Elastic pion scattering on the deuteron in a multiple scattering model

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Pion elastic scattering on deuterium is studied in the Kerman-McManus-Thaler multiple scattering approach developed in momentum space. Using a Paris wave function and the same methods and approximations as commonly used in pion scattering on heavier nuclei excellent agreement with differential cross section data is obtained for pion energies $100 < T_\pi < 250$ MeV. At higher energies and very backward angles, discrepancies appear that are reminiscent of disagreements in pion scattering on ^{3}He, ^{3}H, and ^{4}He. At low energies $T_\pi \lesssim 65$ MeV the model fails to reproduce experimental data at forward direction. In this kinematical region the coupling with the breakup channels has to be included. Polarization observables are studied in detail. While tensor analyzing powers are well reproduced, vector analyzing powers exhibit dramatic discrepancies. [S0556-2813(97)04001-6]

PACS number(s): 25.80.Dj, 11.80.La, 21.45.+v, 24.70.+s

I. INTRODUCTION

In the realm of pion-nuclear physics, the pion interaction with the deuteron is of special interest [1]. On the theoretical side, it is the simplest few-body system in which conventional theories (Faddeev theory, coupled $\pi NN - NN$ equations and multiple scattering theory) can be rigorously tested. The knowledge obtained from these studies can then be extended to many-body systems. On the experimental side, a large set of measurements for differential cross sections and polarization observables are available in πd scattering. Thus, there is sufficient motivation to study pion interaction mechanisms with very light nuclei in detail.

In recent years the three-body approach has been used extensively in describing the pion-deuteron interaction. From the point of view of solving the πNN three-body problem directly this is clearly an important achievement. However, due to the complexity of the three-body formalism the application of the obtained results to the case of pion interaction with heavier nuclei may be more difficult.

On the other hand, a microscopic description of the pion-nuclear interaction in the framework of the multiple scattering theory [2,3] has continuously been improved. More recent developments are due especially to techniques developed to perform calculations in momentum space using the Kerman-McManus-Thaler (KMT) [2] formulation of the multiple scattering theory. In momentum space nonlocalities of the pion-nuclear interaction, off-shell extrapolations of the pion-nucleon scattering amplitudes and exact treatment of Fermi motion can be taken into account [4–7]. The KMT approach is important to avoid double counting of pion rescattering on the same nucleon. With the addition of the phenomenological ρ^2 term [8], which is responsible for real pion absorption and second order effects, the momentum space

formalism was successful in describing the pion-nuclear interaction in the Δ resonance region as well as at low energies for a large set of nuclei with $A = 4$–40. In Refs. [9], this method has been extended successfully to the description of pionic atoms as well.

The aim of the present work is a systematic investigation of pion scattering on the deuteron in the region of pion kinetic energies of $T_\pi = 60$–300 MeV. We employ a multiple scattering framework in momentum space using only elementary amplitudes extracted from πN scattering data and realistic deuteron wave functions. Due to the latter ingredient we are in a good position to fix the nuclear structure input in order to shed more light on the reaction mechanism. Another purpose is to develop a pion deuteron interaction that can be used for pion photoproduction on the deuteron in a straightforward way. Such an approach, applied to pion scattering and pion photoproduction on the trinucleon [7,6] has provided an excellent description of the differential cross section. This has motivated us to extend our momentum space approach for these reactions to the deuteron.

Our study has been divided into two parts. In this paper we investigate pion elastic scattering on the deuteron, testing our multiple scattering approach by comparing with all available experimental data. Coherent π^0 photoproduction is considered in the second part. The main aspects of our formalism based on the KMT multiple scattering approach and coupled-channels method are given in Sec. II. Section III presents our results for elastic pion-deuteron scattering while our conclusions are summarized in Sec. IV.

II. FORMALISM

A. General overview

In multiple scattering theory the pion-nuclear T matrix can be presented as

$$T = \sum_{i=1}^{A} T_i(E), \tag{1}$$

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where

$$T_i(E) = \tau_i(E) + \tau_i(E) G_0(E) \sum_{j \neq i}^{A} T_j(E) \qquad (2)$$

and $\tau_i(E)$ is defined as a solution of the equation

$$\tau(E) = v + v G_0(E) \tau(E). \qquad (3)$$

In Eqs. (2,3), $G_0(E)$ is the Green's function of the noninteracting pion-nuclear system and v is the potential of the pion-nucleon interaction. Note that the τ matrix in Eq. (3) differs from the standard two-body t matrix for free pion-nucleon scattering through the Green's function $G_0(E)$ which contains the many-body nuclear Hamiltonian. For the free pion-nucleon t matrix we have the equation

$$t(\omega) = v + v g_0(\omega) t(\omega) \qquad (4)$$

with the free pion-nucleon Green's function $g_0(\omega)$.

Following the KMT version of multiple scattering theory [2], Eqs. (1–4) are equivalent to the system of integral equations for the auxiliary matrix $T' = [(A-1)/A]T$

$$T'(E) = U'(E) + U'(E) G_0(E) P T'(E), \qquad (5a)$$

$$U'(E) = (A-1)\tau(E) + (A-1)\tau(E) G_0(E) Q U(E), \qquad (5b)$$

$$\tau(E) = t(\omega) + t(\omega)[G_0(E) - g_0(\omega)](P+Q)\tau(E), \qquad (5c)$$

where the operators $P = |0\rangle\langle 0|$ and $Q = \Sigma_{m \neq 0}|m\rangle\langle m|$ project the nuclear state vectors into the ground state $|0\rangle$ and all possible excited states $|m\rangle$, respectively. In the case of the deuteron, the operator Q projects onto the states of the continuum.

One of the main problems which arise in applying these equations to pion deuteron scattering is the correct description of the coupling to the breakup or continuum channels. In this case the Lippmann-Schwinger equation has a noncompact kernel and, therefore, cannot be solved numerically. It is well known that the solution to this problem was found by Faddeev who from the system of Eqs. (5) derived a new set of equations which do not have this inherent shortcoming.

In this paper we demonstrate that multiple scattering theory is able mainly to reproduce existing experimental data using several standard approximations for the solution of the Eqs. (5). This in turn generates a simple pion-deuteron interaction that will be used in our description of coherent π^0 photoproduction, discussed in the following part. As a first step, we *neglect the contribution from the coupling to the breakup channel*. This means that in Eqs. (5b,5c) the contribution from the Q operator is dropped. The motivation for this approximation is that contributions from the diagonal matrix elements dominate due to the large overlap of the nuclear wave functions in the initial and final states. As we will see below this approximation is a 0 starting point for the study of pion-nuclear interaction. However, at low energies where for isoscalar nuclei the contribution from the P operator is very small, the coupling to the breakup channels becomes important.

The next approximation is the special choice for connecting the pion-nucleon energy, ω, with the pion-nuclear one,

E. In general, the energy of the pion-nucleon system is a dynamical variable in the many-body system [10]. Thus there are many possibilities for relation ω and E. Indeed ω can be considered as a parameter of the multiple scattering theory which could be determined using physical or mathematical arguments. One attractive choice is to minimize the contribution from the second term of Eq. (5c) with the help of this parameter. A detailed investigation of this issue was done in Ref. [5]. To be consistent with our previous studies of pion scattering on ^{3}He [6] and on heavier nuclei [5,8] we follow the prescription of Landau and Thomas [4] which is called *modified impulse approximation* with the three-body choice for the reaction energy. The relativistic generalization of this approximation in the case of the deuteron leads to the following expression for the pion-nucleon energy [5]

$$\omega = E + m_\pi + M_N - \left[(m_\pi + M_N)^2 + \frac{(\vec{q}' + \vec{q})^2}{16} \right]^{1/2}$$

$$- \left[M_N^2 + \frac{(\vec{q}' + \vec{q})^2}{16} \right]^{1/2}, \qquad (6)$$

where m_π and M_N are the pion and nucleon masses, respectively. We note that Eq. (6) was evaluated with the value of the nucleon momenta defined in the following by Eq. (14), which gives for the πN system and the residual nucleus the momenta $(q' + q)/4$ and $-(q' + q)/4$, respectively. As shown in Refs. [4,5,11], in the case of weak binding this choice minimizes the contribution from the P operator in the second term of Eq. (5c).

The scattering amplitude $F_{M_f M_i}(\vec{q}', \vec{q})$ is connected with the T' matrix in Eq. (5a) by the relation

$$F_{M_f M_i}(\vec{q}', \vec{q}) = - \frac{\sqrt{\mathcal{M}(q')\mathcal{M}(q)}}{2\pi} \frac{A}{A-1}$$

$$\times \langle \pi(\vec{q}'), f | T'(E) | i, \pi(\vec{q}) \rangle, \qquad (7)$$

where $\vec{q}$ and $\vec{q}'$ are the pion momenta in the initial and final states, respectively, $|i\rangle = |1^+ M_i\rangle$ and $|f\rangle = |1^+ M_f\rangle$ denote the nuclear initial and final states, with the nuclear spin projection M_i in the initial and M_f final states. The pion-nuclear reduced mass is given by $\mathcal{M}(q) = E_\pi(q) E_A(q)/E(q)$, where $E(q) = E_\pi(q) + E_A(q)$ is the total pion-nuclear energy. To shorten the notation in Eq. (7) we skipped the dependence on the energy E for the scattering amplitude. For a given pion energy, E is fixed and in the following we use $E = E(q)$. In the same way we can express the pion-nuclear potential in momentum space $V_{M_f M_i}(\vec{q}', \vec{q})$ via matrix $U'(E)$ from Eq. (5b). Then, in accordance with Eq. (5a) the elastic scattering amplitude can be constructed by solving the integral equation with relativistic kinematics

$$F_{M_f M_i}(\vec{q}', \vec{q}) = V_{M_f M_i}(\vec{q}', \vec{q}) - \frac{a}{(2\pi)^2}$$

$$\times \sum_M \int \frac{d\vec{q}''}{\mathcal{M}(q'')} \frac{V_{M_f M}(\vec{q}', \vec{q}'') F_{M M_i}(\vec{q}'', \vec{q})}{E(q) - E(q'') + i\epsilon}, \qquad (8)$$

where the factor $a = (A-1)/A$ is important to avoid double counting of pion rescattering on the same nucleon, which is

already included in the free pion-nucleon t matrix. Note that due to this factor, Eq. (8) *is not* the standard Lippmann-Schwinger equation.

In accordance with the approximations discussed above and after neglecting the contributions from the second terms in Eqs. (5b) and (5c) the momentum space potential of the pion-nuclear interaction can be given by

$$V_{M_f M_i}(\vec{q}',\vec{q}) = V_{\text{Coul.}}(\vec{q}'-\vec{q};R)\delta_{M_f M_i} + V^{(1)}_{M_f M_i}(\vec{q}',\vec{q}), \quad (9)$$

which contains the Coulomb potential in momentum space, cutoff at radius R. The so-called *first-order potential* of the strong pion-nuclear interaction $V^{(1)}_{M_f M_i}$ is related to the free πN scattering t matrix

$$V^{(1)}_{M_f M_i}(\vec{q}',\vec{q}) = -\frac{\sqrt{\mathcal{M}(q')\mathcal{M}(q)}}{2\pi}$$

$$\times \left\langle \pi(\vec{q}'),f \left| \sum_{j=1}^{A} t_j(\omega) \right| i,\pi(\vec{q}) \right\rangle, \quad (10)$$

where ω is defined by Eq. (6). Thus our first oder potential is related to the first terms of Eqs. (5b) and (5c) with a minimized contribution from the P operator in the second term of Eq. (5c).

The free pion-nucleon scattering t matrix is defined in the following way:

$$t_{\pi N} \equiv \langle \vec{q}',\vec{p}' | t(\omega) | \vec{p},\vec{q} \rangle$$

$$= -\frac{2\pi}{\sqrt{\mu(q',p')\mu(q,p)}} f_{\pi N}(\omega,\theta_\pi^*)\delta(\vec{p}'+\vec{q}'-\vec{p}-\vec{q}), \quad (11a)$$

$$f_{\pi N}(\omega,\theta_\pi^*) = A_0 + A_T \vec{t}\cdot\vec{\tau} + i\vec{\sigma}\cdot[\hat{\vec{q}}_i \times \hat{\vec{q}}_f](A_S + A_{ST}\vec{t}\cdot\vec{\tau}), \quad (11b)$$

where $\mu(q,p) = E_\pi(q)E_N(p)/\omega$ is the pion-nucleon reduced mass, $\vec{p}$ and $\vec{p}'$ are the nucleon momenta in the initial and final states (in pion-nuclear c.m. system). In the pion-nucleon scattering amplitude $f_{\pi N}$, which is in general relativistically invariant, $\hat{\vec{q}}_i$ and $\hat{\vec{q}}_f$ are the unit vectors for the initial and final pion momenta in the πN c.m. system. The vectors $\vec{\tau}$ and $\vec{t}$ are the usual isospin operators for the target nucleon and pion.

The scalar functions $A_i(\omega,\cos\theta_\pi^*)(i=0,S,T,$ and $ST)$, that depend on the total pion-nucleon energy ω and the pion angle θ_π^* in the πN c.m. system, are the usual combinations of partial πN scattering amplitudes $f_{l_\pi}^{(\pm)}(\omega)$ and Legendre polynomials $P_{l_\pi}(\cos\theta_\pi^*)$, where l_π is the pion-nucleon angular momentum. The definition of these amplitudes has been given in our previous work [6]. Note that their on-shell value was obtained from phase-shift analysis of the elastic pion-nucleon scattering data.

The off-shell extrapolation of the πN partial amplitudes was constructed using a separable form

$$f_{l_\pi}^{(\pm)}(\text{off-shell}) = f_{l_\pi}^{(\pm)}(\omega) \frac{v_{l_\pi}^{(\pm)}(q'')v_{l_\pi}^{(\pm)}(q')}{[v_{l_\pi}^{(\pm)}(q)]^2} \quad (12)$$

with πN form factors

$$v_{l_\pi}^{(\pm)}(q) = \frac{q^{l_\pi}}{[1+(r_0 q)^2]^2}. \quad (13)$$

Here we employ the value $r_0 = 0.47$ fm, consistent with the analysis of the separable πN potential of Ref. [12].

Finally, we mention a useful approximation related to the treatment of nucleon Fermi motion. In accordance with the results of Refs. [4,13] the internal nucleon momentum can be replaced by an effective value

$$\vec{p} \to \vec{p}_{\text{eff}} = -\frac{\vec{q}}{2} - \frac{1}{4}(\vec{q}-\vec{q}'). \quad (14)$$

This choice is based on the fact that in the case of Gaussian wave functions, which can reproduce the dominant S-wave part of the deuteron ground state at low momenta, such a replacement treats the linear $\vec{p}/2M$ terms in the pion-nucleon scattering amplitude exactly.

B. Partial wave decomposition and polarization observables

We express the scattering amplitude in terms of partial amplitudes using the representations of total angular momentum j with projection m

$$F_{M_f M_i}(\vec{q}',\vec{q}) = 4\pi \sum (2j+1)i^{L_\pi - L'_\pi}Y^{L'_\pi}_{M'_\pi}(\hat{\vec{q}}')F^j_{L'_\pi L_\pi}$$

$$\times (q',q)Y^{*L_\pi}_{M_\pi}(\hat{\vec{q}})$$

$$\times \begin{pmatrix} L'_\pi & 1 & j \\ M'_\pi & M_f & -m \end{pmatrix} \begin{pmatrix} L_\pi & 1 & j \\ M_\pi & M_i & -m \end{pmatrix}, \quad (15)$$

where the $Y^{L_\pi}_{M_\pi}(\hat{\vec{q}})$ are the spherical harmonics for the pion waves and $L_\pi(L'_\pi)$ is the angular momentum of the incoming (outgoing) pions. Note that due to parity conservation $(-1)^{L_\pi + L'_\pi} = 1$. For the amplitude $V_{M_f M_i}$ we perform an expansion identical to Eq. (15).

Substituting the above expansions of $V_{M_f M_i}$ and $F_{M_f M_i}$ into Eq. (8) we obtain the following system of integral equations for the partial wave amplitudes

$$F^j_{L'_\pi L_\pi}(q',q) = V^j_{L'_\pi L_\pi}(q',q) - \frac{a}{\pi}$$

$$\times \sum_{\tilde{L}} \int \frac{q''^2}{\mathcal{M}(q'')} \frac{V^j_{L'_\pi \tilde{L}}(q',q'')F^j_{\tilde{L}L_\pi}(q'',q)}{E(q)-E(q'')+i\epsilon}dq''. \quad (16)$$

This equation is solved using the matrix inversion method. In our evaluation of the partial amplitudes we also take into

account the Coulomb interaction applying the matching procedure developed by Vincent and Phatak [14].

Expressions for the polarization observables can be obtained using the spherical tensor operator $\tau_{k\kappa}$ which is defined as

$$\langle JM'|\tau_{k\kappa}|JM\rangle = \hat{J}\hat{k}(-1)^{J+M'}\begin{pmatrix} J & J & k \\ M & -M' & \kappa \end{pmatrix}, \quad (17)$$

were we use the notation $\hat{J}=\sqrt{2J+1}$. Analyzing powers can then be expressed as expectation values

$$T_{k\kappa} = \frac{\mathrm{Tr}(F\tau_{k\kappa}F^\dagger)}{\mathrm{Tr}(FF^\dagger)}. \quad (18)$$

There are four polarization observables which can be measured in πd elastic scattering using a pion beam and a polarized deuteron target. These are the vector analyzing power iT_{11}, and the three tensor analyzing powers T_{20}, T_{21}, and T_{22}. The last two can be measured only in linear combinations, such as

$$\tau_{21} = T_{21} + \tfrac{1}{2}(T_{22} + T_{20}/\sqrt{6}), \quad (19a)$$

$$\tau_{22} = T_{22} + T_{20}/\sqrt{6}. \quad (19b)$$

The differential cross section and four polarization observables can be expressed in terms of five amplitudes, called $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D},$ and $\mathcal{E}$, which correspond to our amplitudes $F_{M_f M_i}$ with different initial and final deuteron spin projections. Following *Robson's convention* [15] they are defined as

$$F_{M_f M_i} = \begin{pmatrix} F_{++} & F_{0+} & F_{-+} \\ F_{+0} & F_{00} & F_{-0} \\ F_{+-} & F_{0-} & F_{--} \end{pmatrix} = \begin{pmatrix} \mathcal{A} & \mathcal{B} & \mathcal{C} \\ \mathcal{D} & \mathcal{E} & -\mathcal{D} \\ \mathcal{C} & -\mathcal{B} & \mathcal{A} \end{pmatrix}, \quad (20)$$

where the sign $+, 0, -$ corresponds to the deuteron's spin projection $M_{i(f)} = +1, 0, -1$. Then the five observables can be written as

$$\frac{d\sigma}{d\Omega} = \frac{2q'}{3q}(|\mathcal{A}|^2 + |\mathcal{B}|^2 + |\mathcal{C}|^2 + |\mathcal{D}|^2 + \tfrac{1}{2}|\mathcal{E}|^2) \equiv \frac{q'}{q}a(\theta_\pi), \quad (21a)$$

$$iT_{11} = \sqrt{2/3}\,\mathrm{Im}[\mathcal{D}^*(\mathcal{A}-\mathcal{C}) - \mathcal{B}^*\mathcal{E}]/a(\theta_\pi), \quad (21b)$$

$$T_{20} = \frac{\sqrt{2}}{3}(|\mathcal{A}|^2 + |\mathcal{B}|^2 + |\mathcal{C}|^2 - 2|\mathcal{D}|^2 - |\mathcal{E}|^2)/a(\theta_\pi), \quad (21c)$$

$$T_{21} = -\sqrt{2/3}\,\mathrm{Re}[\mathcal{D}^*(\mathcal{A}-\mathcal{C}) + \mathcal{B}^*\mathcal{E}]/a(\theta_\pi), \quad (21d)$$

$$T_{22} = \frac{1}{\sqrt{3}}[2\mathrm{Re}(\mathcal{A}^*\mathcal{C}) - |\mathcal{B}|^2]/a(\theta_\pi). \quad (21e)$$

Note that due to the relation [15]

$$\mathcal{E} = (\mathcal{A}-\mathcal{C}) - \sqrt{2}(\mathcal{B}+\mathcal{D})\cot\theta_\pi \quad (22)$$

only four amplitudes are independent.

In our numerical calculations we will use a system of coordinates which corresponds to the *Madison Convention*: A right-handed coordinate system in which the positive z axis is along the beam direction (along the initial pion momentum $\vec{q}$) and the y axis is along the vector $[\vec{q}\times\vec{q}']$.

C. Pion-nuclear potential

Here we discuss the first-order potential $V^{(1)}_{M_f M_i}$, defined in Eq. (10). It can be rewritten as

$$V^{(1)}_{M_f M_i}(\vec{q}',\vec{q}) = -2\frac{\sqrt{\mathcal{M}(q')\mathcal{M}(q)}}{2\pi}$$

$$\times \int d\vec{r}\,\Psi^*_{M_f}(\vec{r})\exp\left(\frac{i\vec{Q}\cdot\vec{r}}{2}\right)t_{\pi N}(\omega;\vec{q}',\vec{q})$$

$$\times \Psi_{M_i}(\vec{r}), \quad (23)$$

where $\vec{Q} = \vec{q} - \vec{q}'$ is the momentum transfer.

The deuteron wave function in this expression is given by

$$\Psi_m(\vec{r}) = -\sqrt{3}\sum_{l,m_l}(-1)^l\frac{U_l(r)}{r}$$

$$\times \begin{pmatrix} l & 1 & 1 \\ m_l & m_s & -m \end{pmatrix} Y^l_{m_l}(\hat{\vec{r}})\chi^1_{m_s}, \quad (24)$$

where $l=0$ and 2 denote the S and D components, respectively, $\chi^s_{m_s}$ denotes the spin wave function for the two-nucleon system with total spin $s=1$ and projection m_s. After a multipole decomposition for the exponential, $\exp(i\vec{Q}\cdot\vec{r}/2)$, in Eq. (23) the pion-nuclear potential can be expressed via reduced nuclear matrix elements $M_{SLJ}(Q)$ defined as

$$\left\langle f\left|\sum_{j=1}^2 [Y^L(\hat{\vec{r}}_j)\otimes\sigma^S_j]^J_M j_L\left(\frac{Qr_j}{2}\right)\right|i\right\rangle$$

$$= (-1)^{1-M_f}\begin{pmatrix} 1 & J & 1 \\ -M_f & M & M_i \end{pmatrix} M_{SLJ}(Q), \quad (25)$$

where $j_L(z)$ is the spherical Bessel function, $\sigma^S = 1$ and $\vec{\sigma}$ for $S=0$ and 1, respectively.

Using standard techniques one finds

$$M_{SLJ}(Q) = 3\sqrt{3(S+1)/4\pi}\sum_{l'l}\hat{l}\hat{l}'\hat{L}\hat{j}\begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}$$

$$\times \begin{Bmatrix} 1 & l' & 1 \\ 1 & l & 1 \\ J & L & S \end{Bmatrix} R^{(L)}_{l'l}(Q). \quad (26)$$

The radial integrals $R^{(L)}_{l'l}(Q)$ are defined as

$$R^{(L)}_{l'l}(Q) = \int r U_{l'}(r) j_L\left(\frac{Qr}{2}\right) U_l(r) \quad (27)$$

and satisfy the normalization condition

$$R_{00}^{(0)}(Q) + R_{22}^{(0)}(Q) \stackrel{Q\to 0}{=} 1. \qquad (28)$$

Explicit expressions for the matrix elements $M_{SLJ}(Q)$ are given in the Appendix.

The radial wave functions $U_l(r)$ are parameterized [16] as a discrete superposition of Yukawa-type terms

$$U_0(r) = \sum_{n=1}^{N} C_n \exp(-m_n r) \quad (^3S_1 \text{component}) \quad (29a)$$

$$U_2(r) = \sum_{n=1}^{N} D_n \exp(-m_n r)\left(1 + \frac{3}{m_n r} + \frac{3}{m_n^2 r^2} \right)$$
$$(^3D_1 \text{component}) \qquad (29b)$$

where the 13 coefficients C_n, D_n and masses m_n have been calculated following the prescription given in Ref. [16].

In order to facilitate a qualitative understanding of the behavior of the polarization observables we present expressions for the five amplitudes $\mathcal{A}, \ldots, \mathcal{E}$ in the *plane wave impulse approximation* (PWIA) [the second term in Eq. (8) is dropped]. Taking into account only contributions from the S-wave component and their interference with the D-wave component of the deuteron wave function (neglecting terms with $l' = l = 2$), we obtain

$$\mathcal{A} \cong 2 A_0 W_0 \left[R_{00}^{(0)} - \frac{1}{2\sqrt{2}}(1 - 3\cos\theta_\pi)R_{02}^{(2)} \right], \quad (30a)$$

$$\mathcal{B} \cong -\sin\theta_\pi \left[\sqrt{2} A_S W_S \left(R_{00}^{(0)} + \frac{1}{\sqrt{2}}R_{02}^{(2)} \right) - 3 A_0 W_0 R_{02}^{(2)} \right], \qquad (30b)$$

$$\mathcal{C} \cong -\frac{3}{\sqrt{2}}(1 + \cos\theta_\pi)A_0 W_0 R_{02}^{(2)}, \qquad (30c)$$

$$\mathcal{D} \cong \sin\theta_\pi \left[\sqrt{2} A_S W_S \left(R_{00}^{(0)} + \frac{1}{\sqrt{2}}R_{02}^{(2)} \right) + 3 A_0 W_0 R_{02}^{(2)} \right], \qquad (30d)$$

$$\mathcal{E} \cong 2 A_0 W_0 \left[R_{00}^{(0)} + \frac{1}{\sqrt{2}}(1 - 3\cos\theta_\pi)R_{02}^{(2)} \right]. \qquad (30e)$$

Here, A_0 and A_S are the isoscalar scalar and spin-flip πN-scattering amplitudes from Eq. (11), and $R_{l'l}^{(L)}$ are the radial integrals defined in Eq. (27), the coefficients W_0 and W_S are kinematical factors arising from the Lorentz transformation of the elementary amplitude from the πN c.m. to the πd c.m. system. They can easily be obtained from Eqs. (10,11) and the angle transformation of $[\vec{q}_i \times \vec{q}_f]$ in the elementary amplitude [4]. The complete expressions of the five amplitudes (including the full contribution from the deuteron D state) are given in the Appendix.
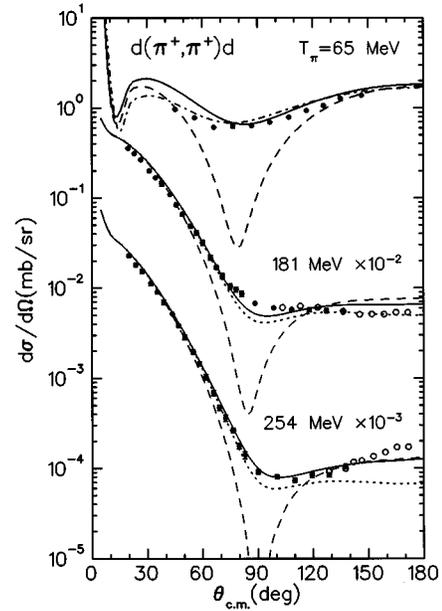

FIG. 1. Differential cross sections for π^+ elastic scattering on the deuteron at pion kinetic energies $T_\pi = 65$, 181, and 254 MeV calculated with (solid curves) and without (dashed curve) spin flip transition. The dotted curves at $T_\pi = 181$ and 254 MeV are the results obtained without the deuteron D state. The dash-dotted curve at $T_\pi = 65$ MeV is the result with second order corrections (32). Experimental data are from Refs. [26,27] (●) and Ref. [28] (o).

III. RESULTS AND DISCUSSION

A. Pion scattering on unpolarized targets

We begin our discussion with some of the main features of the pion-nuclear interaction. The results of our calculations for pion kinetic energies in the lab system, $T_\pi = 65$, 181, and 254 MeV, are shown in Fig. 1. One of the most important properties of the πN interaction in this energy region is the dominance of the p-wave contribution coming from the Δ-isobar excitation. This feature is reflected in the coherent scattering process which is proportional to A (nuclear mass number) and described by the scalar-isoscalar part (A_0) of the πN amplitude. Since the p-wave part of this amplitude has a $\cos\theta_\pi$ dependence, the differential cross section experiences a minimum around $\theta_\pi = 90°$ (see dashed curves). The position of this minimum is slightly shifted from the Lorentz transformation of the pion angle from the πN to the πd c.m. frame and from the s- and p-wave interference. The spin-flip transition coming from the amplitude A_S (which is also mainly of p-wave nature), is proportional to $\sin\theta_\pi$. The corresponding spin-flip contribution fills in the minimum, as shown in Fig. 1.

In the Δ-resonance region ($T_\pi = 180$ MeV) our results are in fairly good agreement with the experimental data and are basically the same as the results of three-body Faddeev calculations [1]. This may be due to the dominance of the Δ-resonance contribution. It had been shown in Ref. [17] that in this case the two-body approach is a good approximation for the more elaborate three-body Faddeev framework.

However, at lower energies ($T_\pi = 65$ MeV) our model fails to reproduce the experimental data in the forward direc-

tion. This is related to the well-known problem of describing the low-energy S-wave pion-nucleon interaction in nuclei with zero isospin. In this case, the contribution from the large isovector πN scattering amplitude A_T cancels and only the small A_0 amplitude remains in the first-order potential. Therefore, in the zero-energy limit our approach gives a small value for the πd-scattering length

$$a_{\pi d}^{(1)} = \lim_{q \to 0} \frac{\tan \delta_0^{(1)}}{q} \approx -0.015 \ \text{fm}, \qquad (31)$$

where $\delta_0^{(1)}$ is the S-wave πd scattering phase shift calculated with the first-order potential. The experimental value is about five times larger: $a_{\pi d}^{\text{expt}} = -(0.073 \pm 0.02)$ fm [18]. In such a situation one expects important contributions from the Q operator in Eq. (5b). A well-known estimate (see for example Refs. [19,20]) for the corresponding second-order correction is

$$a_{\pi d}^{(2)} = 2 \left(A_0^2 - 2A_T^2 \right) \left\langle \frac{1}{r} \right\rangle C^2, \qquad (32)$$

where $\langle 1/r \rangle = 0.46 \ \text{fm}^{-1}$ and $C = (1 + m_\pi/M)(1 + m_\pi/2M)^{-1}$. In this expression the contribution from the isovector amplitude A_T which describes double isovector scattering (including charge exchange) becomes important. Taking into account such a correction for the scattering length we obtain

$$a_{\pi d} \approx -(0.015 + 0.034) \ \text{fm} = -0.049 \ \text{fm}, \qquad (33)$$

which is in better agreement with the experimental value as well as with results from Faddeev calculations: $a_{\pi d}^{\text{Fad}} = 0.046$ fm [21] (for the same set of A_0 and A_T amplitudes). As shown in Fig. 1, this correction factor also improves the agreement with the experimental data for the differential cross section at $T_\pi = 65$ MeV (see dash-dotted curve).

The role of the $^3D_1$ configuration in the deuteron wave function is also illustrated in Fig. 1. Its contribution can be seen by turning off the nuclear matrix elements of Eq. (26) with $L = 2$ (see dotted curves in Fig. 1), thus retaining the normalization condition of Eq. (28). As expected, the D-state contribution becomes significant at large angles, $\theta_\pi > 90°$, for pion energies $T_\pi > 180$ MeV which correspond to momentum transfers of $Q > 1.78 \ \text{fm}^{-1}$.

Figure 2 presents our results for higher pion energies. Note that with increasing pion energies the second-order correction discussed above becomes less important due to the dominance of the pion-nucleon p-wave contribution. However, it is still visible at $T_\pi = 142$ MeV and it improves the description in the forward direction (the corresponding result is not shown in Fig. 2). Figure 2 illustrates the role of pion rescattering contributions coming from the second term of Eq. (8) by comparing the plane wave [without second term of Eq. (8)] calculations (dashed curves) with the full results (solid curves). Pion rescattering becomes very important in the Δ region ($140 < T_\pi < 250$ MeV), especially at large angles. Here it can reduce the plane wave results by a factor of 2. However, with further increase of energy the πN interaction becomes weaker again, and, consequently, the rescattering contributions become smaller.
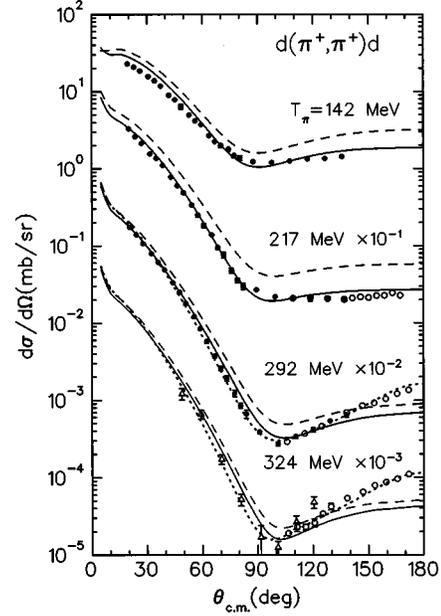

FIG. 2. Differential cross sections for π^+ elastic scattering on the deuteron at pion kinetic energies $T_\pi = 142$–324 MeV. Solid and dashed curves are full and PWIA calculations, respectively. The dotted curves are our best fit with a phenomenological interaction of two intermediate nucleons in the Jennings mechanism. Experimental data are from Ref. [27] (●), Ref. [28] (o), and Ref. [29] (△).

At $T_\pi > 250$ MeV the main problem which arises in our approach (as well as Faddeev calculations) is the discrepancy with experimental data at backward angles. In contrast to Faddeev results which usually overestimate the data in this region by a factor of 2 [1], our calculations tend to underestimate the measurements. We point out that a similar disagreement has been found for pion scattering on ^{3}He and ^{4}He [6,22].

In the framework of Faddeev equations there have been numerous attempts to improve the situation at backward angles, mainly by changing the two-body inputs. One of the more controversial attempts addressed the P_{11} part of the πN amplitude with its division into a pole and nonpole term, summarized in Refs. [1,23]. We only note here that the pole term determines the coupling between the NN and πNN channels. Due to this coupling, pion scattering can also be presented as pion absorption on one nucleon and subsequent emission from another. Such a two-body mechanism is not included in the standard multiple scattering theory. At large momentum transfers this process could be important because of the possibility of momentum sharing. However, as argued by Jennings [24], such a two-body mechanism is almost completely canceled by additional contributions which come from the time ordering of the pion absorption and emission processes.

On the other hand, the issue concerning the so-called P_{11} problem cannot be regarded as settled since the strong cancelation takes place only if there is no interaction between the two nucleons in the intermediate state. Our simple estimates show that if we would include the repulsive part of the NN interaction which involves heavy meson exchange (as was done by Mizutani and Koltun in their study of the πd scattering length [25]), the cancellation disappears. After
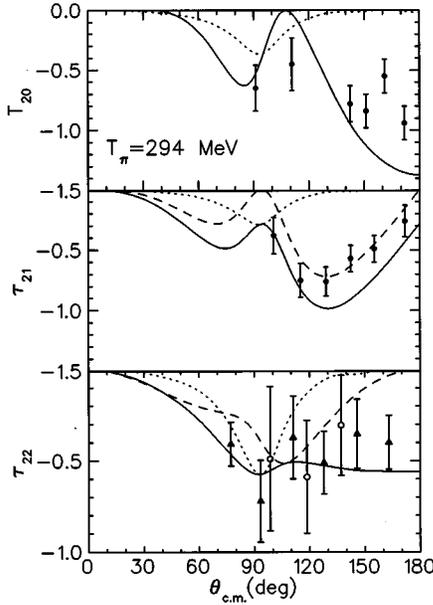
FIG. 3. Tensor analyzing powers $T_{20}, \tau_{21} = T_{21} + \frac{1}{2}(T_{22} + T_{20}/\sqrt{6})$ and $\tau_{22} = T_{22} + T_{20}/\sqrt{6}$ at $T_\pi = 294$ MeV (solid curves). Individual contributions from T_{21} (in τ_{21}) and T_{22} (in τ_{22}) are shown by dashed curves. The dotted curves are the results obtained for T_{20}, τ_{21}, and τ_{22} without the deuteron D state. Experimental data are from Ref. [30] (●), Ref. [31] (black triangles), and Ref. [32] (o).

a modification of the free two-nucleon propagator by adding the mean value of the residual NN interaction $\langle V_{NN}(\rho, \omega, \dots)\rangle$ the corresponding two-body contribution becomes proportional to $\langle V_{NN}\rangle$. Figure 2 illustrates the effects which could come from this two-body mechanism with $\langle V_{NN}\rangle = 40$ MeV. Clearly, a more microscopic calculation is called for, here we want to merely point out that this mechanism can improve the situation at backward angles.

B. Polarization observables

Here we begin our discussion with the results for the tensor analyzing powers. In order to understand the general structure of these observables and their behavior in different kinematical regions we first consider the size of the observables at backward angles $\theta_\pi = 180°$. It is instructive to also neglect the contribution of the deuteron $^3D_1$ component. As an example, Fig. 3 presents T_{20}, τ_{21} and τ_{22} at $T_\pi = 294$ MeV. We remind the reader that, according to Eq. (19), the last two observables are linear combinations of T_{20}, T_{21}, and T_{22}.

It follows from Eq. (30) that at $\theta_\pi = 180°$ the amplitudes $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ vanish. Therefore, at backward angles the observables T_{21} and T_{22} vanish as well, and T_{20} reduces to the following expression:

$$T_{20}(180°) = \frac{\sqrt{2}(|\mathcal{A}|^2 - |\mathcal{E}|^2)}{2|\mathcal{A}|^2 + |\mathcal{E}|^2}. \quad (34)$$

From Eq. (30) we can see that in the region where the radial integral $R_{00}^{(0)} = \sqrt{2}R_{02}^{(2)}$ the contribution from the $\mathcal{A}$ amplitude becomes very small. In this region we then have
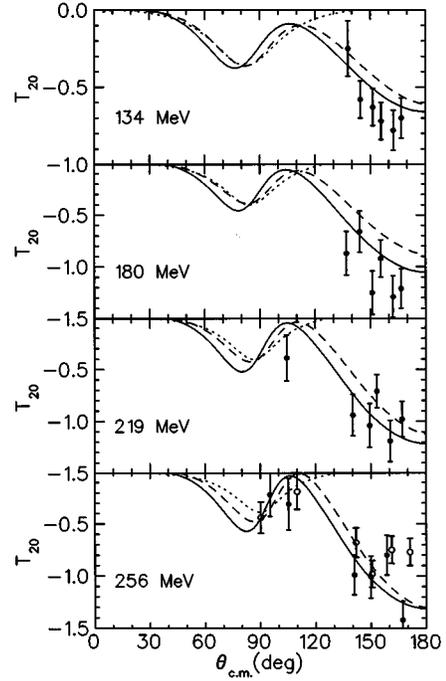

FIG. 4. T_{20} observable at $T_\pi = 134$–256 MeV. Solid and dashed curves are full and PWIA calculations, respectively. The dotted curves are the results obtained without the deuteron D state. Experimental data are from Ref. [33] (●) and Ref. [30] (o).

$$T_{20}(180°) \approx -\sqrt{2}, \quad \tau_{22}(180°) \approx 2\tau_{21}(180°) \approx -\frac{1}{\sqrt{3}}. \quad (35)$$

This result is confirmed experimentally for $T_\pi > 200$ MeV. Note, that at lower energies in the region where the deuteron D-wave contribution is small ($R_{00}^{(0)} \gg R_{02}^{(2)}$) tensor observables at backward angles become proportional to the strength of the deuteron D state. For example, $T_{20}(180°) \approx -4R_{02}^{(2)}/R_{00}^{(0)}$

Another simplified scenario is given by neglecting the pion rescattering contributions and the deuteron D wave. In this case, the $\mathcal{C}$ amplitude vanishes, and $\mathcal{A}^{(S)} = \mathcal{E}^{(S)}$ and $\mathcal{B}^{(S)} = -\mathcal{D}^{(S)}$. Due to these identities, $T_{21}^{(S)} = 0$ and the other observables can be expressed solely via the elementary πN scattering amplitudes

$$T_{20}^{(S)} = -\frac{2\sqrt{2}|A_S|^2\sin^2\theta_\pi}{12|A_0|^2 + 8|A_S|^2\sin^2\theta_\pi}, \quad T_{22}^{(S)} = \sqrt{3/2}T_{20}^{(S)}. \quad (36)$$

As we have shown in the analysis of the differential cross sections, due to the dominance of the p-wave contribution in πN scattering, $A_0(90°) \approx 0$. Therefore, in the $\theta_\pi = 90°$ region where the contribution from the deuteron S-state dominates we have

$$T_{20}^{(S)}(90°) \approx -\sqrt{2}/4, \quad \tau_{22}^{(S)}(90°) \approx 2\tau_{21}^{(S)}(90°) \approx -\frac{1}{\sqrt{3}}. \quad (37)$$

Figure 3 shows that these predictions can already reproduce the experimental measurements fairly well.

The next three figures, Fig. 4, Fig. 5, and Fig. 6, present tensor observables at $T_\pi = 134$, 180, 219, and 256 MeV. In
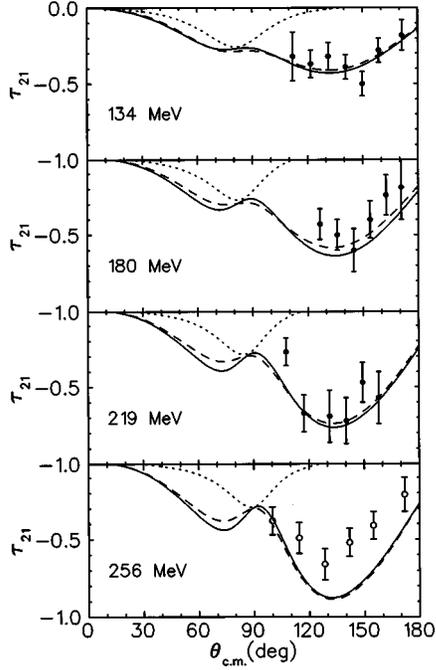
FIG. 5. The same as in Fig. 4 for the observable $\tau_{21}=T_{21}+\frac{1}{2}(T_{22}+T_{20}/\sqrt{6})$. Experimental data are from Ref. [33] ($\bullet$) and Ref. [30] (o).

general, their behavior follows the simple picture described above. At backward angles T_{20} and T_{22} are almost completely given by the $^3D_1$ component of the deuteron wave function. In forward direction (up to $\theta_\pi=100°$) the main contribution comes from the $^3S_1$ component. Here T_{20} and T_{22} contain very little nuclear structure information. T_{21} de-
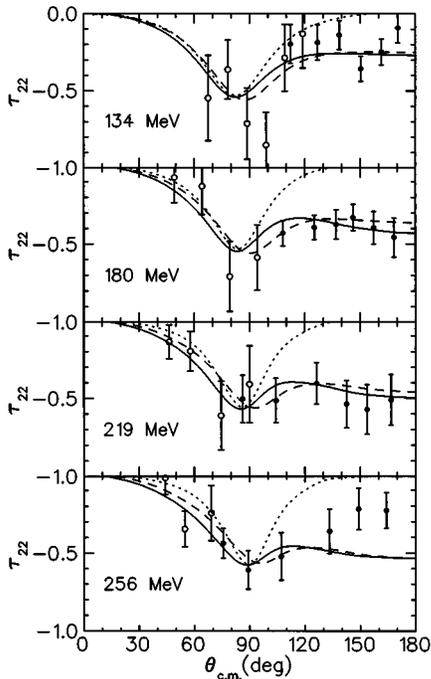
FIG. 6. The same as in Fig. 4 for the observable $\tau_{22}=T_{22}+T_{20}/\sqrt{6}$. Experimental data are from Ref. [31] ($\bullet$) and Ref. [32] (o).

pends entirely on the deuteron D state. In the case of the τ_{21} observable, which is mainly associated with T_{21}, the deuteron S state tends to fill in the minimum around $\theta_\pi=90°$. Note that this minimum appears because of the p-wave dominance of the πN scattering amplitude. Finally, we see that the influence of pion rescattering effects on all tensor analyzing powers is small.

Thus, we can obtain a good description for the tensor observables. This may be related to the fact that they are determined mainly by the absolute values of the $\mathcal{A}-\mathcal{E}$ amplitudes. In contrast, the vector analyzing power iT_{11} depends on the interference between these amplitudes. Therefore, we expect this quantity to be more sensitive to details of the model ingredients. In Fig. 7 we compare our results with some old [33] and more recent [32] measurements. We obtain satisfactory agreement with these data only at $T_\pi=100$ MeV. At all other energies our results fail to reproduce the experimental data, especially for $\theta_\pi>90°$. The failure becomes more dramatic with increasing pion energy. In our full calculation iT_{11} goes through zero around $T_\pi=180$ MeV and becomes negative at $\theta_\pi>90°$. We have found that pion rescattering is mainly responsible for this effect. Experimental measurements show quite the opposite behavior: Above $T_\pi=250$ MeV, iT_{11} changes the sign in forward direction producing a negative dip around $\theta_\pi=70°$ and it assumes large positive values for $\theta_\pi>90°$. This behavior could only be described in simple calculations without pion rescattering (dashed curves). Note that the influence of the deuteron D state on iT_{11} is small. Therefore, in the plane wave approximation the vector analyzing power can be expressed via the elementary πN scattering amplitudes

$$iT_{11}^{(S)}=-\frac{\sqrt{6}\mathrm{Im}(A_S^*A_0)\sin\theta_\pi}{12|A_0|^2+8|A_S|^2\sin^2\theta_\pi}. \qquad (38)$$

In this paper we have not attempted to present a detailed comparison with the Faddeev calculations that have been performed. A review of these results can be found in Ref. [1]. We only mention that all conventional calculations encounter the difficulties in the description of the measured vector analyzing power in the Δ-resonance region.

On a phenomenological level we have studied the effect of modifying the spin-flip amplitude A_S (similar to A_0 at low energies)

$$A_S\rightarrow A_S-B_S\frac{\left\langle d\left|e^{i(\vec{q}+\vec{q'})\cdot\vec{r}/2}\dfrac{e^{-m_\pi r}}{r}\right|d\right\rangle}{\langle d|e^{i(\vec{q}-\vec{q'})\cdot\vec{r}/2}|d\rangle}, \qquad (39)$$

where $|d\rangle$ is the deuteron ground state. The phenomenological term with a complex parameter B_S would be associated with second-order contributions. The sensitivity of the differential cross section and tensor analyzing powers to this correction is small since they are dominated by the coherent contribution from the non-spin-flip amplitude A_0. However, it has a dramatic influence on the vector analyzing power. Using this sensitivity we extracted the B_S parameter from the experimental data for iT_{11}. The result of our fit is shown in Fig. 7 by the dash-dotted curves. It is remarkable that the energy behavior of the real and imaginary parts of B_S fol-
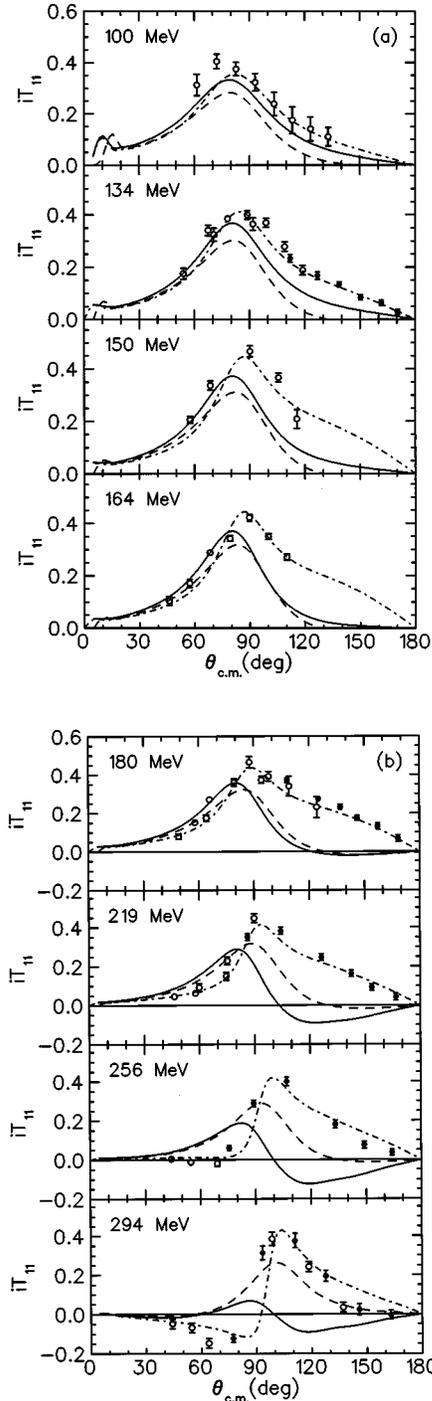
FIG. 8. Energy dependence of the modification for the spin-flip interaction [see Eq. (39)] extracted from the vector analyzing power iT_{11}.

IV. CONCLUSION

In this paper, we have studied the interaction of pions with the deuteron in a multiple scattering approach carried out in momentum space. Our investigation covered the energy region of $T_\pi = 60$–300 MeV, thus covering both the low-energy and the Δ region. Paris wave functions were employed to describe the deuteron. Phase-shift parameterizations were used for the elementary πN amplitudes, along with a separable potential for the off-shell extrapolation. The full spin and isospin dependence of the πN amplitudes were taken into account.

Our framework of multiple scattering theory is clearly less sophisticated than the many Faddeev calculations that have studied pion deuteron scattering over the years. The primary purpose of our investigation was to demonstrate that the approach developed recently for the description of the pion interaction with $A \geq 3$ nuclei can be reliably applied in the case of the deuteron. The results obtained are transparent and can in the following be employed for the study of pion photo- and electroproduction off deuterium.

We found that our approach gives a good description of the differential cross sections and the tensor analyzing powers. At low pion energies $T_\pi \leq 65$ MeV the model fails to reproduce experimental data at forward direction. In this kinematical region the coupling with the breakup channels has to be included. At higher energies and backward angles discrepancies appear that may be related to two-body processes involving heavy meson exchange. The tensor analyzing powers strongly depend on the deuteron D state and can be reproduced satisfactorily. Clearly, the outstanding problem is the explanation of the well-measured vector analyzing power which has defied a number of attempts using more advanced approaches than ours. While there may be a sensitivity regarding a residual ΔN-interaction care has to be taken to include it consistently.

In the following paper (paper II), we will show that the pion-deuteron interaction developed here is adequate to describe the final state interaction in coherent pion photoproduction from deuterium.

ACKNOWLEDGMENTS

FIG. 7. Vector analyzing power iT_{11} at (a) $T_\pi = 100$–164 MeV and (b) $T_\pi = 180$–294 MeV. Solid and dashed curves are full and PWIA calculations, respectively. Dash-dotted curve are the results of our fit with phenomenological term (39). Experimental data are from Ref. [33] ($\bullet$) and Ref. [32] (o).

lows a typical resonance structure (see Fig. 8). This might be an indication for the presence of a residual ΔN interaction. We point out that a similar situation occurs in pion scattering on ^{3}He for the target asymmetry $A_y = \sqrt{2} iT_{11}$ [34]. It seems reasonable to assume that the origin of this phenomenon is similar for both reactions. It therefore becomes very important to find a consistent description of polarization observables in pion scattering on the deuteron and ^{3}He.
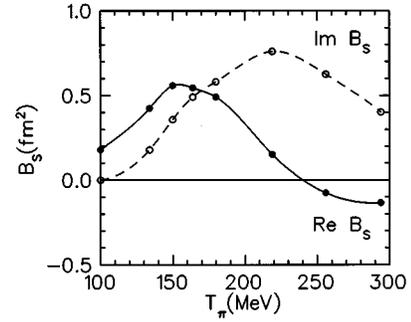
APPENDIX

In this appendix, we give the complete expressions for the *Robson amplitudes* $\mathcal{A}, \ldots, \mathcal{E}$ where the full contributions from the deuteron D state have been taken into account.

First, we express the nuclear matrix elements $M_{SLJ}(Q)$ defined by Eqs. (25), (26) via the radial integrals of Eq. (27)

$$M_{000}(Q) = 2\sqrt{3/4\pi}[R_{00}^{(0)}(Q) + R_{22}^{(0)}(Q)], \qquad \text{(A1)}$$

$$M_{022}(Q) = 4\sqrt{3/4\pi}\left[R_{02}^{(2)}(Q) - \frac{\sqrt{2}}{4}R_{22}^{(2)}(Q)\right], \qquad \text{(A2)}$$

$$M_{101}(Q) = 2\sqrt{6/4\pi}\left[R_{00}^{(0)}(Q) - \frac{1}{2}R_{22}^{(0)}(Q)\right], \qquad \text{(A3)}$$

$$M_{121}(Q) = -2\sqrt{6/4\pi}\left[R_{02}^{(2)}(Q) + \frac{1}{\sqrt{2}}R_{22}^{(2)}(Q)\right]. \qquad \text{(A4)}$$

Then the amplitudes $\mathcal{A}, \ldots, \mathcal{E}$ in the PWIA approach can be written as

$$\mathcal{A} = \sqrt{4\pi/3}A_0W_0\left[M_{000}(Q) - \frac{1}{4\sqrt{2}}(1 - 3\cos\theta_\pi)M_{022}(Q)\right], \qquad \text{(A5)}$$

$$\mathcal{B} = -\tfrac{1}{2}\sqrt{4\pi/3}\sin\theta_\pi[A_SW_SM_S(Q) - \tfrac{3}{2}A_0W_0M_{022}(Q)], \qquad \text{(A6)}$$

$$\mathcal{C} = -\tfrac{3}{4}\sqrt{4\pi/6}(1 + \cos\theta_\pi)A_0W_0M_{022}(Q), \qquad \text{(A7)}$$

$$\mathcal{D} = \tfrac{1}{2}\sqrt{4\pi/3}\sin\theta_\pi[A_SW_SM_S(Q) + \tfrac{3}{2}A_0W_SM_{022}(Q)], \qquad \text{(A8)}$$

$$\mathcal{E} = \sqrt{4\pi/3}A_0W_0\left[M_{000}(Q) + \frac{1}{2\sqrt{2}}(1 - 3\cos\theta_\pi)M_{022}(Q)\right]. \qquad \text{(A9)}$$

In Eqs. (A6) and (A8) we introduced the matrix element

$$M_S(Q) = M_{101}(Q) - \frac{1}{\sqrt{2}}M_{121}(Q). \qquad \text{(A10)}$$

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