Intermittency as a signal of criticality in multifragmentation studies

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Bond percolation model predictions for the behavior of scaled factorial moments (SFM's) of cluster size distributions do not show intermittency for ''near-critical'' events. An intermittentlike signal is observed only for ''overcritical'' events, independent of the size of the lattice. SFM analysis of ALADIN experimental data (600, 800, and 1000 MeV/nucleon Au + Au reactions) is in qualitative agreement with the percolation predictions. [S0556-2813(97)02601-0]

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The power-law behavior of the scaled factorial moments (SFM's), known as intermittency, is a statistical tool for the extraction of nonstatistical fluctuations [1]. Intermittency of the distribution of a physical observable is believed to probe criticality. For example, within the Ising model, the magnetization of the subdomains exhibits intermittency at the critical point [2].

Ploszajczak and Tucholski introduced the SFM analysis for the fragment size distributions in nuclear multifragmentation [3–6]. They showed that, within the bond percolation model, the cluster size distributions for a lattice with 6^3 sites produce a nonzero intermittency signal only within a certain ''optimal'' interval of the bond percolation parameter $q=0.21$–0.28 [3], close to the critical value for an infinite system $q_{\rm cr}^{\infty}\approx 0.25$. They observed a similar behavior also in the experimental data of Waddington and Freier [7] from fragmentation of 1 GeV/nucleon Au projectiles in nuclear emulsion, and concluded that the study of intermittency in nuclear fragmentation is relevant in the search for critical phenomena. Thus, it has been believed that cluster size distributions are intermittent at the critical point.

Many authors have investigated the SFM's of fragment size distributions to search for hints of criticality in experimental data [8–12] and to try and distinguish different multifragmentation models [13–24]. However, no definitive conclusions about the origin of the observed intermittency signal have been drawn so far. Explanations that have no connection to criticality have also been presented [23,9,24]. It has been mentioned that the appearance of an intermittentlike signal can be caused by the large width of the multiplicity distributions [16–18,9,24] and by mixing events of different excitation energies [16,20,9].

Recently, Campi and Krivine [24] reexamined the bond percolation model results of Refs. [3,4], and showed that the intermittency signal in the vicinity of the critical region disappears if events of fixed multiplicity are selected. They also demonstrated that the intermittency signal vanishes when the size of the system goes to infinity. They concluded their investigation stating that the intermittency signal, put forward in the pioneering work of Refs. [3,4], has been wrongly *interpreted*, since an intermittent behavior results from both the power law of the mean size fragment distribution and from the finite width of the multiplicity distribution.

In this work, we also reexamine the results of Refs. [3,4], and show that an intermittency pattern in the cluster size distributions is clearly observed for some events generated with the bond percolation model, but these events *are not the critical ones.*

To find the occurrence of intermittency in a distribution [6–9] one analyzes the (horizontally) scaled factorial moments

$$F_k(\Delta)=\frac{\Sigma_{i=1}^{A_0/\Delta}\langle N_i(N_i-1)\cdots(N_i-k+1)\rangle}{\Sigma_{i=1}^{A_0/\Delta}\langle N_i\rangle^k}. \qquad (1)$$

Within the bond percolation model, A_0 is the total number of sites of the disassembling lattice which is divided in A bins of size Δ; N_i represents the number of clusters with number of sites in the interval $[(i-1)\Delta,i\Delta],i=1,2,\ldots,A_0/\Delta$. The ensemble averaging $\langle\cdots\rangle$ in Eq. (1) is done over all fragmentation events considered. Intermittency is observed if the moments F_k follow a power law as a function of resolution Δ, of the form $F_k\propto\Delta^{-f_k}$, i.e., if the moments show self-similarity for different resolutions $F_k(a\Delta)=a^{-f_k}F_k(\Delta)$.

Percolation models, which exhibit a phase transition for infinite systems, are applicable also to smaller systems. However, finite size effects make the identification of the critical events somewhat obscure for a nuclear lattice consisting of no more than 6^3 sites. The $\langle\gamma_2(m)\rangle$ function [25] is commonly used [18,19,25] to avoid this problem, m standing for the reduced multiplicity (i.e., multiplicity/A_0) of the decay products. We follow this prescription, initially proposed by Campi [25]. The reduced variance $\langle\gamma_2(m)\rangle$ is calculated as

$$\langle\gamma_2(m)\rangle=N_{\rm ev}^{-1}\sum_j\frac{M_0^{(j)}(m)M_2^{(j)}(m)}{[M_1^{(j)}(m)]^2}. \qquad (2)$$

Here M_p^j is the pth mass distribution moment [25] calculated for the jth event. The summation runs over all the events in a sample of $N_{\rm ev}$ events with fixed reduced multiplicity m. Figure 1 shows the $\langle\gamma_2(m)\rangle$ function calculated within the frame of the bond percolation model, for lattices with 6^3, 15^3, and 20^3 sites, and random choice of the percolation

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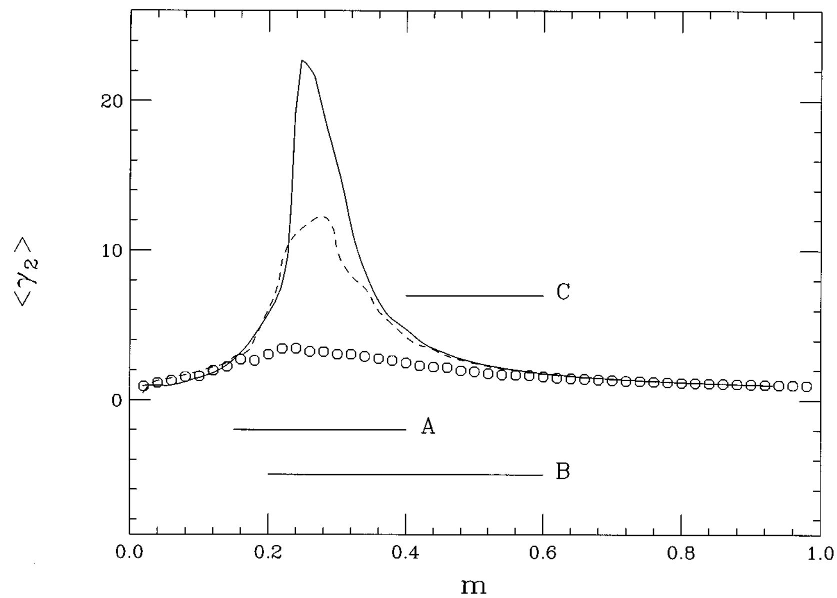

FIG. 1. Reduced variance $\langle \gamma_2(m) \rangle$ as a function of the reduced multiplicity m calculated within the bond percolation model with 6^3 (circles), 15^3 (dashed line), and 20^3 (solid line) sites. Brackets reproduce the m-gating intervals for ''near-critical'' events used in this work (bracket A) and in the original study of Ref. [3] (bracket B). The ''optimal range'' of m gating for the appearance of intermittency is also indicated (bracket C).

parameter[1] q. A peak in $\langle \gamma_2(m) \rangle$, created by the critical events, is clearly observed for large lattices. For the lattice with 6^3 sites, the peak broadens considerably, making the selection of critical events cumbersome. However, ranges of m for selection of ''subcritical'' ($0<m<0.15$), ''near-critical'' ($0.15 \leqslant m \leqslant 0.40$, region A in Fig. 1), and ''overcritical'' events ($0.40<m<1.0$) can still be regarded as a good approximation.

One easily finds that, for all lattices, the slopes of the SFM's do not show any sign of intermittent behavior for the events selected as ''subcritical'' (Fig. 2 top left panel) or ''near-critical'' (Fig. 2 top right panel). Only events denoted as ''overcritical'' [of the peak in $\langle \gamma_2(m) \rangle$] exhibit a strong increase of the SFM's with decreasing bin size, but this increase is nonlinear (Fig. 2 bottom left panel). For a small interval of m within the ''overcritical'' region (denoted as bracket C in Fig. 1), one observes the power-law behavior of the SFM. Figure 2 illustrates this and shows that events selected in the ''optimal range'' ($0.40<m<0.60$, region C in Fig. 1) produce an intermittency signal (bottom right panel in Fig. 2). Note also that, even if both the absolute values and the shape of the SFM's are highly vulnerable to the inclusion of a threshold A for the minimum size of identified clusters, we did not observe the appearance of an intermittency signal for ''near-critical'' events for any thresholds $A=1-6$.

Region B in Fig. 1 indicates the m gating for ''near-critical'' events the corresponding to the q gating of Refs. [3,4] (dispersions of q and m have been taken into account during this procedure). The ''optimal range'' for an intermittentlike signal (region C in Fig. 1) happens to be included in this interval. It is thus a coincidence that the values of q for which the intermittency signal appears in Refs. [3,4] for the lattice with 6^3 sites roughly agree with the critical value for the infinite lattice $q_{\rm cr}^{\infty} \approx 0.25$. And in fact, it has recently been shown [26] that for the lattice with 6^3 sites the critical value is $q_{\rm cr}^{6^3} \approx 0.34$. These results indicate that intermittency of cluster size distributions is not connected to criticality in the percolation theory. Therefore, the use of the intermittency analysis as a method to search for hints of criticality in nuclear multifragmentation is questionable.

Intermittency analysis of the latest projectile fragmentation ALADIN [27–30,32] data from 600, 800, and 1000 MeV/nucleon Au + Au reactions resembles, at least qualitatively, the trends predicted by the percolation model.

Because of the existence of experimental detection thresholds and of the transparency of the experimental setup to neutrons, the ALADIN data are not presented as a function of the reduced multiplicity m, but as a function of the projectile charge that remains bound in fragments (with charge $Z \geqslant 2$) after the interaction. This quantity $Z_{\rm bound}$ has been shown to be a good indicator of the centrality of the interaction, monotonically decreasing with decreasing impact parameter [30]. $Z_{\rm bound}$ is believed to select events from similar initial conditions [29], thus avoiding spurious fluctuations resulting from the mixing of events.

The experimental quantity $\langle \gamma_2(Z_{\rm bound}) \rangle$, rescaled for plotting purposes, is presented in Fig. 3 (stars) for 600 (left panel) and 1000 (right panel) MeV/nucleon Au + Au reactions (a similar plot is obtained for Au + Au reactions at 800 MeV/nucleon). Here $\langle \gamma_2(Z_{\rm bound}) \rangle$ is calculated according to Eq. (2) where M_p^j is now the pth charge distribution moment for the jth event. The largest fragment detected in the event is excluded in an attempt to eliminate the large ''percolating'' cluster [25]. Also $Z=2$ fragments are excluded, motivated by the fact that α particles may originate from processes not related to the fragmentation of the projectile spectator such as preequilibrium emission, coalescence of nucleons, etc. [31]. The summation runs over all the events in a sample of $N_{\rm ev}$ events with fixed $Z_{\rm bound}$.

It is impossible to interpret all the ALADIN experimental data within the frame of the standard bond percolation model used in this work, and modifications would be needed [27].

[1]The calculations for the lattice with 6^3 sites show that $q=0.55-1.23m+1.42m^2-0.75m^3$ is a good approximation for the description of the general relation between these two parameters.

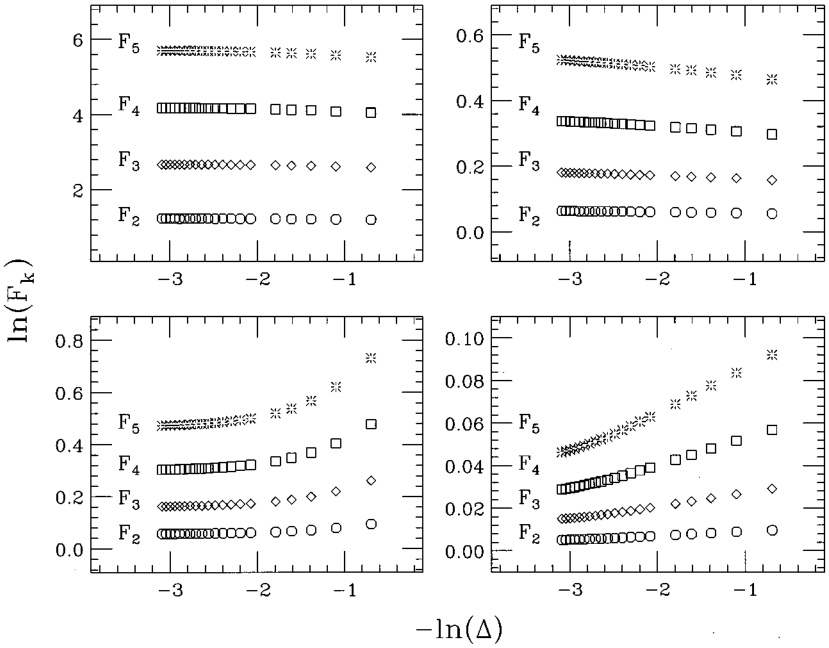

FIG. 2. Double logarithmic plot $\ln F_k$ vs $-\ln\Delta$ ($k=2$–5), for 6^3 percolation events selected as ''subcritical'' $0<m<0.15$ (top left panel), ''near-critical'' $0.15\leqslant m\leqslant 0.40$, region A in Fig. 1 (top right panel), ''overcritical'' $0.40<m<1.0$ (bottom left panel), and ''optimal'' $0.40<m<0.60$, region C in Fig. 1 (bottom right panel).

However, the bond percolation model, for the lattice with 6^3 sites, predicts $Z_{\mathrm{bound}}=80.32+0.41m-279.2m^2+199.2m^3$. One gets this general dependence going over from the size (mass) of the cluster to its charge, by means of an empirical formula [18] and taking into account the efficiency of ALADIN [27]. According to this, the region of ''near-critical'' events ($m=0.15$–0.40) is projected into the interval of $Z_{\mathrm{bound}}=49$–75 which covers the experimentally observed broad maximum of $\langle\gamma_2(Z_{\mathrm{bound}})\rangle$ in Fig. 3. Thus one could believe that this peak is a signature of the ''near-critical'' events in the experimental distributions. ''Overcritical'' events are found to the left of the peak.

We now proceed to explore the occurrence of intermittency by studying the SFM. The occurrence of intermittency is indicated by the following features [20]: $\ln F_k$ are positive for each Δ; a single, positive slope is observed for the double logarithmic plot $\ln F_k$ vs $-\ln\Delta$; the relative ordering of the moments is $\ln F_{k+1}>\ln F_k$.

We observe a clear correlation between the qualitative behavior of the SFM and the width of the fragment ($Z\geqslant 3$) multiplicity distributions [16,9]. For this reason we show the ratio $\sigma^2/\langle n\rangle$ (divided by 10 for plotting purposes) as a solid line in Fig. 3.

When the multiplicity distributions are much narrower than Poisson distributions [i.e., $(\sigma^2/\langle n\rangle)<1$], which occurs for $Z_{\mathrm{bound}}<40$, no intermittency signal is found. The values of $\ln F_k$ are negative everywhere and the relative ordering of the various moments is reversed compared to that of a signal for intermittency.

When the dispersion increases, thus approaching Poisson, we encounter a ''transition region'' where $\ln F_k$ are positive for small bin sizes, but tend to become negative for the larg-

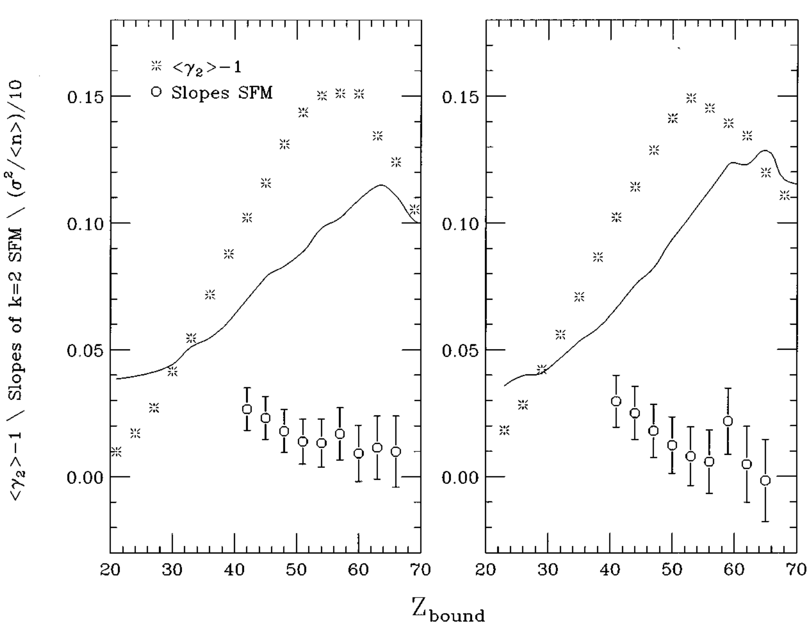

FIG. 3. Plotted as a function of Z_{bound}, from 600 (left panel) and 1000 (right panel) MeV/nucleon Au + Au collisions (ALADIN data): Reduced variance $\langle\gamma_2\rangle-1$ (stars). Variance-to-mean ratio (divided by 10) of fragment multiplicity distributions (solid line). Slopes of the double logarithmic plot $\ln F_k$ vs $-\ln\Delta$ for $k=2$ and $\Delta=1$–20 (circles).

est bin (all fragments in one bin) [16,9]. This ''transition region'' (approximately $40<Z_{\rm bound}<50$), to the left of the critical peak, represents ''overcritical'' events and yields the largest ''intermittency signal,'' in qualitative agreement with the percolation predictions. The positive slopes of straight lines fitted to the plots $\ln F_k$ vs $-\ln\Delta$ are presented in Fig. 3 (circles) for $k=2$ and $\Delta=1$–20. The error bars represent the statistical error only (one standard deviation), but there is also a smaller systematic error originating from the range of binning resolution chosen for the fitting procedure. The range $\Delta=1$–20 used in our analysis yields good values of χ^2 per degree of freedom. However, deviations from linearity of the double logarithmic plot $\ln F_k$ vs $-\ln\Delta$ are observed for increasing Δ, particularly at small values of $Z_{\rm bound}$.

Finally, for multiplicity distributions larger than Poisson, the slopes of the SFM are consistent with zero or negative.[2]

We believe that the intermittentlike signal observed in the ''transition region'' ($40<Z_{\rm bound}<50$) originates from both the constraints introduced by conservation laws [23] and a certain summation over different event classes in the region around $Z_{\rm bound}\approx 50$ [32,33], and is not connected to criticality.

In conclusion, the bond percolation model does not confirm the intermittent pattern of the fragment size fluctuations in the critical region. Therefore, one should not link the experimentally observed intermittency signal to the development of a critical behavior in the reactions. The latest ALADIN experimental data from 600, 800, and 1000 MeV/nucleon Au + Au reactions qualitatively follow the predictions of the bond percolation model.

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[2]For 1000 MeV/nucleon collisions, a weak intermittency signal can be identified in the range $58\leqslant Z_{\rm bound}\leqslant 60$, where the slopes of the SFM are $0.022\pm0.013(k=2), 0.034\pm0.024(k=3)$.

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