Double beta decay of ^{92}Mo: Comparison of the shell model and the quasiparticle random-phase approximation

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The two-neutrino double beta decay of ^{92}Mo to the ground state and excited states of ^{92}Zr is analyzed using the shell-model approach and the quasiparticle random-phase approximation (QRPA). Two different valence spaces and three different Gamow-Teller operators are used in the shell-model analysis. The resulting double beta matrix elements and half-lives are compared with the corresponding QRPA results. [S0556-2813(97)00202-1]

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I. INTRODUCTION

The nucleus provides a convenient laboratory for testing fundamental theories going beyond the standard model [1]. This is achieved by the study of the neutrinoless double beta ($0\nu\beta\beta$) decay and the two-neutrino double beta ($2\nu\beta\beta$) decay of nuclei in which the ordinary beta decay is energetically forbidden. The $2\nu\beta\beta$ decay mode provides an excellent testing ground for the nuclear models due to the significant experimental progress made in the detection of this decay mode for transitions to the ground state and excited states of the final nucleus [2,3].

Since most of the $2\nu\beta\beta$ decaying nuclei are medium heavy or heavy open-shell nuclei the nuclear shell model (SM) has been inapplicable for calculation of the associated nuclear matrix elements. This is why the SM analysis has been restricted to ground-state decay of ^{48}Ca in the past. At present, there are some shell-model codes [4–7] which have potential to treat, in a reasonably realistic way, some of the more heavy double-beta-decaying nuclei.

In the present article we apply the shell-model approach to investigate the $2\nu\beta\beta$ decay of ^{92}Mo to the ground state and 2_1^+ and 0_1^+ excited states in ^{92}Zr. With few exceptions (see, e.g., Haxton and Stephenson [8]), most previous SM studies of the double β decay have concentrated on ground-state transitions. However, the study of the excited-state $2\nu\beta\beta$ transitions has lately gained importance due to its nature of complementing the nuclear-structure information coming from the ground-state transition. Extra motivation for these studies is given by the rapid progress on the experimental side leading already to an experimental half-life value for the 0_1^+ transition in ^{100}Mo [9]. Several other projects are going on to measure such transitions in other double-beta-decay candidates.

Of particular interest in our study is to examine the dependence of the $2\nu\beta\beta$ matrix elements on the configuration space used in the SM calculation. This is accomplished by considering two different models which have recently been employed to study the Gamow-Teller decay of the $N=50$ nuclei. In the first of these [10] the valence protons are placed in the $p_{1/2}$ and $g_{9/2}$ orbitals, while in the second [11] the proton space is enlarged to include the $p_{3/2}$ orbital. In both cases the valence neutrons are distributed in the neighboring orbitals $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$.

The dominant mechanism responsible for the $2\nu\beta\beta$ decay of ^{92}Mo is expressed by the bare Gamow-Teller operator, which transforms a proton in the $g_{9/2}$ orbital to a $g_{7/2}$ neutron. However, most SM calculations in the mass $A\simeq 100$ region [10–13] consider renormalized Gamow-Teller operators in order to account for the observed quenching of the beta-decay strength within the $Q_{\rm EC}$ window [14]. This quenching may result from experimental deficiencies [13] or would indicate that some of the Gamow-Teller strength is pushed above the $Q_{\rm EC}$ window, by means of configuration mixing. Some of the effects of this mixing can be taken into account by enlarging the space of the calculation but others, like those which correspond to the breaking of the $N=50$ core, are more conveniently described by introducing effective Gamow-Teller operators [10–12]. This approach has been used in the present article in the determination of the virtual Gamow-Teller transitions of the ^{92}Mo ground state to the 1^+ states of ^{92}Nb and the following deexcitation of the latter to the ground and excited 0_1^+ and 2_1^+ states of ^{92}Zr. The resulting double Gamow-Teller (DGT) matrix elements of the $2\nu\beta\beta$ decay are compared with the ones obtained using the bare Gamow-Teller operator.

The various SM results (different valence spaces and different Gamow-Teller operators) are compared with the results of the proton-neutron quasiparticle random-phase approximation (pnQRPA) which, since its first applications [15,16], has been used extensively to calculate $2\nu\beta\beta$ and $0\nu\beta\beta$ decay rates to the final ground state. This method was extended in [17–19] to description of the excited-state transitions. Here the method will be called the multiple-

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commutator method (MCM), for short, and will be applied to the analysis of the $2\nu\beta\beta$ decay of ^{92}Mo both to the ground state and to the excited 2_1^+ and 0_1^+ states in ^{92}Zr.

II. FORMALISM

A. The DGT matrix elements and the decay half-life

The $2\nu\beta\beta$-decay half-life is taken to be of the form [20]

$$[T_{1/2}^{2\nu}(0^+\rightarrow J_f^+)]^{-1}=|M_{\rm GT}^{2\nu}(J_f^+)|^2 G_{\rm GT}^{(\alpha)}(J_f^+)\,, \qquad (1)$$

where $G_{\rm GT}^{(\alpha)}(J_f^+)$ denotes the phase-space factor, depending on the final spin J_f^+. Here α denotes the various possible decay modes, namely $2\nu KK$ capture, i.e., the double K-electron capture, $2\nu\beta^+K$ and $2\nu\beta^+\beta^+$, with the obvious notation [20]. Values of these phase-space factors for $J_f^+=0_f^+$ transitions are calculated using the method of Ref. [20]. As indicated in Eq. (1) the same nuclear matrix element is used for different modes α.

The double Gamow-Teller matrix elements (DGT) are given by

$$|M_{\rm GT}^{(2\nu)}(J_f^+)|=\frac{1}{\sqrt{s}}\sum_{m,m'}\frac{(J_f^+\|\Sigma_i\sigma(i)\tau^+(i)\|1_{m'}^+)\langle 1_{m'}^+|1_m^+\rangle(1_m^+\|\Sigma_i\sigma(i)\tau^+(i)\|0_i^+)}{[(\frac{1}{2}Q_{\beta\beta}(J_f^+)+E(1_m^+)-M_i)/m_{\rm e}+1]^s}, \qquad (2)$$

where $s=1+2\delta_{J_f^2}$, $Q_{\beta\beta}(J_f^+)$ denotes the $2\nu\beta\beta$-decay Q value taking into account the excitation energy of the final state $|J_f^+\rangle$, and $E(1_m^+)-M_i$ is the energy difference between the mth intermediate 1^+ state and the initial ground state. The overlap factor in Eq. (2) is trivial for the SM calculation, i.e., $\langle 1_{m'}^+|1_m^+\rangle=\delta_{m'm}$, but for the QRPA it represents matching of the two different sets of 1^+ states, one emerging from the pnQRPA calculation for the initial nucleus (1_m^+) and the other coming from the calculation for the final nucleus $(1_{m'}^+)$.

B. The shell-model formalism

As discussed in Sec. I, the $2\nu\beta\beta$ decay of ^{92}Mo is studied in two different shell-model calculations. These two models, to be hereafter described as Model1 and Model2, differ in the definition of the model space but also in the manner the effective interaction is determined.

In Model1 calculations $^{88}_{38}$Sr is assumed as inert core and the valence protons are placed in the $p_{1/2}$ and $g_{9/2}$ orbitals, while the valence neutrons are distributed in the close-spaced orbitals $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$. The matrix elements of the effective interaction, appropriate for the Model1 space, have been determined by fitting them to observed data [10,21]. Here we use the sets JS1, JS4, and JS6 of Ref. [21] for the proton-proton, proton-neutron, and neutron-neutron parts of the interaction, respectively.

Model2 calculations follow closely the procedure adopted in recent shell-model studies of the properties of the $N=50-52$ nuclei [11,22,23]. In these previous calculations, the double-magic $^{100}_{50}$Sn state is assumed as core and the valence proton holes are placed in the $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$ orbitals, while the valence neutrons, like in the Model1 case, are distributed in the $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ orbitals. The effective two-body interaction has been determined [11,22,23] by considering second-order corrections to the Sussex matrix elements [24] while the single-particle energies have been determined by fitting them to the observed levels of the $N=50$ [22] and $N=51$ [23] nuclei.

In the present calculation we find it more convenient to use particle formalism instead of the particle-hole formalism employed in Refs. [11,22,23]. Thus the Model2 calculations have been performed assuming $^{84}_{34}$Se as core. Single-particle energies with respect to the new core were determined by removing, to first order, the effect of the additional 16 protons of $^{100}_{50}$Sn. On the other hand, we have considered as an approximation, that the three-body effects which arise in a second-order determination of the effective interaction are negligible. In terms of this approximation, we use in the present calculation the two-body interaction of Refs. [11,22,23] except, of course, that we employ the Pandya relation to transform particle-hole matrix elements to particle-particle ones.

As discussed above, Model1 and Model2 have been previously employed to study the Gamow-Teller decay of the $N=50$ nuclei [10,11]. In addition, both these models have been found to give a very satisfactory description of the observed properties of the low-lying states of several $N=51,52$ nuclei [21,23].

As already mentioned, the $2\nu\beta\beta$ decay of ^{92}Mo proceeds mainly through the Gamow-Teller transition $\pi g_{9/2}\rightarrow\nu g_{7/2}$. However, there are other configurations, like those which describe the breaking of the $N=50$ core or the presence of a proton in the $g_{7/2}$ orbital, which also contribute to the $2\nu\beta\beta$ decay. These configurations are outside the model space considered in both Model1 and Model2 calculations. However, the effects of such configurations can be taken into account, at least in lowest order of perturbation, by introducing effective Gamow-Teller operators. The manner in which these effective operators may be determined is demonstrated in Fig. 1.

The diagrams shown in Fig. 1 represent both one-body and two-body corrections. Due to the selectivity of the Gamow-Teller operator, all one-body terms can only connect a $g_{9/2}$ proton to a $g_{7/2}$ neutron. As has been demonstrated in previous shell-model studies of the $N=50$ nuclei [10–12], consideration of diagrams (b) and (c) leads to a drastic quenching of the Gamow-Teller strength from that calculated with only the bare operator [diagram (a)]. In a model space consisting only of the $g_{9/2}$ proton and $g_{7/2}$ neutron orbitals, the strong quenching produced by corrections (b) and (c) is partially cancelled by consideration of the two-body dia-

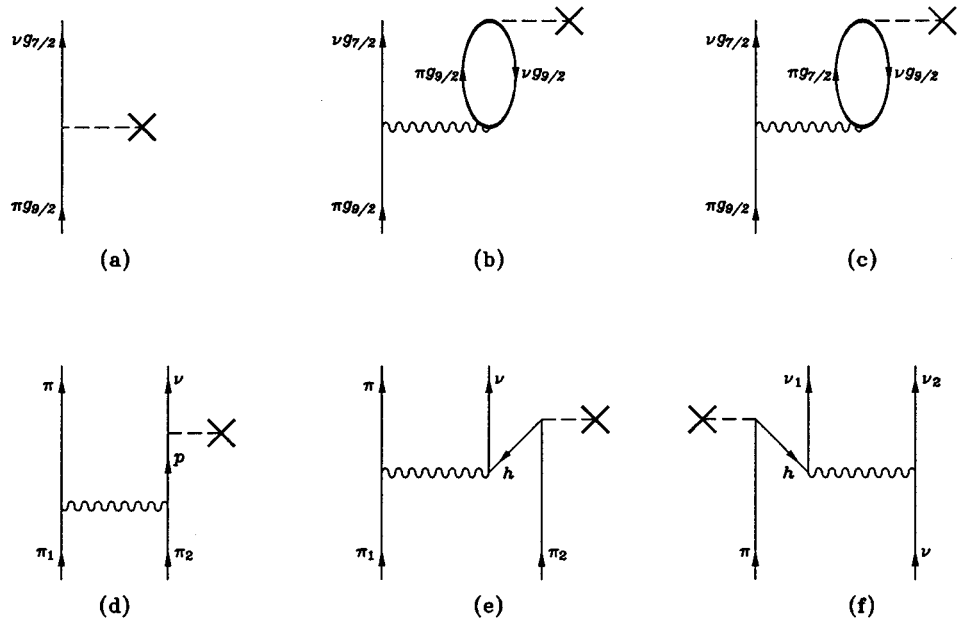

FIG. 1. Diagrams considered in the determination of the effective Gamow-Teller operator. The labels π and ν denote proton and neutron, respectively. Diagram (f) is only involved in the decay of nuclei with $N \geqslant 51$ and thus it does not contribute in the decay of ^{92}Mo.

grams (d) and (e) [10–12]. Generally, however, the role of the two-body diagrams is more complicated since, as Fig. 1 shows, they involve all orbitals of the model space.

C. The QRPA formalism

The reduced matrix elements needed in Eq. (2) can be evaluated by using the usual commutation techniques of the random-phase approximation (RPA). In these matrix elements the 1^+ states of the odd-odd nuclei are calculated using the pnQRPA whereas the excited final states $J_f^+ = 0_1^+, 2_1^+$ are described within the charge-conserving QRPA framework. This combination of the two formalisms is called the multiple-commutator method (MCM) and is described extensively in [17–19].

In the QRPA calculation the two-body matrix elements were obtained from the Bonn potential using G-matrix techniques. For both protons and neutrons the adopted valence space consists of the nine single-particle orbitals in the p-f and s-d-g oscillator shells, supplemented with the intruder orbital $0h_{11/2}$ coming from the oscillator shell above. The single-particle energies were obtained from a Coulomb-corrected Woods-Saxon potential [25] and a subsequent BCS calculation was performed for the initial (^{92}Mo) and final (^{92}Zr) nuclei. The pairing-strength parameters were adjusted as described in [26].

The proton-proton and neutron-neutron two-body G-matrix elements were scaled [17] using the experimental excitation energy of the first 2^+ state in ^{92}Zr. The overall scale of the proton-neutron particle-hole matrix elements, in turn, was set by the Gamow-Teller giant-resonance data. The proton-neutron particle-particle interaction strength [26] $g_{\rm pp}$ is usually left as a free parameter to be determined from some experimental data. In this work the value $g_{\rm pp} = 0.90$ was chosen to represent an upper limit for the physical values of the $2\nu\beta\beta$ matrix elements within the QRPA formalism.

III. RESULTS AND DISCUSSION

In this section we compare the predictions of three different calculations on the $2\nu\beta\beta$ decay of ^{92}Mo. These calculations differ not only in the adopted method, i.e., two follow the shell-model approach while the third the QRPA approach, but also in the definition of the model space, as described in Sec. II. The 1^+ matrix of ^{92}Nb has dimension 11 in the Model1 space compared to 404 of the Model2 case. The corresponding dimensions for the 2^+ state of ^{92}Zr are 77 and 4043. This drastic increase in dimensions clearly indicates that it is practically impossible to include additional orbitals into the model space of the SM calculations.

It is well known that, in the same model space, the QRPA produces a considerably smaller number of states than the SM. Thus QRPA calculations can be performed in large model spaces, which cannot be handled in the SM approach. In the present application of the QRPA method the model space contains ten single-particle orbitals, i.e., the whole fp and sdg shells with the addition of the $0h_{11/2}$ orbital, which are available to both protons and neutrons. From the SM point of view, one expects that certain of these orbitals, namely the $f_{7/2}$, $f_{5/2}$, and the $h_{11/2}$, would have insignificant contribution to the $2\nu\beta\beta$ matrix elements studied here. There are two reasons for this expectation: (a) because these three orbitals have large energy differences from the leading orbitals of the calculation, i.e., the proton $g_{9/2}$ and the neutron $d_{5/2}$. Thus their presence in the wave functions of the low-lying states of the initial ^{92}Mo and final ^{92}Zr nuclei cannot be but very small and (b) because of the selectivity of the Gamow-Teller operator which does not connect the three mentioned orbitals to any of the others in the model space. From the remaining orbitals of the QRPA calculation, the effects of the proton $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ and of the neutron $g_{9/2}$, $p_{1/2}$, and $p_{3/2}$ are taken care in the SM calculations by renormalizing, to first order, the Gamow-Teller operator.

The results of the three calculations are summarized in Table I, which lists values of the total DGT matrix elements for the $2\nu\beta\beta$ decay of ^{92}Mo to the ground state and the 2_1^+ and 0_1^+ excited states of ^{92}Zr. The 2νKK half-lives for the $J_f^+ = 0^+$ states, calculated according to Eq. (1), are also given in the table. The SM results in Table I have been

TABLE I. The double Gamow-Teller matrix element corresponding to the transitions to the ground state and the first excited 2^+ and 0^+ states. The corresponding $2\nu KK$ decay half-life is given whenever the phase-space factor is known. The SM calculation has been performed in the Model1 and Model2 basis with the bare, one-body corrected and one- and two-body corrected Gamow-Teller operators.

	Model2			Model1			
Quantity	Bare	one-body	one + two-body	Bare	one-body	one + two-body	QRPA
$M_{\rm DGT}$(g.s.)	0.075	0.027	0.107	0.153	0.071	0.229	0.290
$t_{1/2}^{(2\nu)}$ (g.s.) [10^{22}y]	44	325	21	10	48	4.6	3.1
$M_{\rm DGT}(2_1^+)$	−0.0019	−0.0007	−0.0027	−0.0015	−0.0005	−0.0019	−0.0036
$M_{\rm DGT}(0_1^+)$	0.0337	0.0124	0.0366	0.0013	0.0147	0.0601	0.0149
$t_{1/2}^{(2\nu)}(0_1^+)$[10^{28}y]	4.6	34	3.9	3100	24	1.5	22

obtained using the three different Gamow-Teller operators discussed in Sec. II B. This helps to estimate the effects of the renormalization procedure on the DGT matrix elements. On the other hand, the QRPA results have been obtained using only the bare Gamow-Teller operator. This is because the configurations leading to the renormalization are directly included in the space of the QRPA calculation.

A more detailed presentation of the results of our calculations is shown in Figs. 2–4. These figures present the cumulative contributions to the DGT matrix elements as functions of the excitation energy (relative to the ^{92}Nb ground state) of the 1^+ states in the intermediate odd-odd nucleus ^{92}Nb.

There are some general features which may be observed in the results shown in Table I and Figs. 2–4. The first is that, despite differences in individual DGT matrix elements, there is an overall agreement in the trend of the results produced by the three calculations. This in turn implies an overall agreement in the description of the wave functions which enter in the determination of the DGT matrix elements. Another general feature in Table I is that the QRPA results are closer to those obtained in the two SM calculations using the fully renormalized Gamow-Teller operator, i.e., the one which includes both one- and two-body corrections. This feature, which is discussed in more detail below, reflects the importance of certain orbitals, like those of the proton $1d$ shell, which are not directly included in the space of the SM calculations. Another observation that can be made in Figs. 2–4 is that the SM distributions obtained with the bare and the effective one-body Gamow-Teller operator have identical patterns. This result is due to the fact, discussed in Sec. II B, that the one-body corrections, such as the bare operator, can only connect a $g_{9/2}$ proton to a $g_{7/2}$ neutron. Finally, one observes in Table I that the one- and two-body corrections give opposite contributions to the values of the DGT matrix elements. Such a feature was also observed in the single β^+ decay of the $N=50$ nuclei [10–12]. As discussed by Towner [12], one of the reasons for this behavior is due to the cancellation of the Pauli violating terms contained in both one-body and two-body terms.

Concerning the ground-state transition, as Fig. 2 shows, the two SM calculations predict a coherent contribution to the DGT matrix element from several 1^+ states of ^{92}Nb. The number of such states and their spreading in the energy spectrum is larger in the Model2 case, due to the stronger configuration mixing involved in the latter calculation. However, the two SM results agree that the 1^+ states, which contribute significantly to the DGT value, appear only in the low-energy part of the ^{92}Nb spectrum, i.e., up to about 2.2 MeV in the Model1 case, or to about 3.5 MeV in the Model2 one.

The contribution of the low-lying 1^+ states of ^{92}Nb to the ground-state transition is represented in the QRPA result by a single state, at about 1 MeV of excitation. The mechanism which is mainly responsible for the contribution of this state, is the $\pi g_{9/2} \rightarrow \nu g_{7/2}$ transition which, as Fig. 2 shows, also

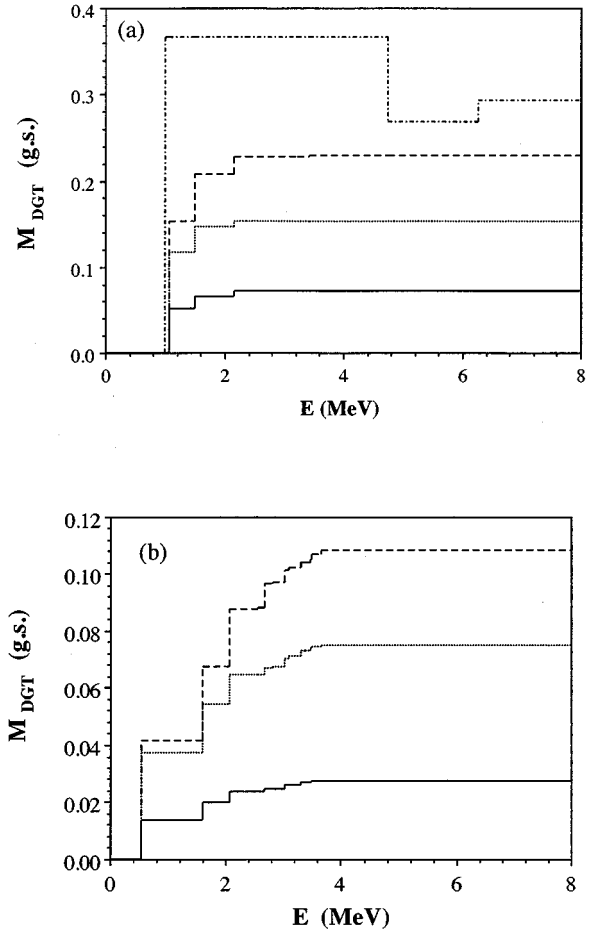

FIG. 2. Cumulative contribution, as a function of the energy of the 1^+ states in ^{92}Nb, to the double Gamow-Teller matrix element for the transition to the ground state. In (a) the QRPA (dash-dotted line) and the Model1 basis results for the SM are shown. (b) contains the results for the SM calculation in the Model2 basis. In (a) and (b) the bare (dotted line), one-body corrected (solid line) and one- and two-body corrected (dashed line) Gamow-Teller operators have been used.

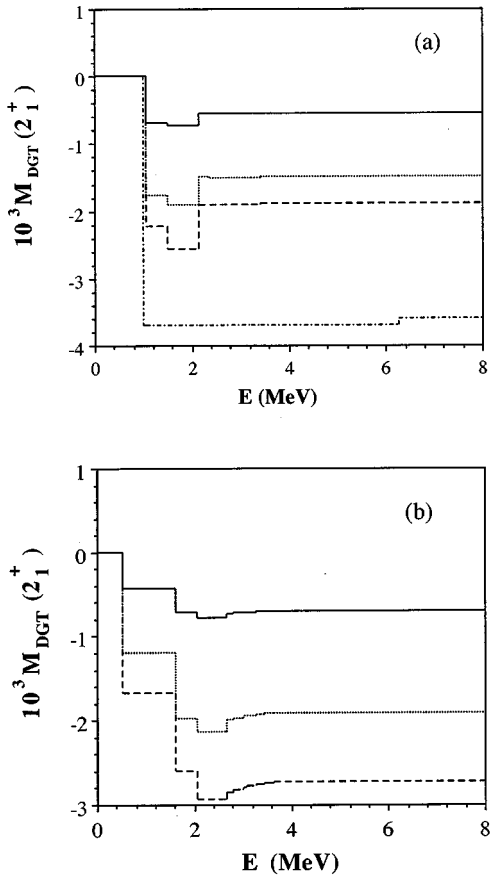

FIG. 3. The same as Fig. 2 for the double Gamow-Teller matrix element describing transition to the first 2^+ state of ^{92}Zr. The matrix element has been multiplied by a factor of 1000.

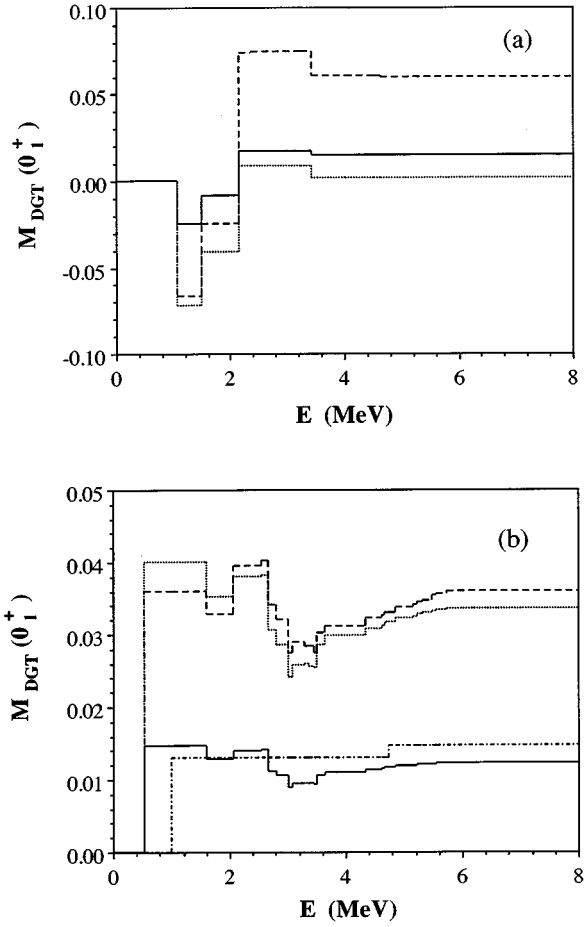

FIG. 4. The same as Fig. 2 for the transition to the first excited 0^+ state of ^{92}Zr, except that the QRPA result (dash-dotted line) is shown together with the Model2 results in (b).

dominates in the SM results. If the QRPA and SM calculations were performed in the same space and with the same effective operators, one would expect the transition strength provided by the QRPA state at 1 MeV to represent the centroid of the SM transition strengths [27].

The lowest 1^+ state provides about 80% of the total DGT value in the QRPA result. The final value is formed by the contribution of two other 1^+ states which, as Fig. 2(a) shows, appear at about 4.8 and 6.3 MeV of excitation in the spectrum of ^{92}Nb. The contribution of these two last states makes the strength distribution of the DGT element quite different from that obtained from the shell-model results. The explanation for this difference is easily obtained by examining the wave functions of the two states at 4.8 and 6.3 MeV. It is found that they correspond to about 55 and 39 % probability, respectively, of finding two quasiparticles in the $1d$ orbitals, i.e., the wave functions of these two states have small projections onto the model space used by the SM calculations.

It is well known that the perturbation approach fails to reproduce the effects caused by the presence of intruder states [28]. Due to this fact, the approach adopted in our SM calculations of renormalizing the GT operator is unsuitable to reproduce the QRPA form of the strength distribution, even if higher order corrections were considered. To produce a strength distribution resembling that of the QRPA, one should clearly include directly into the space of the SM calculations the proton $1d$ orbitals. However, it is interesting to observe in Fig. 2 that the renormalization of the operator moves, in some average manner, the SM results into closer agreement with the QRPA predictions.

As seen from Table I, the magnitude of the DGT matrix element for the 2_1^+ transition is quite small in both the SM and the QRPA results. This is in accordance with the earlier results of Refs. [18,19,29] and does not support the very large 2_1^+ DGT matrix elements obtained in Ref. [30]. As Fig. 3 shows, the two SM calculations do not predict a coherent contribution for the 2_1^+ transition. Thus at some point beyond 2 MeV of excitation, part of the cumulative contribution is cancelled and the magnitude of the 2_1^+ matrix element reduced. This cancellation is much more modest for the QRPA. Thus for the QRPA the magnitude of the 2_1^+ DGT matrix element is almost solely determined by the virtual transitions via the first intermediate 1^+ state. In this case the second-branch virtual transition [Nb $\rightarrow$ Zr(2_1^+)] contains, in addition to the $\pi g_{9/2} \rightarrow \nu g_{7/2}$ single-particle transition, strong transitions between the $1d$ single-particle states. This feature partly explains the difference between the SM and the QRPA results.

As indicated by Table I and already noticed in [29], the DGT matrix element for the 0_1^+ transition is much larger, in magnitude, than for the 2_1^+ transition. Such a feature was also detected in the case of the $2\nu\beta^-\beta^-$ decays, although there it was more pronounced than in the present case [29]. In the SM the cumulative contributions to the 0_1^+ matrix

element behave more irregularly than in the case of the ground-state and 2_1^+ transitions. As seen in Fig. 4, there is a large interference between the low-energy contributions for all types of Gamow-Teller operators used in the SM calculation. As a result of this there is even a sign change of the cumulative DGT matrix element around 2 MeV of intermediate excitation in the Model1 [Fig. 4(a)]. For the bare Gamow-Teller operator this leads to an almost complete cancellation of the 0_1^+ DGT matrix element. For the Model2 the cancellations affect less the magnitude of the DGT matrix element and no sign change results. In the QRPA the first intermediate 1^+ state almost exhausts the value of the 0_1^+ DGT matrix element. In this case the second branch of the virtual transitions is dominated by the $\pi g_{9/2} \rightarrow \nu g_{7/2}$ transition and thus the SM and the QRPA DGT matrix elements are in this case more similar than in the case of the 2_1^+ transition.

IV. CONCLUSIONS

In this paper we studied the $2\nu\beta\beta$ decay of ^{92}Mo leading to the ground state and the first excited 0^+ and 2^+ states of ^{92}Zr. In this study we employed both the SM and the QRPA methods and compared their predictions. In the following we present certain conclusions that may be drawn from this comparison. (i) There is an overall agreement in the trend of results produced by the SM and QRPA calculations. A similar conclusion was drawn by Civitarese *et al.* [27] who compared the SM and QRPA predictions on the single β^+ decay of ^{26}Mg. In fact the conclusion of Ref. [27] is more definite than ours since their SM and QRPA calculations were performed in the same model space and using the same effective interaction. (ii) The differences in the values of individual DGT matrix elements obtained from the SM and QRPA calculations could be attributed to the different model spaces and to the different treatment of effective operators used in the two methods. However, an equally important factor may be considered to be the more detailed structure contained in the SM wave functions. This is evident from the distribution of the DGT strength shown in Figs. 2–4. The same conclusion may also be reached by comparing the results of the two shell-model calculations. It is to be noted that the QRPA results are generally closer to the predictions of the Model1 calculation which involves a smaller number of configurations than Model2. (iii) Certain orbitals included in the QRPA space but not in that of the SM, such as the proton $1d$ orbitals, produce significant contributions to the DGT matrix elements. The effects of these orbitals is taken care in the SM approach by means of renormalizing the Gamow-Teller operator. However, in view of the significance of these orbitals, their treatment in terms of perturbation appears to be questionable and can only be justified by the inability of shell-model calculations to include these orbitals directly in the model space.

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