S-matrix and *R*-matrix determination of the low-energy ^{5}He and ^{5}Li resonance parameters

Attila Csótó[1,2] and G. M. Hale[2]
[1]*National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824*
[2]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
(Received 6 August 1996)

We study the low-energy $3/2^-$ and $1/2^-$ states of ^{5}He and ^{5}Li in a microscopic cluster model. The scattering phase shifts of Bond $(\alpha+n)$ and of Schwandt $(\alpha+p)$, respectively, are well reproduced. We determine the resonance parameters by localizing the poles of the analytically continued *S* matrix at complex energies. Our results differ from conventional *R*-matrix resonance parameters, that were extracted from experimental data using the definition of a resonance based on the positions and widths of reaction cross section peaks. However, they nicely agree with the results of an extended *R*-matrix method that works at complex energies. [S0556-2813(97)05101-7]

PACS number(s): 27.10.+h, 24.30.Gd, 21.60.Gx

Wigner's *R* matrix is one of the most powerful tools in nuclear physics. It is routinely used to analyze experimental data. Just to mention one example, the analysis of the ^{12}C$(\alpha,\gamma)^{16}$O process, which is a key reaction in nuclear astrophysics [1], is heavily based on *R*-matrix fits [2]. The beauty of Wigner's method is that all scattering quantities are parametrized in terms of real, energy-independent quantities. It is true that the values of some of the parameters of the theory, namely the reduced-width amplitudes $\gamma_{\lambda c}$, and energy eigenvalues E_λ, are somewhat arbitrary due to their dependence on the boundary-condition numbers B_c, and channel radii a_c. However, any sufficiently robust description (i.e., one including enough levels) of the scattering data in a given energy region will give stable (independent of a_c, B_c) resonance parameters when they are based on the actual complex-momentum poles and residues of the *S* matrix, as calculated from continuing the *R* matrix into the complex energy plane.

We will call this method [3] based on the actual *S*-matrix pole structure in the complex plane the ''extended'' *R*-matrix prescription for defining resonance parameters, in order to distinguish it from the usual resonance-parameter prescriptions that are defined entirely on the real-energy axis. The real-energy parameters are easier to obtain, especially when they are extracted directly from the measurements, which of course exist only on the real-energy axis of the physical sheet. However, it is sometimes difficult to interpret experimental results based solely on methods that work at real energies. We mention here two examples: the large cross section of the $t(d,n)\alpha$ reaction and the problem of the soft dipole resonance in neutron halo nuclei. Only analyses at complex energies were able to reveal that the large reaction cross section is caused by a shadow pole of the scattering matrix in the former case [3,4], and that the soft dipole resonance does not exist in ^{6}He in the latter one [5].

It is an intriguing question whether or not the results of methods that are not confined to real energies agree with those of conventional analyses for the relatively broad resonances encountered in light systems. As an exploratory investigation, in this paper we study the low-energy $3/2^-$ and $1/2^-$ states of the ^{5}He and ^{5}Li nuclei. The parameters of these states have been determined from conventional *R*-matrix analyses of certain experimental data. In those works the definition of the resonance position and width was based on the positions and widths of peaks in the reaction cross sections.

For narrow isolated single-channel resonances the two definitions of resonance parameters (*S*-matrix pole and cross-section peak, respectively) are consistent with each other, and give the same results. However, for broad resonances the results coming from the two definitions may be different, because only the scattering theoretical quantities defined at complex energies (e.g., the *S* matrix, the Fredholm determinant, and the Jost function on the multisheeted Riemann energy surface) contain the correct dynamical information. Even in the case of a narrow multichannel resonance the results coming from the two definitions can disagree, and studying the complex-energy scattering quantities can give a much deeper insight into the dynamics of the problem [3,4].

The extraction of the $\alpha+N$ resonance parameters from observables that are continued to complex energies was first suggested by Ahmed and Shanley [6]. They pointed out that the determination of the resonance parameters from real energy observables is difficult because, e.g., the $1/2^-$ phase shift does not even pass through 90 degrees. Here we determine these parameters from the analytic continuation of the $\alpha+N$ scattering matrix to complex energies in a microscopic model. We also extract the ^{5}He and ^{5}Li resonance parameters from the extended *R*-matrix method.

Our model is a microscopic $\alpha+N$ resonating group method (RGM) approach to the five-nucleon system. The trial function of the five-body problem reads

$$\Psi=\sum_{i=1}^{N_\alpha}\mathcal{A}\{[[(\Phi^{\alpha_i}\Phi^N)]_S\chi_L^{\alpha_i N}(\boldsymbol{\rho}_{\alpha N})]_{JM}\}, \tag{1}$$

where $\mathcal{A}$ is the intercluster antisymmetrizer, the $\boldsymbol{\rho}_{\alpha N}$ vector is the intercluster Jacobi coordinate, L and S is the total angular momentum and spin, respectively, and $[\cdots]$ denotes angular momentum coupling. While Φ^N $(N=n$ or $p)$ is a neutron or proton spin-isospin eigenstate, the antisymmetrized ground

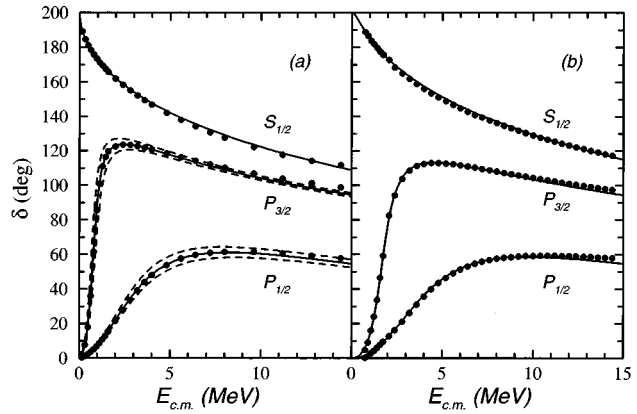

FIG. 1. Phase shifts for $\alpha+n$ (a) and $\alpha+p$ (b) scattering in our RGM model. The dashed lines are results from calculations where the resonance parameters differ from those in Table I by 10%. Experimental data points are taken from [12] (a) and [13] (b), respectively. 20 degrees are added to the $S_{1/2}$ phase shifts, for clarity.

state ($i=1$) and monopole excited states ($i>1$) of the α particle are represented by the wave functions

$$\Phi^{\alpha_i}=\sum_{j=1}^{N_\alpha} A_{ij}\phi^{\alpha}_{\beta_j}, \quad i=1,2,\ldots,N_\alpha. \tag{2}$$

Here $\phi^{\alpha}_{\beta_j}$ is a translationally invariant shell-model wave function of the α particle with size parameter β_j, and the A_{ij} parameters are to be determined by minimizing the energy of the α particle [7]. Putting Eq. (1) into the five-nucleon Schrödinger equation, which contains a two-nucleon strong and Coulomb interaction, we arrive at an equation for the intercluster relative motion functions χ. This equation is solved by utilizing a Kohn-Hulthén variational method for the S matrix, which uses square integrable basis functions matched with the correct scattering asymptotics [8].

The input data for the R-matrix studies are the cross sections and polarizations for all possible reactions involving ^{5}He and ^{5}Li. In the present RGM approach we concentrate on reproducing the $\alpha+N$ scattering phase shifts because they are most closely related to the usual definition of the resonance position and width. So, if our model reproduces the phase shifts, then our resonance parameters are hopefully close to the ones that characterize the reactions. Moreover, in order to analyze cross sections, we should build reaction mechanisms into the model, which would make this model rather phenomenological and ambiguous.

We use the Minnesota effective N–N interaction [9]. The same model (for the ^{5}He and ^{5}Li subsystems) and interaction were used in [10,11,5] to successfully describe the structure and beta delayed deuteron emission of ^{6}He, and the three-body resonances of the $A=6$ nuclei.

In Fig. 1 we show our S and P wave $\alpha+N$ phase shifts, together with the experimental data of Bond [12] for $\alpha+n$ and Schwandt [13] for $\alpha+p$. A rather good agreement is observed, especially in the resonance region. We do not show the higher partial waves because they do not influence our results, and they are practically zero at low energies in agreement with the experiments.

The experimental parameters of the low-lying ^{5}He and ^{5}Li states are listed in Table I. We compare our results with those of Barker [14] and with the compilation [15]. The results of Ref. [6] are also shown.

The $3/2^-$ and $1/2^-$ resonance parameters are determined by analytically continuing the S matrix to complex energies [4]. In practice this is done by solving the Schrödinger equation for the $\alpha-N$ relative motion at complex energies with the following boundary condition for $\rho_{\alpha N}\to\infty$

$$\chi_L^{\alpha_i N}(\varepsilon_i,\rho_{\alpha N})\to H_L^-(k_i\rho_{\alpha N})-\widetilde{S}_L(\varepsilon_i)H_L^+(k_i\rho_{\alpha N}). \tag{3}$$

Here ε_i and k_i are the *complex* energies and wave numbers of the relative motions, and H^- and H^+ are the incoming and outgoing Coulomb functions, respectively. The function $\widetilde{S}$ has no physical meaning, except if it is singular at the energy ε. Then $\widetilde{S}$ coincides with the physical S matrix describing a purely outgoing solution, that is a resonance. So we search for the poles of $\widetilde{S}$ at complex energies. All quantities are defined on the multisheeted Riemann energy surface, and are analytic almost everywhere. The complex Coulomb functions were calculated by using [16].

The positions (E_r) and widths (Γ) of the resonances are extracted from the $\varepsilon=E_r-i\Gamma/2$ complex pole positions of the S matrix. We compare these parameters with those coming from the cross-section peak definition of a resonance, used in conventional R-matrix approaches. We can see in Table I, that our parameters are rather different from the R-matrix results of [14]. It is especially intriguing that the splitting of the $3/2^-$ and $1/2^-$ states is much smaller than in [14], not to mention [15]. We explored the dependence of the

TABLE I. Parameters of the low-energy ^{5}He and ^{5}Li resonances in the center-of-mass frame, coming from the S-matrix pole and cross-section peak definitions, respectively. E_r is the resonance position relative to the $\alpha+N$ threshold, and Γ is the full width at half maximum. All numbers are in MeV.

	^{5}He				^{5}Li			
Method	$E_r(3/2^-)$	$\Gamma(3/2^-)$	$E_r(1/2^-)$	$\Gamma(1/2^-)$	$E_r(3/2^-)$	$\Gamma(3/2^-)$	$E_r(1/2^-)$	$\Gamma(1/2^-)$
Compilation [15]	0.89±0.05	0.60±0.02	4.89±1	4±1	1.96±0.05	≈ 1.5	7–12	5±2
R-matrix, stripping [14]	0.838±0.018	0.645±0.046	2.778±0.46	3.6±1.2	1.76±0.06	1.18±0.13	3.63±0.56	4.1±2.5
R-matrix, pickup [14]	0.869±0.003	0.723±0.019	3.449±0.4	5.3±2.3	1.86±0.01	1.44±0.08	4.54±0.5	6.1±2.8
Scattering ampl. [6]	0.778	0.639	1.999	4.534	1.637	1.292	2.858	6.082
S-matrix, RGM	0.76	0.63	1.89	5.20	1.67	1.33	2.70	6.25
Extended R-matrix	0.80	0.65	2.07	5.57	1.69	1.23	3.18	6.60

TABLE II. Parameters of the low-energy ^{5}He and ^{5}Li resonances in the center-of-mass frame, determined by assuming that $d\delta/dE$ has a maximum at $E=E_r$, and $\Gamma=2/(d\delta/dE)_{E_r}$. E_r is the resonance position relative to the $\alpha+N$ threshold, and Γ is the full width at half maximum. All numbers are in MeV.

	^{5}He				^{5}Li			
Phase shifts	$E_r(3/2^-)$	$\Gamma(3/2^-)$	$E_r(1/2^-)$	$\Gamma(1/2^-)$	$E_r(3/2^-)$	$\Gamma(3/2^-)$	$E_r(1/2^-)$	$\Gamma(1/2^-)$
Expt. data [12,13]	0.77	0.69	2.13	7.26	1.53	1.42	2.77	8.89
RGM	0.76	0.68	2.07	7.18	1.67	1.46	2.92	8.88
R-matrix	0.75	0.85	2.21	7.98	1.67	1.37	3.35	9.40

phase shifts on the resonance parameters by slightly changing the N–N interaction. In Fig. 1(a) the dashed lines show the change of the resonant phase shifts when the resonance parameters of Table I are changed by 10%. We also checked the effect of including the $d+t$ channel in Eq. (1), and found that the resonance parameters are little changed, provided the phase shifts have the same quality as in Fig. 1(a).

We also used the extended R-matrix method [3] to extract resonance parameters for the $A=5$ ground state and first excited states from multichannel R-matrix analyses of reactions in the ^{5}He and ^{5}Li systems. The $A=5$ analyses included the two-body channels $N+\alpha$ and $d+t$ or $d+^3$He, along with pseudo-two-body configurations to represent the breakup channels $n+p+t$ or $n+p+^3$He. Included in the $n-\alpha$ data are the differential elastic scattering cross sections of Morgan [17], Hoop [18], Niiler [19], and Shamu [20]; polarization and analyzing-power measurements by Sawers [21], Broste [22], May [23], and Perkins [24]; and neutron total cross sections measured by Haesner [25]. The $p-\alpha$ data include the differential elastic scattering cross sections of Freier [26], Jarmie [27], Garreta [28], and Plattner [29]; polarization and analyzing-power measurements by Schwandt [13], Plattner [29], and Hardekopf [30]; and polarization-transfer measurements by Keaton [31].

The results are given in Table I. It is remarkable that most of the resonance parameters agree nicely with the RGM results, but differ from the conventional R-matrix results. Perhaps the only exception is the $1/2^-$ state of ^{5}Li, where the agreement between the RGM and extended R-matrix results is not so good. The small differences between the extended R matrix and RGM results in the case of ^{5}He probably come from the fact that our R matrix gives phase shifts that are slightly different from [13] at higher energies. Our RGM results are also in good agreement with [6], where a higher-order scattering amplitude expansion was used.

In addition to the dependence of the resonance parameters of Ref. [14] on the mechanism by which ^{5}He and ^{5}Li are formed, there is also a marked dependence on the channel radius. Barker has argued that this dependence can be used to determine a ''best'' value of channel radius, which for $n+\alpha$ is taken to be 5.5 fm. It is therefore quite interesting that when the $n-\alpha$ R-function parameters from Table 8 of Ref. [14] are used in the S-matrix pole prescription [3], the resulting resonance parameters, $E_{3/2^-}=0.77$ MeV, $\Gamma_{3/2^-}=0.65$ MeV, $E_{1/2^-}=2.10$ MeV, $\Gamma_{1/2^-}=5.37$ MeV, are in good agreement with the RGM values and with the extended R-matrix values, defined for $a_{n-\alpha}=3.0$ fm.

The simplest way to extract resonance parameters is to fit the cross-section or phase-shift data with Breit-Wigner forms. This is, however, an ambiguous procedure for broad resonances where the phase shift is not ''ideal'' (i.e., not going from 0 to 180 degrees within a short energy interval). For an ''ideal,'' isolated, narrow resonance the phase shift, given by scattering theory, behaves like $\tan\delta(E)=0.5\Gamma/(E_r-E)$, which implies that $d\delta/dE$ has a maximum at $E=E_r$, and $\Gamma=2/(d\delta/dE)_{E_r}$. This prescription to extract E_r and Γ is also used for broader resonances [32]. In Table II we show the $3/2^-$ and $1/2^-$ parameters coming from this definition applied to the experimental and model phase shifts. One can see that this simple procedure provides resonance parameters that are close to the RGM results, except that it systematically overestimates the widths.

In a very recent paper [33] the authors extracted the resonance parameters of the ^{5}He and ^{5}Li ground states by determining the pole positions of the S matrix corresponding to ^{3}H$(d,\gamma)^5$He and ^{3}He$(d,\gamma)^5$Li measurements [34]. Their results, $E_{3/2^-}(^5\text{He})=0.8\pm0.02$ MeV, $\Gamma_{3/2^-}(^5\text{He})=0.65\pm0.02$ MeV, and $E_{3/2^-}(^5\text{Li})=1.72\pm0.03$ MeV, $\Gamma_{3/2^-}(^5\text{Li})=1.28\pm0.03$ MeV are in good agreement with our values in Table I.

In summary, we have determined the parameters of the low-energy ^{5}He and ^{5}Li resonances from the complex pole positions of the $\alpha+N$ scattering matrix in a microscopic cluster model. Our results are different from the results coming from a conventional R-matrix method, which define the resonance parameters based on the real-energy properties of cross-section peaks. However, they are in good agreement with the results of an extended R-matrix method that works in the complex energy plane. We emphasize that the extended R-matrix method involves no difference in the way that R-matrix parameters are extracted from experimental data, but only in the way that they are subsequently used to define resonance parameters. However, it is quite clear that the dependence on the channel radius of the usual real-energy prescription for defining resonance parameters from R-matrix parameters is characteristic of that prescription.

The S-matrix pole prescription gives consistent resonance parameters for the ground state and first excited state of the $A=5$ nuclei, which are approximately independent of the method used to describe the nuclear dynamics or the reaction in which the resonance is observed, and in the case of the R-matrix parametrizations, also independent of the channel radii and boundary conditions. We expect this would also be the case for relatively broad levels in other light systems, where different resonance-parameter prescriptions can lead to quite different results [35], and so we recommend using the complex S-pole prescription to specify resonance parameters in all cases.

We thank Ian Thompson for some useful information on his complex Coulomb code, and Fred Barker and Jean Humblet for enlightening discussions. The work of A.C. was supported by Wolfgang Bauer's Presidential Faculty Grant No. PHY92-53505 and by OTKA Grant No. T019834. This work was also supported by NSF Grant No. PHY94-03666 (MSU) and by the U.S. Department of Energy (Los Alamos).

[1] W. A. Fowler, *Les Prix Nobel–1983* (Almqvist & Wiksell, Stockholm, Sweden, 1984), p. 88.
[2] R. E. Azuma *et al.*, Phys. Rev. C **50**, 1194 (1994).
[3] G. M. Hale, R. E. Brown, and N. Jarmie, Phys. Rev. Lett. **59**, 763 (1987).
[4] A. Csótó, R. G. Lovas, and A. T. Kruppa, Phys. Rev. Lett. **70**, 1389 (1993).
[5] A. Csótó, Phys. Rev. C **49**, 3035 (1994).
[6] M. U. Ahmed and P. E. Shanley, Phys. Rev. Lett. **36**, 25 (1976).
[7] P. N. Shen, Y. C. Tang, Y. Fujiwara, and H. Kanada, Phys. Rev. C **31**, 2001 (1985).
[8] M. Kamimura, Prog. Theor. Phys. Suppl. **68**, 236 (1980).
[9] D. R. Thompson, M. LeMere, and Y. C. Tang, Nucl. Phys. **A268**, 53 (1977); I. Reichstein and Y. C. Tang, *ibid.* **A158,** 529 (1970); see also Ref. [10].
[10] A. Csótó, Phys. Rev. C **48**, 165 (1993).
[11] A. Csótó and D. Baye, Phys. Rev. C **49**, 818 (1994).
[12] J. E. Bond and F. W. K. Firk, Nucl. Phys. **A287**, 317 (1977).
[13] P. Schwandt, T. B. Clegg, and W. Haeberli, Nucl. Phys. **A163**, 432 (1971).
[14] C. L. Woods, F. C. Barker, W. N. Catford, L. K. Fifield, and N. A. Orr, Aust. J. Phys. **41**, 525 (1988); F. C. Barker and C. L. Woods, *ibid.* **38**, 563 (1985).
[15] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).
[16] I. J. Thompson and A. R. Barnett, Comput. Phys. Commun. **36**, 363 (1985); J. Comput. Phys. **64**, 490 (1986).
[17] G. L. Morgan and R. L. Walter, Phys. Rev. C **2**, 2034 (1970).
[18] B. Hoop, Jr., and H. H. Barschall, Nucl. Phys. **83**, 65 (1966).
[19] A. Niiler, M. Drosg, J. C. Hopkins, J. D. Seagrave, and E. C. Kerr, Phys. Rev. C **4**, 36 (1971).
[20] R. E. Shamu and J. G. Jenkin, Phys. Rev. **135**, B99 (1964).
[21] J. R. Sawers, G. L. Morgan, L. A. Schaller, and R. L. Walter, Phys. Rev. **168**, 1102 (1968).
[22] W. B. Broste, G. S. Mutchler, J. E. Simmons, R. A. Arndt, and L. D. Roper, Phys. Rev. C **5**, 761 (1972).
[23] T. H. May, R. L. Walter, and H. H. Barschall, Nucl. Phys. **45**, 17 (1963).
[24] R. B. Perkins and C. Glashausser, Nucl. Phys. **60**, 433 (1964).
[25] B. Haesner *et al.*, Phys. Rev. C **28**, 995 (1983).
[26] G. Freier, E. Lampi, W. Sleator, and J. H. Williams, Phys. Rev. **75,** 1345 (1949).
[27] N. Jarmie and J. Jett, Phys. Rev. C **15**, 518 (1976).
[28] D. Garreta, J. Sura, and A. Tarrats, Nucl. Phys. **A132**, 204 (1969).
[29] G. R. Plattner, A. D. Bacher, and H. E. Conzett, Phys. Rev. C **5,** 1158 (1972).
[30] R. A. Hardekopf and G. G. Ohlsen, Phys. Rev. C **15**, 514 (1976).
[31] P. W. Keaton, Jr., D. D. Armstrong, R. A. Hardekopf, P. M. Kurjan, and Y. K. Lee, Phys. Rev. Lett. **29**, 880 (1972).
[32] J. Carlson and V. R. Pandharipande, Nucl. Phys. **A424**, 47 (1984); B. S. Pudliner, V. R. Pandharipande, J. Carlson, and R. B. Wiringa, Phys. Rev. Lett. **74**, 4396 (1995).
[33] V. D. Efros and H. Oberhummer, Phys. Rev. C **54**, 1485 (1996).
[34] M. J. Balbes, G. Feldman, L. H. Kramer, H. R. Weller, and D. R. Tilley, Phys. Rev. C **43**, 343 (1991).
[35] D. R. Tilley, H. R. Weller, and G. M. Hale, Nucl. Phys. **A541,** 1 (1992).