1.3247, -3.1856, -2.1246, -1.0835, -2.1845). Then, the $s_{1/2}^2$ diagonal matrix element is shifted by twice the shift of the $s_{1/2}$ single-particle energies between ^{17}O and ^{17}F (752 keV) plus 147 keV for the difference between the two-body matrix elements of e^2/r for d^2 and s^2 configurations [16], and the new matrix is diagonalized to get $(sd)^2$ wave functions for ^{18}Ne. The resulting energies, wave functions, and intensities of $1s_{1/2}^2$ are given in Table IV. The $s_{1/2}^2$ intensity rises from 15% to 21.7%, an increase of 44% (the squared overlap of the ground-state wave functions is still 0.9925). The increase in $\xi' v$ in Eq. (7) by a factor 1.32 rather than 1.20 for the $s_{1/2}\rightarrow p_{1/2}$ matrix element alone is due to the cancellation effects involving the $d_{3/2}\rightarrow p_{3/2}$ matrix element.

The above calculation, which does succeed in providing an explanation for the measured β-decay rates, is not a consistent one, but clearly indicates the direction in which charge-dependent effects will affect the β-decay branch in ^{17}Ne decay. An explanation of the energy shifts and wave function changes for the 0^+ $T=1$ states of $A=18$ requires that the $4p2h$ configurations be included. A calculation of the energy shifts without wave function changes [14] does rather well, but the ^{18}Ne ground state could do with a ''push'' of the magnitude (163 keV) shown in Table IV. The $(sd)^2$ calculation is actually more applicable to the $2p1h$ states of ^{17}N and ^{17}Ne because the $4p3h$ states are expected

TABLE IV. Results of $(sd)^2$ diagonalizations. Wave function amplitudes are given in columns 4–6. The binding energy of the 0_1^+ state of ^{18}O is chosen as the zero of energy.

	J_n^π	E_x	$d_{5/2}^2$	$s_{1/2}^2$	$d_{3/2}^2$	$\%s_{1/2}^2$
^{18}O	0_1^+	0.000	0.8886	0.3878	0.2448	15.0
	0_2^+	4.320	0.3932	−0.9190	0.0287	84.5
^{18}Ne	0_1^+	−0.163	0.8521	0.4654	0.2394	21.7
	0_2^+	3.588	0.4667	−0.8827	0.0547	77.9

[17] to lie above both states obtained by coupling a $p_{1/2}$ hole to the two lowest $(sd)^2$ 0^+ states. The second of these states is known at 3.663 MeV in ^{17}Ne and is lowered from its position in ^{18}O in large part because the spin-average $p_{1/2}^{-1}s_{1/2}$ $T=1$ particle-hole interaction is less repulsive by $\sim$700 keV than the coresponding $p_{1/2}^{-1}d_{5/2}$ interaction [18,17] and to a lesser extent because of the removal of the influence of the $4p2h$ configuration.

To put the structure of ^{17}N and ^{17}Ne in a broader context, it should be noted that the four particle-hole matrix elements mentioned above can be deduced directly from the binding energies of the lowest four states of ^{16}N (the charge-dependent shifts of the $0d_{5/2}$ and $1s_{1/2}$ orbits, including a dependence on separation energy, can be seen across these $T=1$ multiplets). Within the framework of the same weak-coupling assumption used to deduce the particle-hole matrix elements, the total binding energies and multiplet spacings of the low-lying states of the heavy carbon and nitrogen isotopes which contain one or more sd-shell neutrons can be rather nicely accounted for (of course, small components in the wave functions are important for detailed spectroscopic applications such as first-forbidden β decay). In consistent shell-model calculations which include charge-dependent interactions, the response to changes in T_z on the one hand and to changes in the number of particles or holes on the other strongly restricts the $d_{5/2}/s_{1/2}$ content of the low-lying states. An interesting case in the context of the present study is ^{16}C which has a rank-0 β-decay branch of 0.68% [19] to the lowest 0^- state of ^{16}N. With an extra $p_{1/2}$ proton hole, the energy of the excited 0^+ state has been lowered to 3.02 MeV, implying slightly more $1s_{1/2}^2$ in the ground state than for ^{17}N. The first-forbidden β-decay rate is well accounted for using the same type of shell-model calculation and meson-exchange enhancement as for ^{17}N [12].

A unique first-forbidden β branch of 1.6(5)% [20] to the ground state of ^{17}O is known for the decay of ^{17}N. This branch corresponds to $f^{(2)}=24(8)$. With no change in the single nuclear matrix element involved, the expected branch in ^{17}Ne decay is 0.55(18)%. Charge-dependent effects should lower this value slightly because of a decrease in the $d_{5/2}^2$ component of the ^{17}Ne ground state (Table IV), amplified somewhat by cancellation between $d_{5/2}\rightarrow p_{1/2}$ and $d_{5/2}\rightarrow p_{3/2}$ contributions. Shell-model calculations with the basis of Ref. [12] overpredict $f^{(2)}$ by a little more than a factor of two for either harmonic oscillator or Woods-Saxon wave functions. This is quite consistent with a similar overestimate for the unique first-forbidden decay of ^{16}N for a correspondingly small shell-model basis. This problem is resolved in calculations using a very large shell-model basis with all configurations up to $4\hbar\omega$ [8,21]. The rank-0 matrix elements are also reduced in such calculations [8], but by a lesser amount due to a cancellation between contributions from $2p2h$ admixtures induced by central and tensor forces. The experimental β-decay rates can then be reproduced using values for $\varepsilon_{\rm mec}$ close to the theoretical value of about 1.6 [8].

In conclusion, the use of realistic (e.g., Woods-Saxon) radial wave functions is essential for evaluating first-forbidden β-decay matrix elements [8,12], particularly for $1s_{1/2}\leftrightarrow 0p_{1/2}$ transitions for which the $1s_{1/2}$ nucleon is loosely bound, as is the case for the decay of ^{17}N and ^{17}Ne to the first-excited $1/2^+$ states of ^{17}O and ^{17}F. However, radial wave function differences do not account for the strong asymmetry observed for these decays. Rather, plausible T_z-dependent differences in the $1s_{1/2}$ occupancy for the initial states can account for the asymmetry. Furthermore, the very small separation energy for the $1s_{1/2}$ proton in ^{17}F is not germane to the problem since this proton is a spectator in the β-decay process. In fact, from the way in which the parentage expansion is made and separation energies determined, the spectator $1s_{1/2}$ proton forms part of a ^{16}F core where it is unbound for the physical core states (by 535 keV for the 0^- state). Substantial asymmetries have also been observed for the allowed decays of ^{17}N and ^{17}Ne [22]. While overlap factors for radial wave functions bound at different energies now play a role because the Gamow-Teller operator has no spatial structure, it again seems likely that the observed asymmetries are largely due to T_z-dependent mixing of various shell-model configurations. For the $2p1h$ configurations with $T=1/2$, the mixing of configurations with $T=0$ and $T=1$ for the $(sd)^2$ configurations determines both the overall spatial symmetry and the relative contributions to the Coulomb energy from p and sd orbits. There are also low-lying $4p3h$ configurations (one $1/2^-$ and two $3/2^-$) which have their own Coulomb energy shifts and mix strongly with the $2p1h$ configurations. Thus there should be significant T_z-dependent mixing in both the initial and final states for the Gamow-Teller decays. A beautiful demonstration of this type of T_z-dependent mixing is seen in changes of the ratio of Gamow-Teller strengths for the lowest two 2^+; $T=1$ states reached via (n,p), (p,p'), and (p,n) reactions on ^{14}N [23]. Here, the near degeneracy of $2h$ and $2p4h$ configurations [24], with Coulomb energies that differ by $\sim$700 keV across the multiplet, leads to very different wave functions for each nucleus.

This research was supported by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016 with Brookhaven National Laboratory.

[1] M. J. B. Borge, J. Deding, P. G. Hansen, B. Jonson, G. Martinez Pinedo, P. Møller, G. Nyman, A. Poves, A. Richter, K. Riisager, and O. Tengblad, Phys. Lett. B **317**, 25 (1993).

[2] A. R. Poletti and J. G. Pronko, Phys. Rev. C **8**, 1285 (1973).

[3] D. E. Alburger and D. H. Wilkinson, Phys. Rev. C **13**, 835 (1976).

[4] A. Ozawa, M. Fujimaki, S. Fukuda, S. Ito, T. Kobayashi, S. Momota, T. Suzuki, I. Tanihata, K. Yoshida, H. Kitigawa, G. Kraus, and G. Münzenberg, Report RIKEN-AF-NP-238.

[5] D. J. Millener, D. E. Alburger, E. K. Warburton, and D. H. Wilkinson, Phys. Rev. C **26**, 1167 (1982).

[6] H. Behrens and W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon, Oxford, England, 1982).

[7] K. Kubodera, J. Delorme, and M. Rho, Phys. Rev. Lett. **40**, 755 (1978).
[8] E. K. Warburton, I. S. Towner, and B. A. Brown, Phys. Rev. C **49**, 824 (1994).
[9] E. K. Warburton and I. S. Towner, Phys. Rep. **243**, 103 (1994).
[10] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 501 (1985).
[11] I. S. Towner and J. C. Hardy, Nucl. Phys. **A179**, 489 (1972).
[12] D. J. Millener and E. K. Warburton, *Nuclear Shell Models*, edited by M. Vallieres and B. H. Wildenthal (World Scientific, Singapore, 1985), p. 365.
[13] G. R. Satchler, *Direct Nuclear Reactions* (Clarendon, Oxford, England, 1983), Secs. 16.4 and 17.4.
[14] A. V. Nero, E. G. Adelberger, F. S. Dietrich, and G. E. Walker, Phys. Rev. Lett. **32**, 623 (1974); A. V. Nero, E. G. Adelberger, and F. S. Dietrich, Phys. Rev. C **24**, 1864 (1981).
[15] B. H. Wildenthal, *Progress in Particle and Nuclear Physics*, edited by D. H. Wilkinson (Pergamon, Oxford, England, 1984), Vol. 11, p. 5.
[16] S. H. Kahana, Phys. Rev. C **5**, 63 (1972).
[17] E. K. Warburton and D. J. Millener, Phys. Rev. C **39**, 1120 (1989).
[18] F. C. Barker, Aust. J. Phys. **37**, 17 (1984).
[19] C. A. Gagliardi, G. T. Garvey, N. Jarmie, and R. G. H. Robertson, Phys. Rev. C **27**, 1353 (1983).
[20] M. G. Silbert and J. C. Hopkins, Phys. Rev. **134**, B16 (1964).
[21] E. K. Warburton, B. A. Brown, and D. J. Millener, Phys. Lett. B **293**, 7 (1992).
[22] D. R. Tilley, H. R. Weller, and C. M. Cheves, Nucl. Phys. **A564**, 1 (1993).
[23] K. P. Jackson for the CHARGEX Collaboration, private communication.
[24] E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 (1960).