Ambiguities in the elastic scattering potentials for 350 MeV ^{7}Li ions on ^{12}C and ^{28}Si

M. T. Bennett and K. Amos
School of Physics, University of Melbourne, Parkville 3052, Victoria, Australia
(Received 16 October 1996)

The differential cross sections from the scattering of 350 MeV ^{7}Li ions from ^{12}C and from ^{28}Si have been analyzed by both a global and a numerical inverse scattering theory. Statistically significant fits to both high quality data sets have been found from which the associated effective local interaction potentials between the colliding ions have been deduced. The interactions obtained by the different approaches are radically different. *A priori* physical information about the nature of realistic effective interactions is required to ascertain which, if any, is the most reasonable one for use in analyses of other reaction data from these systems. [S0556-2813(97)06202-X]

PACS number(s): 25.70.Bc, 24.10.−i

Inversion procedures [1] to analyze elastic scattering data are ambiguous, be they of numerical form as with the conventional optical model (OM) scheme or of global form such as the Lipperheide-Fiedeldey (LF) schemes [2] that we use herein. In OM studies there are always equivalent parameter sets defining potentials from which identical scattering phase shifts are obtained. In global inversion procedures, the phase shifts given by a fit to the measured data are not unique, even if unitarity can be used as a constraint [3]. But often those ambiguities result because the data set is not sensitive enough to discriminate between competing interaction forms. However, there are now high quality data sets, i.e., many data over a wide range of scattering angles and with small statistical errors. But with them, it has proved very difficult to obtain fits having statistical significance, especially with numerical inversion by standard optical model search schemes. In the present study, we report a case in which the desired quality of fit to an extensive data set has been found, not only by using the global (LF) inverse scattering theory, but also with a conventional OM search program. Thus we may compare extracted complex local heavy ion interactions that are equally established so far as the quality of fit to the experimental data is concerned.

The alternative approach is to analyze the scattering using ''direct'' means such as by double folding an appropriate effective two-nucleon interaction with the density matrices of the colliding systems to specify an effective optical potential [4]. To date, such attempts have not been successful in so far as a stringent quality of fit to the data is concerned. Most double folding specifications of the internuclear (heavy ion) interaction use oversimplistic forms for the effective two-nucleon interaction and usually resort to phenomenological forms for the absorptive component. By so doing those studies, like the purely phenomenological optical model approach, are then examples of numerical inversion.

Whether ''direct'' or ''inverse'' (global or numerical) approaches are taken to analyze quantum scattering data, the measure of the fit often chosen is the total value of chi square, χ^2, where, for $\mathcal{N}$ data points $\sigma(\theta_i);i=1,\mathcal{N}$ with errors, $d\sigma(\theta_i)$ and calculated results $\sigma^{\rm th}(\theta_i)$,

$$\chi^2=\sum_{i=1}^{\mathcal{N}}\frac{[\sigma(\theta_i)-\sigma^{\rm th}(\theta_i)]^2}{[d\sigma(\theta_i)]^2}.$$

Sometimes the chi square per datum $\chi^2/\mathcal{N}$ is used, but it is best to specify a fit by the chi square per degree of freedom χ^2/F, where the degrees of freedom we identify as the number of data points ($\mathcal{N}$) less the number of free parameters (M) adjusted in the process to give the fit to the data. A result is said to be statistically significant if it is associated with a value of χ^2/F of order unity. When that is so it has proved possible, in some cases, to make an error analysis of the final result and so place confidence limits upon the radial values of the (local) complex effective heavy ion interaction [5]. Such limits are subject to the reliability of the normalization of the measured data. Frequently that uncertainty is set as 10%. Some investigations then have used that figure as the experimental error in the evaluations of χ^2 whence small values for that can result. But then the result is not guaranteed to be a sensitive one. We retain, in preference, the statistical error reported for the experiment.

As noted above, we have found a case in which the desired quality of fit to an extensive data set has been achieved, not only by using the global (LF) inverse scattering theory, but also with a conventional OM search program, and with use of the reported (small) statistical errors. Specifically we have analyzed the differential cross section obtained by the scattering of 350 MeV ^{7}Li ions off of ^{12}C [6]. A companion data set of 350 MeV ^{7}Li scattering from ^{28}Si has also been analyzed, but for which we were unable to find an OM fit to meet the stringent statistically significant requirement.

A central feature of most studies of quantal (elastic) scattering is the S matrix, S_ℓ, or, equivalently, the phase shifts for partial wave ℓ identified by $S_\ell=e^{2i\delta_\ell}$. For most scattering systems however, the processes by which those phase shifts are extracted from data (for inverse scattering theories) are inherently ambiguous. But so also are predictions by direct solution of the Schrödinger equations with any specified interaction potential, as there are always phase shift equivalent forms associated with supersymmetric partners or on addition of ''transparent'' potentials. To use the LF inverse scattering theories, one seeks S functions defined in the entire complex angular momentum plane and in the form of a rational function of the complex angular momentum (λ),

$$S(\lambda)=S_{\rm ref}(\lambda)\prod_{n=1}^{N}\left(\frac{\lambda^2-\beta_n^2}{\lambda^2-\alpha_n^2}\right), \qquad (1)$$

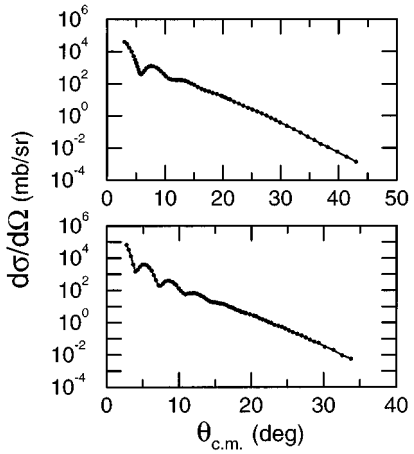

FIG. 1. The differential cross sections for the elastic scattering of 350 MeV ^{7}Li ions from ^{12}C (top) and from ^{28}Si (bottom) compared with the results calculated from solutions of the Schrödinger equations with the LF inversion potentials.

where, with η being the Sommerfeld parameter,

$$S_{\rm ref}(\lambda)=\exp[i\eta\ln(\lambda^2+\lambda_c^2)] \tag{2}$$

is a reference (modified Coulomb) S function. With λ_c appropriately chosen to define forward scattering results, the minimal set of (complex) pole/zero pairs, $(\{\alpha_n,\beta_n\})$, are found by optimally fitting the cross-section data. For scattering at energies below the first nonelastic threshold, whence the S function is unitary, the pole/zeros form a set of complex conjugate values. Above that threshold, that constraint is broken. Allowing flux loss then there are $4N+1$ free parameters in the search process. For the ^{12}C data, a fit with χ^2/F of 0.98 was found by using 21 parameters (5 pole/zero pairs and $\lambda_c=1.4$). The ^{28}Si cross section was fit similarly with a value of χ^2/F of 1.01, but required 9 pole/zero pairs and $\lambda_c=2.8$. Those fits are displayed in Fig. 1. In fact, shown therein are the fits resulting from use of the (LF) inversion potentials we have obtained in calculating solutions of the Schrödinger scattering equations. The original fit results are equivalent on the scales shown. The displayed results are very similar also to those we have obtained using phenomenological OM potentials. In those cases, fits to the quoted data were found having measures (χ^2/F) of 1.07 and 7.17 for the ^{12}C and ^{28}Si cases, respectively. The basic optical potential chosen had the seven-parameter form

$$V_{\rm OM}(r)=V_{\rm Coul}(r)+V_0f(x_0)+iW_cf(x_c), \tag{3}$$

where $V_{\rm Coul}(r)$ was taken as the potential of a uniformly charge sphere of radius $r_cA^{1/3}$ and the form factors have Woods-Saxon form, viz.,

$$f(x_i)=\left[1+\exp\left(\frac{r-r_iA^{1/3}}{a_i}\right)\right]^{-1}. \tag{4}$$

That interaction sufficed to give the excellent result for the ^{12}C cross section. To find the fit for the ^{28}Si data, however, it was necessary to extend that base potential by ''surface'' terms, i.e., to use

TABLE I. Optical model parameter values for the fits to the differential cross sections from the elastic scattering of 350 MeV ^{7}Li ions from ^{12}C and from ^{28}Si.

	^{7}Li-^{12}C	^{7}Li-^{28}Si		^{7}Li-^{12}C	^{7}Li-^{28}Si
V_0 (MeV)	107.507	61.041	W_c (MeV)	39.303	2.347
r_0 (fm)	1.364	1.564	r_c (fm)	1.640	2.466
a_o (fm)	0.859	0.759	a_c (fm)	0.764	0.970
V_s (MeV)		14.612	W_S (MeV)		24.177
r_s (fm)		1.352	r_S (fm)		1.391
a_s (fm)		0.283	a_S (fm)		0.646

$$V_{\rm OM}(r)=V_{\rm Coul}(r)+V_0f(x_0)+4A_sV_s\frac{df(x_s)}{dr}+i\left(W_cf(x_c)+4a_SW_S\frac{df(x_S)}{dr}\right). \tag{5}$$

The parameter values are given in Table I. For the ^{12}C data, those parameters are so similar to the ones determined by Nadasen *et al.* [6] that we can identify the result as their ''unique'' OM potential. With the ^{28}Si data, if we restricted the search to the seven-parameter model, we obtained a best fit with parameter values very similar again to those found by Nadasen and his colleagues. But the fit measure is poor with χ^2/F being 14.3. Note that if we use 10% errors on the data, then the value drops to 3.8 consistent with the result quoted in Ref. [6]. But a better result can be found by extending the OM potential form to the 13-parameter function specified in Eq. (5). With the parameter values listed in Table I, the data is fit with $\chi^2/F=7.17$; a fit that is still not statistically significant (unlike the LF inversion we have obtained with this data) but nevertheless much improved upon the previous results. This poorer fit (compared to the ones for the ^{7}Li-^{12}C case) is due mainly to a misfit at small scattering angles, and notably to the four data points around the minimum at 7.4°.

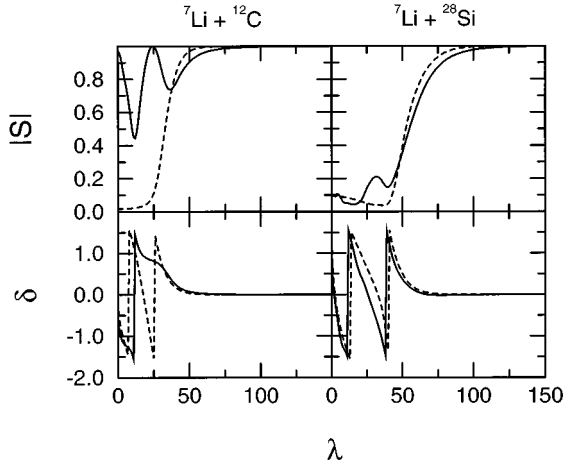

FIG. 2. The S functions (for real λ) given in magnitude (top) and phase (bottom) and which were found with the rational function fits to the data (solid curves) and by the OM search procedures (dashed curves). The phases of the S functions are plotted modulo π to facilitate viewing.

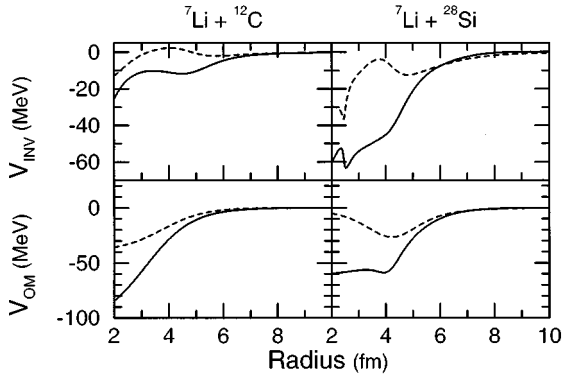

FIG. 3. The complex (local) potentials found by LF inversion (top) of, and by the OM search procedures (bottom) on, the elastic scattering differential cross sections for 350 MeV ^{7}Li ions from ^{12}C (left panel) and from ^{28}Si (right panel). The refractive and absorptive components are displayed by the solid and dashed curves, respectively.

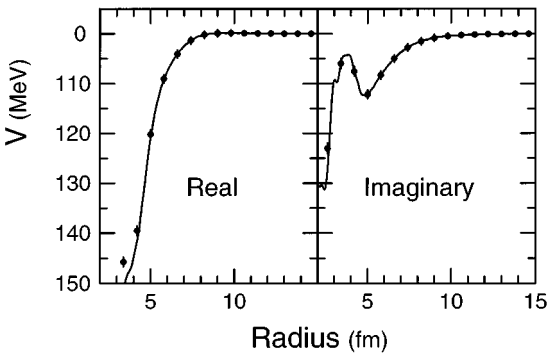

FIG. 4. The (WKB) confidence levels and the LF inversion potential from the analysis of the ^{7}Li-^{28}Si cross-section data.

Thus we have statistically significant fits with both the LF inversion and the OM S functions to the ^{7}Li-^{12}C scattering cross section, but only with the LF inversion S function for the ^{7}Li-^{28}Si data. The OM fit to the ^{28}Si data is quite good nevertheless, and leads to a surprise indicated first when we compare the various S functions on the real angular momentum axis. Such is done in Fig. 2. Therein the S functions are shown, moduli in the top panels and phases (modulo π) in the bottom ones with the LF rational S function forms displayed by the solid curves, while the OM ones are shown by the dashed curves. The two ^{12}C scattering results are radically different. The LF rational form is not of strong absorption type, never having a modulus less than 0.4, and the associated phase never exceeds 4 radians. In contrast, the OM result is typical with the low partial wave contributions of very little significance. The two model results for the ^{28}Si case however are quite similar. One could consider the LF fit result to be the natural improvement needed with the OM fixed form interaction to give a statistically significant fit to the data.

The potentials are compared in Fig. 3; the real and imaginary components being shown by the solid and dashed curves, respectively. The results are only portrayed from 2 fm as the data are not sensitive to shorter range properties of either interaction. The differences between the LF inversion and the OM potentials for the ^{7}Li-^{12}C scattering are as marked as the associated S functions. But recall that both give statistically significant fits to that data. The LF inversion result is weakly refractive and weakly absorptive with a slight long wavelength oscillatory character in both attributes. Such, and even more pronounced, oscillatory behavior has been noted with other data analyses [5] and may reflect the strong nonlocality that one should expect of a more physical representation of heavy ion interactions. On the other hand, the OM result is strongly refractive and absorptive.

In contrast, the LF inversion and the OM potentials from the analyses of the ^{7}Li-^{28}Si data are quite similar, at least from about 4 fm. They are strongly refractive and moderately absorptive with some oscillatory behavior. But there are still noticeable differences between these ^{28}Si results. The LF inversion potential is shallower on the whole and concommitantly less absorptive. Those differences are significant though since the quality of the fit obtained by the LF inversion approach permits us to make an error analysis of the resultant interaction values. Allen and McCarthy [7] have shown how the LF rational form of S function can be used easily within the WKB approximation to place confidence limits on the (WKB) inversion potentials at each radius. Such confidence levels signify the 67% probability bounds within which any potential (of that class) would lie if it were to fit the same data with $\chi^2/F \sim 1$. Recently the scheme enabled us [5] to find very narrow bands surrounding LF inversion potentials from analyses of diverse, high quality, nuclear scattering cross-section data. With the ^{7}Li-^{28}Si result of current interest, those confidence levels are displayed in Fig. 4. The relevant components of the WKB inversion potential are shown therein by the dots. They concur very well with the quantal (LF) inversion potential components (solid curves) at all radii, but the smallest. The confidence limits, portrayed by the error bars on those dots, are small at all radii shown. The OM results do not lie within those bands. In fact, they are not even within a few standard deviations. So despite pictorial similarity, it seems unlikely that an OM form which improves upon our result will reduce to the LF potential.

Clearly, the quality data from the 350 MeV ^{7}Li scattering experiments on ^{12}C and ^{28}Si do not fix a unique complex local potential form for the effective interaction between the ions. In the case of scattering from ^{12}C, the LF inversion and our OM potentials are radically different. They are two different classes of Schrödinger potentials with which the data can be fit with statistical significance. They do not satisfy a supersymmetry given the quite different S functions (and so phase shifts) with which they are associated. That is likely to be the case also with the ^{28}Si scattering, despite the similarity (graphically) of the S functions and potentials, given the very narrow confidence bands that the WKB error analysis placed on the LF inversion results.

Therefore, *a priori* information is needed with any analysis to identify the ''most physical'' effective interaction between heavy ions given that such interactions are input to analyses of nonelastic reaction data.

[1] K. Chadan and P. S. Sabatier, *Inverse Problems in Quantum Scattering Theory*, 2nd ed. (Springer, Berlin, 1989).
[2] R. Lipperheide and H. Fiedeldey, Z. Phys. A **286**, 45 (1978); **301**, 81 (1981).
[3] H. Huber, D. R. Lun, L. J. Allen, and K. Amos, Phys. Rev. A **54**, 1363 (1996); (to be published).
[4] D. T. Khoa, G. R. Satchler, and W. von Oertzen, Phys. Rev. C **51**, 2069 (1995).
[5] M. T. Bennett, C. E. Steward, L. J. Allen, and K. Amos, Phys. Rev. C **54**, 822 (1996).
[6] A. Nadasen *et al.*, Phys. Rev. C **52**, 1894 (1995).
[7] L. J. Allen and I. E. McCarthy, Phys. Rev. A **36**, 2570 (1987).