Scalar-isoscalar meson exchange in the calculation of the nucleon-nucleon interaction

C. M. Shakin* and Wei-Dong Sun
Department of Physics and Center for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210
(Received 15 April 1996)

We provide a unified description of (i) scalar-isoscalar exchange in the nucleon-nucleon interaction, (ii) the pion-nucleon sigma term, and (iii) the scalar form factor of the nucleon. Our analysis requires that we specify a parameter that appears in the description of a nucleon valence-quark ''core.'' Other parameters are fixed either by our analysis of the Nambu–Jona-Lasinio model, or with reference to a recent lattice simulation of QCD in which the scalar form factor of the nucleon was calculated. We find that our model has some predictive power. Once the parameters are fixed, we find that we reproduce the values of the scalar form factor of the nucleon, as determined in the lattice simulation. We also predict the strength of the scalar-isoscalar *NN* potential for the particular one-boson-exchange model considered here, where the effects of (virtual) Δ excitation are treated in an explicit fashion. However, the overall strength of the force obtained in this work is sensitive to the approximations used in the calculation. [S0556-2813(97)01802-5]

PACS number(s): 21.30.Cb, 12.39.Fe, 12.39.Ki

I. INTRODUCTION

This paper is mainly concerned with the role of a low-mass σ meson in nuclear physics. Such a meson plays an important role in the one-boson-exchange model of the nucleon-nucleon interaction, where it is usually assigned a mass of about 550 MeV [1]. A low-mass σ is needed to provide the intermediate-range attraction in the *NN* interaction. It also plays an important role in the Walecka model [2] and in relativistic Brueckner-Hartree-Fock theory [3]. When these many-body theories are used to describe nuclear matter, a mean scalar field of about -400 MeV is obtained. (There is also a large repulsive vector field due to ω exchange, with a magnitude of approximately 300 MeV.) It has been suggested that the large scalar field in matter is an order parameter describing the reduction of the $\overline{q}q$ condensate from its vacuum value in the presence of matter [4]. With that interpretation one can readily make contact with the body of work that deals with QCD sum rules in matter [5–7].

One approach to the calculation of the intermediate-range attraction in the nucleon-nucleon force is through the use of dispersion relations. In that work the σ appears as an *effective* meson that is introduced to represent the effects of correlated two-pion exchange [8]. While that may be correct, the dispersion-relation formalism does not relate the effective σ to the $\overline{q}q$ condensate, so that the basis found for the σ in correlated two-pion exchange does not make contact with the work on QCD sum rules in matter [5–7], nor with the suggestion of partial restoration of chiral symmetry at finite baryon density [4].

Further evidence for the need for a low-mass scalar is seen in the recent work of Furnstahl, Serot, and Tang [9]. Their goal was to create an effective chiral Lagrangian that is consistent with the symmetries of QCD. In that regard, it is natural to adopt the nonlinear σ model, since that model forms the basis for chiral perturbation theory and many other applications of chiral symmetry in particle physics [10]. However, in Ref. [9] it was necessary to introduce an additional low-mass scalar of the type that appears naturally in the linear σ model. Thus, we are back to the problem of describing the origin and nature of such a scalar field. We will argue in the following that the difficulty of interpretation of the low-mass scalar is made less severe when we recognize that the meson momenta in nuclear structure studies are *spacelike*.

While the above remarks indicate the importance of a low-mass scalar-isoscalar meson in various aspects of nuclear physics, one has the problem that such a meson has not appeared in the data tables until quite recently. Now one finds a f_0 meson of mass between 400 and 1200 MeV in the latest list of elementary particles and resonances [11]. More to the point, we have the recent work of Törnqvist and Roos, who find strong evidence in their theoretical study for a σ meson with Breit-Wigner parameters m_σ=860 MeV and Γ_σ=880 MeV [12]. As discussed in Ref. [12], the σ field is an extremely important degree of freedom, since it plays the role of the ''Higgs meson'' in QCD. That remark is based upon the relation of the σ field to the $\overline{q}q$ condensate which, in turn, is responsible for a large part of the mass of the constituent quark and the nucleon, for example [13]. [Törnqvist and Roos also claim that the very broad σ resonance is responsible for generating most of the pion-pion phase shift $\delta_0^0(L=0,T=0)$ for energies less than 900 MeV.]

Now, if we consider the Nambu–Jona-Lasinio (NJL) model [14] with SU(2) flavor, one finds a scalar-isoscalar state, with $m_\sigma^2=4m_q^2+m_\pi^2$. Here, m_q is the constituent quark mass. For m_q=260 MeV, we have m_σ=540 MeV. That is certainly of interest, since it corresponds to the low-mass σ meson needed for nuclear physics studies. As we will see, upon bosonization of the NJL model, the σ field is directly related to the expectation value of the scalar quantity $\overline{q(x)}q(x)$. (In vacuum the σ field has the value f_π.)

One problem has been to reconcile the absence of a low-mass σ meson in the data tables with the mode found in the NJL model. In our work, we have seen that the inclusion of a model of confinement in the NJL model moves the energy of the σ excitation upward by about 300 MeV. It then has an

*Electronic address: casbc@cunyvm.cuny.edu

energy quite consistent with that found by Törnqvist and Roos [12]. However, it is important to remember that the momenta of mesons used in nuclear structure physics are spacelike ($q^2 \leqslant 0$). We find that the physical state at 860 MeV does not govern the dynamics in the spacelike region, where the (effective) σ mass is still 540 MeV. (We have seen, in an earlier work, that the coupling of the σ to the two-pion continuum becomes progressively less important as one moves further into spacelike domain for the meson momentum, that is, as $-q^2$ increases [15].) We, therefore, suggest that we need not consider the properties of the σ for timelike values of the momentum and that we can make use of the results of a study of the NJL model when the meson momenta are spacelike. One way to carry out this program is to study the quark-quark scattering amplitude in the NJL model. We find in such a study that, if we limit consideration to spacelike scalar-isoscalar exchange, the T matrix behaves exactly as if there were a pole in the timelike region with $m_\sigma \simeq 540$ MeV.

In this work we will use some information gained from a recent QCD lattice simulation [16], where a study was made of the pion-nucleon sigma term and the scalar form factor of the nucleon. One of our goals is to study the scalar-isoscalar component of the nucleon-nucleon interaction, making use of the extended version of the Nambu–Jona-Lasinio model that we have developed [17–19]. In particular, we are interested in understanding the parametrization of the one-boson-exchange (OBE) model of the nucleon-nucleon interaction [1] in the case of scalar exchange.

The organization of our work is as follows. In Sec. II we review a calculation of the pion-nucleon σ term σ_N made earlier and also define the scalar form factor of the nucleon $F_S(q^2)$. We also show how to rewrite the quark-quark T matrix in terms of meson masses and momentum-dependent coupling constants. This is a kind of ''momentum space bosonization.'' In Sec. III we discuss the role of sigma exchange in the one-boson-exchange model of the nucleon-nucleon force and also provide a fit to the scalar form factor of the nucleon obtained in the lattice QCD study [16]. Finally, Sec. IV contains some further discussion and conclusions.

II. NUCLEON SCALAR FORM FACTOR AND THE PION-NUCLEON σ TERM

It is useful to start our discussion with a review of our calculation of the nucleon scalar-isoscalar form factor at zero momentum transfer [5]. Let us define the scalar form factor $F_S(q^2)$, such that

$$F_S(q^2)\overline{u(\mathbf{p}+\mathbf{q},s')}u(\mathbf{p},s)\,\delta_{\tau\tau'} = \langle N,\mathbf{p}+\mathbf{q},s',\tau'|\overline{q(0)}q(0)|N,\mathbf{p},s,\tau\rangle. \quad (2.1)$$

[Here $q(x)$ is the quark field and $u(\mathbf{p},s)$ is a Dirac spinor.] In our earlier work, we calculated $F_S(0)$ in a σ-dominance model [20]. (See Fig. 1.) In Fig. 1 we show the various ingredients of the calculation. The operator $\overline{q(0)}q(0)$ is represented by a large filled circle. The single lines represented (constituent) quarks, while the string of $q\overline{q}$ loops may be expressed in terms of the basic quark-antiquark loop integral of the NJL model $J_S(q^2)$ [17,20].

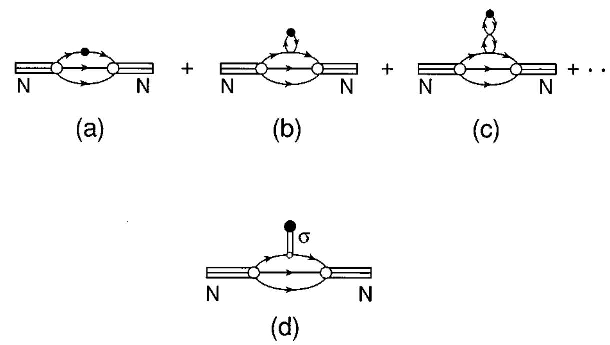

FIG. 1. Calculation of the scalar form factor of the nucleon. The operator $\overline{q(0)}q(0)$ is denoted by the large filled circle. The single lines represent quarks or antiquarks. (a) The valence contribution is shown; (b) and (c) A series of quark-antiquark loop diagrams are shown; (d) A σ dominance model representing the diagrams shown in (a), (b), (c), etc. There the small open circle represents $g_{\sigma qq}$.

The first diagram gave a contribution to $F_S(0)$ that we may denote as $\langle N|\overline{q}q|N\rangle_{\rm val}$, where the subscript indicates that we have separated the form factor into a valence part and a part due to the ''meson cloud'' [16,20]. Thus

$$F_S(0) = \langle N|\overline{q}q|N\rangle_{\rm val} + \langle N|\overline{q}q|N\rangle_C. \quad (2.2)$$

The value obtained for $F_S(0)$ in the QCD simulation was $F_S(0) \simeq 10.0$, with 85% of this value coming from the up and down quark contribution. It was also found that $\langle N|\overline{q}q|N\rangle_{\rm val} = 3.02$ [16]. (See Table I.) The diagram representing the valence contribution in Ref. [16] was the same as the diagram of Fig. 1(a). In Ref. [16] that diagram represented a connected amplitude, while the remaining (meson cloud) contribution was given by a disconnected diagram. It was also found that the meson cloud contribution was about twice the valence contribution. It is of interest to see that our σ-dominance calculation has the same feature. To understand that comment, we present the result of the analysis given in Ref. [20]. There we had

TABLE I. Results of lattice simulation and of the SU(2) NJL model. [We identify $F_S^{\rm val}(0)$ with $g_{S,\rm con}$ of Ref. [16].]

	Ref. [16]	This work
$\left(\frac{m_u^0+m_d^0}{2}\right)$	5.84 (13)	5.50 MeV
$\langle p\|\overline{u}u\|p\rangle$	4.55 (16)	
$\langle p\|\overline{d}d\|p\rangle$	3.92 (16)	
$\langle N\|\overline{s}s\|N\rangle$	1.53 (7)	0
$\langle N\|\overline{u}u+\overline{d}d\|N\rangle$	8.47 (24)	9.42
$F_S(0)=\langle N\|\overline{u}u+\overline{d}d+\overline{s}s\|N\rangle$	10.00 (25)	9.42
$\Delta\sigma_N=\sigma_N(2m_\pi^2)-\sigma_N(0)$	6.62 (59) MeV	~6.6 MeV
$\sigma_N(0)$	49.7 (2.6) MeV	51.8 MeV
$F_S^{\rm val}(0)$	3.02 ± 0.09	3.02 (taken from Ref. [16])
$y=\frac{2\langle N\|\overline{s}s\|N\rangle}{\langle N\|\overline{u}u+\overline{d}d\|N\rangle}$	0.36 (3)	0

TABLE II. The parameters of the NJL model (Λ_E, m_q^0, G_S) used in Ref. [20] and the quantities calculated in Ref. [20] are given.

Λ_E	1.0 GeV
m_q^0	5.5 MeV
G_S	7.91 GeV^{-2}
m_q	260 MeV
m_σ	538 MeV
$1-G_SJ_S(0)$	0.321
$[1-G_SJ_S(0)]^{-1}$	3.12
$g_{\sigma qq}$	2.58
$g_{\pi qq}$	2.68
f_π	93 MeV
m_π	138 MeV
$-2m_q^0\langle 0\vert\overline{u}u\vert 0\rangle$	1.76×10^8 MeV^4
$\langle 0\vert\overline{u}u\vert 0\rangle^{1/3}$	-252 MeV
σ_N	51.5 MeV

$$\langle N|\overline{q}q|N\rangle = \frac{1}{1-G_SJ_S(0)}\langle N|\overline{q}q|N\rangle_{\text{val}}, \tag{2.3}$$

where the factor $[1-G_SJ_S(0)]^{-1}$ serves to generate the series depicted in Figs. 1(a), 1(b), 1(c), etc. We may write Eq. (2.3) as

$$\langle N|\overline{q}q|N\rangle = \langle N|\overline{q}q|N\rangle_{\text{val}} + \left(\frac{1}{1-G_SJ_S(0)} - 1\right)\langle N|\overline{q}q|N\rangle_{\text{val}}, \tag{2.4}$$

where the second term is the ''meson cloud'' contribution.

We used $G_S=7.91$ GeV^{-2} and found $[1-G_SJ_S(0)]^{-1}=3.12$ in Ref. [20]. (See Table II.) Thus, since we had taken $\langle N|\overline{q}q|N\rangle_{\text{val}}=3.0$, we obtained $\langle N|\overline{q}q|N\rangle=9.36$. In Eq. (2.4) the meson cloud contribution is 6.36, which is 2.12 times the valence contribution. While it is not clear whether our meson cloud contribution is directly related to the disconnected (meson cloud) contribution of Ref. [16], the numerical values obtained are remarkably similar. Note further that we may define the pion-nucleon σ term for $q^2=0$, with $\overline{q}q=\overline{u}u+\overline{d}d$, as

$$\sigma_N = \left(\frac{m_u^0+m_d^0}{2}\right)\langle N|\overline{q}q|N\rangle, \tag{2.5}$$

where m_u^0 and m_d^0 are the current quark masses. In our earlier work we found that the average current quark mass was 5.50 MeV, if we were to obtain $m_\pi=138$ MeV, when we used a Euclidean cutoff of $\Lambda_E=1000$ MeV. (See Table II.) Thus, we had $\sigma_N=51.5$ MeV, which is very close to the value obtained in the QCD lattice simulation: $\sigma_N=49.7$ (2.6) MeV. (Here 2.6 MeV represents a measure of the theoretical uncertainty.) Our result is also consistent with the analysis of Vogl and Weise [21] that is based upon the use of the Feynman-Hellman theorem [22,23] in the calculation of σ_N. Vogl and Weise calculate a pion-quark σ term and then multiply by 3 to obtain the pion-nucleon σ term. It was found that [21,24]

$$\sigma_N = 3m_\pi^2\left(\frac{m_u}{4m_u^2+m_\pi^2}\right). \tag{2.6}$$

Our analysis of the NJL model found $m_q=260$ MeV for $\Lambda_E=1.0$ GeV. Therefore, from Eq. (2.6), we have $\sigma_N=51.3$ MeV, which is quite close to the value $\sigma_N=51.5$ MeV obtained from our study of the scalar form factor [20]. We remark that, in the work of Vogl and Weise [21], up and down quark masses of about 360 MeV are used. Use of $m_u=360$ MeV in Eq. (2.6) leads to $\sigma_N=38$ MeV. That value is considered to be too small and, therefore, other features (such as diquark correlations in the nucleon) that enhance the calculated value of σ_N are studied [21].

In order to introduce the σ ''meson'' into the analysis, it is useful to write

$$\frac{G_S}{1-G_SJ_S(q^2)} = -\frac{g_{\sigma qq}^2(q^2)}{q^2-m_\sigma^2} \tag{2.7}$$

for $q^2<0$. Equation (2.7) serves to define the momentum-dependent meson-quark coupling parameter $g_{\sigma qq}(q^2)$. For values of $-q^2$ that are positive, and not too large, we may write [21]

$$\frac{G_S}{1-G_SJ_S(q^2)} \simeq -\frac{g_{\sigma qq}^2}{q^2-m_\sigma^2}, \tag{2.8}$$

$$\frac{1}{1-G_SJ_S(q^2)} = -\left(\frac{g_{\sigma qq}}{G_S}\right)\frac{1}{q^2-m_\sigma^2}\,g_{\sigma qq}, \tag{2.9}$$

where $g_{\sigma qq}=2.58$ and $m_\sigma=540$ MeV [20]. Thus, the factor $(-g_{\sigma qq}/G_S)$ in Eq. (2.9) may be understood as arising from the bosonization relation

$$\sigma(x) = -\frac{G_S}{g_{\sigma qq}}\,\overline{q}(x)q(x) \tag{2.10}$$

that appears in the simplest bosonization scheme used for the NJL model. [In vacuum, Eq. (2.9) may be identified with the Goldberger-Treiman relation $f_\pi=m_q/g_\pi$, where for exact chiral symmetry $g_\pi\equiv g_{\sigma qq}=g_{\pi qq}$.] Equation (2.8) and Eq. (2.3) may be used to generate the diagrams of the σ-dominance model, such as that shown in Fig. 1(d).

While the comments made above are suggestive, we still have to describe how these ideas may be used to discuss σ exchange in the nucleon-nucleon interaction. That discussion will be taken up in the next section.

III. σ EXCHANGE IN THE OBE MODEL OF THE NUCLEON-NUCLEON INTERACTION

In earlier work we have discussed a relation between the OBE model of the nucleon-nucleon interaction and the NJL model [25–27]. To describe that relation, it is useful to define a parameter λ_σ that governs the momentum dependence of the *valence* scalar form factor of the nucleon:

$$F_S^{\text{val}}(q^2) = F_S^{\text{val}}(0)\left(\frac{\lambda_\sigma^2}{\lambda_\sigma^2-q^2}\right), \tag{3.1}$$

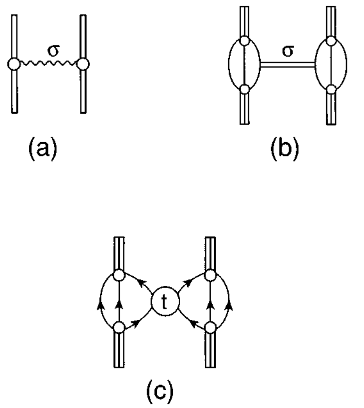

FIG. 2. (a) The OBE amplitude due to σ exchange between nucleons. The large open circles denote the vertex cutoffs of the OBE model [1]. (b) The representation of σ exchange in the NJL model based upon the use of the valence form factor seen in Fig. 1(a). (c) The nucleon-nucleon interaction is related to a quark-quark T matrix. A σ dominance model of the T matrix is shown in (b).

with $F_S^{\mathrm{val}}(0)=\langle N|\overline{q}q|N\rangle_{\mathrm{val}}$. We put $F_S^{\mathrm{val}}(0)=3.02$ in accordance with the analysis of Ref. [16]. (See Table I.)

In the OBE model there are vertex cutoffs that are dependent upon a parameter Λ_σ. Thus the OBE force in the scalar channel is [1]

$$V_\sigma^{\mathrm{OBE}}(q^2)=g_{\sigma NN}^2\left(\frac{\Lambda_\sigma^2-m_\sigma^2}{\Lambda_\sigma^2-q^2}\right)^2\frac{1}{q^2-m_\sigma^2}. \tag{3.2}$$

(We suppress reference to Dirac and isospin matrices for simplicity.) In Table B.1 of Ref. [1], we find $\Lambda_\sigma=1.5$ GeV, $g_{\sigma NN}^2/4\pi=6.32$, and $m_\sigma=550$ MeV. (These parameters are given for the case where the effects of the virtual excitation of the delta are treated explicitly.) It is useful to define

$$\frac{(G_{\sigma NN}^{\mathrm{OBE}})^2}{4\pi}=\frac{g_{\sigma NN}^2}{4\pi}\left(\frac{\Lambda_\sigma^2-m_\sigma^2}{\Lambda_\sigma^2}\right)^2. \tag{3.3}$$

With the values given above, we find $G_{\sigma NN}^{\mathrm{OBE}}=7.71$.

The corresponding amplitude in the NJL model is given in terms of the quark-quark T matrix $t_{qq}(q^2)$, such that

$$V_\sigma^{\mathrm{NJL}}(q^2)=t_{qq}(q^2)[F_S^{\mathrm{val}}(0)]^2\left(\frac{\lambda_\sigma^2}{\lambda_\sigma^2-q^2}\right)^2 \tag{3.4}$$

$$=\frac{g_{\sigma qq}^2}{q^2-m_\sigma^2}[F_S^{\mathrm{val}}(0)]^2\left(\frac{\lambda_\sigma^2}{\lambda_\sigma^2-q^2}\right)^2. \tag{3.5}$$

[See Fig. 2(b).] For consistency we should have $G_{\sigma NN}^{\mathrm{NJL}}=G_{\sigma NN}^{\mathrm{OBE}}$, with

$$G_{\sigma NN}^{\mathrm{NJL}}=g_{\sigma qq}F_S^{\mathrm{val}}(0). \tag{3.6}$$

Evaluation of the right-hand side of Eq. (3.6) with $g_{\sigma qq}=2.58$ and $F_S^{\mathrm{val}}(0)=3.02$ yields $G_{\sigma NN}^{\mathrm{NJL}}=7.79$, so that $G_{\sigma NN}^{\mathrm{OBE}}/G_{\sigma NN}^{\mathrm{NJL}}=0.99$. Thus, we see that the strength of the force is predicted accurately for this choice of parameters.

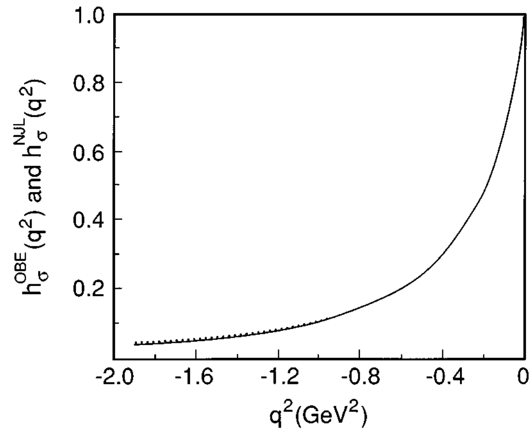

FIG. 3. The values of $h_\sigma^{\mathrm{OBE}}(q^2)$ (dotted line) and $h_\sigma^{\mathrm{NJL}}(q^2)$ (solid line) are shown. Here $\Lambda_\sigma=1.5$ GeV and $\lambda_\sigma=1.10$ GeV. For the OBE amplitude $m_\sigma=0.550$ GeV. [See Eqs. (3.7) and (3.8)].

As in our previous work, we determine λ_σ by equating $V_\sigma^{\mathrm{NJL}}(q^2)$ and $V_\sigma^{\mathrm{OBE}}(q^2)$. We find that $\lambda_\sigma=1.1$ GeV yields an excellent fit. For example, we may put

$$V_\sigma^{\mathrm{NJL}}(q^2)=V_\sigma^{\mathrm{NJL}}(0)h_\sigma^{\mathrm{NJL}}(q^2) \tag{3.7}$$

and

$$V_\sigma^{\mathrm{OBE}}(q^2)=V_\sigma^{\mathrm{OBE}}(0)h_\sigma^{\mathrm{OBE}}(q^2), \tag{3.8}$$

where $h_\sigma^{\mathrm{NJL}}(0)=h_\sigma^{\mathrm{OBE}}(0)=1$. We compare the values of $h_\sigma^{\mathrm{OBE}}(q^2)$ and $h_\sigma^{\mathrm{NJL}}(q^2)$ in Fig. 3 for the case $\lambda_\sigma=1.1$ GeV. It is seen that the fit is excellent for the chosen value of λ_σ.

We now ask whether our formalism has any further predictive power. To that end, we may now calculate our values for the scalar form factor of the nucleon. We see that in our σ-dominance model

$$F_S(q^2)=\frac{1}{1-G_SJ_S(q^2)}F_S^{\mathrm{val}}(0)\left(\frac{\lambda_\sigma^2}{\lambda_\sigma^2-q^2}\right), \tag{3.9}$$

where $F_S(0)=3.02$, as determined in Ref. [16]. (See Table I.)

We now define

$$f_S(q^2)=\frac{F_S(q^2)}{F_S(0)} \tag{3.10}$$

$$=\frac{[1-G_SJ_S(0)]}{[1-G_SJ_S(q^2)]}\left(\frac{\lambda_\sigma^2}{\lambda_\sigma^2-q^2}\right). \tag{3.11}$$

In Fig. 4, the solid line represents $F_S(q^2)$, while the "data points" are the result of the QCD lattice simulation of the scalar form factor [16]. Again, we find a good fit for our choice of $\lambda_\sigma=1.1$ GeV.

IV. DISCUSSION

In our work we have stressed that the linear σ model is quite useful for nuclear physics studies. However, that does not argue against the use of the nonlinear σ model. For example, the *physical* σ mass has been given as 860 MeV, with a large width of 880 MeV [12]. Therefore, if one wishes to

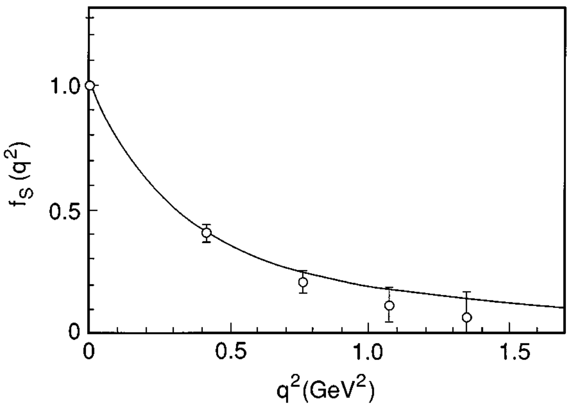

FIG. 4. The values calculated for $f_S(q^2) = F_S(q^2)/F_S(0)$ are shown. [See Eqs. (3.9) and (3.11).] Here $\lambda_\sigma = 1.10$ GeV. The circles with error bars are results of the QCD lattice simulation of Ref. [16].

write a Lagrangian describing meson-meson interactions, the neglect of the scalar octet is probably a good approximation.

On the other hand, if one limits oneself to the consideration of only *spacelike* momenta, the linear σ model is a useful effective theory. (We note that the linear σ model has been studied by Ko and Rudaz, who have also included vector mesons in the effective Lagrangian [28]. They attempt to relate the σ to observed scalar states and therefore need to find a mechanism for reducing the large width of the σ for $\sigma \to \pi + \pi$. They find such a mechanism when including $\pi - a_1$ coupling. This model is therefore quite different than that of Ref. [12], where the σ has a very large width (880 MeV) and is not identified with any scalar-isoscalar mesons previously known.)

We have seen that we have obtained an excellent fit to the strength of the OBE scalar-isoscalar force and to the q^2 dependence of that force. We also fit the values of the scalar form factor of the nucleon that were obtained in a QCD lattice simulation [16]. However, some cautionary remarks are in order. Our results are sensitive to the value obtained for $[1 - G_S J_S(0)]$. Various modifications may be considered. For example, if we include a model of confinement, $J_S(0)$ is replaced by $\hat{J}_S(0)$ which is about 6 percent smaller than $J_S(0)$ [25,26]. However, we may also include coupling to the two-pion continuum, as discussed in our earlier work [15,17]. If both effects are included, $[1 - G_S J_S(0)]$ is replaced by $\{1 - G_S[\hat{J}_S(0) + \hat{K}_S(0)]\}$, where $\hat{K}_S(q^2)$ describes the coupling of the $q\bar{q}$ states to the two-pion continuum. We remark that $\hat{K}_S(0)$ is positive and about 10% of the value of $J_S(0)$. Therefore, our results are only slightly modified in a first approximation, if we include such effects. These corrections suggest that the rather precise agreement found for $G^{\rm NJL}_{\sigma NN}$ and $G^{\rm OBE}_{\sigma NN}$ may be somewhat fortuitous. However, small modifications of the value of $[1 - G_S J_S(0)]$ will not change the general success of our analysis. We have also seen that the value of the enhancement factors, $[1 - G_S J_S(0)]^{-1} = 3.12$, found in Ref. [20] and used here, provides a satisfactory value for σ_N. The associated value of the constituent quark mass, $m_q = 260$ MeV, also yields a good phenomenological description of the nucleon scalar form factor and the OBE potential in the scalar-isoscalar channel. In a future work we hope to extend our model to study the SU(3)-flavor version of the NJL model so that we can make a more detailed comparison to the results of Ref. [16].

ACKNOWLEDGMENTS

This work is supported in part by a grant from the National Science Foundation and by PSC-CUNY Faculty Research Program.

[1] R. Machleidt, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1989), Vol. 19.

[2] B. D. Serot and J. D. Walecka, in *Advances in Nuclear Physics*, edited by J. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.

[3] L. S. Celenza and C. M. Shakin, *Relativistic Nuclear Physics: Theories of Structure and Scattering* (World Scientific, Singapore, 1986).

[4] B. Goulard, L. S. Celenza, and C. M. Shakin, Phys. Rev. D **24**, 912 (1981); L. S. Celenza, A. Pantziris, C. M. Shakin, and Wei-Dong Sun, Phys. Rev. C **45**, 2015 (1992); L. S. Celenza, A. Pantziris, L. S. Celenza, and Wei-Dong Sun, *ibid.* **46**, 571 (1992).

[5] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. Lett. **67**, 961 (1991).

[6] C. M. Shakin, Phys. Rev. C **50**, 1129 (1994).

[7] R. J. Furnstahl, D. K. Griegel, and T. D. Cohen, Phys. Rev. C **46**, 1507 (1992); X. Jin, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, *ibid.* **47**, 2882 (1993).

[8] J. W. Durso, M. Saavela, G. E. Brown, and B. J. Verwest, Nucl. Phys. **A278**, 445 (1997); J. W. Durso, A. D. Jackson, and B. J. Verwest, *ibid.* **A345**, 471 (1980); W. Lin and B. D. Serot, *ibid.* **A512**, 637 (1990); G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976).

[9] R. J. Furnstahl, B. D. Serot, and Hua-Bin Tang, Ohio State University Report No. OSU-96-612, nucl-th/9608035 1996.

[10] A useful introduction to chiral perturbation theory and the linear and nonlinear σ model is found in J. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992); For an earlier discussion of the linear and nonlinear σ model see V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973), Chap. 5. Recent work on the dynamical generation of the gauged linear σ model may be found in R. Delbourgo and M. D. Scadron, Mod. Phys. Lett. A **10**, 251 (1995).

[11] R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996). See the listing for the f_0 (400–1200).

[12] N. A. Törnqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).

[13] B. I. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981).

[14] Reviews of the NJL model are found in U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991); S. P. Klevan-

sky, Rev. Mod. Phys. **64**, 649 (1992); T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 223 (1994).
[15] L. S. Celenza, C. M. Shakin, and J. Szweda, Int. J. Mod. Phys. E **2**, 437 (1993).
[16] S. J. Dong, J.-F. Lagaë, and K. F. Liu, University of Kentucky Report No. UK/95-12 (1995).
[17] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Int. J. Mod. Phys. E **2**, 603 (1993).
[18] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Phys. Rev. D **51**, 3636 (1995).
[19] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Ann. Phys. (N.Y.) **241**, 1 (1995).
[20] Nan-Wei Cao, C. M. Shakin, and Wei-Dong Sun, Phys. Rev. C **46**, 2535 (1992).
[21] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991).
[22] R. P. Feynman, Phys. Rev. **56**, 340 (1939).
[23] H. Hellman, *Einführung in die Quantenchemie* (Deutsche Verlag, Leipzig, 1937).
[24] U. Vogl, Z. Phys. A **337**, 191 (1990).
[25] C. M. Shakin, Wei-Dong Sun, and J. Szweda, Phys. Rev. C **52**, 3353 (1995).
[26] Shun-fu Gao, L. S. Celenza, C. M. Shakin, Wei-Dong Sun, and J. Szweda, Phys. Rev. C **53**, 1936 (1996).
[27] L. S. Celenza, C. M. Shakin, and Wei-Dong Sun, Phys. Rev. C **54**, 487 (1996).
[28] P. Ko and S. Rudaz, Phys. Rev. D **50**, 6877 (1994).