Regularization, renormalization, and range: The nucleon-nucleon interaction from effective field theory

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Regularization and renormalization is discussed in the context of low energy effective field theory treatments of two or more heavy particles (such as nucleons). It is desirable to regulate the contact interactions from the outset by treating them as having a finite range. The low energy physical observables should be insensitive to this range provided that the range is of a similar or greater scale than that of the interaction. Alternative schemes, such as dimensional regularization, lead to paradoxical conclusions such as the impossibility of repulsive interactions for truly low energy effective theories where all of the exchange particles are integrated out. This difficulty arises because a nonrelativistic field theory with repulsive contact interactions is trivial in the sense that the S matrix is unity and the renormalized coupling constant zero. Possible consequences of low energy attraction are also discussed. It is argued that in the case of large or small scattering lengths, the region of validity of effective field theory expansion is much larger if the contact interactions are given a finite range from the beginning. [S0556-2813(97)01401-5]

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I. INTRODUCTION

One common issue in particle physics is the existence of phenomena on widely differing energy scales. In studying the low energy phenomenology in such situations, the techniques of effective field theory (EFT) have proven extremely useful [1]. They allow one to include systematically only those effects of the short range physics which contribute to the long range phenomena up to some given level of accuracy. The philosophy underlying this is that one can integrate the short wavelength degrees of freedom, i.e., those degrees of freedom whose momenta are larger than some separation scale μ out of the functional integral. Of course, in doing this one obtains an effective action which is nonlocal. However, the nonlocality is on the scale of the degrees of freedom which have been integrated out. At scales far below this it is legitimate to expand this in the form of a derivative expansion. It is often the case that one cannot, in fact, carry out this partial functional integration of the underlying fundamental theory either because it is technically intractable or because one does not know the underlying theory in detail. In this case, one can use a knowledge of the form of the symmetries of the underlying theory to develop an effective field theory with phenomenological coefficients which corresponds to the derivative expansion of the full theory. A classic example of this approach is chiral perturbation theory which has been used to describe the interactions of pseudo-Goldstone bosons with each other [2].

Several years ago, Weinberg suggested that the technology of EFT—when properly modified—could be used to describe low energy nuclear phenomena such as nucleon-nucleon scattering and bound states and the interaction of nuclei with pions and photons [3]. The key to this approach was the development of a formalism based on a systematic power counting scheme describing the interactions of heavy particles (where ''heavy'' means that the mass is very large compared to the momentum scale being probed). The fundamental insight is that the power counting should apply to n-particle irreducible graphs (i.e., potentials) and not to the full amplitudes. The full amplitudes are obtained by iteration of these potentials. The approach is implemented via an effective Lagrangian containing explicit light degrees of freedom (e.g., pions) along with contact interactions whose coupling constants serve to parametrize the effects of shorter range physics. Weinberg's suggestion has inspired a considerable amount of research on effective field theoretic approaches to low energy nuclear phenomena [4–16].

In this paper it will be shown that great care must be exercised when renormalizing this effective theory. A version of the formalism elucidated by Weinberg has a rather perverse feature which can be traced to the renormalization scheme: The approach is apparently incapable of describing systems whose low energy interactions are repulsive in the limit of very low energy scattering, i.e., the limit where the momenta are much less than all of the masses in the problem (so that in the nuclear case one could integrate out the pion). In such a case, as discussed in Refs. [3] and [16], one can integrate out all of the light degrees of freedom to obtain an effective Lagrangian with contact interactions only. To lowest order in the power counting, the T matrix for s-wave scattering of heavy fermions (e.g., nucleons) in Weinberg's treatment [3] depends on only a single parameter which corresponds to a particular combination of spin-independent and spin-dependent contact interactions whose renormalized value is fixed by the scattering length a:

$$T_0(p',p)=\frac{4\pi/M}{1/a+i\sqrt{ME+i\epsilon}}, \tag{1}$$

where M is the mass of the particles and p is the magnitude of the momentum of the nucleon in the center-of-mass frame. The subscript 0 indicates that this T matrix was derived from the contact interaction with no derivatives. The

energy of the state, E, is p^2/M for scattering states and the $i\epsilon$ fixes the boundary conditions in extrapolations to negative energies.

The difficulty is easy to see from Eq. (1). Elementary considerations show that a negative value of a necessarily corresponds to attraction. On the other hand, a positive value of a can either correspond to repulsion or to attraction with at least one bound state. Bound states give rise to poles in the T matrix for negative energies. Purely repulsive interactions always correspond to a T matrix without negative energy poles. From the form of the T matrix in Eq. (1), however, it is apparent that when a is positive, there is always a pole in the T matrix at $E=-1/(Ma^2)$. Thus, regardless of the sign of a, the T matrix in Eq. (1) corresponds to an attractive interaction.

How serious a problem is this? One might argue that the problem is purely formal and is of no phenomenological concern. After all, in nuclear physics the potential *is* attractive at low energies; the inability to describe repulsion may simply not be relevant. On the other hand, the EFT methods used to derive Eq. (1) are not particular to nuclear physics and never explicitly use the fact the interaction is attractive—if the arguments are valid they ought to apply equally well to cases where the interaction is repulsive. Nothing in Weinberg's power counting scheme depends on the sign of the interaction. Thus, the inability to describe repulsion suggests that something is seriously wrong with the formalism. As will be seen in this paper the difficulty can ultimately be traced to the fact that the interaction in the effective Lagrangian has zero range. The only way which an explicit range can enter into the dynamics in this approach is through regulation and renormalization prescriptions. The general issues of regulation and renormalization are clearly important in the attractive case.

It will be shown here that the problem is technical and is related to the renormalization scheme used in the derivation of Eq. (1). It should be recalled that the contact terms in an effective Lagrangian do not, in fact, describe zero range physics. Rather, they serve to parametrize the effects of physics of shorter range than the separation scale. Ultimately, the contact terms lead to divergences which necessitate some regularization prescription and an associated renormalization of the coefficients in the Lagrangian. The regularization prescription should be consistent with the fact that the interactions are, in fact, of finite range. For example, one can introduce a regulator into the contact interaction, thus making it a finite range interaction. The range of this interaction should not be taken to be zero in any intermediate step of the calculation. At the end of the calculation, the regulator parameter should be fixed by the separation scale μ. As will be discussed here, it must correspond to a larger range than the typical range of the potential (e.g., the effective range). If there is a true separation of scales in the problem, one will find that low energy physical observables will be insensitive to the precise choice of the separation scale and the form of the regulator.

In the derivation of Eq. (1), however, it was implicitly assumed that the range is, in fact, zero. That is, at various points in the calculation the contact interaction is treated literally, as opposed to merely serving to parametrize some short range physics. It has been known for some time that the repulsive δ function interaction in nonrelativistic quantum mechanics is trivial—the renormalized coupling constant must be zero and the S matrix, unity [17]; this is a consequence of Friedman's theorem [18]. Thus, it is not surprising that Eq. (1) fails to describe repulsion. Treating the contact terms literally is incompatable with the derivation of the effective field theories from an underlying theory since integrating out short range physics yields a nonlocal theory. Of course, in most applications of EFT this inconsistency is innocuous in that errors induced by it are small and can be systematically corrected at higher orders. However, in the case of two heavy particles where one must iterate the potential to all orders the problem can be serious. The inability to describe repulsion should be viewed as an artifact of this inconsistent treatment.

It is important to use a consistent regularization scheme even in the case of attractive interactions. For example, as recently noted in Ref. [16], the convergence of Weinberg's scheme is controlled by the scattering length; as the scattering length diverges the region of validity of the expansion tends to zero. In nature the scattering length is quite large, implying a very limited regime of applicability of the approach. As will be discussed briefly in this paper and in more detail in a subsequent work this is also a consequence of a renormalization scheme based on true zero range interactions. The central point of this paper is that if one wishes to use EFT methods in nuclear interactions, it is essential to regulate the contact interactions from the outset by giving them a finite range.

It is worth noting that excepting work based on a new expansion scheme proposed in Ref. [16], numerical studies of the NN force based on effective field theories and chiral counting do not employ the renormalization prescription used in the derivation of Eq. (1). Rather, they cut off the integrals in the momentum-space Schrödinger equation which effectively gives a finite range to the interactions. Thus the problems discussed here do not afflict the calculations in Refs. [4,7,13].

II. LOW ENERGY *T* MATRIX

Before discussing the problem of repulsion in any detail it is useful to review how the T matrix in Eq. (1) emerges in an effective field theory treatment. In order to use effective field theories one needs a systematic power counting scheme. Traditionally in effective field theory treatments this power counting is for a Feynman amplitude. However, as pointed out in Ref. [3], such a scheme fails for the situation where two or more heavy particles interact strongly at low energy. The difficulty is that if the particles typically have a momentum Q, the free propagator goes as M/Q^2 and becomes large in the limit of small Q, destroying simple power counting in Q/Λ. The solution to this is quite simple—instead of using power counting for the Feynman amplitude itself, one develops a systematic power counting only for the n-particle irreducible graphs—i.e., for potentials. The details of the power counting argument will not be given here as it is well described in Ref. [3].

To obtain scattering amplitudes, one can iterate these potentials to all orders which corresponds to solving the Schrödinger equation for these potentials. Recently Kaplan, Sav-

age, and Wise (KSW) have proposed a different resummation [16], in which the lowest order potential is summed to all orders as a Schrödinger equation and subsequently the inverse of the real part of the Feynman amplitude is expanded systematically. This apparently greatly improves the convergence of the expansion when the scattering length is large. However, the problem discussed here applies to the lowest order calculation of the scattering amplitude and it affects both the Weinberg and the KSW schemes.

In the case where all of the particles are treated as heavy, including all exchanged bosons, it is trivial to write down the potentials to some order. They are given in terms of an effective Lagrangian which consists entirely of contact interactions with various numbers of derivatives. This effective Lagrangian is

$$\mathcal{L}=N^\dagger i\partial_t N-N^\dagger\frac{\nabla^2}{2M}N-\frac{1}{2}C_S(N^\dagger N)^2-\frac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2\cdots, \tag{2}$$

where the ellipsis indicates contact terms with two or more derivatives. Such terms are higher order in the power counting. Isoscalar s-wave scattering only depends on the combination $C\equiv(C_S-3C_T)$.

The next step is to solve the Schrödinger equation with appropriate boundary conditions for scattering and thus determine the T matrix. This is done most naturally in the form of the Lippmann-Schwinger equation $T=V+VG_0T$, where $G_0=1/(E-p^2/M+i\epsilon)$ and p is the relative momentum operator. Clearly, this corresponds to iterating the potential to all orders. As written above, the Lippmann-Schwinger equation is an operator equation; in momentum space, it is an integral equation:

$$T(p,p')=V(p,p')+(2\pi)^{-3}\int d^3p''V(p,p'')G_0(p'';E)\times T(p'',p'), \tag{3}$$

where $G_0(p;E)=1/(E-p^2/M+i\epsilon)$. For an arbitrary V one must solve this equation via standard numerical means.

For the present case the zeroth order potential is simply a delta function in configuration space and therefore a constant in momentum space, $V_0(k,k')=C$. Formally, it is straightforward to solve the Lippmann-Schwinger equation with this potential. Since V_0 is a constant, the equation becomes algebraic; the solution is

$$T_0(p,p')=\frac{1}{1/C-(2\pi)^{-3}\int d^3p''G_0(p'',E)}. \tag{4}$$

Unfortunately, the solution is only formal since $(2\pi)^{-3}\int d^3p''G_0(p'',E)$ diverges, and so as written the solution is meaningless. This is hardly surprising—it is well known that in 3+1 dimensions, delta function potentials with finite strength are sufficiently singular as to have no well-behaved solutions.

Thus, to make sense of Eq. (4) one must renormalize. The bare parameter C must go to zero, but must do so in such a way that the T matrix remains finite. Weinberg introduces a renormalized coupling C_R given by

$$1/C_R=1/C-(2\pi)^{-3}\int d^3p''G_0(p'',E=0). \tag{5}$$

In terms of C_R the T matrix is given by

$$T_0(p,p')=\frac{1}{1/C_R-(2\pi)^{-3}\int d^3p''[G_0(p'',E)-G_0(p'',E=0)]}=\frac{4\pi}{4\pi/C_R+iM\sqrt{ME+i\epsilon}}. \tag{6}$$

The second equality is easily obtained since the integral is now convergent. Finally, identifying the zero energy T matrix as $4\pi a/M$ immediately gives a renormalization condition that $C_R=4\pi a/M$ and yields Eq. (1). It is also worth observing at this stage Weinberg's renormalization scheme is completely equivalent to dimensional regularization with the $\overline{\text{MS}}$ renormalization scheme as discussed in KSW.

III. REGULARIZATION, RENORMALIZATION, RANGE, AND REPULSION

This section addresses the question of why the calculation based on the renormalization prescription discussed in Sec. II cannot describe repulsion. As mentioned in the Introduction, this occurs because the calculation implicitly assumes that the range of the interaction is zero and not simply shorter than some separation scale. One indication that Eq. (1) is based on a true zero range interaction is the absence of any dependence on a regulator mass in the final expression for the T matrix. Indeed, in Weinberg's derivation no regularization scheme is explicitly introduced. In fact, the regulator mass has implicitly been taken to infinity at two distinct places in this calculation. The first is the derivation of Eq. (4); had a finite range been given to the interaction via any form of a regulator, one could not obtain the simple result of Eq. (4). Instead one would have had to solve an integral equation. The second place where the regulator mass was implicitly taken to infinity is in the second equality in Eq. (6).

KSW reproduce Weinberg's result using dimensional regularization. This is to be expected, since by construction dimensional regularization introduces no regulator mass. In principle, a scale can enter the problem through renormalization but, as noted by KSW, at this order the renormalization scale dependence is trivial:

$$\mu\partial_\mu(1/C_R)=0. \tag{7}$$

The KSW result is the same as Weinberg's and suggests that the lack of a regulator in the derivation of Eq. (4) is sufficient for the system to lose the information that the range of the interaction is finite.

To see that Eq. (1) does correspond to a truly zero range interaction one should study finite range interactions and then show that Eq. (1) is the zero range limit. Consider a regularization prescription where one replaces the δ function potential by a finite range potential at the beginning of the problem. If one is in the regime in which the effective field

theory is valid, then the results are insensitive to the precise form of the regulator and the precise value of the regulator mass.

For simplicity, consider a simple form for the regulated δ function—a square well of radius $1/\mu$:

$$\delta_R(\vec{x};\mu)=\frac{3\mu^3\theta(1/\mu-|x|)}{4\pi}, \tag{8}$$

where μ is the regulator mass. In coordinate space, the potential is just

$$V_0(\vec{x})=C(\mu)\delta_R(\vec{x};\mu). \tag{9}$$

The bare coefficient is written as $C(\mu)$ to indicate that the value of the coupling depends on the regulator mass μ through a renormalization condition.

It is an elementary exercise to find the T matrix associated with this potential. The phase shifts satisfy

$$p\cot(\delta)=\frac{\kappa\cot(\kappa/\mu)+p\tan(p/\mu)}{1-\kappa/p\cot(\kappa/\mu)\tan(p/\mu)}, \tag{10}$$

with

$$\kappa=\sqrt{p^2-\frac{3C(\mu)M\mu^3}{4\pi}}. \tag{11}$$

This expression is valid for both attractive and repulsive interactions. For repulsive interactions and sufficiently small p, κ becomes imaginary. The on-shell T matrix is related to $\cot(\delta)$ by

$$T(p)=\frac{-2\pi}{Mp(\cot(\delta)-i)}. \tag{12}$$

The expression for the phase shift in Eqs. (10) and (11) depends on the bare coupling $C(\mu)$. It is useful to express this in terms of a physical observable. This amounts to picking a renormalization condition for $C(\mu)$. The most natural choice is to use the scattering length which is related to the phase shifts near $p=0$,

$$\lim_{p\to 0} p\cot(\delta)=-1/a, \tag{13}$$

to fix $C(\mu)$. Using Eqs. (10), (11), and (13), one finds the following renormalization condition for $C(\mu)$:

$$\sqrt{-\frac{3C(\mu)M\mu}{4\pi}}\cot\left(\sqrt{-\frac{3C(\mu)M\mu}{4\pi}}\right)=\frac{1}{1-a\mu}. \tag{14}$$

It is straightforward to demonstrate that for attractive interactions in the limit of $\mu\to\infty$, one recovers Eq. (1). The key point is that in this limit $-C(\mu)\mu^3\to\infty$ and thus κ also diverges. Moreover, as $-C(\mu)\mu^3\to\infty$, $\kappa\to[-3C(\mu)\mu^3/4\pi]^{1/2}$ which is independent of p. Although $C(\mu)\mu^3$ diverges, $C(\mu)\mu$ can remain finite. Moreover $p/\mu\to 0$. Imposing the limit, one finds that Eq. (10) becomes

$$\lim_{\mu\to\infty,C(\mu)\mu\ \text{fixed}} p\cot(\delta)=\frac{[-3C(\mu)M\mu^3/4\pi]^{1/2}\cot\{[-3C(\mu)M\mu/4\pi]^{1/2}\}}{1-[-3C(\mu)M\mu/4\pi]^{1/2}\cot\{[-3C(\mu)M\mu/4\pi]^{1/2}\}}, \tag{15}$$

where the right-hand side of Eq. (15) is independent of p. Imposing the renormalization condition in Eq. (14) on the expression in Eq. (15) one sees that $p\cot(\delta)=-1/a$; Eq. (1) immediately follows. The conclusion of this analysis is that, as expected, Eq. (1) corresponds to an interaction of literally zero range.

Now consider what happens for a repulsive potential with $C(\mu)>0$. Formally, Eq. (15) still applies. There is a difficulty, however, in implementing the renormalization condition. For $C(\mu)>0$, Eq. (14) becomes

$$\sqrt{\frac{3C(\mu)M\mu}{4\pi}}\coth\left(\sqrt{\frac{3C(\mu)M\mu}{4\pi}}\right)=\frac{1}{1-a\mu}. \tag{16}$$

For repulsive interactions, $C(\mu)>0$ and the left-hand side of Eq. (16) is positive so that the renormalization condition can only be satisfied if

$$\mu<1/a. \tag{17}$$

Thus, when describing repulsion, one cannot take the regulator mass to infinity while still describing the correct scattering length. Indeed, when one lets $\mu\to\infty$ one is forced to have $a\to 0$ which implies a zero cross section; as $\mu\to 0$, all effects of the repulsive interaction must vanish.

Of course, the preceding analysis is just an alternative demonstration of the triviality of the repulsive delta function interaction discussed in the context of the nonrelativistic limit of ϕ^4 field theories by Bég and Furlong [17]. A rigorous mathematical proof of this was provided by Friedman [18].

There is no great mystery here. A regulated delta function of the form in Eq. (8), with an infinite strength repulsive interaction, is simply a hard core interaction of radius $1/\mu$. The scattering length for a hard core potential is just the radius of the hard core. Thus, no matter how strong the repulsion in the regulated δ function, one cannot get a scattering length greater than $1/\mu$. It is very clear why this happens, as $\mu\to\infty$, $C(\mu)$ gets large. The effect of a potential which has a large positive value over some finite region is simply to exclude the wave function from that region. As $\mu\to\infty$, how-

ever, the size of the region over which the wave function is excluded goes to zero and the effect of the repulsion vanishes.

It is worth stressing that Friedman's theorem guarantees that the inability to describe repulsion when one takes the regulator mass to infinity is a general feature and not simply a peculiar feature of the square-well regulator. This can be explicitly verified by choosing various alternative forms. For example, the regulated delta function can be chosen to be a surface delta function on a shell of radius $1/\mu$:

$$\delta_R(\vec{x};\mu)=\frac{\mu^2}{4\pi}\delta(|x|-1/\mu). \tag{18}$$

Taking $V_0(\vec{x})=C(\mu)\delta_R(\vec{x};\mu)$, calculating the T matrix, and using the scattering length to fix $C(\mu)$ gives the following renormalization condition:

$$C(\mu)=\frac{4\pi a}{M(1-\mu a)}. \tag{19}$$

As in the case of the square well regulator, one can satisfy the renormalization condition for repulsive interactions [which of necessity have $C(\mu)>0$ and $a>0$] only for $\mu<1/a$.

IV. ATTRACTIVE INTERACTIONS AND THE CONVERGENCE OF THE EFT EXPANSION

The preceding section showed that, in order to describe repulsion in an effective field theory with all exchanged particles integrated out, it was necessary to regulate the theory by giving the contact interactions a finite range. Moreover, it was seen that it was not possible to let the regulator parameter go to infinity. This section briefly discusses possible consequences of taking the regulator mass to infinity for attractive interactions. It is easy to see that the problems arise with such a scheme when the scattering length is either very large or very small. The case of large scattering length is of particular importance since in the nuclear physics case the scattering length in the singlet channel is very large. This situation was discussed by KSW who point out that Weinberg's scheme, when implemented with dimensional regularization and $\overline{\text{MS}}$ renormalization, breaks down at a momentum scale set by the scattering length. As the scattering length goes to infinity, Weinberg's approach breaks down for lower and lower momenta; if a were infinite, Weinberg's expansion would break down for arbitrarily small p and thus be of no utility.

KSW suggest that this breakdown is a consequence of strong correlations between the coefficients of contact terms at different orders in the EFT expansion of the potential. They propose to avoid this difficulty by expanding $p\cot\delta$ rather than by expanding the potentials and iterating to all orders as proposed by Weinberg. At first glance the explanation for the breakdown of Weinberg's scheme seems quite unnatural; it depends on a conspiracy among the higher order terms. On the other hand, one might argue that generically the scattering length should be of order $1/\Lambda$ and that having a very long scattering length—one much longer than $1/\Lambda$—is, in itself, unnatural. Thus, one might expect that to describe such a situation an *a priori* unlikely correlation among various terms in the expansion is not absurd. However, even if there are correlations of the form postulated by KSW, there is still a problem. The conventional power counting scheme requires that the contribution of V_2, the two-derivative contact interaction, to the T matrix be down by a power of p^2/Λ^2, compared to the effect of V_0; this should hold up to momenta of order Λ. KSW show explicitly that this fails for large a when dimensional regularization and $\overline{\text{MS}}$ renormalization is used. This raises a thorny question since there is no obvious flaw with conventional power counting arguments and the power counting does not obviously depend on the scattering length being small.

In this section, an alternative explanation for the breakdown of Weinberg's scheme at low p for large a will be explored. It will be argued that the breakdown is another consequence of taking the regulator mass to infinity and is not an intrinsic defect in the expansion.

In many ways, this problem is quite analogous to the difficulty of describing repulsion. In the repulsion case, the range of the interaction was intrinsic to the description—the scattering length was always smaller than the range of the potential. Thus any scheme which treats the range as being zero is destined to fail. The effective range in the case of infinite scattering length is similar. Recall that the effective range r_0 is defined in terms of an expansion of $p\cot(\delta)$,

$$p\cot(\delta)=-1/a+\tfrac{1}{2}r_0p^2+\cdots. \tag{20}$$

Suppose, for example, that the underlying dynamics were in fact a square well. Then it is trivial to show from Eqs. (10), (11), and (20) that when the scattering length is infinite, the effective range is just the radius of the well. Thus, the physical size of the well is an essential part of the physics of the effective range when a is infinite. It will hardly be surprising if it turns out not to be possible to describe this by a zero range interaction.

Consider the treatment of the physics of the effective range in Weinberg's scheme. Clearly it depends on V_2, the two-derivative contact term in the effective Lagrangian. Formally, the effects of this are order p^2/Λ^2 suppressed relative to V_0. Although there are several terms in the Lagrangian of this order, only one linear combination plays a role in the singlet s-wave channel and one can write V_2 as

$$V_2(\vec{p}\,',\vec{p})=\frac{C_2}{2}(p^2+p'^2). \tag{21}$$

Iterating this potential, using dimensional regularization and $\overline{\text{MS}}$ renormalization, and using the scattering length r_0 to fix the renormalized C_2 gives the following T matrix [16]:

$$T_2(p',p)=\frac{4\pi/M}{(a+\frac{1}{2}a^2r_0p^2)^{-1}+i\sqrt{ME+i\epsilon}}. \tag{22}$$

The subscript 2 indicates that this T matrix includes the effects of contact interactions with up to two derivatives.

By conventional power counting one expects $T_2=T_0[1+\mathcal{O}(p^2/\Lambda^2)]$. However, expanding Eqs. (22) and comparing with Eq. (1) one sees that

$$T_2(p',p)=T_0(p',p)[1+\tfrac{1}{2}ar_0p^2+\mathcal{O}(p^3a^2r_0)]. \quad (23)$$

Thus, for $a \gg r_0$, the effects of V_2 becomes comparable to the the effects V_0 when $p \sim (ar_0)^{-1/2}$. This is a signature of the breakdown of the power counting argument. If $a \to \infty$, the momentum scale at which the power counting breaks down goes to zero.

This problem can be avoided quite simply if the δ-function interactions are regulated from the beginning. The basic strategy is to exploit the freedom in choosing μ. If one begins with regulated δ functions, the strength of both C_0 and C_2 depends on both the renormalization conditions (fixed by a and r_0) and the regulator mass μ. In principle, all physical results should be independent of μ since it is an artificial parameter introduced only for convenience. However, the full theory is not being solved; within a given approximation scheme, results do depend on μ, albeit only weakly. One can exploit the freedom in choosing μ to improve the convergence of the approximation scheme. An optimal choice of μ is one which minimizes the errors associated with truncating the expansion. Thus, for example, in perturbative QCD treatments of deep inelastic scattering one chooses the factorization scale μ to be of order Q^2 in order to avoid large logarithms in the higher order corrections. In an analogous fashion, for the present problem one can fix μ so as to minimize the higher order corrections of the EFT expansion. In particular, one can choose μ so that $C_2=0$. This is possible for any reasonable regulator since one can fix r_0 and a from the range and depth of the regulated δ function of V_0. With this optimal regulator $T_2=T_0$ for all p and the difficulty of the expansion breaking down at low p is avoided. More generally, one expects that if a nonoptimal regulator mass μ comparable to or less than $1/r_0$ were chosen, then $T_2=T_0[1+\mathcal{O}(p/\mu)]$. This will be studied in a subsequent publication.

There is also a problem with this treatment in the limit $a \to 0$. This corresponds to a zero T matrix at zero energy. This situation can occur in a nontrivial way if the underlying potential has both attraction and repulsion whose effects cancel at zero energy; it can also occur in an attractive potential with a sufficiently deeply bound state. In general, for scattering problems with nonzero potentials and $a=0$, the T matrix is zero only for zero energy. For a generic interaction tuned to give $a=0$, normal power counting would lead one to expect that $T \sim p^2/\Lambda^2$. In contrast, consider Eq. (22). As a goes to zero, T_2 goes to zero for all p violating the conventional power counting arguments. Again this represents a serious difficulty since nothing in the conventional power counting depends in an obvious way on a being nonzero. This problem is also an artifact of imposing an infinite cutoff.

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