Interpretation of the ^{14}C fine structure in the decay of ^{223}Ra

Raymond K. Sheline
Departments of Chemistry and Physics, Florida State University, Tallahassee, Florida 32306

Ingemar Ragnarsson
Department of Mathematical Physics, Lund Institute of Technology, P.O. Box 118, S-221 00 Lund, Sweden
(Received 27 June 1996; revised manuscript received 8 November 1996)

Assuming that the wave function of the $3/2^+$ ground state of ^{223}Ra is quadrupole-octupole deformed with β_2, β_4, β_6, β_3, and β_5 having the optimum values of 0.129, 0.075, 0.004, 0.10, and 0.01, respectively, its amplitude in the wave functions of the $g_{9/2}$, $i_{11/2}$, $j_{15/2}$, $d_{5/2}$, $s_{1/2}$, and $g_{7/2}$ shell model states of ^{209}Pb has been calculated. The systematics of the amplitudes as a function of the octupole deformation, β_3, provides physical insight into the failure to observe ^{14}C radioactive decay to the $j_{15/2}$ state and the very different hindrance factors populating the $g_{9/2}$ and $i_{11/2}$ states in ^{209}Pb. [S0556-2813(97)04002-8]

PACS number(s): 23.70.+j, 21.60.Cs, 21.60.Ev, 27.80.+w

Since the discovery [1] of the exotic ^{14}C radioactivity of ^{223}Ra previously predicited [2], a large number of C, O, F, Ne, Mg, and Si exotic radioactivities have been observed. An excellent summary of both the theoretical work and the experimental results has been given by Price [3].

These exotic activities with masses from 14 to 32 may be considered as intermediate between alpha decay and fission. In that connection it is clear that an odd-*A* nucleus like ^{223}Ra may have a detectable ^{14}C fine structure like that observed in alpha decay. It is obvious that the fine structure in odd-*A* nuclei should be much easier to observe, in both alpha and exotic radioactivities, than in even-even nuclei.

An important step forward in these studies was the experimental detection [4] of fine structure in the energy spectrum of ^{14}C ions emitted by ^{223}Ra. This experiment revealed the possibilities of a detailed spectroscopic interpretation of ^{14}C decay as has been so useful in alpha decay.

In analogy with alpha decay it is possible to make a Geiger-Nuttal-type plot of the ^{14}C radioactivity of ^{222}Ra, ^{224}Ra, and ^{226}Ra. This plot of the $\ln_{10}T_{1/2}$ vs $Q^{-1/2}$ ($T_{1/2}$ being the half-life and Q being the energy in MeV for the ^{14}C decays) leads to a straight line as shown in Fig. 1. When the points for ^{223}Ra are added, they lie above the line (Fig. 1) as indicated and correspond to hindrance factors (HF's) just like the corresponding situation in alpha decay.

There has, however, been some experimental and theoretical ambiguity in connection with fine structure in the ^{14}C radioactivity of ^{223}Ra. The lowest-lying states in the daughter nucleus ^{209}Pb are the $g_{9/2}$ ground state, the 779 keV $i_{11/2}$ state, and the 1423 keV $j_{15/2}$ state. The initial experimental work [4] indicated that all three of these states were populated in the ^{14}C radioactive decay of ^{223}Ra, the ground state with high HF and the 779 and 1423 keV states with low HF's. Theoretical interpretation [5] of the experimental work [4] suggested a good agreement between experiment and theory for the population of these three states in the ^{14}C radioactivity of ^{223}Ra. However, subsequent theoretical work [6] involving the calculation of the overlap of the octupole deformed ^{223}Ra ground state with low-lying states in ^{209}Pb indicated that the $j_{15/2}$ state at 1423 keV should be weakly populated with a high HF. Very recently an elegant experiment [7] maximizing the resolution of the ^{14}C ions [90 keV full width at half maximum (FWHM)] has proved conclusively that the population of the 1423 keV state has a HF >89. Thus only one ^{223}Ra excited state is shown in Fig. 1.

The ^{14}C radioactivity of ^{223}Ra affords a unique opportunity of studying the overlaps (and therefore the relatedness) of the octupole deformed ^{223}Ra ground state and various shell model states in ^{209}Pb. We have already shown [6] that, although the ^{223}Ra ground state configuration can be thought

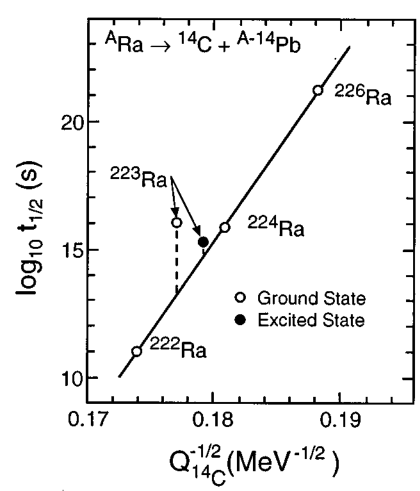

FIG. 1. Geiger-Nuttal diagram of the ^{14}C radioactive decay of Ra isotopes. The logarithm of the experimental half-lives in seconds is plotted against $Q^{-1/2}$ with Q being energy of the decay in MeV. The distance the ^{223}Ra points are above the straight line through the even-even Ra isotopes is used to determine the hindrance factors.

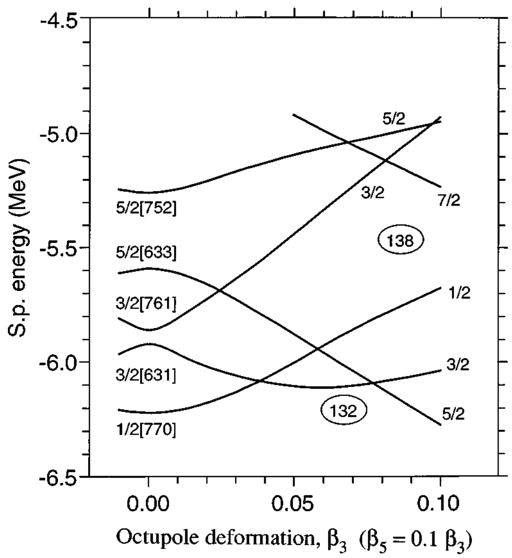

FIG. 2. Calculated energies of the quadrupole-octupole deformed neutron orbitals with β_2, β_4, and β_6 having the optimum values 0.129, 0.075, and 0.004, respectively, as a function of β_3. The orbitals are labeled for convenience with the Nilsson quantum numbers which are actually appropriate only when $\beta_3=0$. Therefore when discussing these orbitals where β_3 is not equal to zero, the Nilsson labels are put in quotes. Neutron numbers are encircled.

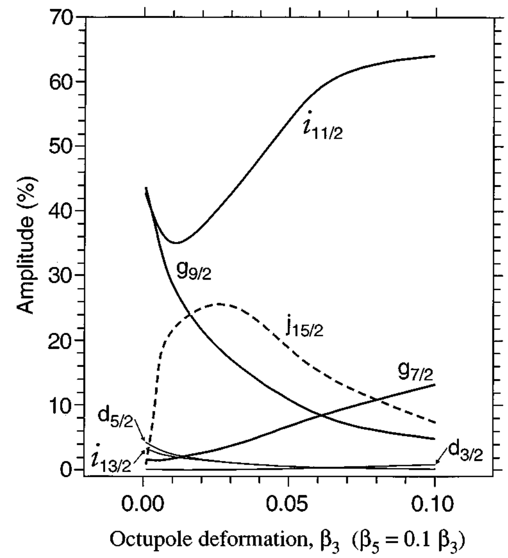

FIG. 3. Calculated values of the amplitude of the ^{223}Ra "3/2[631]" neutron orbital (Fig. 2) in various ^{209}Pb neutron shell model states with increasing octupole deformation β_3.

of as 3/2[761] coupled to 3/2[631] [8], and that these Nilsson configurations arise from the $j_{15/2}$ and $i_{11/2}$ shell model states at zero quadrupole and octupole deformation, the $j_{15/2}$ state cannot be strongly populated in ^{14}C radioactive decay.

We now need to understand this fact and simultaneously see if we can understand the unique way in which an octupole deformed configuration can describe the HF's in ^{14}C radioactive decay.

With this purpose in mind we have carried out similar calculations as in Ref. [6], using a Woods-Saxon potential to study single-particle energies and corresponding wave functions of the Nilsson orbitals at different deformation. According to Ref. [8], a good description of the properties of ^{223}Ra are obtained at the deformations $\beta_2=0.129$, $\beta_3=0.10$, $\beta_4=0.075$, $\beta_5=0.01$, and $\beta_6=0.004$. Here we will refer to these parameters, which were also used in Ref. [6], as optimal. In order to study the importance of octupole deformation we have kept the β_2,β_4, and β_6 deformations at these optimal values and varied the β_3 parameter between 0 and its optimal value 0.10. Here β_5 is given the value $0.1\beta_3$. The single-particle orbitals along this path are given in Fig. 2. It should be noted that the labeling of the orbitals is approximately correct only at $\beta_3=0$, but is used for convenience. Elsewhere the orbitals have mixed parity. In Fig. 3 the amplitudes of the "3/2[631]" neutron orbital in various neutron shell model states along the path of increasing β_3 are given.

Several insights into ^{14}C decay probabilities to various ^{209}Pb shell model orbitals are immediately obvious in Fig. 3. One notes, for example, that at small β_3 values there are two close-lying 3/2 states in Fig. 2. Because they are close together, they will mix strongly and give large amplitudes with both the $i_{11/2}$ and $j_{15/2}$ orbitals, while at larger values of β_3 the $j_{15/2}$ amplitude becomes much smaller as the 3/2 states move away from each other. This is clearly shown in Fig. 3 and gives physical meaning to the experimentally observed low amplitude with the $j_{15/2}$ orbital at the expected higher values of β_3.

In a similar way the amplitudes of the "3/2[631]" orbital with the $g_{9/2}$ and $i_{11/2}$ neutron shell model orbitals are similar and large at small values of β_3. However, with increasing β_3, these amplitudes diverge, becoming very large with the $i_{11/2}$ orbital and very small with the $g_{9/2}$ orbital. This is consistent with the hindrance factors which can be extracted from the experimental half-lives plotted in Fig. 1, 2.2 and 380 for the decay to the $11/2^+$ state at 779 keV and to the $9/2^+$ ground state, respectively.

Finally, it should be noted that the observation of one ^{14}C ion at the position of the $1/2^+(s_{1/2})$ state at 2032 keV in the experiment [7] cannot be explained with these calculations. The $3/2^+$ ^{223}Ra ground state orbital (in our model) can have no amplitude in the $s_{1/2}$ orbital and is forbidden. At higher order it is possible that small amounts of a $1/2^+$ orbital could mix into the $3/2^+$ orbital. However, at most an extension of our calculations suggests a highly forbidden ^{14}C transition to the $1/2^+$ 2032 keV state, whereas the one observed count may suggest a HF as low as 4. Therefore additional experimental work would be of considerable interest to see if this single ^{14}C event corresponds to degradation of its energy along the path in the spectrometer or, in fact, to the $1/2^+$ excited state in ^{209}Pb.

One of us (R.K.S.) would like to thank the National Science Foundation for support under Contract No. PHY92-07336 with Florida State University, while I.R. acknowledges the Swedish Natural Science Research Council for financial support.

[1] H. J. Rose and G. A. Jones, Nature (London) **307**, 245 (1984).
[2] A. Sandulescu, D. N. Poenaru, and W. Greiner, Sov. J. Part. Nucl. **11**, 528 (1980).
[3] P. B. Price, Annu. Rev. Nucl. Part. Sci. **39**, 19 (1989).
[4] L. Brillard, A. G. Elayi, E. Hourany, M. Hussonnois, J. F. Le Du, L. H. Rosier, and L. Stab, C.R. Acad. Sci. A **309**, 1105 (1989). See also M. Hussonnois, J. F. Le Du, L. Brillard, J. Dalmasso, and G. Ardisson, Phys. Rev. C **43**, 2599 (1991).
[5] M. Hussonnois, J. F. Le Du, L. Brillard, and G. Ardisson, J. Phys. G **16**, 177 (1990); Phys. Rev. C **42**, R495 (1990); **44**, 2884 (1991); M. Hussonnois and G. Ardisson, Z. Phys. A **349**, 311 (1994).
[6] R. K. Sheline and I. Ragnarsson, Phys. Rev. C **43**, 1476 (1991); **44**, 2886 (1991).
[7] E. Hourany, G. Berrier-Ronsin, A. Elayi, P. Hoffmann-Rothe, A. C. Müller, L. Rosier, G. Rotbard, G. Renou, A. Liѐbe, and D. N. Poenaru, Phys. Rev. C **52**, 267 (1995). See also E. Hourany, L. Rosier, G. Berrier-Ronsin, A. Elayi, A. C. Müller, G. Rappenecker, G. Rotbard, G. Renou, A. Liѐbe, L. Stab, and H. L. Ravn, *ibid.* **44**, 1424 (1991).
[8] R. K. Sheline, Y. S. Chen, and G. A. Leander, Nucl. Phys. **A486**, 306 (1988).