Flavor asymmetry of the nucleon sea

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We reexamine the effects of antisymmetry on the antiquarks in the nucleon sea arising from gluon exchange and pion exchange between confined quarks. While the effect is primarily to suppress down relative to up antiquarks, this is numerically insignificant for the pion terms. [S0556-2813(97)04802-4]

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I. INTRODUCTION

Since the precise measurement of the Gottfried sum rule [1] by the New Muon Collaboration (NMC) [2] in 1991, the possibility that $\overline{u}(x)\neq\overline{d}(x)$ in the proton has been extensively studied. The reason for this attention is that, in the quark-parton model, the Gottfried sum rule is expressed as

$$S_G=\int_0^1[F_2^p(x)-F_2^n(x)]\frac{dx}{x}=\frac{1}{3}+\frac{2}{3}\int_0^1[\overline{u}(x)-\overline{d}(x)], \tag{1}$$

where charge symmetry between the proton and the neutron was used. If SU(2) flavor symmetry $\overline{u}=\overline{d}$ is also assumed, S_G is reduced to 1/3.

In order to test the Gottfried sum rule experimentally, the NMC [2] performed deep inelastic muon scattering on hydrogen and deuterium and measured the deuteron/proton cross-section ratio. To extract the proton and neutron structure functions, they used the simple relation $F_2^p(x)-F_2^n(x)$ $F_2^p(x) - F_2^n(x) = 2F_2^d(x)[1 - F_2^n(x)/F_2^p(x)]/[1+F_2^n(x)/F_2^p(x)]$, with $F_2^n(x)/F_2^p(x)=2F_2^d(x)/F_2^p(x)-1$. A parametrization [2,3] for $F_2^p(x)$ based on NMC, SLAC, and BCDMS data was used. The NMC reported the following value for the Gottfried sum rule:

$$S_G(0.004\leqslant x\leqslant 0.8)=0.221\pm0.008(\text{stat})\pm0.019(\text{syst}),$$

$$S_G=0.235\pm0.026, \tag{2}$$

where the contribution for $x>0.8$ was estimated using a smooth extrapolation of F_2^n/F_2^p and for the region $x<0.004$ a Regge-like behavior (ax^b with $a=0.2\pm0.03$ and $b=0.59\pm0.06$) was assumed. It is worth noticing that in the extraction of F_2^n/F_2^p, target mass corrections and nuclear effects were not included; nor were higher twist corrections included in the sum rule as a whole. However, a detailed examination of binding, shadowing, and meson exchange (antishadowing) corrections [4] does not greatly change their conclusions.

The experimental result from the NMC yields

$$\int_0^1 dx[\overline{u}(x)-\overline{d}(x)]=-0.147\pm0.039. \tag{3}$$

Theoretically, there has been some indication for $\overline{u}\neq\overline{d}$ since the work of Field and Feynman [5], where the Pauli principle was invoked to justify that $u\overline{u}$ pair creation is suppressed relative to $d\overline{d}$. Later, Ito *et al.* [6] measured continuum dimuon production and determined the sea quark distribution in the context of the Drell-Yan description of dilepton production. From an analysis of the logarithmic derivative of the measured cross section, they inferred that the Drell-Yan model, assuming a symmetric sea, underestimated the measured slope. This fact was interpreted as an indication of broken symmetry in the sea, with $\overline{u}<\overline{d}$. Somewhat later, Thomas [7], following Sullivan's suggestion [8] that the pion cloud can contribute to the nucleon structure function, realized that pion dressing of the nucleon naturally predicts an excess of $\overline{d}$ over $\overline{u}$. This is because, for instance, the proton has a greater probability to emit a π^+ than a π^0 or a π^-. In fact, this observation was used [9] to interpret the results of Ito *et al.* on dilepton production. Since then, the idea of pions producing an asymmetric sea has been widely explored [10] as a possible interpretation of the NMC results. Further theoretical evidence for asymmetry in the sea was found in the calculation of quark distributions of Signal and Thomas [11]. The matrix elements of the quark distribution involve intermediate states where the photon scatters on a valence quark and also intermediate states where the photon oscillates into a $q\overline{q}$ pair and a quark or antiquark is inserted in the nucleon. Again, following the idea of Field and Feynman, it is easier to insert a d than a u quark into the proton.

Even with all this evidence, until the NMC experiment, the nucleon continued to be seen as having a symmetric sea. There is a simple explanation for this. The whole set of data available could be described by parametrizations of parton distributions where it was assumed that $\overline{u}(x)=\overline{d}(x)$, with the price of a slightly odd behavior of the valence quark distribution when $x\rightarrow0$ [12]. Along this line of thought, it was also proposed that the NMC result could still be reproduced with flavor and charge symmetry but with a modified behavior for $F_2(x)$ at small x, in such way that the integral $[F_2^p(x)-F_2^n(x)]/x$ saturates the naive expectation 1/3 [13].

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However, the NA51 Collaboration [14] recently carried out an experiment proposed by Ellis and Stirling [15] to distinguish between a symmetry-breaking effect and an odd small x behavior of the parton distributions. The NA51 results indicate a strong flavor symmetry breaking at $x=0.18$, thus reinforcing the idea that the sea is indeed flavor asymmetric. In a recent reanalysis [16] of the NA51 result, this conclusion was reinforced, although the exact size of the flavor asymmetry is dependent on the extent of the charge symmetry breaking between the proton and the neutron. Finally, calculations based on meson cloud models [17] are also able to produce an asymmetry compatible with what is measured by the NA51 collaboration.

We then see that the Pauli principle and pion dressing of the nucleon are two powerful tools to understand flavor symmetry breaking. If a bare proton is seen as having three valence quarks, two up and one down, then all the antiquarks have their origin in gluonic and/or pionic effects. For instance, pair creation through gluon emission of one of the valence quarks will produce an intermediate state of four quarks and one antiquark. The flavor of the antiquark is certainly restricted by the valence structure of the proton via the Pauli principle. However, one should also account for the antisymmetrization between the various quarks. In this case, the valence structure will also interfere in the number of antiquarks created. In order to understand the interplay of these effects, we study in some detail the relation between the Pauli principle and quark antisymmetrization in pair creation inside a nucleon.

In the pionic case the same reasoning applies. Of course, the pion is complicated by the fact that it is a pseudo-Goldstone boson. Thus its internal structure will be quite complicated, and the probability to find just a q-$\overline{q}$ pair with the quark in a $1s$ state would be expected to low. As antisymmetrization with the valence quarks in the nucleon will only effect quarks in the $1s$ state (of the nucleon bag) we expect that antisymmetry should have a rather small effect on pion loops. Nevertheless, it is important to quantify this prejudice and that is the main purpose of the present calculation.

II. PAULI PRINCIPLE IN THE PROTON SEA

The most natural idea to account for the observed discrepancy between theory and experiment is to invoke the Pauli principle. Field and Feynman [5] were the first ones to use this idea: If the proton is composed of two valence u quarks and one valence d quark, then the creation of a quark-antiquark pair through gluon emission will tend to give more $d\overline{d}$ pairs than $u\overline{u}$ pairs. This is easy to understand because there are five empty states for the d quark and four for the u quark. Although this idea is attractive and essentially correct, there is one other effect to consider. We will see that we also have to consider graphs containing interference between the sea quarks generated in the gluon emission and the remaining quarks in the nucleon. This effect will, in fact, hide the excess of $\overline{d}$ over $\overline{u}$ due to the Pauli principle.

To illustrate our discussion, we begin by reviewing the pair creation through gluon exchange, following the calculations of Donoghue and Golowich [18]. The assumption made is that the bare proton is composed of two valence u quarks, one valence d quark. Its color, flavor, and spin wave function is given by

$$|p\rangle_0=\frac{1}{\sqrt{18}}\epsilon^{\alpha\beta\gamma}[b^\dagger(u,\uparrow,\alpha)b^\dagger(d,\downarrow,\beta) - b^\dagger(u,\downarrow,\alpha)b^\dagger(d,\uparrow,\beta)]b^\dagger(u,\uparrow,\gamma)|0\rangle. \tag{4}$$

The sea quarks are generated through a quark gluon interaction given by

$$H_I(x)=g\overline{\psi(x)}\gamma^\mu\frac{\lambda^a}{2}\psi(x)A^a_\mu(x), \tag{5}$$

where g is the coupling constant, A^a are the gluon fields, and ψ the quark field. We are not going to worry about the form of the spatial part of these operators but will concentrate only on the color, spin, and flavor parts of the proton dressed with a quark-antiquark pair:

$$|p\rangle=|p\rangle_0+\frac{1}{E_0-H_0}H_I\frac{1}{E_0-H_0}H_I|p\rangle_0+\cdots, \tag{6}$$

with H_0 the free Hamiltonian. (Note that we have omitted the term corresponding to a single gluon with the three valence quarks.)

The vector coupling between quarks and gluons allows for vertices where the quark that emits the gluon either changes its spin or not. Another feature of the pair creation process is that, since a particle has opposite intrinsic parity to an antiparticle, at least one quark (or the antiquark) or the two quarks and the antiquark have to be in a state of odd parity in order to conserve the parity of the proton. The proton wave function is then written as

$$|p\rangle=|p\rangle_0+C_s|\psi_s\rangle+C_v|\psi_v\rangle, \tag{7}$$

with $|\psi_v\rangle$ the wave function for the case where the quark that emits the gluon can change its spin and $|\psi_s\rangle$ the wave function for the case that the quark emitting the gluon does not change its spin. The factors C_s and C_v depend on the particular form for the spatial wave functions, according to Eq. (6). We then have

$$\langle p|p\rangle=1+|C_s|^2\langle\psi_s|\psi_s\rangle+|C_v|^2\langle\psi_v|\psi_v\rangle+C_s^*C_v\langle\psi_s|\psi_v\rangle + C_v^*C_s\langle\psi_v|\psi_s\rangle. \tag{8}$$

The wave function involved in the vector coupling is calculated from Eqs. (5) and (6):

$$\begin{aligned}|\psi_v\rangle&=(-1)^{\tilde{n}-1/2}b^\dagger(s,n,\rho)\sigma^l_{n\tilde{n}}\lambda^a_{\rho\tilde{\rho}}d^\dagger(\overline{s,}-\tilde{n},\tilde{\rho})b^\dagger(v,m,\delta)\\&\quad\times\sigma^l_{m\tilde{m}}\lambda^a_{\delta\tilde{\delta}}b(\tilde{v},\tilde{m},\tilde{\delta})|p\rangle_0\\&=\frac{(-1)^{\tilde{n}-1/2}}{\sqrt{18}}\sigma^l_{n\tilde{n}}\sigma^l_{m\tilde{m}}\lambda^a_{\rho\tilde{\rho}}\lambda^a_{\delta\tilde{\delta}}\{\epsilon^{\alpha\beta\gamma}(\delta_{\tilde{v}u}\delta_{\tilde{\delta}\alpha}A^\dagger\\&\quad-\delta_{\tilde{v}d}\delta_{\tilde{\delta}\beta}B^\dagger+\delta_{\tilde{v}u}\delta_{\tilde{\delta}\gamma}C^\dagger)|0\rangle\},\end{aligned} \tag{9}$$

where σ^l are the Pauli spin matrices, λ^a are the Gell-Mann color matrices, and the spin and flavor indices are summed. We also have

TABLE I. Amplitudes of probabilities to find a quark in the sea for the various possible states of the sea quark and the valence quark that emits the gluon. These probabilities are based only on the spin-flavor-color wave function.

State	$\langle \psi_s \| \psi_s \rangle$	$\langle \psi_v \| \psi_v \rangle$	$\langle \psi_s \| \psi_v \rangle$
$v=s=g$	0	$4608+1792\delta_{su}-1024\delta_{sd}$	0
$v=g,\ s\neq g$	0	4608	0
$v\neq g,\ s=g$	$1152\ +\ 640\delta_{su}+320\delta_{sd}$	$3456+1280\delta_{su}-320\delta_{sd}$	$-320\delta_{su}-640\delta_{sd}$
$v=s\neq g$	$1152\ +\ 128\delta_{su}+64\delta_{sd}$	$3456-384\delta_{su}-192\delta_{sd}$	$384\delta_{su}+192\delta_{sd}$
$v\neq s,\ v\neq g,\ s\neq g$	1152	3456	0

$$A^\dagger = b^\dagger(s,n,\rho)d^\dagger(\overline{s,-\tilde{n},\tilde{\rho}})[b^\dagger(v,m,\delta)b^\dagger(d,\downarrow,\beta)\delta_{\tilde{m}\uparrow} - b^\dagger(v,m,\delta)b^\dagger(d,\uparrow,\beta)\delta_{\tilde{m}\downarrow}]b^\dagger(u,\uparrow,\gamma),$$

$$B^\dagger = b^\dagger(s,n,\rho)d^\dagger(\overline{s,-\tilde{n},\tilde{\rho}})[b^\dagger(v,m,\delta)b^\dagger(u,\uparrow,\alpha)\delta_{\tilde{m}\downarrow} - b^\dagger(v,m,\delta)b^\dagger(u,\downarrow,\alpha)\delta_{\tilde{m}\uparrow}]b^\dagger(u,\uparrow,\gamma),$$

$$C^\dagger = b^\dagger(s,n,\rho)d^\dagger(\overline{s,-\tilde{n},\tilde{\rho}})b^\dagger(v,m,\delta) \times \delta_{\tilde{m}\uparrow}[b^\dagger(u,\uparrow,\alpha)b^\dagger(d,\downarrow,\beta) - b^\dagger(u,\downarrow,\alpha)b^\dagger(d,\uparrow,\beta)]. \quad (10)$$

For the scalar coupling the wave function is the same except that the Pauli spin matrices are omitted and $n=\tilde{n}$, $m=\tilde{m}$.

The calculation of the wave function overlap is very long and tedious. The results are displayed in Table I, where v refers to the state of the valence quark after the gluon emission, s corresponds to the state where the quark in the sea is created, and g refers to the ground state. Notice that there is no interference between the second and third lines of the table because of angular momentum conservation. These calculations agree with the results of Donoghue and Golowich [18].

The most striking result to be read from Table I is that the probability to find the sea with a $u\bar{u}$ pair is bigger than the sea composed of a $d\bar{d}$ pair. This conclusion is the opposite of the experimental result collected by the NMC [2] where the measured Gottfried sum rule is smaller than 1/3, a result that implies $\bar{d}>\bar{u}$. How can we then understand that the intrinsic sea in the proton generated by gluons favors $u\bar{u}$ pairs over $d\bar{d}$ pairs? In principle, this result also contradicts the intuitive picture introduced by Field and Feynman [5], based on the Pauli principle. For each flavor there are six empty states (two from spin and three from color). As the bare proton has two u quarks and one d quark, there are four available states for insertion of u quarks and five available states for insertion of d quarks. Although this reasoning is correct, one also cannot forget that antisymmetrization between quarks is an additional complication. The same excess of u valence quarks that prevents $u\bar{u}$ pair creation in comparison with $d\bar{d}$ pair creation also produces extra contributions because of antisymmetrization between the u quarks that does not exist for the d quarks.

To understand this effect in more detail, we consider the case where the quark that emits the gluon does not change its spin but goes to an excited state and the sea is created in the ground state or in a P state, according to parity conservation. This case was chosen because it is the simplest, as can be seen from Table I. In Fig. 1 we show the graphs containing $u\bar{u}$ pairs and in Fig. 2 the graphs containing $d\bar{d}$ pairs.

Diagrams (a), (c), and (d) for $u\bar{u}$ are the analogs of diagrams (a), (c), and (d) for $\bar{d}$. Their values are, respectively, the same because the created sea does not mix with the valence quarks and, for this reason, it is not possible to distinguish the sea flavor. Thus, in the case where the sea contracts with itself, the values of these graphs are the same for any flavor. Of course, the heavier the quark, the smaller its contribution through these diagrams, but this is an effect related solely to mass and not to spin statistics. The computed value is $\langle \psi_s|\psi_s\rangle=384$ for (a), $\langle \psi_s|\psi_s\rangle=128$ for (c), and $\langle \psi_s|\psi_s\rangle=64$ for (d). Diagram (b) for $\bar{u}$ is the analog of diagram (b) for $\bar{d}$ and they only exist when $u\bar{u}$ and $d\bar{d}$ pairs are created. Their values are $\langle \psi_s|\psi_s\rangle=4\times 160/3$ for (b) of Fig. 1 and $\langle \psi_s|\psi_s\rangle=6\times 160/3$ for (b) of Fig. 2. Generally speaking, the set of diagrams just mentioned are the only ones which are comparable. From them, one easily sees the Pauli principle in action and from it an excess of $\bar{d}$ over $\bar{u}$ as expected. However, we also have to include graphs (e) and (f) of Fig. 1 for $\bar{u}$. They appear because there is an excess of u valence over d valence quarks. If graphs (e) and (f) are included, as they should be, then the results change because these graphs give positive contributions: $\langle \psi_s|\psi_s\rangle=320$ for (e) and $\langle \psi_s|\psi_s\rangle=2\times 160/3$ for (f). With these graphs included the probability to find a $\bar{u}$ antiquark in the proton is larger than the probability to find a $\bar{d}$ in the same proton.

One could doubt this interpretation of the relation between the Pauli principle and antisymmetrization effects if one were to count only those graphs where a d quark emits a gluon. In this case, where a d quark emits a gluon, we would have six empty states for a d quark and only four empty states for the u quark (if the sea is created in the ground state). On the other hand, from Fig. 1 we see that graphs (b), (c), and (d) [also graph (f) if we include the one coming exclusively from the excess of u valence quarks] give a larger contribution than the corresponding graphs (c) and (d) of Fig. 2. As graphs (b), (c), and (d), in principle, are not related to the excess of u valence quarks, there is an apparent contradiction with the simple counting of states and the whole argument of the role of the interference graphs presented by us before. In reality, this contradiction is only apparent.

To better understand what is happening, consider a proton made of only one u and one d quark. We consider again the case where the quark that emits the gluon goes to an excited state. In this case, if a d quark emits a gluon, we would have in principle six empty states for the insertion of a d quark

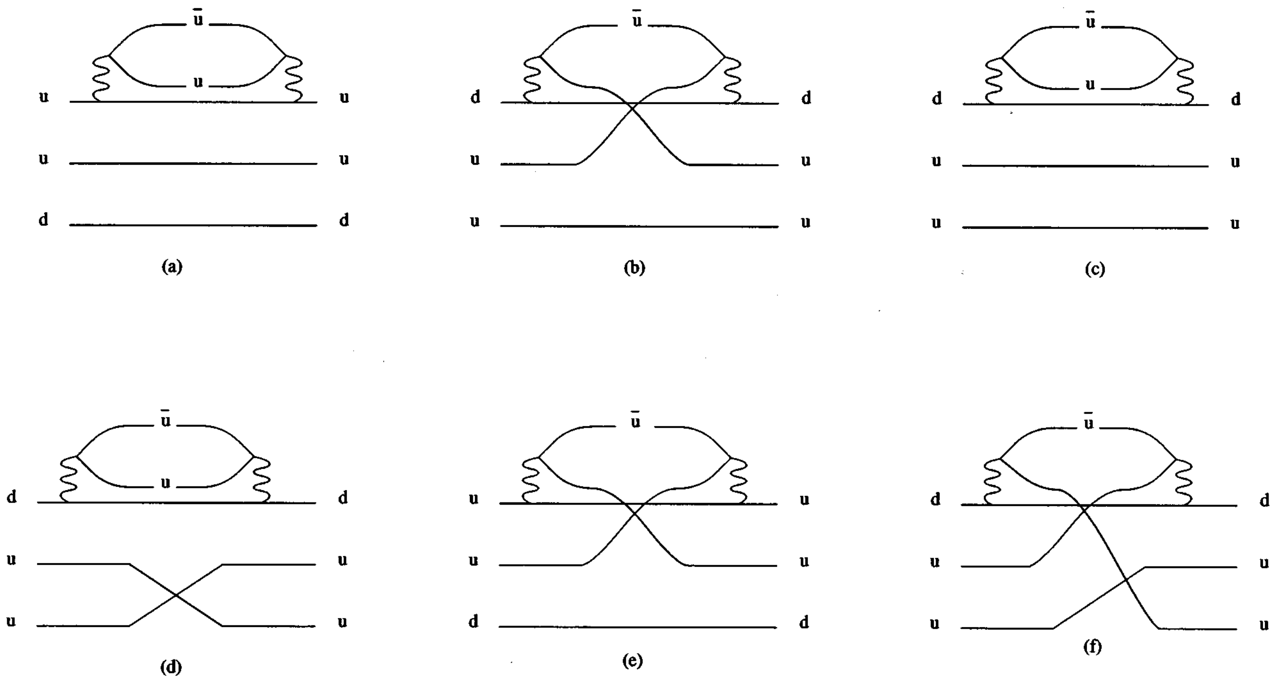

FIG. 1. Graphs containing $u\bar{u}$ pairs in the case where the valence quark emitting the virtual gluon goes to an excited state (i.e., $s=g, v\neq g$).

and five for the insertion of a u quark. The possible graphs would be the analog of (b) and (c) from Fig. 1 for a $u\bar{u}$ in the sea and the analog of graph (c) from Fig. 2 for a $d\bar{d}$ in the sea. Again, we would have more $u\bar{u}$ pairs than $d\bar{d}$ pairs and now it is clear why that happens: This is because there is one free valence u quark that can be exchanged with the sea and there is no such free valence d quark to be exchanged (in the case of a $d\bar{d}$ sea). The opposite situation happens when the u quark emits the gluon such that the sum of all diagrams, gluon emission from u and d valence quarks, renders an equal probability for a $u\bar{u}$ and $d\bar{d}$ pair creation, as expected in a proton containing only one quark of each flavor. The lesson is that we cannot treat the gluon emission in the proton from different flavors separately, and expect the Pauli

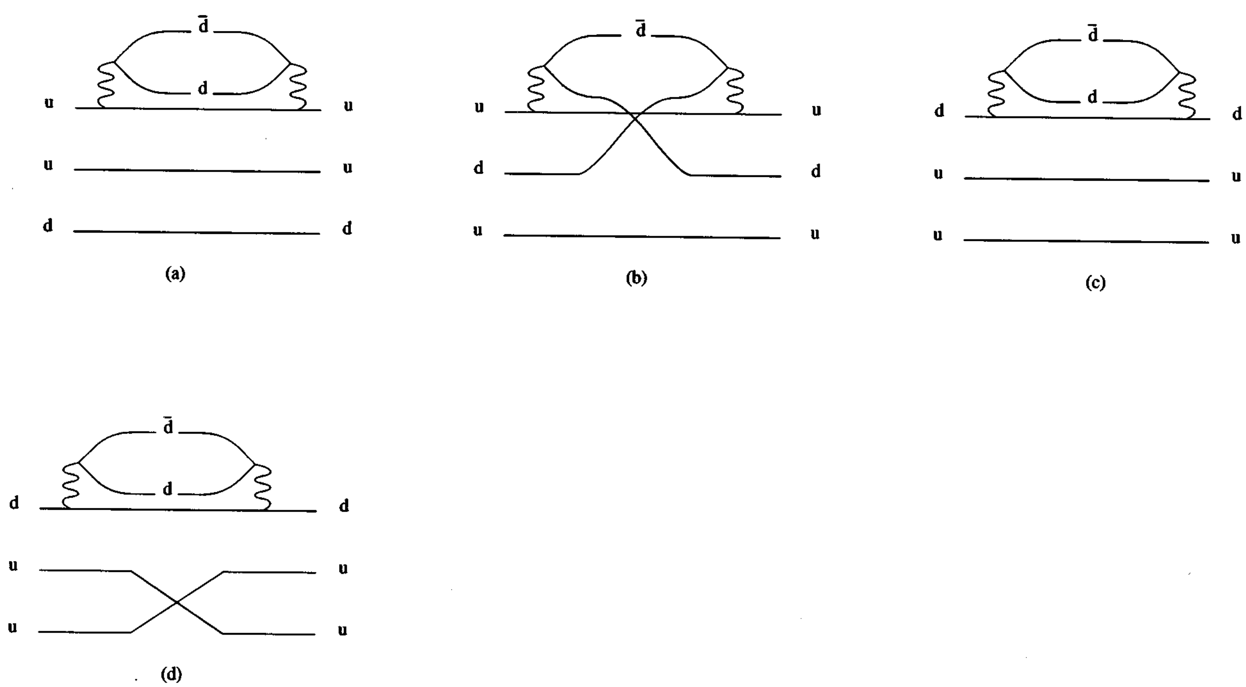

FIG. 2. Graphs containing $d\bar{d}$ pairs, in the case where the valence quark emitting the virtual gluon goes to an excited state (i.e., $s=g, v\neq g$).

principle to work free of any other effects.

For the combined result, the probability to find a $\bar{u}$ is larger than the probability to find a $\bar{d}$. One then can say that interference terms overcome the naive expectation of the Pauli principle. This result is extremely important because it also says that if we have to antisymmetrize the sea quarks with valence quarks in the case where the sea quarks are generated through pions, then the whole set of conclusions about the importance of the pions to the Gottfried sum rule might need to be revised. In Sec. III we investigate whether this is in fact the case.

A. Discussion of the role of antisymmetry

Before discussing pions it is of interest to relate the findings which we have just reported to a similar issue in lattice QCD. Figures 2(a), 2(c), and 2(d) are a very simple example of the class of diagrams known in lattice QCD as disconnected insertions (DI's) [19]. While the DI's in lattice calculations include an infinite number of gluon exchanges, the simple example which we have just presented illustrates an important principal. The intermediate $4q$ states in Figs. 2(a), 2(c), and 2(d) are unphysical in the sense that their wave function is not totally antisymmetric. Only when the exchange process, shown in Fig. 2(b), is included do we have a physically meaningful result. Within lattice terminology the latter are connected insertions (CI's). Thus we find that neither the CI's, nor the DI's are physically meaningful alone. This must be taken into account when one is trying to interpret the physical mechanisms behind lattice results for flavor breaking in the nucleon sea, the σ commutator, and so on.

III. ANTISYMMETRIZATION OF THE QUARKS OF THE PION

We now wish to investigate the effect of antisymmetrization on the internal structure of the pion. As already noted in the Introduction, the pion (as a pseudo-Goldstone boson) is expected to have a complicated internal structure. The effect of antisymmetrization on this internal structure can only affect components of the wave function where a quark is in the $1s$ state of the nucleon bag. We consider the case where the pion couples to just one q-$\bar{q}$ pair with the extra quark in the $1s$ state. In order to estimate the amplitude for this five-quark configuration we use the cloudy bag model in which the pion-quark coupling is dictated by chiral symmetry. Motivated by the findings in the gluon case, we wish to check whether the $\bar{u}$ antiquark is favored over the $\bar{d}$ antiquark. If this turns out to be true, the asymmetry in favor of $\bar{d}$ (from the process $u\to d\pi^+$) over $\bar{u}$ (from $d\to u\pi^-$) may be at risk.

The relevant diagram to study antisymmetrization in relation to the pion cloud is the one shown in Fig. 3(b), involving two loops. However, we will be mainly interested in the relative importance of the two-loop process in comparison with that involving only one loop. To this end we also need to compute diagram (a) of Fig. 3, which we do as a warm-up exercise. The calculations are going to be done at the quark level ($q\to\pi q$), which means that we need an interaction Hamiltonian between quarks and pions. To fix the calculation in a specific scheme, we shall use the interaction between quarks and pions as given by the cloudy bag model

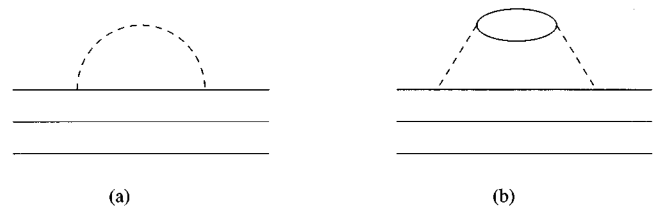

FIG. 3. The one- and two-loop graphs involved in pion emission.

[20–22], where the interaction is totally specified by chiral symmetry:

$$H_I=\frac{i}{2f}\int d^3x\,\overline{\psi(x)}\,\gamma_5\tau^i\psi(x)\,\phi^i\,\delta(r-R), \tag{11}$$

with R the bag radius and

$$\vec{\phi}(\vec{x})=\frac{1}{(2\pi)^{2/3}}\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(\vec{a}_{\vec{k}}e^{i\vec{k}\cdot\vec{x}}+\vec{a}^{\dagger}_{\vec{k}}e^{-i\vec{k}\cdot\vec{x}}),$$

$$\psi^s(\vec{x})=\frac{N}{(4\pi)^{1/2}}\sum\left\{\begin{pmatrix} ij_0(\omega r/R)\\ -j_1(\omega r/R)\vec{\sigma}\cdot\hat{r}\end{pmatrix}\chi_m b(f,m,\alpha)+\begin{pmatrix} -ij_1(\omega r/R)\vec{\sigma}\cdot\hat{r}\\ j_0(\omega r/R)\end{pmatrix}\chi_m d^{\dagger}(f,m,\alpha)\right\},$$

$$\psi^p(\vec{x})=\frac{N}{(4\pi)^{1/2}}\sum\left\{\begin{pmatrix} ij_1(\omega r/R)\vec{\sigma}\cdot\hat{r}\\ j_0(\omega r/R)\end{pmatrix}\chi_m b(f,m,\alpha)+\begin{pmatrix} ij_0(\omega r/R)\\ j_1(\omega r/R)\vec{\sigma}\cdot\hat{r}\end{pmatrix}\chi_m d^{\dagger}(f,m,\alpha)\right\}. \tag{12}$$

Here $N^2=\omega^3/[2R^3(\omega-1)\sin^2\omega]$, ω is the frequency associated with a given principal quantum number and orbital angular momentum, and j_0 and j_1 are Bessel functions subject to the condition $j_0=j_1$ at the bag surface, $r=R$. We wrote the explicit forms for the s and p waves for a quark inside a cavity because they are going to be used later on.

Three vertices are relevant to our diagrams: $q\to\pi q$, $\pi\to q\bar{q}$, and $0\to\pi q\bar{q}$. As in the gluon case, there are some restrictions due to the conservation of parity that can be helpful. As is well known, for fermions a particle has opposite intrinsic parity to the antiparticle. By convention, a particle has parity $+1$ and an antiparticle parity -1. Also, the parity of one particle relative to a set of others particles is given by $(-1)^l$, with l the orbital angular momentum of the particle in question. If $\bar{q}$ has parity -1, then one of the quarks (or the same antiquark) has to be in a state $l=1$, or a p wave, so that the proton parity is conserved. In general, the system must always be in a state of even parity.

We now write down the explicit form for the interaction Hamiltonian for both vertices. It happens that their form, besides the creation operator for a quark or for an antiquark, is the same for both processes $q\to\pi q$ and $0\to\pi q\bar{q}$:

$$
\begin{aligned}
H_I^{q^{\tilde f}\to \pi q^f} &= H_I^{0\to \pi q^f \bar q^{\tilde f}} \\
&= \frac{i}{2f(2\pi)^{3/2}}\left(\frac{\omega^a\omega^b}{(\omega^a-1)(\omega^b-1)}\right)\int \frac{d^3k}{(2\omega_k)^{1/2}} \\
&\quad \times \chi_m^\dagger \vec{\sigma}\cdot\vec{k}\chi_{\tilde m}\frac{j_1(kR)}{kR} \\
&\quad \times b^\dagger(f,m,\alpha)\,\tau^i_{f\tilde f}\begin{matrix} b(\tilde f,\tilde m,\tilde\alpha) \\ d^\dagger(\tilde f,\tilde m,\tilde \alpha)\end{matrix} a_k^{\dagger i}. \qquad (13)
\end{aligned}
$$

For the case $\pi\to q\bar q$ the interaction is just slightly different:

$$
\begin{aligned}
H_I^{\pi\to q^f\bar q^{\tilde f}} &= \frac{-i}{2f(2\pi)^{3/2}}\left(\frac{\omega^a\omega^b}{(\omega^a-1)(\omega^b-1)}\right)\int \frac{d^3k}{(2\omega_k)^{1/2}} \\
&\quad \times \chi_m^\dagger \vec{\sigma}\cdot\vec{k}\chi_{\tilde m}\frac{j_1(kR)}{kR}b^\dagger(f,m,\alpha) \\
&\quad \times \tau^i_{f\tilde f}d^\dagger(\tilde f,\tilde m,\tilde\alpha)a^i_k. \qquad (14)
\end{aligned}
$$

In expressions (13) and (14) the indices a and b refer to the spatial wave functions of the quarks and/or antiquarks involved in a specific reaction.

Once we have the interaction Hamiltonian, it is easy to calculate the quantities in which we are interested. As announced, we first calculate the probability to find a pion in the nucleon. To be specific, we set $\omega=\omega^a=\omega^b$ in Eq. (13), which means that the quark remains in the ground state after it emits the pion. In this case we get

$$
\begin{aligned}
\frac{1}{E_0-H_0}H_I|p\rangle_0 &= \frac{-i}{2f(2\pi)^{3/2}}\frac{\omega}{(\omega-1)}\int\frac{d^3k}{(2\omega_k)^{1/2}}\frac{1}{-\omega_k} \\
&\quad \times \chi_m^\dagger\vec\sigma\cdot\vec k\chi_{\tilde m}\frac{j_1(kR)}{kR}b^\dagger(v,m,\alpha) \\
&\quad \times \tau^i_{v\tilde v}b(\tilde v,\tilde m,\tilde\alpha)a_k^{\dagger i}|p\rangle_0, \qquad (15)
\end{aligned}
$$

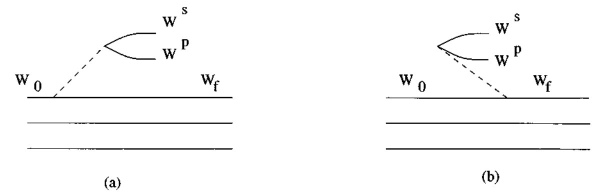

FIG. 4. The two-loop graph in time order.

where we used $(E_0-H_0)(H_I|p\rangle_0)=[E_0-(E_0+\omega_k)](H_I|p\rangle_0)$. The probability to find a pion is then given by

$$
\begin{aligned}
P_\pi &= \left({}_0\langle p|H_I\frac{1}{E_0-H_0}\right)\left(\frac{1}{E_0-H_0}H_I|p\rangle_0\right) \\
&= \frac{\pi}{6f^2(2\pi)^3}\frac{\omega^2}{(\omega-1)^2}\int_0^\infty\frac{k^4dk}{\omega_k^3}\frac{j_1^2(kR)}{(kR)^2}{}_0\langle p|b^\dagger(\tilde v',\tilde m',\tilde\delta') \\
&\quad \times \tau^i_{\tilde v'v'}\sigma^j_{\tilde m'm'}b(v',m',\delta')b^\dagger(v,m,\delta) \\
&\quad \times \tau^i_{v\tilde v}\sigma^j_{m\tilde m}b(\tilde v,\tilde m,\tilde\delta)|p\rangle_0. \qquad (16)
\end{aligned}
$$

After some calculation, the expectation value between bare proton states is found to be 57. In terms of the pion components, this result reads $57=22\delta_{\pi^+}+19\delta_{\pi^0}+16\delta_{\pi^-}$. This means that, as expected, the probability to find a $\bar d$ in the nucleon is larger than to find a $\bar u$. Using the identity $\omega/f(\omega-1)=\sqrt{4\pi}18f_{\pi NN}/5m_\pi$, where m_π is the physical pion mass and $f_{\pi NN}$ the pion nucleon nucleon coupling constant, expression (16) is rewritten as

$$
P_\pi=\frac{57}{25}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2\frac{3}{\pi}\int_0^\infty\frac{k^4dk}{\omega_k^3}\left(\frac{3j_1(kR)}{(kR)}\right)^2. \qquad (17)
$$

The next step is the evaluation of the two-loop graph. Its calculation is a straightforward application of Eqs. (6), (13), and (14). We start with graph (a) of Fig. 4. The process is pion creation with subsequent decay into a $q\bar q$ pair:

$$
\begin{aligned}
&\frac{1}{E_0-H_0}H_I^{\pi\to q\bar q}\frac{1}{E_0-H_0}H_I^{q\to\pi q}|p\rangle_0 \\
&= \frac{1}{(2f)^2(2\pi)^3}\sqrt{\frac{\omega_f\omega_0\omega^s\omega^p}{(\omega_f-1)(\omega_0-1)(\omega^s-1)(\omega^p-1)}}\int d^3k'\int d^3k\frac{1}{(2\omega_{k'}2\omega_k)^{1/2}}\frac{R}{\omega_0-\omega_f-R\omega_k}\frac{R}{\omega_0-\omega_f-\omega^s-\omega^p} \\
&\quad \times \chi_n^\dagger\vec\sigma\cdot\vec k'\chi_{\tilde n}\frac{j_1(k'R)}{k'R}b^\dagger(s,n,\rho)\tau^j_{s\tilde s}d^\dagger(\tilde s,\tilde n,\rho)a^j_{k'}\chi_m^\dagger\vec\sigma\cdot\vec k\chi_{\tilde m}\frac{j_1(kR)}{kR}b^\dagger(v,m,\delta)\tau^i_{v\tilde v}b(\tilde v,\tilde m,\delta)a_k^{\dagger i}|p\rangle_0. \qquad (18)
\end{aligned}
$$

The energies of the intermediate states are clearly indicated in the figure. The notation is the following: ω_0 and ω_f are, respectively, the quark frequencies before and after pion emission or absorption, and ω^s and ω^p are, respectively, the frequencies of the s-wave quark and of the p-wave antiquark.

Similarly to case (a), we calculate the contribution from graph (b), where a pion and a $q\bar q$ pair are created and the pion is subsequently absorbed by a quark:

$$\frac{1}{E_0-H_0}H_I^{q\pi\to q}\frac{1}{E_0-H_0}H_I^{0\to\pi q\bar{q}}|p\rangle_0$$
$$=\frac{1}{(2f)^2(2\pi)^3}\sqrt{\frac{\omega_f\omega_0\omega^s\omega^p}{(\omega_f-1)(\omega_0-1)(\omega^s-1)(\omega^p-1)}}\int d^3k'\int d^3k\frac{1}{(2\omega_{k'}2\omega_k)^{1/2}}\frac{R}{\omega_0-\omega_f-\omega^s-\omega^p}\frac{R}{-\omega^s-\omega^p-R\omega_k}$$
$$\times\chi_n^\dagger\vec{\sigma}\cdot\vec{k}'\chi_{\tilde{n}}\frac{j_1(k'R)}{k'R}b^\dagger(v,n,\delta)\tau^i_{v\tilde{v}}b(\tilde{v},\tilde{n},\delta)a^i_{\vec{k}'}\chi_m^\dagger\vec{\sigma}\cdot\vec{k}\chi_{\tilde{m}}\frac{j_1(kR)}{kR}b^\dagger(s,m,\rho)\tau^j_{s\tilde{s}}d^\dagger(\tilde{s},\tilde{m},\rho)a^{\dagger j}_{\vec{k}}|p\rangle_0. \quad (19)$$

The total amplitude for the two-loop process is then given by the sum of graphs (a) and (b). To better deal with this sum, we rewrite the product of operators in Eq. (19) as

$$b^\dagger(v,n,\delta)\tau^i_{v\tilde{v}}b(\tilde{v},\tilde{n},\delta)a^i_{\vec{k}'}b^\dagger(s,m,\rho)\tau^j_{s\tilde{s}}d^\dagger(\tilde{s},\tilde{m},\rho)a^{\dagger j}_{\vec{k}}|p\rangle_0=b^\dagger(s,n,\rho)\tau^j_{s\tilde{s}}d^\dagger(\tilde{s},\tilde{n},\rho)b^\dagger(v,m,\delta)\tau^i_{v\tilde{v}}b(\tilde{v},\tilde{m},\delta)a^i_{\vec{k}'}a^{\dagger j}_{\vec{k}}|p\rangle_0$$
$$+\delta_{\tilde{v}s}\delta_{\tilde{n}m}\delta_{\delta\rho}b^\dagger(v,n,\delta)\tau^i_{v\tilde{v}}\tau^j_{s\tilde{s}}d^\dagger(\tilde{s},\tilde{m},\rho)a^i_{\vec{k}'}a^{\dagger j}_{\vec{k}}|p\rangle_0. \quad (20)$$

We then use $a^i_{\vec{k}'}a^{\dagger j}_{\vec{k}}|0\rangle=\delta^{ij}\delta(\vec{k}'-\vec{k})|0\rangle$ to write the sum of graphs (a) and (b) as

$$\frac{1}{E_0-H_0}H_I^{\pi\to q\bar{q}}\frac{1}{E_0-H_0}H_I^{q\to\pi q}|p\rangle_0+\frac{1}{E_0-H_0}H_I^{q\pi\to q}\frac{1}{E_0-H_0}H_I^{0\to\pi q\bar{q}}|p\rangle_0$$
$$=\frac{1}{(2f)^2(2\pi)^3}\sqrt{\frac{\omega_f\omega_0\omega^s\omega^p}{(\omega_f-1)(\omega_0-1)(\omega^s-1)(\omega^p-1)}}\int d^3k\frac{(\vec{\sigma}\cdot\vec{k})_{n\tilde{n}}(\vec{\sigma}\cdot\vec{k})_{m\tilde{m}}}{2\omega_k}\frac{j_1^2(kR)}{(kR)^2}\frac{R^2}{\omega_0-\omega_f-\omega^s-\omega^p}$$
$$\times\left\{\left(\frac{1}{\omega_0-\omega_f-R\omega_k}+\frac{1}{-\omega^s-\omega^p-R\omega_k}\right)b^\dagger(s,n,\rho)\tau^i_{s\tilde{s}}d^\dagger(\tilde{s},\tilde{n},\rho)b^\dagger(v,m,\delta)\tau^i_{v\tilde{v}}b(\tilde{v},\tilde{m},\delta)\right.$$
$$\left.+\frac{1}{-\omega^s-\omega^p-R\omega_k}\delta_{\tilde{v}s}\delta_{\tilde{n}m}\delta_{\delta\rho}b^\dagger(v,n,\delta)\tau^i_{v\tilde{v}}\tau^i_{s\tilde{s}}d^\dagger(\tilde{s},\tilde{m},\rho)\right\}|p\rangle_0. \quad (21)$$

We are now ready to compute the probability $P_{\pi q\bar{q}}$ to find a $q\bar{q}$ pair in the nucleon. It is given by the square of the amplitude (21):

$$P_{\pi q\bar{q}}=\left(\frac{1}{(2f)^2(2\pi)^3}\right)^2\frac{\omega_f\omega_0\omega^s\omega^p}{(\omega_f-1)(\omega_0-1)(\omega^s-1)(\omega^p-1)}\left(\frac{4\pi}{3}\right)^2\int_0^\infty\frac{k^4dk}{2\omega_k}\frac{j_1^2(kR)}{(kR)^2}$$
$$\times\int_0^\infty\frac{k'^4dk'}{2\omega_{k'}}\frac{j_1^2(k'R)}{(k'R)^2}\left(\frac{R^2}{\omega_0-\omega_f-\omega^s-\omega^p}\right)^2\left\{\left(\frac{1}{\omega_0-\omega_f-R\omega_k}+\frac{1}{-\omega^s-\omega^p-R\omega_k}\right)\right.$$
$$\times\left(\frac{1}{\omega_0-\omega_f-R\omega_{k'}}+\frac{1}{-\omega^s-\omega^p-R\omega_{k'}}\right){}_0\langle p|b^\dagger(\tilde{v}',\tilde{m}',\delta')\tau^i_{\tilde{v}'v'}\sigma^j_{\tilde{m}'m'}b(v',m',\delta')d(\tilde{s}',\tilde{n}',\rho')$$
$$\times\tau^i_{\tilde{s}'s'}\sigma^j_{\tilde{n}'n'}b(v',m',\delta')b^\dagger(s,n,\rho)\tau^k_{s\tilde{s}}\sigma^l_{n\tilde{n}}d^\dagger(\tilde{s},\tilde{n},\rho)b^\dagger(v,m,\delta)\tau^k_{v\tilde{v}}\sigma^l_{m\tilde{m}}b(\tilde{v},\tilde{m},\delta)|p\rangle_0$$
$$+\frac{1}{-\omega^s-\omega^p-R\omega_k}\frac{1}{-\omega^s-\omega^p-R\omega_{k'}}{}_0\langle p|d(\tilde{s}',\tilde{m}',\rho')\tau^i_{\tilde{s}'s'}\tau^i_{s'v'}\sigma^j_{\tilde{m}'m'}\sigma^j_{m'n'}b(v',n',\rho')$$
$$\left.\times b^\dagger(v,n,\rho)\tau^k_{vs}\tau^k_{s\tilde{s}}\sigma^l_{nm}\sigma^l_{m\tilde{m}}d^\dagger(\tilde{s},\tilde{m},\rho)|p\rangle_0\right\}. \quad (22)$$

The last quantities to be calculated are the expectation values between the bare proton states. It is a very long but straightforward calculation. We calculated only the case where the valence quark that emits or absorbs the pion remains in the same orbital state. The results are

$${}_0\langle p|b^\dagger(\tilde{v}',\tilde{m}',\delta')\tau^i_{\tilde{v}'v'}\sigma^j_{\tilde{m}'m'}b(v',m',\delta')d(\tilde{s}',\tilde{n}',\rho')\tau^i_{\tilde{s}'s'}\sigma^j_{\tilde{n}'n'}b(v',m',\rho')$$
$$\times b^\dagger(s,n,\rho)\tau^k_{s\tilde{s}}\sigma^l_{n\tilde{n}}d^\dagger(\tilde{s},\tilde{n},\rho)b^\dagger(v,m,\delta)\tau^k_{v\tilde{v}}\sigma^l_{m\tilde{m}}b(\tilde{v},\tilde{m},\delta)|p\rangle_0$$
$$=684-110\delta_{\tilde{s}\bar{u}}-181\delta_{\tilde{s}\bar{d}}$$
$$\times{}_0\langle p|d(\tilde{s}',\tilde{m}',\rho')\tau^i_{\tilde{s}'s'}\tau^i_{s'v'}\sigma^j_{\tilde{m}'m'}\sigma^j_{m'n'}b(v',n',\rho')b^\dagger(v,n,\rho)\tau^k_{vs}\tau^k_{s\tilde{s}}\sigma^l_{nm}\sigma^l_{m\tilde{m}}d^\dagger(\tilde{s},\tilde{m},\rho)|p\rangle_0$$
$$=972-54\delta_{\tilde{s}\bar{u}}-81\delta_{\tilde{s}\bar{d}}. \quad (23)$$

TABLE II. The two- to one-loop ratio for various bag radii.

Integral	$R=0.6$ fm	$R=0.8$ fm	$R=1$ fm
I_1	1.6848	1.24065	0.980306
I_2	0.1946	0.1734430	0.157762
I_3	4.92548	2.65299	1.63279
$P_{\pi q\bar{q}}/P_\pi$	0.0217	0.0124	0.0081

The results contained in expression (23) are quite surprising. They say that, if the quark structure of the pion is important and if the quarks from the pion are allowed to antisymmetrize with the quarks from the parent proton, then the probability for the antiquark in the pion to be a $\bar{u}$ is larger than $\bar{d}$. The above result is independent of the particularities of the given model, in the sense that expression (23) is a direct consequence of the bare proton wave function, Eq. (4). They are also a consequence of assuming a pion quark interaction.

To better determine how important these second order effects could be, we will calculate the ratio of probabilities, $P_{\pi q\bar{q}}/P_\pi$. To this end we need to perform the integrals over k and k' in Eqs. (17) and (22). These integrals are dependent on the particular value of the bag radius and here we will display the results for $R=0.6$, 0.8, and 1 fm. We set $\omega_0=\omega_f$ and define the integrals

$$I_1=\frac{1}{2}\int_0^\infty \frac{dx}{m_\pi^2R^2+x^2}\,\frac{\omega^s+\omega^p+2(m_\pi^2R^2+x^2)^{1/2}}{\omega^s+\omega^p+(m_\pi^2R^2+x^2)^{1/2}}\times\left(\frac{\sin^2 x}{x^2}+\cos^2 x-\frac{\sin x\cos x}{x}\right),$$

$$I_2=\frac{1}{2}\int_0^\infty \frac{dx}{(m_\pi^2R^2+x^2)^{1/2}}\,\frac{1}{\omega^s+\omega^p+(m_\pi^2R^2+x^2)^{1/2}}\times\left(\frac{\sin^2 x}{x^2}+\cos^2 x-\frac{\sin x\cos x}{x}\right),$$

$$I_3=\int_0^\infty \frac{dx}{(m_\pi^2R^2+x^2)^{3/2}}\left(\frac{\sin^2 x}{x^2}+\cos^2 x-\frac{\sin x\cos x}{x}\right), \tag{24}$$

where we use $x=kR$. The numerical values of these integrals are displayed in Table II.

With these definitions, we rewrite expression (22) as

$$P_{\pi q\bar{q}}=\left(\frac{1}{(2Rf)^2(2\pi)^3}\right)^2\frac{\omega_0^2}{(\omega_0-1)^2}\,\frac{\omega^s\omega^p}{(\omega^s-1)(\omega^p-1)}\times\left(\frac{4\pi}{3}\right)^2\left(\frac{1}{\omega^s+\omega^p}\right)^2\{I_1^2(684-110\delta_{\tilde{s}\bar{u}}-181\delta_{\tilde{s}\bar{d}})+I_2^2\ (972-54\delta_{\tilde{s}\bar{u}}-81\delta_{\tilde{s}\bar{d}})\}. \tag{25}$$

The ratio $P_{\pi q\bar{q}}/P_\pi$ is then easily expressed:

$$\frac{P_{\pi q\bar{q}}}{P_\pi}=\frac{1}{(2Rf)^2(2\pi)^3}\,\frac{\omega^s\omega^p}{(\omega^s-1)(\omega^p-1)}\,\frac{4\pi}{3}\times\left(\frac{1}{\omega^s+\omega^p}\right)^2\left\{\frac{I_1^2}{I_3}\,\frac{(684-110\delta_{\tilde{s}\bar{u}}-181\delta_{\tilde{s}\bar{d}})}{57}+\frac{I_2^2}{I_3}\,\frac{(972-54\delta_{\tilde{s}\bar{u}}-81\delta_{\tilde{s}\bar{d}})}{57}\right\}. \tag{26}$$

The value of this ratio for different sizes of the bag is also displayed in Table II, where we used $m_\pi=140$ MeV for the pion mass and $f=93$ MeV for the pion decay constant. There is a strong dependence of the calculated ratio on the bag size but, even for the worst scenario (the case of a small bag), the size of the two-loop contribution is just 2% of the one-loop term [22].

IV. FINAL REMARKS

Our primary interest has been to investigate the effect of antisymmetry between the valence quarks in the nucleon and the internal structure of the pion. Within the cloudy bag model, in which the coupling of the pion to quarks is dictated by chiral symmetry, we found that it appears to be safe to neglect possible antisymmetrization effects between pion quarks and the nucleon valence quarks. Of course, when writing the nucleon wave function one would have to add the contributions from all possible states:

$$|N\rangle=Z^{1/2}[|N\rangle_0+|N\pi\rangle_0+\sum_{q\ \text{states}}|N\pi q\bar{q}\rangle_0+\cdots], \tag{27}$$

where the sum of all quark states ($1s$, $2s$, etc.) was particularly emphasized. If the quark that emits the pion remains in the ground state, the only contribution from antisymmetry is the one in Table II. As we have to sum over all other possible states for this quark, it turns out that the antisymmetry effects are further diluted. However, we will also have some other contributions even in this case, because if the quark in the pion is in the ground state, it can antisymmetrize with the two spectator valence quarks. Also, if the quark in the pion is in an excited state, it can combine with a particular excited state of the valence quark that emitted the pion in the first place, as in the gluon example. We did not make these calculations because of their level of complexity. Our goal was to examine the behavior of the dominant contribution, displayed in Table II, and our results indicate that this number is itself already very small.

Of course, as the sum over quark states in Eq. (27) is infinite, we cannot rule out the possibility that eventually the antisymmetry graphs could be important. However, we believe that this is improbable, as the contributions from graphs which would be unaffected by antisymmetry would grow even faster. In summary, the major purpose of the present calculation was to check whether the antisymmetrization effects in pion emission can be safely disregarded—they can.

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