Preferred modes of decay in nuclear fragmentation

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(Received 12 August 1996)

Recent experimental studies show a characteristic energy dependence of the different fragmentation modes in heavy-ion reactions at intermediate energies. In this work we study this dependence and find that, in multifragmentation, just like in low-energy fission, some modes of decay are more probable than the rest. We argue that these high-probability mass partitions are ultimately responsible for the observed energy dependence. [S0556-2813(97)01602-6]

PACS number(s): 25.70.Pq, 24.10.−i

I. INTRODUCTION

Recent studies show that the number of fragments produced in heavy-ion reactions at intermediate energies has a very peculiar dependence on the energy used to produce the breakup. The characteristic energy dependence was first observed experimentally in ^{197}Au+^{27}Al, ^{51}V, and $^{\text{nat}}$Cu at 60 MeV/nucleon by Moretto and co-workers [1], and in ^{16}O+^{197}Au at 50 MeV/nucleon by Pouliot and co-workers [2].

The observed relationship is based on the number of times a given reaction (at a given excitation energy E) produces N intermediate-mass fragments compared to the number of times it produces only two fragments. That is, denoting by P_N the probability for producing N fragments, the logarithm of the ratio of P_N to P_2 has a strong linear dependence on $1/\sqrt{E}$, with a negative slope.

Under the assumption of a simultaneous production of fragments, Moretto and co-workers argued that the decays of a hot nuclear system into several pieces must be governed by the barriers generated by the fragments. They initially took these barriers as independent of the mass partition [1]. Using $B_2,\ldots,B_N$ to denote the ''average barriers'' associated with binary and higher decay modes, they took the decay probability for each channel as proportional to the level density of the system, ρ, at an excitation energy equal to the available energy minus the barrier, i.e., $P_N(E)\propto\rho(E-B_N)$. Then, using a Fermi-gas level density, and in the limit $E\gg B_N$, they obtained

$$P_N(E)\propto e^{2\sqrt{a(E-B_N)}}\approx e^{2\sqrt{aE}}e^{-B_N\sqrt{a/E}},$$

where a is the level density parameter. Then the ratio of the N-fragment events to the binary events yields

$$\ln\{P_N/P_2\}\propto-\sqrt{a/E}(B_N-B_2),$$

in excellent agreement with the observed linear dependence of $\ln\{P_N/P_2\}$ on $E^{-1/2}$.

[A second decay possibility treated in [1] was a sequence of binary decays. In this case the authors again assumed the existence of mass-independent generic barriers $b_1,\ldots,b_m$ for the successive splits. Using similar arguments as before, the probability to obtain N fragments in a sequence of decays is then

$$P_N(E)\propto k(N)e^{-b_1/T_1}e^{-b_2/T_2}\ldots\approx k(N)e^{-B_N/T}=k(N)e^{-B_N\sqrt{a/E}},\quad(1)$$

where $B_N=b_1+b_2+\cdots$, T is the temperature (assuming $T\approx T_1\approx T_2\cdots$), and $k(N)$ is a combinatorial factor. Again a linear dependence of $\ln\{P_N/P_2\}$ on $E^{-1/2}$ is obtained for a sequential decay.]

In spite of the nice agreement with the data, it is easy to understand that the simplifying assumptions made in deriving these results are not correct. Binary fission barriers, for instance, vary strongly with the decay channel (i.e., with the mass partition), and multifragment-fission barriers depend even on the relative position of the fragments (i.e., spatial configuration of the fragments) [3,4]. Likewise, recent results [3,4] contradict the assumption that the preexponential factor $k(N)$ of Eq. (1) does not depend on the mass partition.

In a later work [5] the authors addressed this point by introducing a distribution of barriers in the binary decays to average the functional form of $P_N(E)$ over the mass asymmetry. Although the straight-line dependence was again recovered, the parametrization used for the mass dependence of the barrier was not justified and cannot be generalized for multifragment fissions.

The question still remains: How can one reconcile the straight-line dependence of $\ln\{P_N/P_2\}$ on $E^{-1/2}$ with the varying barriers? It is this intriguing question that motivates the present study. A correct analysis of the fragmentation process should include the appropriate barriers for the different decays. Of the models treating simultaneous multifragmentation, only the *transition state theory* (TST) of nuclear fragmentation [3,4] takes into account the interfragment barriers as well as the relative position of the fragments during the breakup stage. Here we use the TST to calculate the multifragment fission widths and study their energy dependence. We start with a brief description of the technique.

II. TRANSITION STATE THEORY FOR FRAGMENTATION

The transition state method which deals with binary fission was crafted by Bohr and Wheeler over 50 years ago [6]. For the case of simultaneous multifragment fission, the approach was generalized by López and Randrup [3,4]. (See

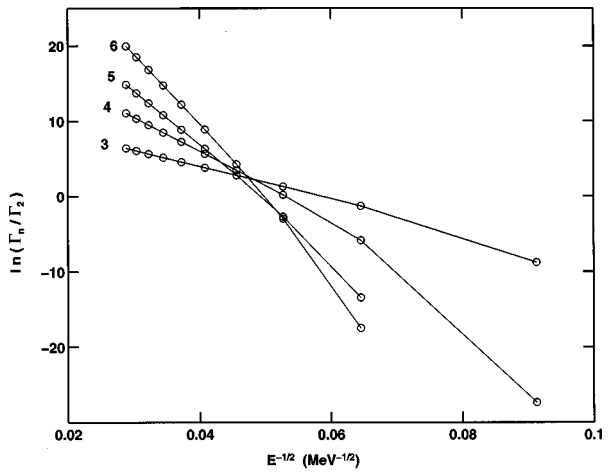

FIG. 1. Energy dependence of $\ln\{\Gamma_N/\Gamma_2\}$ for the decay of ^{120}Sn into several multiplicities. The circles show the calculated points and the numbers label the multiplicity N.

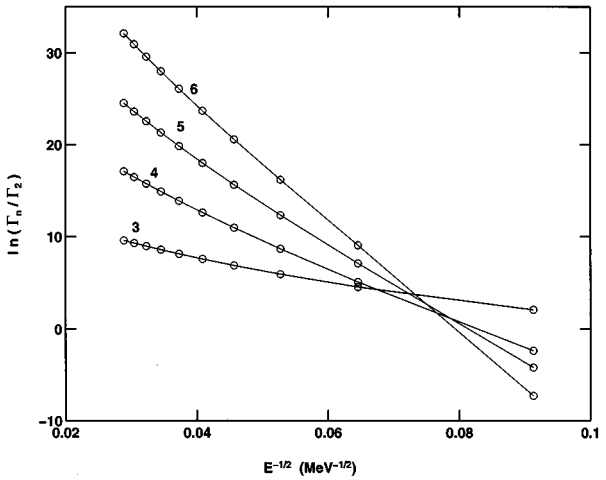

FIG. 2. Energy dependence of $\ln\{\Gamma_N/\Gamma_2\}$ for the breakup of ^{240}Pu. As in the previous figure, the circles represent the calculated points and the numbers the multiplicity N.

references listed in [7] for recent results and a summary of the technique.)

Considering an excited compound system with total nucleon number A and total excitation energy E (omitting charge number and angular momentum for simplicity), any particular manifestation of the system is described as a number of N interacting prefragments with masses $\{m_n\}$ (or mass numbers $\{A_n\}$) and positions $\{r_n\}$.

The evolution of this system as it goes from compound to a disassembled system is determined by a generalized potential produced by the prefragments. This potential varies as a function of the relative position of the prefragments developing a barrierlike shape much like that of regular fission. To trace this evolution, a generalized fission coordinate q can be used to measure the spatial size of the system. Parametrizing the potential in terms of q, it is possible to localize a generalized transition state in this case of multifragment fission. [Near the barrier top [4], the fission coordinate is given by $q^2=(1/m_0)\Sigma_n m_n(r_n^2+3R_n^2/5)$, where $R_n=r_0A_n^{1/3}$ is the radius of the prefragment n and $m_0=\Sigma_n m_n$.]

The partial width for the system to pass over the barrier and disassemble is given by

$$\Gamma_{A_1\cdots A_N}(E)\approx\frac{\sqrt{4\pi}}{\rho(A,E)\Gamma((3N-3)/2)}\times\left\langle\left(\frac{m_0q_N^2\overline{\tau}}{2\hbar^2}\right)^{3N-2/2}\rho_N(\varepsilon_N)\overline{\tau}\right\rangle', \quad (2)$$

where $\rho(E)$ and $\rho_N(\varepsilon_N)$ are the level density of the compound system and at the transition state, respectively, with ε_N denoting the available excitation energy at the transition state (i.e., the total excitation energy minus the multifragment barrier). Likewise, $\overline{\tau}$ is the corresponding temperature, q_N is the disassembly coordinate value, and the brackets $\langle\,\rangle'$ indicate an average over all possible fragment positions $\{r_n\}$ describing multifragment configurations. Expression (2) will now be used to calculate the rate at which an excited compound nucleus fissions into any channel of N prefragments of ten nucleons or more.

III. ENERGY DEPENDENCE ACCORDING TO TST

To explore the energy dependence predicted by the transition state theory, we study, as an illustration, the decay of ^{120}Sn and ^{240}Pu at excitation energies from 1 to 10 MeV/nucleon.

Expression (2) was used to calculate the fission width of decays into specific mass partitions. The configurational average $\langle\,\rangle'$ was done over 40 different space arrangements of the prefragments. The total width for breakups into N fragments, Γ_N, was then obtained by summing the individual widths for decays into *all* mass partition of N fragments. Out of these calculations we obtained $\Gamma_2,\ldots,\Gamma_6$, and formed the ratios $\ln\{\Gamma_N/\Gamma_2\}$ to study their energy dependence.

The results obtained for the decay of ^{120}Sn are shown in Fig. 1 and for ^{240}Pu in Fig. 2. Surprisingly, the curves are almost straight lines for all multiplicities. These curves are equivalent to those in Fig. 2 in Ref. [3], and show that at higher energies, large multiplicities are more probable than smaller ones, and vice versa.

The variation of the barriers appears to have little impact on the observed energy dependence. This unexpected result makes the question even more intriguing: What obscures the effect of the varying barriers on the energy dependence of $\ln\{\Gamma_N/\Gamma_2\}$? Next we study this dependence qualitatively with the transition state theory.

IV. UNDERSTANDING THE ENERGY DEPENDENCE

We now use the TST formalism to extract the functional energy dependence of $\ln\{\Gamma_N/\Gamma_2\}$. Using expression (2) for multiplicities 2 and N (labeled with primes), the ratio is readily reduced to

$$\frac{\Gamma_N}{\Gamma_2}=\frac{\sum_N\Gamma(\tfrac{3}{2})(m_0/2\hbar^2)^{3N-3/2}\langle(q_N^2\overline{\tau'})^{3N-2/2}\overline{\tau'}\rho_N\rangle'}{\sum_2\Gamma((3N-3)/2)\langle q_2^2\overline{\tau^2}\rho_2\rangle'}, \quad (3)$$

where Σ_i denotes the sum over all mass partitions of multiplicity i. To simplify this expression we take, as in Ref. [4], Fermi-gas-type energy level densities

$$\rho_2=\frac{\kappa\sqrt{4\pi\overline{\tau E}_1}}{\sqrt{A_1}}e^{2\sqrt{a_1E_1}}\frac{\kappa\sqrt{4\pi\overline{\tau E}_2}}{\sqrt{A_2}}e^{2\sqrt{a_2E_2}},$$

$$\rho_N=\frac{\kappa\sqrt{4\pi\overline{\tau'}E'_1}}{\sqrt{A'_1}}e^{2\sqrt{a'_1E'_1}}\cdots\frac{\kappa\sqrt{4\pi\overline{\tau'}E'_N}}{\sqrt{A'_N}}e^{2\sqrt{a_N'E_N'}},$$

where the E_i's and E_i's are the energies carried out by the prefragments in the dinuclear and multifragmented systems, respectively, and κ is a constant independent of the masses and energies [4]. Then, using these level densities in Eq. (3), and putting the constant aside, we get

$$\frac{\Gamma_N}{\Gamma_2}\sim\frac{\Sigma_N\langle q_N^{3N-4}\overline{\tau'}^{2N-1}(E'_1\cdots E'_N/A'_1\cdots A'_N)e^{2\sqrt{a'_1E'_1}}\cdots e^{2\sqrt{a'_NE'_N}}\rangle'}{\Sigma_2\langle q_2^2\overline{\tau}^3(E_1E_2/A_1A_2)e^{2\sqrt{a_1E_1}}e^{2\sqrt{a_2E_2}}\rangle'}.$$

To further reduce this we use $E_i=(E-B_N)A_i/A$ as the fragment energies and the Fermi-gas temperature for $\overline{\tau}$ to get

$$\overline{\tau'}^{2N-1}\sqrt{E'_1\cdots E'_N}=\left(\frac{E-B_N}{A}\right)^{(2N-1)/2}\left[\frac{A'_1}{A}(E-B_N)\cdots\frac{A'_N}{A}(E-B_N)\right]^{1/2}\sim(A'_1\cdots A'_N)^{1/2}(E-B_N)^{(3N-1)/2}$$

and

$$\overline{\tau}^3\sqrt{E_1E_2}=\left(\frac{E-B_2}{A}\right)^{3/2}\left[\frac{A_1}{A}(E-B_2)\frac{A_2}{A}(E-B_2)\right]^{1/2}\sim(A_1A_2)^{1/2}(E-B_2)^{5/2}.$$

With this, the functional form of the ratio is then simplified to

$$\frac{\Gamma_N}{\Gamma_2}\sim\frac{\Sigma_N\langle q_N^{3N-4}(E-B_N)^{(3N-1)/2}e^{2\sqrt{a'_1E'_1}}\cdots e^{2\sqrt{a'_NE'_N}}\rangle'}{\Sigma_2\langle q_2^2(E-B_2)^{5/2}e^{2\sqrt{a_1E_1}}e^{2\sqrt{a_2E_2}}\rangle'},\tag{4}$$

which looks like an obstacle, unless we can get rid of the sum over mass partitions in some way.

To proceed further, we can, as Moretto and co-workers did, replace B_N and B_2 by average values, and assume that the preexponential factors do not depend on the mass partition to eliminate them. Although this approach takes us immediately to the wanted linear relationships, it involves the use of assumptions we know are not true. We take a different route and investigate the sum itself.

In particular we are concerned with the sum over all mass partitions. Since the terms included in this sum carry different weights, the possibility that a few of these terms can dominate the total sum is a real one. We verify this suspicion by checking the fission width distribution in a test case.

Figure 3 shows the logarithm of the fission widths obtained for all possible mass partitions of ^{120}Sn decaying at 3 MeV/nucleon into three fragments. For convenience, the widths have been normalized to have positive logarithms by dividing all values by the smallest width. The axes of the contour plot are the mass numbers of two of the fragments, and the labels denote the values of the logarithm at the equilevel curves. Clearly seen are the high-probability regions at around masses 55, 55, and 10. By symmetry this peak appears at points (10,55), (55,10), and (55,55).

Figure 4 depicts the same information in a three-dimensional plot. For clarity only one-third of the width distribution is shown. From this graph, we can see that the weight carried by a few mass partitions can in fact obscure the contribution from the rest. It is worth mentioning that Fig. 4 shows the *logarithm* of the widths; in a linear plot, the peak of the distribution would stand up as a pole.

With this information on hand a clearer picture emerges. Even though the barriers and fission coordinates vary widely, the fission widths of some mass partitions clearly dominate the sum. This fact, which has been known for binary decays, allows us then to replace the sum by the dominant term. Using the values of the most probable mass partitions to

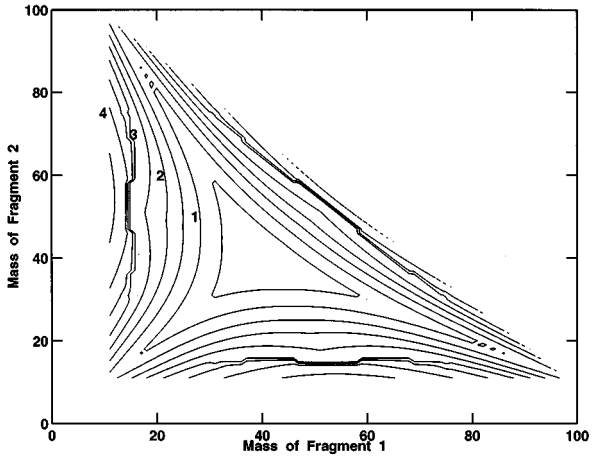

FIG. 3. Distribution of $\log_{10}(\Gamma_3)$ for the decay of ^{120}Sn. The contour plot shows the values of the logarithm of the fission widths of all possible decays of ^{120}Sn into three fragments at 3 MeV/nucleon. The numbers label the values of the equilevel curves.

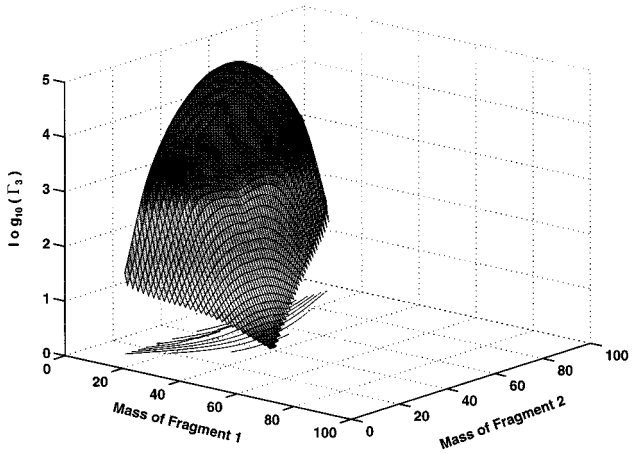

FIG. 4. Section of the distribution of $\log_{10}(\Gamma_3)$ for the decay of ^{120}Sn into all possible channels of multiplicity three at 3 MeV/nucleon. The projection on the x-y plane is the contour plot corresponding to the surface shown.

eliminate the sums of Eq. (4) we thus get

$$\frac{\Gamma_N}{\Gamma_2} \sim \frac{q_N^{3N-4}(E-B_N)^{(3N-1)/2}}{q_2^2(E-B_2)^{5/2}} \frac{e^{2\sqrt{a_1'E_1'}} \cdots e^{2\sqrt{a_N'E_N'}}}{e^{2\sqrt{a_1E_1}} e^{2\sqrt{a_2E_2}}}, \quad (5)$$

where now all the variables are those of the most probable mass partition (averaged over spatial configurations). Now using $a_i = A_i/8.5$ for the level density parameter we notice that

$$e^{2\sqrt{a_1'E_1'}} \cdots e^{2\sqrt{a_N'E_N'}}$$

$$= e^{2\sqrt{A_1'^2(E-B_N)/(8.5A)}} \cdots e^{2\sqrt{A_N'^2(E-B_N)/(8.5A)}}$$

$$= e^{2\sqrt{(E-B_N)/(8.5A)}(A_1' + \cdots + A_N')} = e^{2\sqrt{a(E-B_N)}},$$

and with similar manipulation for the exponential in the numerator of Eq. (5), the functional form of the ratio can be reduced to

$$\frac{\Gamma_N}{\Gamma_2} \sim \frac{e^{2\sqrt{a(E-B_N)}}}{e^{2\sqrt{a(E-B_2)}}} \approx \frac{e^{-\sqrt{a/E'}B_N}}{e^{-\sqrt{a/E}B_2}} = e^{-\sqrt{a/E}(B_N - B_2)}.$$

Finally we get

$$\ln\left\{\frac{\Gamma_N}{\Gamma_2}\right\} \sim \frac{1}{\sqrt{E}},$$

as observed experimentally.

V. CONCLUSIONS

In this work, we examined the peculiar energy dependence of nuclear fragmentation first found by Moretto and co-workers. We studied this dependence using the theoretically sound framework of the transition state theory and determined that the observed linear relationship between $\ln\{P_n/P_2\}$ and $1/\sqrt{E}$ should be expected for simultaneous fragmentations. We concluded that this is a direct consequence, not of the existence of characteristic fragmentation barriers, but of preferred modes of decay in multifragment fission.

This finding is analogous to the existance of high-probability breakup modes in binary fission (for instance, symmetric splits), and it is also in agreement with experimental findings and molecular dynamics simulations [8,9] that show that some values of multiplicities are favored at certain excitation energies. In a future study we will examine the preferred mass partitions for several multiplicities and excitation energies.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under Grant No. PHY-9600038, NSF's AMP program, the Society of Physics Students, and the Artemio de la Vega Foundation. We thank Jaime Morales for his kind help with some of the drawings.

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