Configuration mixing calculation in the framework of the interacting boson model (2) in even $^{112-118}$Sn isotopes

A. K. Singh, G. Gangopadhyay, and D. Banerjee*
Department of Physics, University College of Science, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India
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In this paper, the normal and the intruder 2p-2h bands in even $^{112-118}$Sn isotopes have been studied through configuration mixing calculation in the framework of IBM-2. The states belonging to the normal and the intruder bands have been calculated separately and then allowed to mix by using a simple band-mixing Hamiltonian. Energy values, $E2$ transition probabilities, and quadrupole moments of the first 2^+ state have been calculated and compared with the experimental values.
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The structure of even-even Sn nuclei has been investigated earlier using different models, e.g., in the BCS approach using neutron two-quasiparticle excitation [1], broken-pair or generalized seniority schemes [2], etc. Existence of a collective band starting from 0^+ level [3] is a common feature in the $Z=50$ region. These intruder states have their origin in excitations across the proton shell closure at $Z=50$ [4]. Wenes *et al.* [5] have investigated these collective bands in even Sn isotopes through a model taking into account both the pure quadrupole vibrational excitations of doubly even nuclei as well as the proton 2p-2h configurations coupled with quadrupole vibrational excitations.

Ring and Schuck have described the lowest 2^+ state in ^{116,118,120}Sn as vibrational states [6]. Coulomb excitation study in even Sn isotopes has shown that the levels 2_2^+, 2_3^+, and 4_1^+ show partly a vibrational character [7]. For example, the value of $B(E2;\ 4_1^+ \rightarrow 2_1^+)$ observed for ^{116,118}Sn agrees with the value $2B(E2;2_1^+ \rightarrow 0_1^+)$ expected for a two phonon vibrational state. Also the crossover transitions $2_2^+ \rightarrow 0_1^+$ and $2_3^+ \rightarrow 0_1^+$ are strongly retarded. Wenes *et al.* have also interpreted the low-lying structure of Sn isotopes as evidence of strong mixing between the vibrational and the rotational structures. Recently we have tried to interpret the low-lying levels of $^{115-119}$Sn isotopes in terms of $U^{BF}(5)\otimes SU^F(4)$ dynamical symmetry of U(6/20) super Lie group [8]. The intruder bands in neighboring Cd [9] and Te [10] nuclei have been well described in IBM-2 using a configuration mixing calculation. These studies indicate a strong mixing between the normal and the intruder configurations. In Sn isotopes the experimental $B(E2)$ values of some of the transitions also indicate a strong mixing between the two coexisting configurations.

The purpose of this work is to investigate the ground-state band up to the two phonon triplets and the collective bands in even-even $^{112-118}$Sn nuclei in the framework of IBM-2 mixed configuration calculation in which the 2p-2h band is coupled with the anharmonic quadrupole vibration of the ground band. Investigation of the consequences in the Sn isotopes is also important in the context of I spin [11].

In the present calculation we follow the procedure of Duval and Barrett [12]. The IBM-2 Hamiltonian describing both normal ($N_\pi=0$) and intruder ($N_\pi=2$) configurations is given by

$$H=\epsilon_d(d_\pi^\dagger \tilde{d}_\pi + d_\nu^\dagger \tilde{d}_\nu) + V_{\pi\pi} + V_{\nu\nu} + \kappa(Q_\pi \cdot Q_\nu) + M_{\pi\nu}, \quad (1)$$

where

$$V_{\rho\rho}=\sum_{L=0,2,4} \frac{1}{2} C_{L\rho}\sqrt{2L+1}[(d_\rho^\dagger d_\rho^\dagger)^L(\tilde{d}_\rho \tilde{d}_\rho)^L]^0 \quad (2)$$

represents the interaction between like bosons. Here ρ refers to π (proton) or ν (neutron) bosons.

The expression $Q_\pi \cdot Q_\nu$, where

$$Q_\rho=(s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger s_\rho)^2 + \chi_\rho (d_\rho^\dagger \tilde{d}_\rho)^2, \quad (3)$$

stands for the quadrupole-quadrupole interaction between the neutron and proton bosons.

$$M_{\pi\nu}=-\sum_{k=1,3} 2\xi_k (d_\pi^\dagger d_\nu^\dagger)^k \cdot (\tilde{d}_\pi \tilde{d}_\nu)^k + \xi_2 (d_\pi^\dagger s_\nu^\dagger - s_\pi^\dagger d_\nu^\dagger)^2 \cdot (\tilde{d}_\pi s_\nu - s_\pi \tilde{d}_\nu)^2 \quad (4)$$

is the Majorana force operator. Both the normal and the intruder configurations are calculated separately in terms of the above Hamiltonian. The two configurations are then admixed using the mixing operator

$$H_{\rm mix}=\alpha(s_\pi^\dagger s_\pi^\dagger + s_\pi s_\pi)^0 + \beta(d_\pi^\dagger d_\pi^\dagger + \tilde{d}_\pi \tilde{d}_\pi)^0, \quad (5)$$

where α and β are adjustable strength parameters for the interaction between the two configurations. In our calculation, the basis consisting of the lowest four eigenstates of each configuration has been used to diagonalize $H+H_{\rm mix}$. The energy gap parameter Δ between the two configurations is calculated from the relation [12],

*Corresponding author.

TABLE I. IBM parameters used in the present calculation. All the parameters are in MeV except χ_ρ and e_2/e_0, which are dimensionless and e_ρ, which are in e fm^2.

Parameters	$A=112$	$A=114$	$A=116$	$A=118$
	Normal Configuration			
ϵ_d	1.25	1.30	1.25	1.20
$C_{0\nu}$	−0.3	−0.3	−0.4	−0.3
$C_{2\nu}$	−0.15	−0.15	−0.20	−0.10
χ_ν	0.74	0.63	1.92	0.89
	Intruder Configuration			
ϵ_d	0.60	0.57	0.55	0.59
κ	−0.167	−0.140	−0.147	−0.158
χ_ν	−0.2	−0.15	−0.10	−0.05
χ_π	0.4	0.4	0.4	0.4
$C_{0\nu}$	0.0	0.0	−0.15	−0.05
$C_{2\nu}$	−0.06	−0.10	−0.05	−0.15
$C_{4\nu}$	−0.03	0.0	0.03	0.015
ξ_1	0.24	0.24	0.24	0.24
ξ_2	0.04	0.04	0.04	0.04
ξ_3	0.24	0.24	0.24	0.24
e_2/e_0	1.2	1.2	1.2	1.2
α	0.2	0.2	0.2	0.2
e_ν	9.0	8.5	7.0	7.5
e_π	12	12	12	12

$$\Delta = BE(Z,N) - BE(Z-2,N) - BE(Z+2,N) + BE(Z,N) + 4V_{\rm ph}. \quad (6)$$

Here BE refers to the binding energy of the ground state and $V_{\rm ph}$ is the particle hole interaction energy.

The mixed wave functions are used to calculate $B(E2)$ values of observed transitions and quadrupole moments. The $E2$ transition operator is given by

$$T(E2) = e_0(e_{\pi 0}Q_{\pi 0} + e_{\nu 0}Q_{\nu 0}) + e_2(e_{\pi 2}Q_{\pi 2} + e_{\nu 2}Q_{\nu 2}), \quad (7)$$

where Q_ρ were defined in Eq. (3) and e_j and $e_{\rho j}$ $(j=0,2)$ are adjustable parameters. The suffixes 0 and 2 refer to the normal and the intruder configurations respectively.

In Sn isotopes it is very difficult to identify the high-energy members of the normal ground-state band. Even the two phonon triplets in these isotopes mix strongly with the 2p-2h band and the neutron two quasiparticle levels. In the present calculation we have considered levels up to lowest 4^+ of the normal band and up to 12^+ state of the intruder band. Since there are no proton bosons in the normal configuration of the Sn isotopes, terms in the Hamiltonian involving only neutron bosons will contribute to the energy value calculation. The three parameters used in the normal band calculation are ϵ_d, C_0, and C_2 and their adopted values are given in Table I.

It is expected that the intruder configuration has a gamma-soft structure. In this calculation the intruder band has been chosen to have E_4/E_2 ratio around 2.6, which is close to O(6) limit. The adopted values of the parameter ϵ_d show a smooth variation with the neutron number and becomes minimum at midshell. The value of κ increases from $N=62$ to $N=64$ and then gradually decreases with increasing neutron number. Similar type of neutron number dependence has been observed for the parameter κ in the calculation on Te isotopes. The value of χ_π has been kept constant in all the intruder bands in Sn isotopes. The parameter χ_ν for intruder configuration has been varied. The adopted values of this parameter are slightly less than zero. The values of the Majorana force parameter ξ_1, ξ_2, and ξ_3 are taken from the calculation on the neighboring Cd isotopes [9]. The two configurations are calculated separately and they are admixed by the mixing operator $H_{\rm mix}$. Jolie and Lehmann [13] have shown that for an U(5)-O(6) mixing using Eq. (5), there is only one free parameter as the matrix elements of the two terms involving the two types of bosons are not independent. Therefore, we have used only one parameter α, which we have not varied with mass number. The energy-gap parameter Δ between the two configurations has been calculated using Eq (6). The value of $V_{\rm ph}$ for $Z=50$ region has been found to be -540 KeV [14]. The necessary experimental binding energy values have been calculated from [15]. All the other parameters are taken to be zero.

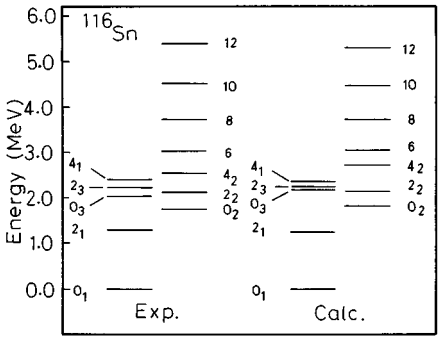

FIG. 1. Comparison of experimental and calculated energy levels of ^{116}Sn isotopes. The suffixes to the spin value refers to the relative position of the spin states occurring more than once in the level scheme.

The calculated energy spectra of ^{116,118}Sn isotopes have been compared with experiment in Figs. 1 and 2. The experimental energy values are taken from Ref. [16] and [17], respectively. Similar agreement between calculated and experimental energy spectra of ^{112}Sn and ^{114}Sn nuclei has also been obtained.

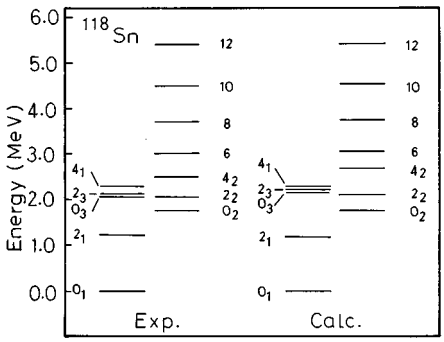

FIG. 2. Comparison of experimental and calculated energy levels of ^{118}Sn isotopes. See caption of Fig. 1 for details.

TABLE II. Comparison of experimental and calculated $B(E2)$ values of low-lying states in ^{112,114,116,118}Sn isotopes.

Mass	Transitions $I_i \to I_f$	E_γ(KeV)	$B(E2)(e^2$ fm$^4)$ Expt.	Calc.
112	$2_1 \to 0_1$	1257	512(12)[a]	500
	$2_2 \to 0_1$	2151	1.3(4)[a]	0.003
	$0_2 \to 2_1$	933	$\leqslant$ 290[b]	885
	$2_2 \to 2_1$	894	370(150)[a]	875
	$2_3 \to 0_1$	2476	$<$1.9[a]	0.4
	$2_3 \to 2_1$	1219	$<$14[a]	0.05
	$4_1 \to 2_1$	991	180(30)[a]	869
114	$2_1 \to 0_1$	1300	500(100)[a]	524
	$0_2 \to 2_1$	653	700(300)[b]	710
	$2_2 \to 0_1$	2240	$<$3[a]	0.003
	$0_3 \to 2_1$	856	$<$160[b]	450
	$2_2 \to 2_1$	940	$<$130[a]	555
	$4_1 \to 2_1$	887	210(60)[a]	895
	$4_2 \to 2_1$	1314	$<$100[a]	73
	$4_2 \to 2_2$	374	$<$2500[a]	1556
116	$2_1 \to 0_1$	1294	417(13)[c]	408
	$0_2 \to 2_1$	463	605(101)[c]	756
	$0_3 \to 2_1$	733	17(2)[c]	220
	$2_2 \to 0_3$	85	1143(437)[c]	446
	$2_2 \to 0_2$	355	739(267)[c]	917
	$2_2 \to 0_1$	2112	2.0(8)[c]	0.001
	$2_2 \to 2_1$	818	131(50)[c]	398
	$2_3 \to 0_1$	2225	1.7(10)[c]	0.002
	$2_3 \to 2_1$	931	101(54)[c]	381
	$2_3 \to 0_2$	468	34(20)[c]	812
	$2_3 \to 0_3$	198	$<$48[a]	37
	$4_1 \to 2_1$	1097	1277(706)[c]	705
	$4_1 \to 2_2$	279	3025(1680)[c]	607
	$4_2 \to 2_1$	1235	$>$0.005[c]	82
	$4_2 \to 2_2$	417	$>$403[c]	1600
	$4_2 \to 2_3$	304	$>$7.4[c]	260
118	$2_1 \to 0_1$	1230	416(17)[d]	414
	$0_2 \to 2_1$	528	653(103)[d]	670
	$2_2 \to 0_2$	285	1341(241)[d]	1236
	$2_2 \to 2_1$	813	237(34)[d]	281
	$2_2 \to 0_1$	2043	2.6(4)[d]	0.02
	$0_3 \to 2_1$	827	$>$7.2[d]	313
	$4_1 \to 2_1$	1050	585(103)[d]	739
	$4_1 \to 2_2$	237	481(241)[d]	600
	$4_2 \to 2_1$	1259	$<$96[d]	45
	$4_2 \to 2_2$	446	$<$8600[a]	1757

[a]Reference [7].
[b]Reference [18].
[c]Reference [16].
[d]Reference [17].

The energy values are not very sensitive to the wave function. Therefore, it is necessary to calculate the transition probabilities, which are very sensitive to the wave function of the system. $B(E2)$ values have been calculated by evaluating matrix elements of the $T(E2)$ operator between the states. For the normal configuration of the Sn isotopes only the neutron part of the first term of the $T(E2)$ operator will contribute. The parameters used in evaluating the matrix elements of the first term of $T(E2)$ operator are only e_ν and χ_ν. The adopted values of these parameters are given in Table I. Matrix elements of the second term of this operator have been calculated in the space of two proton bosons and different number of neutron bosons for these Sn isotopes. The parameter χ_ν and χ_π are the same as used in the energy-value calculation. The adopted values of the effective neutron bosonic charge show a smooth variation with the neutron number, becoming minimum for $N=66$. A similar type of variation has also been observed in the calculation on the neighboring Te isotopes [10]. The effective bosonic charge for neutron bosons has been kept unchanged for both the configurations. The $E2$ matrix elements are very sensitive to the ratio of the parameters e_2/e_0 which has been kept constant at 1.2 for all Sn isotopes. The parameters used in $B(E2)$ calculation are given in Table I.

The experimental and calculated $B(E2)$ values for ^{112,114,116,118}Sn isotopes have been compared in Table II. We discuss some of the differences between the experimental observations and the theoretical predictions. In ^{116}Sn, the $E2$ transition probabilities of $2_3^+ \to 2_1^+$ and $0_3^+ \to 2_1^+$ transitions are overpredicted. This indicates that the present model may have underestimated the contributions of the intruder band to the 2_3^+ and 0_3^+ states. The large experimental $B(E2)$ value of $2_2^+ \to 0_3^+$ transition supports this argument. However, weaker transition probability for $2_3^+ \to 0_2^+$ transition cannot be explained with stronger mixing. Not many experimental data are available on ^{112,114}Sn isotopes. The experimental value of the transition $4_1^+ \to 2_1^+$ is surprisingly very small compared to the neighboring ^{116,118}Sn isotopes, and cannot be explained on the basis of the present model. One reason may be that in this 4_1^+ state there is a large contribution from the neutron two quasiparticle structure resulting in a weaker transition probability. The experimental $B(E2)$ value of the transition $0_2^+ \to 2_1^+$ in ^{112}Sn nucleus is nearly half the value in ^{114,116,118}Sn isotopes, a feature not reproduced by this calculation. Except for these discrepancies, almost all the other transitions are explained on the basis of this simple model. We believe that one of the major sources of error is traceable to neglect of the contribution of two quasiparticle states.

In addition to the transition probabilities, we have also calculated the static quadrupole moment of the first 2^+ state. The calculated values are compared with the experimental observations in Table III.

TABLE III. Comparison of experimental and calculated quadrupole moments of 2_1^+ states in ^{112,114,116,118}Sn isotopes.

Mass	state	Quadrupole moment (e b) $Q_{\text{expt.}}$	$Q_{\text{calc.}}$
112	2_1^+	$-0.03(11)$	-0.11
114	2_1^+		-0.14
116	2_1^+	$-0.17(4)$	-0.21
118	2_1^+	$-0.05(14)$	-0.10

In this paper, we have studied both normal and intruder configurations in the framework of IBM-2 mixed configuration calculation. Both normal ($N_\pi=0$), and intruder ($N_\pi=2$) bands are well reproduced by this calculation. However, the differences between some of the experimental $B(E2)$ values and theoretical predictions indicate a much more complex structure for the low-lying states, especially in the two phonon triplets in the Sn isotopes. The neglect of the neutron two-quasiparticle structure in this calculation may also have influenced these results.

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