$f_{N\pi N}$: From quarks to the pion derivative coupling

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We study the $N\pi N$ coupling, in the framework of a QCD-inspired confining Nambu-Jona-Lasinio model. A simple relativistic confining and instantaneous quark model is reviewed. The Salpeter equation for the qqq nucleon and the $q\bar{q}$ boosted pion is solved. The $f_{n\pi n}$ and $f_{n\pi\Delta}$ couplings are calculated and they turn out to be reasonably good. The sensibility of $f_{n\pi n}$ and $f_{n\pi\Delta}$ to confinement, chiral symmetry breaking, and Lorentz invariance is briefly discussed. [S0556-2813(97)00902-3]

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I. INTRODUCTION

It is widely accepted that the low energy phenomenology can be understood in terms of the mechanism of chiral symmetry spontaneous breaking. In this sense any microscopic theory (including QCD) which has a ''correct'' chiral limit will fare equally well in describing low energy hadronic phenomena. There is, however, an extra ingredient, the typical hadronic size, which should also play an important role in scattering. To see that this scale is important it is sufficient to consider exotic reactions like K-n scattering, for this scale controls the extent of overlap between the bare nucleon and the kaon. In turn this overlap, when taken together with color saturation and a generic spin-spin interaction for the quarks, accounts for the generic hadronic central repulsion, a feature which is notorious in the n-n system [1] and one which can also be measured in the K-n exotic s-wave phase shifts [2]. Exotic reactions like K-n, where the $\bar{s}$ quark cannot annihilate with any of the quarks intervening in the reaction, can be thought as an effective K-n-K-n vertex without the exchange of s-channel resonances. Nor does it have t-channel exchange of pions due to G parity and therefore constitutes an ideal reaction to probe the low energy content of both the nucleon and the kaon wave functions, i.e., the overlap kernel.

On the other hand, the bare rho-pion mass difference is completely controlled by the extent of chiral symmetry breaking and this mechanism, when understood in terms of quarks, amounts to a specific connection between the chiral condensate and the strength of the microscopic hyperfine interaction $\vec{s} \cdot \vec{s}$ [3–5]. It happens that the obtained strength of this $\vec{s} \cdot \vec{s}$ when considered in the exotic K-n s-wave system yields good phase shifts provided we have a small bare nucleon core. At this stage it should be understood that the size of bare hadrons and the extent of the chiral condensate are not independent quantities but instead they are related to each other. This is a consequence of the fact that in hadronic reactions the mechanism of spontaneous chiral symmetry breaking is self-consistently exerted in two separate sectors: in the Salpeter amplitudes of the intervening hadrons and in the modifications it introduces in the quark-quark effective potentials.

It is therefore interesting to see if it is possible to describe in a unified way this central repulsion (linked with nonannihilating quark amplitudes) together with quark-antiquark annihilating amplitudes which are not only at the origin of hadronic attraction, the other prominent feature of hadronic scattering and one which can be associated with n-n peripheral attraction, but also at the origin of other effects like strong decay rates.

The n-n peripheral attraction is a central issue at the crossroads between nuclear physics and hadronic physics. Several attempts to describe this attraction in terms of Skyrmion Lagrangians have been made but the overall result remains inconclusive. In the Skyrmion picture, although being quite adapted in describing n-n repulsion in terms of topological numbers and despite containing the physics of the pion interaction, it is not simple to accommodate with the Yukawa picture of n-n attraction in terms of mesonic exchange and several mechanisms have been proposed in the literature to provide for this attraction [6]. On the other hand, the physics of the central n-n repulsion lies outside the domain of chiral perturbation theory and therefore we cannot use this theory to describe the n-n scattering.

Any effective theory attempting to describe the n-n attraction must also be able to describe the physics of meson-nucleon systems among which we have the πn system. In this respect the exotic K-n scattering is much simpler to study than the πn reaction which, being nonexotic, has now quark-antiquark annihilating amplitudes which were absent in the exotic K-n. Also we are forced to consider the quasi-Goldstone nature of the pion which can be translated in the Salpeter language by the existence of two almost degenerate amplitudes (the so-called E-spin amplitudes) to describe the pion wave function in contrast with the kaon case which is well described by one Salpeter amplitude.

The Nambu-Jona-Lasinio (NJL) effective Lagrangian [7] with a nonlocal potential [3,4,8–11] offers then a simple framework to describe in a unified way (in the present case with only one parameter for the potential strength besides the current quark masses) not only the bulk of hadronic spectroscopy but also hadronic scattering phenomena among which we have the coupled n-n, the Δ-n, and the Δ-Δ scattering processes. It is clear that these processes, when seen from the quark microscopic point of view embodied in the NJL Lagrangian, should correspond to different scattering processes described by the *same* set of Lagrangian parameters. In this paper we will use N to represent either the Δ or the nucleon in the cases where it does not matter and reserve the letter

n for the cases which are specific to the nucleon alone.

When dealing with potentials, Lorentz covariance becomes a problem not unrelated to the problem of a relativistic description of bound states. Covariant generalizations of potentials are available in the literature [12,13] but at this stage we will ignore this issue which will be essential for quantitative predictions but certainly not for the qualitative picture which as we will show can be understood in terms of chiral symmetry and color confinement.

In nuclei the necessity of having a microscopic description of the nucleon in terms of quarks is already felt when considering Coulomb and magnetic form factors for s-shell and p-shell nuclei like ^{6}Li and ^{16}O [14]. But the simple fact that chiral symmetry is spontaneously broken through the appearance of quark-antiquark condensates and considering not only that these new chiral vacua are in turn affected by the presence of quark sources (the physics of chiral restoration and enhancement) [15], but also that we might have excitations of these vacua along the ''Mexican hat'' valley of a continuum of chiral states connected with the ''true'' chiral vacuum, should allow us to use the formalism of this paper to investigate and predict the existence of signatures of these states in the nuclear environment. For this we must have a model accurate enough both in the N-N sector and in the sector of hadron spectroscopy. This paper constitutes a first step towards this goal.

The remainder of this paper is divided into four sections and three appendixes. Section II is devoted to the introduction of the specific Nambu-Jonas-Lasinio effective model that we will be using. The pion and nucleon Salpeter equations will be studied in Sec. III. The overlap diagrams for the couplings $f_{\pi NN}$ are presented in Sec. IV. Finally in Sec. VI we discuss the results. We have three appendixes. In Appendix A some diagrams contributing to the $f_{\pi NN}$ are presented and discussed in detail. The color overlap results are given in Appendix B. In Appendix C we discuss, both for the nucleon and the delta, the relevant flavor-spin overlaps.

II. SIMPLE CONFINING QUARK MODEL WITH CHIRAL SYMMETRY BREAKING

Our quarks are Dirac fermions that interact with a simple effective two-body interaction [8] such as to simulate color confinement together with the introduction of a scale responsible for the actual particle sizes,

$$H = \int d^3x [H_0(\mathbf{x}) + H_I(\mathbf{x})], \tag{1}$$

where H_0 is the Hamiltonian density of the Dirac field, and H_I an effective interaction term,

$$H_0(\mathbf{x}) = \psi^\dagger(\mathbf{x})(m_q\beta - i\vec{\alpha}\cdot\nabla)\psi(\mathbf{x}),$$

$$H_I(\mathbf{x}) = \frac{1}{2}\int d^3y\, V_I(\mathbf{x}-\mathbf{y})\bar{\psi}(\mathbf{x})\frac{\lambda^a}{2}\gamma^0\psi(\mathbf{x})\bar{\psi}(\mathbf{y})\gamma^0\frac{\lambda^a}{2}\psi(\mathbf{y}),$$

$$V_I(\mathbf{x}-\mathbf{y}) = -\frac{3}{4}[K_0^3(\mathbf{x}-\mathbf{y})^2 - U]\delta(t_x - t_y). \tag{2}$$

The λ^a's are the Gell-Mann color matrices. U is an arbitrarily large constant. In Ref. [4] we have shown that physical processes involving color singlets are independent of U whereas any colored objects get an infinite mass when $U\to\infty$.

Different and more complex interactions have also been used in the literature. Within the Wilson loop context, results for the effective microscopic interaction among heavy quarks are available [16] but unfortunately these results cannot be extended to the light quark sector due precisely to the problem of consistency with chiral symmetry. Nevertheless, it is customary to make the approximation of dividing the quark effective potential in two additive terms, one dominated by the Coulomb force due to one-gluon exchange which is responsible for the short distance interaction among quarks and one which plays an important role in the heavy quark spectroscopy, and another term—the confining term—which is responsible for the long distance force among quarks. In fact this approximation is also effectively used when extracting the heavy quark potentials from the Wilson loop.

Although there is a great flexibility in the choice of the effective confining potential for the light quark mass sector, it is, however, not completely arbitrary. First it must not only comply with the requirements of the Ward identities but also provide finite results in the colorless sector while pushing the masses of the colored states to infinity. This is illustrated when we consider a more general Dirac structure like

$$V = V_I(\mathbf{x}-\mathbf{y})\left(s\;\; 1\otimes 1 + p\,\gamma_5\otimes\gamma_5 + v\,\gamma^\mu\otimes\gamma_\mu\right.$$

$$\left. + a\,\gamma^\mu\gamma_5\otimes\gamma_\mu\gamma_5 + \frac{t}{2}\sigma^{\mu\nu}\otimes\sigma_{\mu\nu}\right). \tag{3}$$

Then the axial and vectorial Ward identities would have constrained the parameters s, p, a, v, and t to obey the equations [5]

$$s + p + 6t = 0, \quad s - p = 0. \tag{4}$$

Equation (4) implies that the scalar (s), pseudo-scalar (p), and tensor (t) interactions do not contribute either to chiral symmetry breaking or to the structure of the ground state hadrons. Finally we must also have

$$4v - 4a = -2v - 2a, \quad a = 0, \tag{5}$$

in order to have both the Goldstone pion and the other color singlets infrared independent. As for the shape of the potential, the linear potential has also been extensively studied. However, it seems to yield a much too large hadronic radius.

The potential of Eqs. (1) and (2) has been used to study the charmonium spectroscopy with a potential strength of $K_0 = 290$ MeV. Although the theoretical results did not differ too much from the experimental ones, it was shown in Ref. [5] in the context of a more general potential that we will still need a Coulomb force if we want to get a correct value for the R parameter (which is related with the mass splittings of the ^3P_J triplet). And this happened despite the fact that, *individually*, the theoretically obtained masses were quite good when compared with the experimental ones. It turns out that the charmonium spectroscopy [4] obtained with the simpler potential of Eq. (2) does not fare too badly either, when

we consider individual masses, so that although no one disputes the fact that for short distances a Coulomb force is needed, we decided (also for the sake of mathematical simplicity) to discard this force as a first step towards a quantitative description of the pion-nucleon coupling.

Therefore the model embodied in Eqs. (1) and (2) not only seems to sufficiently meet all the above general requirements but also yields reasonable results besides allowing relatively simple calculations. A covariant version of this model has been developed in Ref. [13] and so far it has only been applied to the study of the pion and kaon. For the simple harmonic confining potential the authors obtained a substantial improvement on the value of f_π although still far from a quantitative agreement. Coupled channels should constitute another sizable correction not only to f_π but also to the hadronic decay widths and masses. Even without covariance we have shown [2] that coupled channels provide a very large correction to f_π, thereby substantiating the hope that covariance plus coupled channels might bring the f_π close to its experimental value. The present calculation constitutes a preliminary step in the microscopic and Lorentz-invariant calculation of the $f_{N\pi N}$. In Eq. (2), the field operator $\psi(\mathbf{x})$ is defined as

$$\psi(\mathbf{x}) = \int \frac{d^3 p}{(2\pi)^{3/2}} [u_s(\mathbf{p})b(\mathbf{p}) + v_s(\mathbf{p})d^\dagger(-\mathbf{p})]e^{i\mathbf{p}\cdot\mathbf{x}}. \quad (6)$$

b and d are, respectively, the quark and antiquark Fock space operators and they carry indices for flavor, spin, and color. Summation over repeated indices is assumed. The spinors u and v, together with the Fock space operators, differ from those used in free Dirac theory and are given by

$$u_s(\mathbf{p}) = \frac{1}{\sqrt{2}}\{[1+\sin\varphi(p)]^{1/2}+[1-\sin\varphi(p)]^{1/2}\hat{\mathbf{p}}\cdot\vec{\alpha}\}u_s^0,$$

$$v_s(\mathbf{p}) = \frac{1}{\sqrt{2}}\{[1+\sin\varphi(p)]^{1/2}-[1-\sin\varphi(p)]^{1/2}\hat{\mathbf{p}}\cdot\vec{\alpha}\}v_s^0.$$

$$(7)$$

In Eq. (7), u_s^0 and v_s^0 are spinor eigenvectors of γ_0 corresponding to eigenvalues ± 1. The function $\varphi(p)$ is called the *chiral angle* and indexes the different Fock spaces compatible with the Pauli principle. This chiral angle has been studied in Refs. [8,9] and is a solution of the *mass gap equation*

$$(k^2\varphi')' = 2k^3\sin(\varphi) - \sin(2\varphi), \quad (8)$$

where k is a dimensionless quantity in units of $K_0 = 1$. This unit will be used in the remainder of this paper. In Fig. 1 we plot $\sin[\varphi(k)]$ as a function of k.

Once this mass gap equation is solved, the quark and antiquark propagators can be diagonalized and it turns out that it is simpler to work in the spin representation rather than in the Dirac representation. As will be clear this representation will turn out to be the simplest representation if we want to study the πn, or $\pi\Delta$ couplings. In the spin representation we have, for the quark energy $E(k)$,

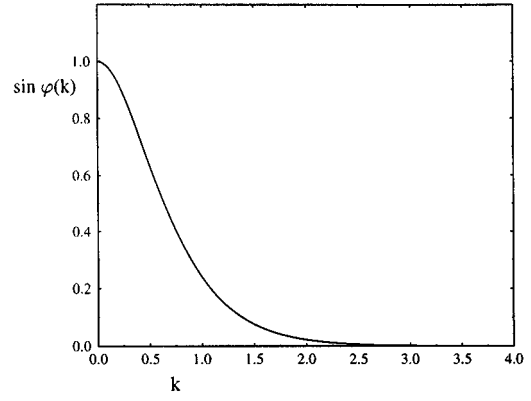$$E(k) = k\cos(\varphi) - \frac{\varphi'^2}{2} - \frac{\cos^2(\varphi)}{k^2}. \quad (9)$$

FIG. 1. The chiral angle as a function of k.

The Feynman rules for the spin representation are given in Fig. 2. We have, for the vertices,

$$u^\dagger{}_{s1}(\mathbf{k}1)u_{s2}(\mathbf{k}2) = \tfrac{1}{2}\{[\sqrt{1+S1}\sqrt{1+S2}$$
$$+\sqrt{1-S1}\sqrt{1-S2}(\hat{\mathbf{k}}1\cdot\hat{\mathbf{k}}2)]\delta_{s1,s2}\}$$
$$+\sqrt{1-S1}\sqrt{1-S2}(i\vec{\sigma}\cdot\hat{\mathbf{k}}1\times\hat{\mathbf{k}}2)_{s1s2}\},$$

$$v^\dagger{}_{s1}(\mathbf{k}1)v_{s2}(\mathbf{k}2) = \tfrac{1}{2}\{[\sqrt{1+S1}\sqrt{1+S2}$$
$$+\sqrt{1-S1}\sqrt{1-S2}(\hat{\mathbf{k}}1\cdot\hat{\mathbf{k}}2)]\delta_{s1,s2}\}$$
$$+\sqrt{1-S1}\sqrt{1-S2}(i\vec{\sigma}\cdot\hat{\mathbf{k}}1\times\hat{\mathbf{k}}2)^*_{s1s2}\},$$

$$u^\dagger_{s1}(\mathbf{k}1)v_{s2}(\mathbf{k}2) = -\tfrac{1}{2}\{[\sqrt{1-S1}\sqrt{1+S2}\hat{\mathbf{k}}1$$
$$-\sqrt{1+S1}\sqrt{1-S2}\hat{\mathbf{k}}2](i\vec{\sigma}\sigma_2)_{s1s2}\},$$

$$v^\dagger_{s1}(\mathbf{k}1)u_{s2}(\mathbf{k}2) = \tfrac{1}{2}\{[\sqrt{1-S1}\sqrt{1+S2}\hat{\mathbf{k}}1$$
$$-\sqrt{1+S1}\sqrt{1-S2}\hat{\mathbf{k}}2](i\vec{\sigma}\sigma_2)^+_{s1s2}\}.$$

$$(10)$$

$S1$ and $S2$ stand, respectively, for $\sin[\varphi(k1)]$ and $\sin[\varphi(k2)]$. As usual $\vec{\sigma}$ represents the Pauli matrices vector $\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$. The subscripts $s1$ and $s2$ stand for the spin projections of the spinors and it is not hard to see that they can be put in a one-to-one relationship with the matrix elements of $i\vec{\sigma}\sigma_2$ or, for that matter, with any other such vector. Notice that the last two vertices which represent q-$\bar{q}$ pair creation or annihilation (the last vertex) are homogeneous functions of $\vec{\sigma}$. This fact alone will entail a derivative coupling of the type $\vec{\sigma}\cdot\mathbf{P}$ for the $N\pi(\mathbf{P})N$ coupling. For the propagators we have

$$\mathcal{S}_q(\mathbf{k},w) = \mathcal{S}_{\bar{q}}(\mathbf{k},w) = \frac{i}{w - E(k) + i\epsilon}. \quad (11)$$

Equipped with these rules we can now proceed to construct the Salpeter amplitudes for arbitrary hadronic processes. Before concluding this section we would like to make a few remarks concerning some aspects of the physical picture em-
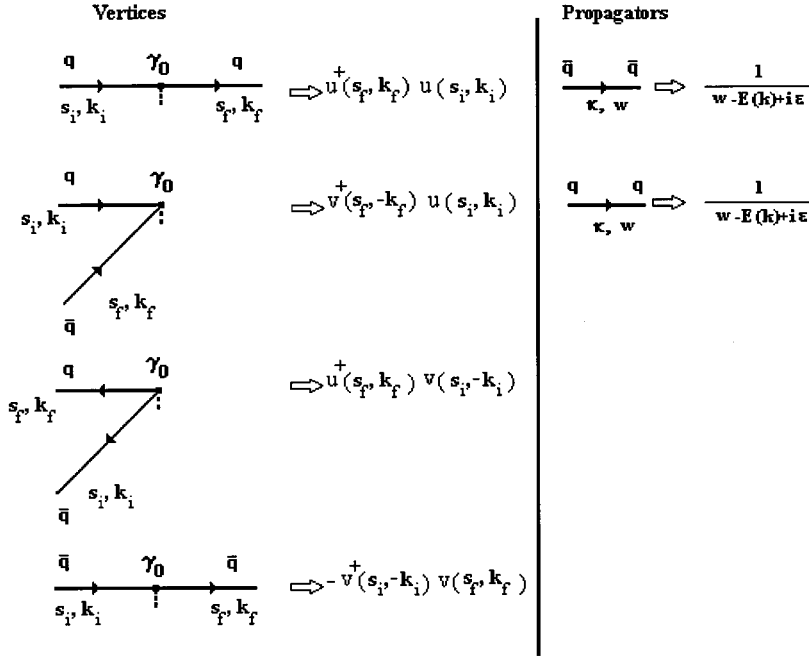
Vertices

FIG. 2. Feynman rules in the spin representation.

bodied in the Hamiltonian (2) and the Valatin-Bogoliubov transformed Dirac spinors of Eq. (7). In many respects it is a picture similar to the BCS theory of superconductivity. For details on how to construct a BCS-like vacuum as a coherent superposition of $^3P_0$ quark-antiquark pairs see Ref. [9]. Here as an illustrative example let us consider the quasiquark creation operator $b^\dagger(\mathbf{p})$ with definite quantum numbers in spin, flavor, and color. As in BCS theory, it represents in the usual Fock space (with a fermion empty vacuum) a superposition of a quark with those quantum numbers with a coherent state made of "Cooper-like" $^3P_0$, color singlet, quark-antiquark pairs. And because of the Pauli principle, this coherent state is made of all the possible $^3P_0$ quark pairs with the single exception of that $^3P_0$ pair which would contain a quark with the same quantum numbers. In the same manner a bound state (a meson or a baryon) will be a superposition of quarks (and/or antiquarks) with $^3P_0$ coherent states built in such a way as to satisfy the Pauli exclusion principle. The usefulness of the Valatin-Bogoliubov transformation stems from the fact that it allows us to "forget" the details of the physical vacuum and therefore to treat complicated quark bound states (which are quite different from ordinary pure bound states) as normal bound sates of quasiquarks with the information on the details of the physical vacuum stored inside the Dirac spinors in the form of a *chiral angle*. The Dirac structure of the quark spinors does not change this picture in any essential way provided we use Bethe-Salpeter equations to study these quasiquarks bound states. For simplicity, the word "quark" will be used in the remainder of this paper to mean a quasiquark.

III. PION, NUCLEON, AND DELTA SALPETER EQUATIONS

The hadronic Salpeter equations can be thought as the generating equations for the Dyson series of the correspondent hadronic T matrix (see Ref. [10]). In Fig. 3 we depict this correspondence for one single meson q-$\bar{q}$ bound state. The associated Salpeter equation for an arbitrary meson $\Phi=(\Phi^+,\Phi^-)$ with four-momentum $(E,\mathbf{P})$ is given by two coupled equations, one for the positive-energy component of the Salpeter amplitude Φ^+,

$$\Phi^+_{s1,s2}(\mathbf{k}-\mathbf{P})=\int\frac{d^3k'\,dw}{(2\pi)^4}\mathcal{S}_q\left(\mathbf{k}'+\frac{\mathbf{P}}{2},\frac{E}{2}+w\right)\mathcal{S}_{\bar{q}}\left(-\mathbf{k}'+\frac{\mathbf{P}}{2},\frac{E}{2}-w\right)[-iV(\mathbf{k}-\mathbf{k}')]$$

$$\times\left\{\left[u^+_{s1}\left(\mathbf{k}+\frac{\mathbf{P}}{2}\right)u_{s3}\left(\mathbf{k}'+\frac{\mathbf{P}}{2}\right)\right]\left[-v^+_{s4}\left(\mathbf{k}'-\frac{\mathbf{P}}{2}\right)v_{s2}\left(\mathbf{k}-\frac{\mathbf{P}}{2}\right)\right]\Phi^+_{s3,s4}(\mathbf{k}'-\mathbf{P})\right.$$

$$\left.-\left[u^+_{s1}\left(\mathbf{k}+\frac{\mathbf{P}}{2}\right)v_{s4}\left(\mathbf{k}'+\frac{\mathbf{P}}{2}\right)\right]\left[u^+_{s3}\left(\mathbf{k}'-\frac{\mathbf{P}}{2}\right)v_{s2}\left(\mathbf{k}-\frac{\mathbf{P}}{2}\right)\right]\Phi^-_{s3,s4}(\mathbf{k}'-\mathbf{P})\right\},\tag{12}$$
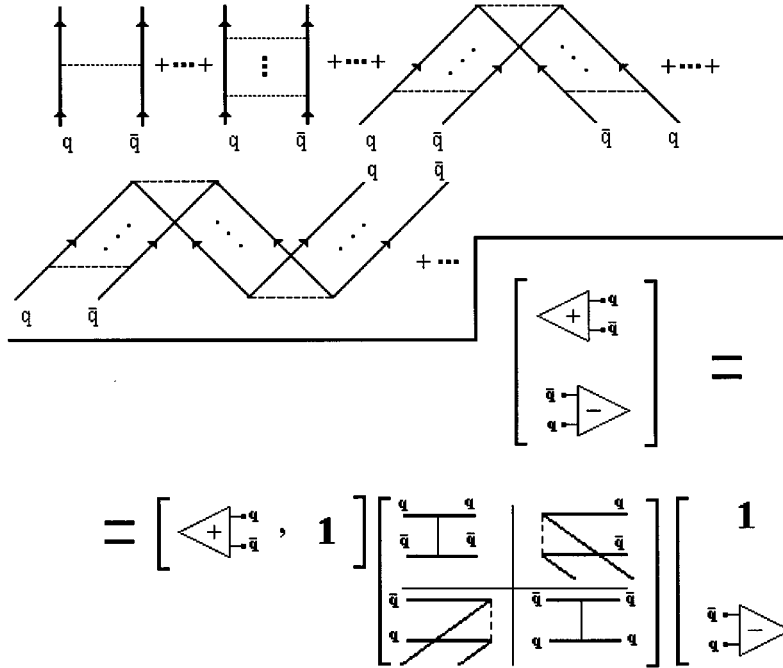
and a similar equation for the negative-energy component Φ^-,

FIG. 3. Dyson series for q-$\bar{q}$ bound states.

$$\Phi_{s1,s2}^-(\mathbf{k},\mathbf{P}) = \int \frac{d^3k'\,dw}{(2\pi)^4} \mathcal{S}_q\left(\mathbf{k}'+\frac{\mathbf{P}}{2},\frac{E}{2}+w\right)\mathcal{S}_{\bar{q}}\left(-\mathbf{k}'+\frac{\mathbf{P}}{2},\frac{E}{2}-w\right)[-iV(\mathbf{k}-\mathbf{k}')]$$

$$\times\left\{\left[u_{s3}^+\left(\mathbf{k}'+\frac{\mathbf{P}}{2}\right)u_{s1}\left(\mathbf{k}+\frac{\mathbf{P}}{2}\right)\right]\left[-v_{s2}^+\left(\mathbf{k}-\frac{\mathbf{P}}{2}\right)v_{s4}\left(\mathbf{k}'-\frac{\mathbf{P}}{2}\right)\right]\Phi_{s3,s4}^-(\mathbf{k}',\mathbf{P})\right.$$

$$\left.-\left[v_{s4}^+\left(\mathbf{k}'-\frac{\mathbf{P}}{2}\right)u_{s1}\left(\mathbf{k}+\frac{\mathbf{P}}{2}\right)\right]\left[v_{s2}^+\left(\mathbf{k}-\frac{\mathbf{P}}{2}\right)u_{s3}\left(\mathbf{k}'-\frac{\mathbf{P}}{2}\right)\right]\Phi_{s3,s4}^+(\mathbf{k}'-\mathbf{P})\right\}. \tag{13}$$

In what follows we denote by $\Phi(\mathbf{P})$ the energy-spin doublet $(\Phi^+(k,\mathbf{P}),\Phi^-(k,-\mathbf{P}))$, therefore omitting the q-$\bar{q}$ internal momentum k and the spins. We will also denote the Taylor series in $\mathbf{P}$ of a given function $F(\mathbf{k},\mathbf{P})$ by ΣF_n instead of the usual $(1/n!)\Sigma \mathbf{f}_n\cdot[\mathbf{P}]^n$, with $[\mathbf{P}]^n$ being a shorthand notation for a tensor in P of degree n.

Notice that in Eqs. (12) and (13) we can integrate out the quark and antiquark propagators,

$$\int \frac{dw}{2\pi} \mathcal{S}_q\left(\mathbf{k},\frac{\pm E}{2}+w\right)\mathcal{S}_{\bar{q}}\left(-\mathbf{k},\frac{\pm E}{2}-w\right) = \frac{i}{\pm E - E_q(\mathbf{k}) - E_{\bar{q}}(\mathbf{k})}, \tag{14}$$

therefore allowing us to rewrite Eqs. (12) and (13) as

$$[H(\mathbf{k},\mathbf{P})-E_{\text{meson}}(\mathbf{P})I]\begin{bmatrix}\Phi^+(\mathbf{k},\mathbf{P})\\\Phi^-(\mathbf{k},-\mathbf{P})\end{bmatrix} = 0, \quad I=\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, \tag{15}$$

were $H(\mathbf{k},\mathbf{P})$ is a 2×2 matrix,

$$H(\mathbf{k},\mathbf{P})=\begin{bmatrix}H^{++}(\mathbf{k},\mathbf{P}) & H^{+-}(\mathbf{k},\mathbf{P})\\H^{-+}(\mathbf{k},\mathbf{P}) & H^{--}(\mathbf{k},\mathbf{P})\end{bmatrix}, \tag{16}$$

which depends in the center-of-mass momentum P. The bare masses of mesons are just the eigenvalues $E_0=M$ of $H(0)=H_0$. We have, for $H^{(+;-)(+;-)}(\mathbf{k},\mathbf{P})$,

$$H^{++}=H^{--}=E(p1)+\frac{1}{4}\left[\left(\varphi'(p1)^2+\frac{4(1-S1)}{p1^2}\right)+\frac{1}{6}\frac{(1-S1)(1-S2)}{p1^2p2^2}\mathbf{p}1\cdot\mathbf{p}2\right](\vec{\sigma}_{\text{qk1}}\cdot\vec{\sigma}_{\text{qk2}})+\mathbf{p}1\rightarrow\mathbf{p}2+\text{h.r.t.},$$

$$H^{+-}=H^{-+}=-\frac{1}{12}\left[\left(\varphi'(p1)^2+\frac{C1}{p1}\right)\left(\varphi'(p2)^2+\frac{C2}{p2}\right)\frac{(\mathbf{p}1\cdot\mathbf{p}2)^2}{p1^2p2^2}-\frac{1}{3}\left(\varphi'(p1)+\frac{C1}{p1}\right)\frac{C2}{p2}+\frac{C1C2}{2p1p2}\right]$$

$$\times(\vec{\sigma}_{\mathrm{qk1}}\cdot\vec{\sigma}_{\mathrm{qk2}})+\mathbf{p}1\rightarrow\mathbf{p}2+\mathrm{h.r.t.},\tag{17}$$

with $\mathbf{p}1=\mathbf{k}+\mathbf{P}/2$ and $\mathbf{p}2=-\mathbf{k}+\mathbf{P}/2$ being, respectively, the momenta of the quark and the antiquark and $\mathbf{P}$ the momentum of the pion. "h.r.t." stands for higher rank tensors which will not contribute for the pion wave function and $S1$ represents $\sin[\varphi(p1)]$, $C2$ represents $\cos[\varphi(p2)]$, and so on. $\vec{\sigma}_{\mathrm{qk1}}$ and $\vec{\sigma}_{\mathrm{qk2}}$ stand for two Pauli matrices vectors acting, respectively, in quark 1 and antiquark 2. In the case of the Goldstone pion we have, for $[H_0-E_0]|\phi\rangle$,

$$\left\{\left(-\frac{d^2}{dk^2}+2E(k)\right)\begin{bmatrix}1&0\\0&1\end{bmatrix}+\left(\frac{\varphi'^2}{2}+\frac{\cos(\varphi)}{k^2}\right)\begin{bmatrix}1&1\\1&1\end{bmatrix}-M\begin{bmatrix}1&0\\0&-1\end{bmatrix}\right\}\begin{bmatrix}\nu^+(k)\\\nu^-(k)\end{bmatrix}=0,\tag{18}$$

where $\nu^\pm=k\Phi^\pm$. In the limit of vanishing current quark masses we have that $\Phi_0^\dagger I\Phi_0=0$. In this limit we also have that $M=0$ and that $\Phi_0^\dagger H_1\Phi_0=0$.

Therefore in order to find the energy and the norm of the Goldstone pion to order P, it turns out to be necessary to expand Eq. (15) to second order in the pion momentum $\vec{P}$. We obtain

$$(H_0-E_0I)\Phi_0=0,$$

$$(H_0-E_0I)\Phi_1+(H_1-E_1I)\Phi_0=0,$$

$$(H_0-E_0I)\Phi_2+2(H_1-E_1I)\Phi_1+(H_2-E_2I)\Phi_0=0.\tag{19}$$

Then we can obtain both the pion energy E_1, correct up to first order in P,

$$\Phi_0^\dagger(H_1-E_1I)\Phi_1+\Phi_0^\dagger H_2\Phi_0=0,\tag{20}$$

and the pion Salpeter wave function normalization which is given by

$$\mathcal{N}^2=\Phi_1^\dagger I\Phi_0+\Phi_0^\dagger I\Phi_1.\tag{21}$$

To obtain the desired pion Salpeter amplitude, we need to apply the above general formalism for our particular model. Using the Feynman rules defined in Sec. II, we are able to obtain the matrix elements of Eq. (16):

$$H_0^{++}=H_0^{--}=2E-\left\{\Delta-\frac{1}{2}\left(\varphi'^2+4\frac{1-S}{k^2}\right)-\frac{1}{3}\frac{(1-S)^2}{k^2}\right.$$

$$\left.\times\vec{\sigma}_{\mathrm{qk1}}\cdot\vec{\sigma}_{\mathrm{qk2}}^t+\mathrm{h.r.t.}\right\},$$

$$H_0^{+-}=H_0^{-+}=\frac{1}{3}\left(\frac{\varphi'^2}{2}+\frac{C^2}{k^2}\right)\vec{\sigma}_{\mathrm{qk1}}\cdot\vec{\sigma}_{\mathrm{qk2}}^t+\mathrm{h.r.t.},\tag{22}$$

$$H_1^{++}=H_1^{--}=-\left\{\frac{1}{4}\frac{(1-S)^2}{k^3}\mathbf{P}\cdot\hat{\mathbf{k}}\times(\vec{\sigma}_{\mathrm{qk1}}\times\vec{\sigma}_{\mathrm{qk2}}^t)\right.$$

$$+\frac{1}{2}\left(\frac{C\varphi'}{k}+2\frac{1-S}{k^2}\right)\mathbf{P}\cdot\hat{\mathbf{k}}[i\hat{\mathbf{k}}\cdot(\vec{\sigma}_{\mathrm{qk1}}-\vec{\sigma}_{\mathrm{qk2}}^t)]$$

$$\left.-\frac{1-S}{2k^2}\mathbf{P}\cdot i(\vec{\sigma}_{\mathrm{qk1}}-\vec{\sigma}_{\mathrm{qk2}}^t)\times\nabla_\mathbf{k}\right\},$$

$$H_1^{+-}=H_1^{-+}=\frac{1}{4k}\left(\varphi'+\frac{C}{k}\right)^2\mathbf{P}\cdot\hat{\mathbf{k}}\times(\vec{\sigma}_{\mathrm{qk1}}\times\vec{\sigma}_{\mathrm{qk2}}^t),$$

$$H_2^{++}=H_2^{--}=\sum\frac{d^2H^{++}}{(dP_i)^2}P_i^2\quad(i=1,2,3),$$

$$H_2^{+-}=H_2^{-+}=\sum\frac{d^2H^{+-}}{(dP_i)^2}P_i^2\quad(i=1,2,3).\tag{23}$$

As usual S stands for $\sin[\varphi(\mathbf{k})]$ and C for $\cos[\varphi(\mathbf{k})]$. k represents the $q\text{-}\bar{q}$ relative momentum and $\mathbf{P}$ the momentum of the pion. Finally we solve the above equations to obtain the boosted pion Salpeter wave function, correct up to first order in P,

$$\Phi^+=\mathcal{N}^{-1}(S+E_1f_1+ig_1\mathbf{P}\cdot\hat{\mathbf{k}}\times\vec{\sigma})\chi_\pi\mathcal{S}_{\mathrm{color}},$$

$$\Phi^-=\mathcal{N}^{-1}(-S+E_1f_1-ig_1\mathbf{P}\cdot\hat{\mathbf{k}}\times\vec{\sigma})\chi_\pi\mathcal{S}_{\mathrm{color}},$$

$$\mathcal{N}^2=4E_1\int\frac{d^3k}{(2\pi)^3}Sf_1=E_1a^2,$$

$$E_1=\frac{2P}{a}\sqrt{\int\frac{d^3k}{(2\pi)^3}S^2\sum\left(\frac{d^2H^{++}}{(dP_i)^2}-\frac{d^2H^{+-}}{(dP_i)^2}\right)}\quad(i=1,2,3).\tag{24}$$

In Eq. (24), χ_π and $\mathcal{S}$ describe, respectively, the spin-flavor pion function given in Eq. (C5) and the usual mesonic color singlet wave function. It is important to notice that the pion normalization goes as $\sqrt{(E_1)}$. This fact will be crucial to retrieve the well-known one-pion exchange potential (OPEP) in the static approximation—see Eq. (33). The constant a equals $0.16K_0$.

It is convenient to introduce the notation $\Phi^\pm=\mathcal{N}^{-1}(\phi_0^\pm+\phi_1^\pm)$ to be able to separate the contributions of the pion at rest and its associated boost correction to the $N\pi N$ coupling. From Eq. (24) we see that in the rest frame the pion Salpeter space-wave function ϕ_0 is simply given by the $\sin(\varphi)$ whereas $\phi_1=\mathcal{N}(E_1f_1+ig_1\mathbf{P}\cdot\hat{\mathbf{k}}\times\vec{\Sigma})$. f_1 and g_1 are the solutions of the differential equations

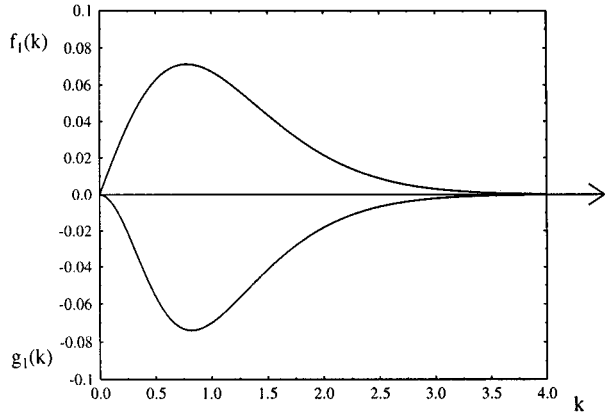FIG. 4. The f_1 and g_1 amplitudes contributing to the boosted pion.

$$\left[-\frac{d^2}{dk^2} + 2kC \right] f_1 = kS,$$

$$\left[-\frac{d^2}{dk^2} + 2kC + \frac{2S^2}{k^2} \right] g_1$$

$$= \frac{i}{2k^3} [2kC\varphi' + S(2S^2 - 2S - k^2\varphi'^2 - 4kC\varphi')].$$

$$(25)$$

The solutions f_1 and g_1 of Eq. (25) are given in Fig. 4. This result is the same as that obtained in Ref. [8], provided we perform the change

$$g_1 = -i \left(f_2 - \frac{\sin[\varphi(k)]}{2} \right). \qquad (26)$$

Surprisingly the baryon Salpeter equation turns out to be simpler than the corresponding pion Salpeter equation. This happens because in the baryon case, or indeed any other baryon, the associated Dyson series for the corresponding S matrix with an instantaneous interaction does not have negative-energy channels (which would correspond to an antibaryon component of the baryon propagator and therefore

of negligible importance for the nucleon ground state in sharp contrast with the pion case); nor does it have couplings to negative-energy quark-positive energy diquark or positive-energy quark-negative energy diquark channels due to color confinement. We also make the approximation that the baryon Salpeter wave function does not change for small boosts due to its heavy mass. In Fig. 5 we show the N associated Salpeter equation. In what follows we will use N to represent either the nucleon or the delta. When the need arises to distinguish the nucleon from the delta we will reserve n to denote the nucleon. The N Salpeter equation is then

$$[M - 3E(p1)]\chi_{s_1 s_2 s_5} \psi(\mathbf{p1,p2,p3})$$

$$= -2 \int d^3q V(\mathbf{q})$$

$$\times [u_{s_1}^\dagger(\mathbf{p1}) u_{s_3}(\mathbf{p1} - \mathbf{q}) u_{s_2}^\dagger(\mathbf{p2}) u_{s_4}(\mathbf{p2} + \mathbf{q})]$$

$$\times \chi_{s_3 s_4 s_5} \psi(\mathbf{p1} - \mathbf{q}, \mathbf{p2} + \mathbf{q}, \mathbf{p3}), \qquad (27)$$

with $\mathbf{p1} + \mathbf{p2} + \mathbf{p3} = 0$. This equation can be solved variationally,

$$\delta_\alpha[\langle N_\alpha | M - \mathcal{H} | N_\alpha \rangle] = 0, \qquad (28)$$

where $\mathcal{H}$ is a shorthand notation representing all the terms of Eq. (27) except for the eigenvalue M, and $\chi_{s_1 s_2 s_5}$ is given in Eqs. (C1) and (C4) in Appendix C.

For the momentum representation of the nucleon wave function $\langle \mathbf{p1,p2,p3} | N_\alpha \rangle$ we used

$$\langle \mathbf{p1,p2,p3} | N \rangle = \frac{e^{-(\rho^2 + \lambda^2)/2\alpha^2}}{\mathcal{N}},$$

$$\boldsymbol{\rho} = \frac{\mathbf{p1} - \mathbf{p2}}{\sqrt{2}},$$

$$\boldsymbol{\lambda} = \frac{\mathbf{p1} + \mathbf{p2} - 2\mathbf{p3}}{\sqrt{6}}, \qquad (29)$$
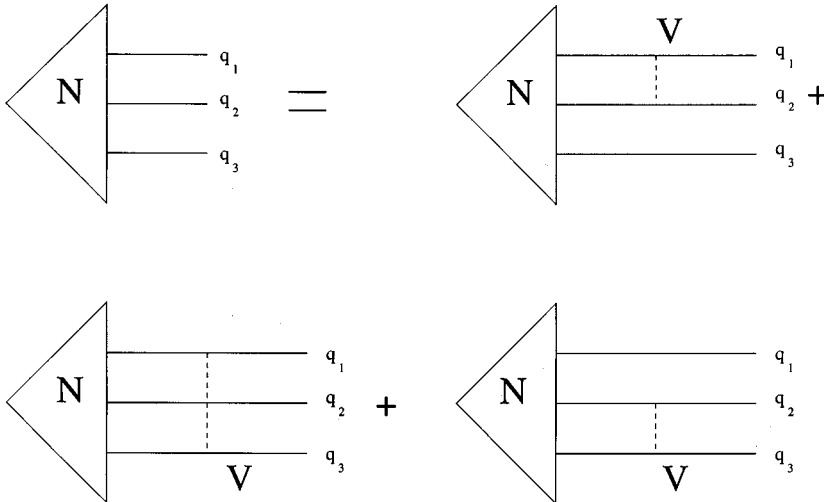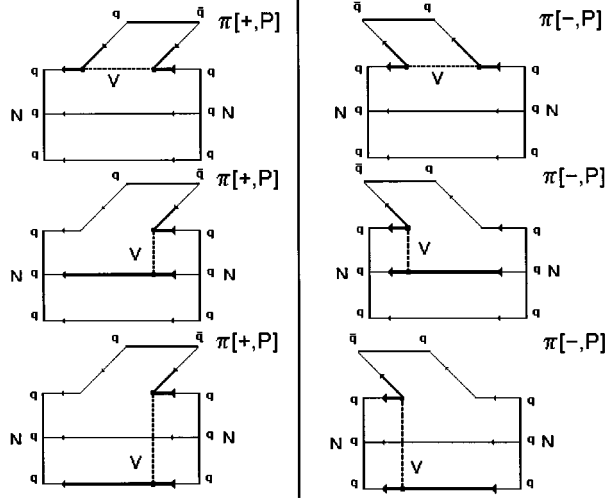

FIG. 5. The salpeter equation for the nucleon and the Δ.

FIG. 6. Diagrams contributing to the $N\pi N$ coupling.

with ρ and λ the appropriate Jacobian variables. α, the inverse baryon radius, turns out to be the same for the nucleon and the delta. The total n, Δ wave function Ψ_N is then given by

$$\Psi_N = \langle \mathbf{p}1, \mathbf{p}2, \mathbf{p}3 | N_a \rangle \langle \text{flavor} | N \rangle \langle \text{color} | N \rangle, \qquad (30)$$

where $\langle \text{flavor} | N \rangle$ stands for the appropriated spin-flavor content of either the nucleon or the delta [see Eqs. (C1)– (C4)] and $\langle \text{color} | N \rangle$ for the usual normalized color Slater determinant. The spin representation is then the natural representation for the reduction of the Salpeter equation to the Schrodinger-like equation (27).

The case of the linear confining potential has also been studied in Ref. [2] and for completeness we simply quote the final conclusions: We obtained for the same nucleon bare mass a larger nucleon core in the case of the linear confining potential than in the harmonic case. This result was then shown to be difficult to accommodate with the k-N exotic scattering which seems to favor a small nucleon core which is precisely what happened with the harmonic force.

IV. OVERLAP DIAGRAM FOR THE PION COUPLING

When studying meson-baryon scattering we need to consider nonexotic scattering amplitudes which are induced by quark-antiquark annihilation (or creation) amplitudes. The Feynman diagrams contributing to the $n\pi n$ and $\Delta\pi n$ vertices are given in Fig. 6. We need to evaluate

$$f_{N\pi N} = \frac{m_\pi}{a} \vec{v}_{N\pi N} \cdot \vec{\mathcal{T}}, \qquad (31)$$

where $\vec{v}_{N\pi N}$ and $\vec{\mathcal{T}}$ are defined in Appendixes C [Eq. (C16)] and A [Eqs. (A13) and (A15)], respectively. The constant a pertaining to the pion Salpeter normalization was introduced in Eq. (24). It is important to notice that the remainder of the pion normalization, $\mathcal{N}$, $\sqrt{E_1}$, is absorbed in the pion propagator

$$\mathcal{P} = \frac{i}{w - E_\pi}. \qquad (32)$$

We have that in this approximation $E_\pi = E_1$, and because in the OPEP we have two $N\pi N$ vertices, we get an overall energy dependence $1/E_1^2$ which is in accordance with the OPEP in the static approximation [17],

$$V_3^p = -\left(\frac{f_{N\pi N}}{m_\pi}\right)^2 (P^2 - m_\pi^2)^{-1} \vec{\tau}_1 \cdot \vec{\tau}_2 (\vec{\sigma}_{N1} \cdot \mathbf{P})(\vec{\sigma}_{N2} \cdot \mathbf{P}), \qquad (33)$$

where $\vec{\sigma}_{N1}$ represents the vector of Pauli matrices acting in nucleon $N1$ as a whole. The same applies to σ_{N2}.

$(\vec{v}_{n\pi n} \cdot \vec{\mathcal{T}})(\vec{\sigma}_n \cdot \mathbf{P})\tau_n$ and $(\vec{v}_{n\pi\Delta} \cdot \vec{\mathcal{T}})(\vec{S} \cdot \mathbf{P})\vec{T}$ are examples of the overlap $\mathcal{O} = \langle N | H_I | (\phi_0 + \phi_1)N \rangle$, where H_I is the Hamiltonian of Eq. (1). The only contribution to $\mathcal{O}$ is provided by the term of H_I with a single q-$\bar{q}$ annihilation—see Fig. 6. We can expand $\mathcal{O}$ up to first order in the pion momentum to obtain

$$\mathcal{O} = \langle N | h_0 + h_1 | (\phi_0 + \phi_1)N \rangle,$$

$$\mathcal{O} = \mathcal{O}_a + \mathcal{O}_b,$$

$$\mathcal{O}_a = \langle N | h_1 | \pi_0 N \rangle, \quad \mathcal{O}_b = \langle N | h_0 | \pi_1 N \rangle, \qquad (34)$$

where h stands for H_I. Notice that $\langle N | h_0 | \phi_0 N \rangle = 0$. That is, at rest the pion decouples from the nucleons. The reason for this is quite simple. From Eq. (10) the q-$\bar{q}$ annihilation vertex is of the form $\vec{\sigma} \cdot \vec{k}$, $\vec{k}$ being one of the momenta flowing in either the quark or the antiquark leg. In turn these momenta are sums of internal loop momenta $\mathbf{k}^i = \{\mathbf{k}, \mathbf{k}', \mathbf{k}''\}$ and the external momenta which in this case turns out to be $\mathbf{P}$, the pion momentum. Then upon integration in the internal momenta loops all the terms which are of the type $\vec{\sigma} \cdot \mathbf{k}^i$ will disappear while the terms homogeneous in $\vec{\sigma} \cdot \mathbf{P}$ are the only surviving terms. In the appendixes we derive $\mathcal{O}_a$ both for $n\pi n$ and $n\pi\Delta$. Because of the smallness of g_1, $\mathcal{O}_b$ turns out to be too small and it will be omitted henceforth. In Appendix A it is shown that in the case of $n\pi\Delta$ we have an extra contribution for $\mathcal{O}_a$ which is absent in the case of $n\pi n$. We will denote it by $\mathcal{O}'$. It will also turn out to be quite small.

We obtained the following values for the adimensional quantities $\mathcal{O}_{fs}$ and $\mathcal{O}'_{fs}$:

$$\mathcal{O}_{fs} = 0.54, \quad \mathcal{O}'_{fs} \simeq 0, \qquad (35)$$

where $\mathcal{O}_{fs}$ and $\mathcal{O}'_{fs}$ stand for $\mathcal{O}$ and $\mathcal{O}'$ amplitudes with the spin-flavor terms amputated—see Eqs. (A13) and (A15).

V. COMPARISON WITH EXPERIMENT AND DISCUSSION

The experimental values for $f_{n\pi n}$ and $f_{n\pi\Delta}$ are, respectively,

$$f_{n\pi n} = 1.0, \quad f_{n\pi\Delta} \simeq 2.1. \qquad (36)$$

We can use the results of Eq. (31) to write the set of equations

$$\frac{f_{n\pi n}}{m_\pi} = \frac{5}{9}\frac{\mathcal{O}_{fs}}{a},$$

$$\frac{f_{n\pi\Delta}}{m_\pi} = \frac{2\sqrt{2}}{3}\frac{\mathcal{O}_{fs}}{a} + \sqrt{2}\frac{\mathcal{O}'_{fs}}{a}. \tag{37}$$

Using the values of Eq. (35) for $\mathcal{O}$, $\mathcal{O}'$, and a we are able to get, for $f_{n\pi n}$ and $f_{n\pi\Delta}$,

$$f_{n\pi n} = 1.9\frac{m_\pi}{K_0}, \quad f_{n\pi\Delta} = 3.2\frac{m_\pi}{K_0}. \tag{38}$$

If we use the value of $K_0 = 247$ MeV of Ref. [8] we obtain the theoretical results

$$f_{n\pi n} = 1.0, \quad f_{n\pi\Delta} = 1.8. \tag{39}$$

These results are surprisingly good.

We have seen that in order to explain the K-n exotic phase shifts we needed a smaller bare nucleon and therefore a larger $K_0 \simeq 400$ MeV. With this K_0 we would get 60% of the values just obtained. However, we still have to consider the effects of Lorentz covariance and the contribution of the pion cloud around the bare nucleon. The study of these two effects will constitute a necessary step in this program of obtaining a quantitative microscopic description of low energy hadronic phenomena. Nevertheless, it is already remarkable that such a simple model (with only one scale K_0 apart from the quark masses) should yield results (ranging from hadronic spectroscopy to the coupling of pions to nucleons) which are not obviously wrong. This is more so if we take in consideration that this model is able to unify in the same description, essentially depending in the chiral angle φ, the (exotic) hadronic repulsion like, for instance, the nucleon-nucleon repulsive core (which is of the same nature than the k-n exotic repulsion) and the n-n peripheral attraction mediated by pions. Retrospectively it is not hard to understand why the "naive" $^3P_0$ model [quark pair creation model (QPCM) [18]] for strong decays should perform so well. It is the minimal model which contains overlap kernels and satisfies parity conservation. In this sense any microscopic model (like the present one) which produces a pion derivative coupling can be simulated by the QPCM. The pion momenta $\mathbf{P}$ which is present in the hadron-pion coupling $\vec{\sigma}\cdot\mathbf{P}$ and which forces a P-wave pion-hadron outgoing relative wave function can be made to have originated in an "incoming" cluster P-wave q-$\bar{q}$ bound state. Parity conservation will force it to be a $^3P_0$ q-$\bar{q}$ bound state. Its amplitude of occurrence, in the literature denoted by γ, can afterward be fitted to data. This is clearly seen if we use the graphical rules [19,2] to evaluate these overlap kernels. Of course with such a minimal model one loses any connection with spectroscopy (hadron bare masses) and the physics of chiral symmetry breaking $S\chi SB$.

The N-$\bar{N}$ scattering constitutes another area where the present model could and should be tested. At this stage we can already anticipate that the present model will again produce results which can be simulated by the QPCM. To see this it is sufficient to notice that from the point of view of overlap kernels and as a qualitative guide we can lump together either the two spectator $\bar{q}$ or the two spectator quarks as an effective extended quark or antiquark, respectively, and therefore understand the N-$\bar{N}$ scattering as a modified N-π scattering. The present calculation will therefore constitute a prerequisite to the calculation of the more complicated N-$\bar{N}$ scattering. The spectroscopy and scattering reactions for higher angular momenta will constitute another class of stringent tests notably in what concerns the old problem of van der Waals forces which we feel can only be realistically compared with experiment in the framework of covariant improvement of this model (retardation).

APPENDIX A: THE EVALUATION OF $\mathcal{O}$, AN EXAMPLE

First a note on notation. We denote by $(N\pi)_{ij}^+$ a N-π-N diagram for positive pion E energy with a potential insertion between quarks i and j. For the negative-E-energy pion, $N\pi N$ will be denoted by $(N\pi)_{ij}^-$. In this appendix we will evaluate in detail the diagrams $(N\pi)_{11}^+$ and $(N\pi)_{11}^-$. The other diagrams can be evaluated in a similar fashion and its derivation will be omitted. The Fourier transform of the potential $K_0^3 r^2$ is given by

$$V(\mathbf{k}) = -(2\pi)^3 K_0^3 \Delta_\mathbf{k}\delta(\mathbf{k}). \tag{A1}$$

The quark momenta we used are

$$\text{incoming Nuc. } \mathbf{p}1 = \mathbf{k}' + \mathbf{k}'' - \frac{\mathbf{P}}{2}, \quad \mathbf{p}2 = -\mathbf{k}' + \mathbf{k}'' - \frac{\mathbf{P}}{2}, \quad \mathbf{p}3 = -2\mathbf{k}'',$$

$$\text{outgoing Nuc. } \mathbf{q}1 = \mathbf{k}' + \mathbf{k}'' + \frac{\mathbf{P}}{2}, \quad \mathbf{q}2 = -\mathbf{k}' + \mathbf{k}'' + \frac{\mathbf{P}}{2}, \quad \mathbf{q}3 = -2\mathbf{k}'',$$

$$\text{pion } \mathbf{p}4 = -\mathbf{k} - \mathbf{k}'' + \frac{\mathbf{P}}{2}, \quad \mathbf{q}4 = \mathbf{k} + \mathbf{k}'' + \frac{\mathbf{P}}{2}. \tag{A2}$$

We also discard the terms depending on g_1 because they are negligible and therefore will not affect the final result.

Integrating by parts, we can get rid of the δ in the potential (A1) and we have only to consider the effect of the Laplacian and gradients on the vertices. We have the following cases to consider:

$$\mathcal{O}_{11}^{+}=\langle\lambda\cdot\lambda\rangle_{11}\int\frac{dk^3dk'^3dk''^3}{(2\pi)^9}(\{v_{s3}^{\dagger}(\mathbf{p}4)\Delta_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]+2v_{s3}^{\dagger}(\mathbf{p}4)\nabla_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]\cdot\nabla_{\mathbf{k}'}\}\delta_{s1''s1'}$$

$$+2\{v_{s3}^{\dagger}(\mathbf{p}4)\nabla_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]\}\cdot\{\nabla_{\mathbf{k}'}[u_{s1''}^{\dagger}(\mathbf{q}1)]u_{s1'}(\mathbf{q}4)\})\times\delta^3(\mathbf{k}-\mathbf{k}')[N(\mathbf{q}1,\mathbf{q}2,\mathbf{q}3)^{\dagger}\Phi N(\mathbf{p}1,\mathbf{p}2,\mathbf{p}3)],\qquad\text{(A3)}$$

with

$$v_{s3}^{\dagger}(\mathbf{p}4)\Delta_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]|_{k=k'}=\frac{1}{2}\left[\varphi''(p1)+\frac{2\varphi'(p1)}{p1}+\frac{2\cos[\varphi(p1)]}{p1^2}\right]\vec{\Sigma}_{s3s1}^{*}\cdot\hat{\mathbf{p}}1,$$

$$v_{s3}^{\dagger}(\mathbf{p}4)\nabla_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]|_{k=k'}=\frac{1}{2}\left[\varphi'(p1)+\frac{\cos[\varphi(p1)]}{p1}\right](\vec{\Sigma}_{s3s1}^{*}\cdot\hat{\mathbf{p}}1)\hat{\mathbf{p}}1-\frac{\cos[\varphi(p1)]}{p1}\vec{\Sigma}_{s3s1}^{*},$$

$$\nabla_{\mathbf{k}'}[u_{s1''}^{\dagger}(\mathbf{q}1)]u_{s1'}(\mathbf{q}4)|_{k=k'}=-\frac{i}{2}\frac{1-\sin[\varphi(q1)]}{q1}(\vec{\sigma}_{s1''s1'}\times\hat{\mathbf{q}}1).\qquad\text{(A4)}$$

$N(\mathbf{p}1,\mathbf{p}2,\mathbf{p}3)$ is given by Eqs. (29) and (30) and ϕ stands for the pion wave function of Eq. (24).

The overlap $\mathcal{O}_{11}^{-}$ is given by

$$\mathcal{O}_{11}^{-}=\langle\lambda\cdot\lambda\rangle_{11}\int\frac{dk^3dk'^3dk''^3}{(2\pi)^9}[(\{u_{s3}^{\dagger}(\mathbf{p}4)\Delta_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]+2u_{s3}^{\dagger}(\mathbf{p}4)\nabla_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]\cdot\nabla_{\mathbf{k}'}\}\delta_{s1''s1'}$$

$$+2\{u_{s3}^{\dagger}(\mathbf{p}4)\nabla_{\mathbf{k}'}[u_{s1}(\mathbf{p}1)]\}\cdot\{\nabla_{\mathbf{k}'}[u_{s1''}^{\dagger}(\mathbf{q}1)]v_{s1'}(\mathbf{q}4)\})\times\delta^3(\mathbf{k}-\mathbf{k}')[N(\mathbf{q}1,\mathbf{q}2,\mathbf{q}3)^{\dagger}\Phi N(\mathbf{p}1,\mathbf{p}2,\mathbf{p}3)].\qquad\text{(A5)}$$

For the Laplacian it is not hard to see that the contributions of $(N\pi)_{11}^{+}+(N\pi)_{12}^{+}+(N\pi)_{13}^{+}$ add to zero. The same happens for the negative-energy pion amplitude. This is due to the fact that

$$\lambda\cdot(\lambda_1+\lambda_2+\lambda_3)f(q)=0\qquad\text{(A6)}$$

for any color singlet nucleon $N(\mathbf{p}1,\mathbf{p}2,\mathbf{p}3)$. We also have with all generality that the contributions of $(N\pi)_{12}^{[+,-]}$ are identical to the contributions of $(N\pi)_{13}^{[+,-]}$. This is a consequence of the antisymmetric properties of the incoming and outcoming nucleon wave functions.

Using the results of Appendix B and after summing in the color degree of freedom we have the following

$$\text{Term(a)}:\{v_{s3}^{\dagger}(\mathbf{p}1)\nabla[u_{s1}(\mathbf{p}1)]\}\cdot\nabla(N\phi^{+}N),$$

terms$[+]$: $(N\pi)_{11}^{+}+(N\pi)_{12}^{+}+(N\pi)_{13}^{+}$

$$\times\left(-\frac{1}{\sqrt{3}}\right)\frac{1}{2}\left[\varphi'(p1)+\frac{\cos[\varphi(p1)]}{p1}\right](\Sigma_{s3s1}^{*}\cdot\hat{\mathbf{p}}1)\hat{\mathbf{p}}1-\frac{\cos[\varphi(p1)]}{p1}\vec{\Sigma}_{s3s1}^{*}\cdot[\nabla(N_{\text{out}})\Phi^{+}N_{\text{in}}].$$

terms$[-]$: $(N\pi)_{11}^{-}+(N\pi)_{12}^{-}+(N\pi)_{13}^{-}$

$$\times\left(-\frac{1}{\sqrt{3}}\right)\frac{1}{2}\left[\varphi'(q1)+\frac{\cos[\varphi(q1)]}{q1}\right](\vec{\Sigma}_{s1's1''}\cdot\hat{\mathbf{q}}1)\hat{\mathbf{q}}1-\frac{\cos[\varphi(q1)]}{q1}\vec{\Sigma}_{s1's1''}\cdot[N_{\text{out}}\Phi^{-}\nabla N_{\text{in}}].$$

$$\text{Term(b)}:2\{v_{s3}^{\dagger}(\mathbf{p}1)\nabla_{\mathbf{k}'}[u_{s1}(\mathbf{p}')]\}\cdot\{\nabla_{\mathbf{k}'}[u_{s1''}(\mathbf{p}')]u_{s1'}(\mathbf{p}1)\},$$

terms$[+]$: $(N\pi)_{11}^{+}$

$$\times 2\left[-\frac{i}{2}\frac{1-\sin[\varphi(q1)]}{q1}\right]\left\{\frac{1}{2}\varphi'(p1)+\frac{\cos[\varphi(p1)]}{p1}(\vec{\Sigma}_{s3s1}^{*}\times\vec{\sigma}_{s1''s1'})\cdot\left[\frac{-1}{2}\hat{\mathbf{p}}1\times(\hat{\mathbf{p}}1\times\hat{\mathbf{q}}1)\right]-\frac{\cos[\varphi(p1)]}{p1}\right.$$

$$\left.\times(\vec{\Sigma}_{s3s1}^{*}\times\vec{\sigma}_{s1''s1'})\cdot\hat{\mathbf{q}}1\right\}\left(-\frac{1}{\sqrt{3}}\right).$$

terms$[-]$: $(N\pi)^-_{11}$

$$\times 2\left[+\frac{i}{2}\frac{1-\sin[\varphi(p1)]}{p1}\right]\left\{\frac{1}{2}\varphi'(q1)+\frac{\cos[\varphi(q1)]}{q1}(\vec{\Sigma}_{s1''s1'}\times\vec{\sigma}_{s3s1})\cdot\left[\frac{-1}{2}\hat{\mathbf{q}}1\times(\hat{\mathbf{q}}1\times\hat{\mathbf{p}}1)\right]-\frac{\cos[\varphi(q1)]}{q1}\right.$$

$$\left.\times(\vec{\Sigma}_{s1''s1'}\times\vec{\sigma}_{s3s1})\cdot\hat{\mathbf{p}}1\right\}\left(-\frac{1}{\sqrt{3}}\right). \tag{A7}$$

In term (b) the contributions of $(n\pi)_{12}$ and $(n\pi)_{13}$ are zero. This is not so in the case of the $n\pi\Delta$ coupling where we will have an extra contribution coming from the sum of the diagrams $(N\pi)^-_{12}$ and $(N\pi)^+_{12}$ which survive the cancellation mechanism of Eq. (A6) and which will contribute along the same lines as before to a new overlap $\mathcal{O}'$,

term (b' +),

$$\left[-\frac{1}{2}\left(\varphi'(p1)+\frac{C1}{p1}\right)\hat{\mathbf{p}}1\times(\hat{\mathbf{p}}1\times\hat{\mathbf{p}}2)-\frac{C1}{p1}\hat{\mathbf{p}}2\right]\left[\frac{1-S2}{2p2}\right]\cdot(i\vec{\sigma}_{qk1}\times\vec{\sigma}_{qk2})\left(-\frac{1}{\sqrt{3}}\right),$$

term (b' −),

$$\left[-\frac{1}{2}\left(\varphi'(q1)+\frac{C1}{q1}\right)\hat{\mathbf{q}}1\times(\hat{\mathbf{q}}1\times\hat{\mathbf{q}}2)-\frac{C1}{q1}\hat{\mathbf{q}}2\right]\left[\frac{1-S2}{2q2}\right]\cdot(i\vec{\sigma}_{qk1}\times\vec{\sigma}_{qk2})\left(-\frac{1}{\sqrt{3}}\right), \tag{A8}$$

where $C1$ and $S2$ stand, respectively, for $\cos[\varphi(p1)]$ and $\sin(\varphi(p2))$ for term (b' +) and $\cos[\varphi(q1)]$ and $\sin[\varphi(q2)]$ for term (b' −). In expression (A8) as well as in the remainder of this paper σ_{qk1} represents the Pauli matrix operator acting in quark 1. A similar notation is used for quark 2.

In order to obtain the results of Eqs. (A7) and (A8) we have made use of the identity

$$(\vec{\Sigma}^*_{s3s1}\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot[\vec{\sigma}_{s1''s1'}\times\hat{\mathbf{q}}])=-\frac{1}{2}(\vec{\Sigma}^*_{s3s1}\times\vec{\sigma}_{s1''s1'}\cdot(\hat{\mathbf{p}}\times[\hat{\mathbf{p}}\times\hat{\mathbf{q}}]), \tag{A9}$$

which is valid both for the nucleon and delta cases because the matrix elements of the tensor operator $\langle N|T_2(\vec{\sigma}_1,\vec{\sigma}_2)\cdot T_2(\mathbf{p}1,\mathbf{p}2)|nN\rangle$ are zero.

Finally we have also to consider the spin wave function of the pion, $i[\sigma_2]_{s1',s3}/\sqrt{2}$, and using the simple relations

$$(i\sigma_2)\vec{\Sigma}^*=\vec{\sigma},$$

$$\vec{\Sigma}(i\sigma_2)=-\vec{\sigma}, \tag{A10}$$

we are able to write

term(a +),

$$\left\{\varphi'(p1)\vec{\sigma}\cdot\nabla N_{\text{out}}+\left(\varphi'(p1)+\frac{\cos[\varphi(p1)]}{p1}\right)\vec{\sigma}\cdot\hat{\mathbf{p}}1\times(\hat{\mathbf{p}}1\times\nabla N_{\text{out}})\right\}\Phi^+_s N_{\text{in}},$$

term(a−),

$$N_{\text{out}}\Phi^-_s\left\{\varphi'(q1)\vec{\sigma}\cdot\nabla N_{\text{out}}+\left(\varphi'(q1)+\frac{\cos[\varphi(q1)]}{q1}\right)\vec{\sigma}\cdot\hat{\mathbf{q}}1\times(\hat{\mathbf{q}}1\times\nabla N_{\text{in}})\right\},$$

term (b+),

$$\frac{1-\sin[\varphi(q1)]}{2q1}\left\{2\frac{\cos[\varphi(p1)]}{p1}\vec{\sigma}\cdot\hat{\mathbf{q}}1+\left(\varphi'(p1)+\frac{\cos[\varphi(p1)]}{p1}\right)\vec{\sigma}\cdot\hat{\mathbf{p}}1\times(\hat{\mathbf{p}}1\times\hat{\mathbf{q}}1)\right\}N_{\text{out}}\Phi^+_s N_{\text{in}},$$

term(b−),

$$-\frac{1-\sin[\varphi(p1)]}{2p1}\left\{2\frac{\cos[\varphi(q1)]}{q1}\vec{\sigma}\cdot\hat{\mathbf{p}}1+\left(\varphi'(q1)+\frac{\cos[\varphi(q1)]}{q1}\right)\vec{\sigma}\cdot\hat{\mathbf{q}}1\times(\hat{\mathbf{q}}1\times\hat{\mathbf{p}}1)\right\}N_{\text{out}}\Phi^+_s N_{\text{in}}, \tag{A11}$$

where Φ_s represents the pion Salpeter amplitude amputated of its spin wave function factor and we have omitted the factor $(-1/\sqrt{3})$ of the color overlap.

At this stage it is convenient to introduce the vector $\vec{v}_{N-\pi-N}$, summarizing the spin-flavor overlap both for the $n\pi n$ and $n\pi\Delta$ cases obtained in Appendix C [Eq. (C16)],

$$\vec{v}_{n\pi n} = \tfrac{1}{2}\left\{ \tfrac{5}{9}, 0 \right\},$$

$$\vec{v}_{n\pi\Delta} = \frac{1}{2}\left\{ \frac{2\sqrt{2}}{3}, \sqrt{2} \right\}. \tag{A12}$$

The factor of $\tfrac{1}{2}$ stands for the spin-flavor overall normalization of the pion wave function. The next step is to expand $\mathcal{O}$ and $\mathcal{O}'$ up to first order in P and to integrate both in $\mathbf{k}''$ and in the solid angle of $\mathbf{k}$. It is also convenient to introduce the vector $\vec{\mathcal{T}} = \{\mathcal{O}_{fs}, \mathcal{O}'_{fs}\}$, which correspond to the generic $N\pi N$ overlaps with both the spin flavor and $\vec{\sigma}_n \cdot \mathbf{P}\vec{\tau}_N$ (for the $n\pi n$ case) or $\vec{S}\cdot\mathbf{P}\vec{T}$ (for the $n\pi\Delta$ case) factors amputated. We have

$$\mathcal{O}_{fs} = \sqrt{3}\frac{\int k^2 dk[\mathcal{C}\sin(\varphi) + \mathcal{G}]e^{-3k^2/2\alpha^2}}{\int k^2 dk\, e^{-3k^2/2\alpha^2}},$$

$$\mathcal{C} = \left\{ -\frac{2}{\alpha^2}\left[\frac{\varphi'}{6} - \frac{k^2\sin(\varphi)}{2} + \frac{3\sin(\varphi)\cos(\varphi) - 5\cos(\varphi)}{6k}\right] \right.$$

$$\left. - \frac{[1 - \sin(\varphi)]\varphi'}{k^2} \right\},$$

$$\mathcal{G} = \left\{ -\frac{2}{\alpha^2} \right\}\frac{k\varphi'}{2}g_1, \tag{A13}$$

and we use the approximate relations

$$\frac{\cos\varphi}{k} \simeq \frac{1 - \sin\varphi}{k^2} \simeq 2.05 e^{-2k^2/\alpha^2}, \tag{A14}$$

valid for the chiral angle solution of the mass gap equation (8) depicted in Fig. 1 to obtain

$$\mathcal{O}'_{fs} = \sqrt{3}\frac{\int k^2 dk\{\mathcal{D}\sin[\varphi(k)]\}e^{-3k^2/2\alpha^2}}{\int k^2 dk\, e^{-3k^2/2\alpha^2}},$$

$$\mathcal{D} \simeq -\frac{1.4}{2\sqrt{2}}\left[\frac{1}{2} + \frac{5k^2}{4\alpha^2}\right]e^{-9k^2/4\alpha^2}, \tag{A15}$$

so that the final overlap is given by

$$f_{N\pi N} = \frac{m_\pi}{a}\vec{v}_{N\pi N}\cdot\vec{\mathcal{T}}. \tag{A16}$$

APPENDIX B: COLOR OVERLAPS

When performing color calculations we have to attend to the following rules:

(I) one quark,

$$\frac{\lambda}{2}\frac{\lambda}{2} = \frac{4}{3},$$

(II) one nucleon $N(1,2,3)$,

$$(\lambda_1 + \lambda_2 + \lambda_3)^2 = 0 \rightarrow \left(\frac{\lambda}{2}\right)_i\left(\frac{\lambda}{2}\right)_j = -\frac{2}{3}\bigg|_{i,j=1,2,3},$$

(III) one meson $Mq-\bar{q}$,

$$\frac{\lambda_q}{2}\frac{\lambda_{\bar{q}}}{2} = -\frac{4}{3},$$

(IV) quark λ $\overline{\text{quark}}$ vertex,

$$\bar{q}_\alpha\lambda q_\beta = \bar{q}_\alpha(q_\alpha\lambda q_\beta) = -(\bar{q}_\alpha\lambda\bar{q}_\beta)q_\beta,$$

(V) quark exchange,

$$\langle N(1,2,3)M(4,\bar{5})|\mathcal{P}^{i4}|N(1,2,3)M(4,\bar{5})\rangle = \tfrac{1}{3}, \tag{B1}$$

so that we have

$$\langle N|\left(-\frac{3}{4}\right)\frac{\lambda}{2}\frac{\lambda}{2}|N\rangle = \frac{1}{2},$$

$$\langle M|\left(-\frac{3}{4}\right)\frac{\lambda}{2}\frac{\lambda}{2}|M\rangle = 1. \tag{B2}$$

In the table below we summarize the color matrix elements for the N-π-N coupling $\mathcal{O}$.

Diagram	$-\dfrac{3}{4}\dfrac{\lambda}{2}\dfrac{\lambda}{2}$	Diagram	$-\dfrac{3}{4}\dfrac{\lambda}{2}\dfrac{\lambda}{2}$
$(N\pi)_{11}^+$	$\dfrac{-1}{\sqrt{3}}$	$(N\pi)_{11}^-$	$\dfrac{-1}{\sqrt{3}}$
$(N\pi)_{12}^+$	$\dfrac{1}{2\sqrt{3}}$	$(N\pi)_{12}^-$	$\dfrac{1}{2\sqrt{3}}$
$(N\pi)_{13}^+$	$\dfrac{1}{2\sqrt{3}}$	$(N\pi)_{13}^-$	$\dfrac{1}{2\sqrt{3}}$

APPENDIX C: SPIN AND FLAVOR OVERLAPS

The spin-flavor content of the nucleon and delta wave functions is given by

$$|n\rangle = \frac{1}{\sqrt{2}}|D_{\text{spin}}D_{\text{flavor}} + F_{\text{spin}}F_{\text{flavor}}\rangle. \tag{C1}$$

It is sufficient to workout the spin overlaps being the flavor overlaps identical. The spin D wave functions are given by

$$|n\uparrow\rangle = \sqrt{\tfrac{2}{3}}|1\ 1, \tfrac{1}{2}-\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}|1\ 0, \tfrac{1}{2}\tfrac{1}{2}\rangle,$$

$$|n\downarrow\rangle = -\sqrt{\tfrac{2}{3}}|1-1, \tfrac{1}{2}\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}|1\ 0, \tfrac{1}{2}-\tfrac{1}{2}\rangle, \tag{C2}$$

where the first two numbers of each ket represent the spin content of the quark pair 1-2. Similarly we have, for the F wave functions,

$$|n\uparrow\rangle = |0\ 0, \tfrac{1}{2}\ \tfrac{1}{2}\rangle,$$

$$|n\downarrow\rangle = |0\ 0, \tfrac{1}{2}\ -\tfrac{1}{2}\rangle. \tag{C3}$$

For the delta we have

$$|\Delta\rangle = |\tfrac{3}{2}\ s\rangle_{\text{spin}} \otimes |\tfrac{3}{2}f\rangle_{\text{flavor}}. \tag{C4}$$

The pion spin-flavor wave function is given by

$$\chi_\pi = \left\{ \frac{i}{\sqrt{2}}\sigma_2 \right\}_{\text{spin}} \otimes \left\{ \frac{1}{\sqrt{2}}\sigma_3, \sigma_\pm \right\}_{\text{flavor}}, \tag{C5}$$

where σ_i stands for the appropriate Pauli matrix. $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$. From now on we will follow the usual notation in what concerns isospin and denote $\sigma_{\text{flavor}} = \tau$.

For the $N\pi N$ overlaps we need to consider the expectation value of the operators σ_{qk1} and $\sigma_{qk1} \otimes \sigma_{qk2}$. We calculate a few examples to illustrate the general method of calculating these operator matrix elements. The other cases are obtained in the same manner. A general matrix element looks like

$$\langle B^\dagger | \vec{\sigma}_{qk1} \otimes \vec{\sigma}_{qk2} | A \rangle = \text{trac}\{B^\dagger \vec{\sigma}_{qk1} \otimes A \vec{\sigma}_{qk2}^t\}, \tag{C6}$$

where the generic form for the spin wave functions B and A of two quarks is given by

$$A = (a_0 + \vec{a}_1 \cdot \vec{\sigma})(i\sigma_2), \quad B = (b_0 + \vec{b}_1 \cdot \vec{\sigma})(i\sigma_2). \tag{C7}$$

The pion spin wave function is, for instance, $a_0(i\sigma_2)$ whereas the vector ρ is given by $(\vec{b}_1 \cdot \vec{\sigma})(i\sigma_2)$. It is also convenient to write the spin-flavor content of the nucleon wave function as

$$(n\uparrow, n\downarrow)_{\text{spin nuc}} = (\uparrow, \downarrow)_{qk3}$$

$$\times \left(\frac{1}{\sqrt{2}}\frac{F_f}{\sqrt{2}} 1_{(qk3,\text{spin nuc})} \right.$$

$$\left. -\frac{1}{\sqrt{6}}\frac{D_f}{\sqrt{2}}\sigma_{(qk3,\text{spin nuc})} \right), \tag{C8}$$

which maps the spin content of the nucleon to the spin of the third quark. Next we use the identities

$$\text{trac}\{\sigma_i\sigma_j\} = 2\delta_{ij},$$

$$\text{trac}\{\sigma_i\sigma_b\sigma_j\} = 2i\epsilon_{ibj},$$

$$\text{trac}\{\sigma_i\sigma_b\sigma_j\sigma_a\} = 2(\delta_{ib}\delta_{ja} - \delta_{ij}\delta_{ba} + \delta_{ia}\delta_{jb}), \tag{C9}$$

to obtain

$$\langle B^\dagger | \sigma_{qk1_i}\sigma_{qk2_j} | A \rangle = -2b_0^\dagger a_0 \delta_{ij} + 2ib_0^\dagger a_{1_k}\epsilon_{ijk}$$

$$-2ia_0 b_{1_k}^\dagger \epsilon_{ijk} - 2b_{1_i}^\dagger a_{1_j} - 2b_{1_j}^\dagger a_{1_i}$$

$$+2\vec{b}_1 \cdot \vec{a}_1 \delta_{ij}, \tag{C10}$$

We have to consider three cases of tensors,

$$\langle B^\dagger | \vec{\sigma}_{qk1} \cdot \vec{\sigma}_{qk2} | A \rangle = -6b_0^\dagger a_0 + 2\vec{b}_1 \cdot \vec{a}_1,$$

$$\langle B^\dagger | i\vec{\sigma}_{qk1} \times \vec{\sigma}_{qk2} | A \rangle = -4(b_0^\dagger \vec{a}_1 - a_0 \vec{b}_1^\dagger). \tag{C11}$$

Then it is simple to evaluate, for instance,

$$\langle F_s | \vec{\sigma}_{qk1} | D_{s'} \rangle = 2\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\vec{\sigma}_{s,s'},$$

$$\langle D_s | \vec{\sigma}_{qk1} | D_{s'} \rangle = -2i\frac{-1}{\sqrt{6}}\frac{-1}{\sqrt{6}}(\vec{\sigma}_{qk1} \times \vec{\sigma}_{qk2})$$

$$= \frac{2}{3}\sigma_{s,s'}. \tag{C12}$$

Similar calculations for the other matrix elements so that for the $n\pi n$ overlap we have the spin matrix elements

	$\langle DD \rangle$	$\langle FD \rangle$	$\langle DF \rangle$	$\langle FF \rangle$
σ_{qk1}	$\frac{2}{3}\vec{\sigma}_n$	0	$-\frac{1}{\sqrt{3}}\vec{\sigma}_n$	$-\frac{1}{\sqrt{3}}\vec{\sigma}_n$
$i\vec{\sigma}_{qk1} \times \vec{\sigma}_{qk2}$	0	0	$-\frac{2}{\sqrt{3}}\vec{\sigma}_n$	$\frac{2}{\sqrt{3}}\vec{\sigma}_n$

where σ_n represents the Pauli matrix acting in spin of the nucleon as a whole. We can repeat this calculations in the flavor space, using the appropriate flavor representation for the pion wave function given in Eq. (C5) so that we have, for the nn spin-flavor overlap,

$$\langle n | \{\vec{\sigma}_{qk1}, i\vec{\sigma}_{qk1} \times \vec{\sigma}_{qk2}\} \otimes \vec{\tau}_{qk1} | n \rangle = \{\tfrac{5}{9}\vec{\sigma}_n \otimes \vec{\tau}_n, 0\}. \tag{C13}$$

For the $n\pi\Delta$ overlap we will need the spin matrix elements

	$\langle \Delta D \rangle$	$\langle \Delta F \rangle$
$\vec{\sigma}_{qk1}$	$\frac{1}{\sqrt{3}}\vec{S}$	$\vec{S}$
$i\vec{\sigma}_{qk1} \times \vec{\sigma}_{qk2}$	0	0

Again we repeat similar calculations, this time in the flavor space, so that we have, for the Δn spin-flavor overlap,

$$\langle \Delta | \{\vec{\sigma}_{qk1}, i\vec{\sigma}_{qk1} \times \vec{\sigma}_{qk2}\} \otimes \vec{\tau}_{qk1} | n \rangle = \left\{ \frac{2\sqrt{2}}{3}\vec{S} \otimes \vec{T}, \sqrt{2}\vec{S} \otimes \vec{T} \right\}, \tag{C14}$$

where $\vec{S}$ and $\vec{T}$ are given in Ref. [17] and are defined by

$$\langle \tfrac{3}{2} \, \nu_\Delta | (S_\lambda^\dagger, T_\lambda^\dagger) | \tfrac{1}{2} \, \nu_N \rangle = \langle \tfrac{3}{2} \, \nu_\Delta | 1 \, \lambda \, \tfrac{1}{2} \, \nu_N \rangle. \qquad (C15)$$

Finally from Eqs. (C13) and (C14) we can construct the vectors $\vec{v}_{N\pi N}$,

$$\vec{v}_{n\text{-}\pi\text{-}n} = \tfrac{1}{2} \left\{ \tfrac{5}{9}, 0 \right\},$$

$$\vec{v}_{n\pi\Delta} = \frac{1}{2} \left\{ \frac{2\sqrt{2}}{3}, \sqrt{2} \right\}. \qquad (C16)$$

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