Analysis of recent π^+p low-energy differential cross-section measurements

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The recent π^+p differential cross-section measurements between 20 and 100 MeV (pion lab kinetic energy) have been analyzed in the context of an extended threshold expansion of the K matrix. The analysis shows that the bulk of the database is self-consistent. Our results in the S_{31} channel disagree with the ones obtained in the Karlsruhe phase-shift analyses (which have been based exclusively on the old πN database); we report differences of (approximately) one degree in the corresponding phase-shift values and of almost 25% in the s-wave scattering length a_3 (our value: $a_3 = -0.077 \pm 0.003\ m_\pi^{-1}$). These differences reflect the extent of the discrepancy between the old and the new low-energy π^+p database; they cannot be attributed to the method of the analysis. A comparison of the present result for the a_3 with the values of the πN scattering lengths, recently obtained from measurements of the $1s$ energy-level shift and width in pionic hydrogen and deuterium, leads to the conclusion that, within the present (experimental and theoretical) accuracies, the various results are not incompatible with the isospin symmetry of the strong interaction. [S0556-2813(97)05901-3]

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I. INTRODUCTION

Thanks to the construction of the three meson factories (LAMPF, PSI, and TRIUMF), the pion-nucleon (πN) interaction has been subjected to a considerable experimental survey during the past two decades. The existence of these high-quality measurements allows one, for the first time, to enter the question of the violation of fundamental symmetries of the underlying theory (isospin symmetry and chiral symmetry) quantitatively. At low energies, the theory describing the interaction—the quantum chromodynamics (QCD)–is highly nonperturbative. Due to this reason, an effective-field approach (respecting the properties of the QCD Lagrangian) has been put forward (for a recent review, see Ref. [1]) to account for the low-energy hadronic phenomena: the chiral-perturbation theory (χPT). This approach, which is a powerful tool in the hands of theoretical physics, has recently achieved predictions for the isovector πN scattering length b_1 [2].

The interest in the field of pion physics has gradually shifted to the low-energy domain. This is due to several reasons. First of all, the lower the energy is, the better χPT is expected to work (the effective Lagrangian is expanded in a Taylor series in the momenta involved). Additionally, the lower the energy is, the more important the isospin-breaking effects are expected to become; this is due to the fact that the (kinetic) energies, associated with the interaction, become then comparable to the u and d quark-mass difference (which is the source of the isospin violation). The amount of the breaking of isospin in the πN system, along with a comparison with chiral-symmetry predictions, will eventually lead to the determination of the u and d quark masses [1]. It is evident that the lower (in energy) the experimental data extend, the higher the precision of such a determination will be.

The Karlsruhe analyses (KH80 [3] and KA85 [4]), the results of which have traditionally been used in several applications, are exclusively based on measurements conducted before 1980. With two exceptions (i.e., the experimental data of Bertin *et al.* [5] for the π^+p reaction down to pion lab kinetic energy $T_\pi = 20.8$ MeV and those of Bussey *et al.* [6] for both elastic processes down to 88.5 MeV), the input data (to these analyses) came from energies above 100 MeV. Using the techniques of the dispersion analysis and theoretical constraints, predictions for the low-energy region (below 100 MeV) have been obtained. It was quite surprising to discover (in the early eighties) that these predictions do not match the results of the meson-factory low-energy experiments. This mismatch implied a significant disagreement between the old (essentially meaning the measurements of Ref. [5], which, apart from one exception, are well reproduced by the Karlsruhe analyses) and the new low-energy database; although the subject has been brought forward many times (e.g., in contributions to the πN Newsletter [7]), the discussion about the source of the disagreement does not seem to be particularly constructive [8]. It is quite unfortunate that, despite the abundance and the quality of the recently obtained low-energy experimental data, no analysis of these measurements exclusively, in terms of phase shifts and of scattering lengths and volumes, has so far been performed.

The investigation of the question of the isospin-symmetry breaking in the πN system necessitates the separate analysis of the three possible low-energy experimentally-accessible reactions, i.e., of the two elastic processes $\pi^\pm p$ and of the single-charge-exchange (SCX) reaction $\pi^- p \to \pi^0 n$. As a first step, in the present work, we analyze the recent π^+p low-energy differential cross sections [9–15]. The self-consistency of the experimental data is investigated. New phase-shift values will be extracted exclusively on the basis of these measurements and a new s-wave scattering length a_3 will be deduced. Our results will be compared with the ones obtained in Refs. [3] and [4].

The choice of the elastic π^+p process has been made on the following basis. (1) The interaction is simpler since it involves only the isospin $I=3/2$ amplitudes. (2) Most of the

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discrepancy in the low-energy region is associated with the π^+p channel. (3) The scattering lengths, corresponding to the elastic π^-p and the SCX reaction, have recently been obtained experimentally [16], as has the scattering length for the elastic π^-d reaction (which is related to the isoscalar πN scattering length b_0) [17]. It will be interesting to combine these results with ours in order to check the isospin-symmetry conservation in the πN system at threshold.

There are good reasons to perform the analysis of the π^+p elastic-scattering reaction with the low-energy data exclusively. Above 100 MeV (e.g., in the Δ_{33}-resonance region), the p wave dominates over the s wave. Extending the database to this region could diminish the sensitivity of the isospin-symmetry test at threshold. Moreover, the πN interaction is simpler in the low-energy region; the contributions of the higher resonances (other than the Δ_{33}) are small [18], the inelasticities are negligible and the higher partial waves (i.e., other than s and p) are tiny.

II. THE METHOD

The relation between the K matrix and the corresponding center-of-mass (c.m.) πN partial-wave amplitude reads as

$$F_\alpha(\epsilon)=\frac{K_\alpha(\epsilon)}{1-ikK_\alpha(\epsilon)}, \tag{1}$$

where the index α stands for the total isospin, orbital and total angular momentum of the particular channel [e.g., see Eq. (3.13) in Ref. [18]]; k and ϵ denote the pion c.m. momentum and kinetic energy, respectively.

For elastic scattering (which is the case for the energy interval considered in the present analysis), the partial-wave amplitude $F_\alpha(\epsilon)$ is related to the corresponding phase shift $\delta_\alpha(\epsilon)$ via the formula

$$F_\alpha(\epsilon)=\frac{e^{2i\delta_\alpha(\epsilon)}-1}{2ik}. \tag{2}$$

Combining Eqs. (1) and (2), we deduce the relation

$$k\cot[\delta_\alpha(\epsilon)]=K_\alpha^{-1}(\epsilon). \tag{3}$$

The π^+p interaction proceeds exclusively through the isospin $I=3/2$ channels. In the energy region concerned in the present work, only the s and p waves are of importance.[1] Thus, only three partial-wave amplitudes are relevant: the ones corresponding to the channels S_{31}, P_{33}, and P_{31} [the first subscript denotes the isospin ($2I$) and the second one stands for the total angular momentum ($2J$) of the π^+p system].

In the S_{31} channel, the standard threshold expansion of $K_\alpha^{-1}(\epsilon)$ contains constant and linear terms in ϵ (effective-range expansion). In the present work, a quadratic term has also been considered. Thus,

$$K_{S31}^{-1}(\epsilon)\equiv\frac{1}{a}+b\epsilon+c\epsilon^2. \tag{4}$$

In the previous expression, the parameter a is the scattering length. The scattering length, corresponding to the S_{31} channel, is usually denoted by a_3.

In the P_{31} channel, a smooth dependence of the K-matrix element on ϵ has been assumed:

$$K_{P31}(\epsilon)\equiv d\epsilon+e\epsilon^2. \tag{5}$$

In the P_{33} channel, the existence of the Δ-isobar resonance at $\epsilon=127$ MeV suggests the inclusion of a resonant piece in the K matrix; a Breit-Wigner formula with an energy-dependent width [19] has been added on top of the background term [described by the sum of a linear and a quadratic term in ϵ in the form of Eq. (5)]. The resonant piece is given by the formula

$$[K_{P33}(\epsilon)]_{\rm res}=\Gamma\frac{M_\Delta^2}{k_\Delta^3 W}\frac{k^2}{M_\Delta^2-W^2}, \tag{6}$$

where Γ is the width of the resonance at the resonance position, M_Δ stands for the mass of the Δ resonance (constant), k_Δ is the pion c.m. momentum at the resonance position, and W denotes the total c.m. energy of the π^+p system. Note that the resonant piece $[K_{P33}(\epsilon)]_{\rm res}$ does not introduce any free parameters.

The values of the physical constants have been obtained from Ref. [20]; m_π was fixed to the charged-pion mass.

The electromagnetic effects have been treated according to the NORDITA algorithm [21]. The corrections to the phase shifts and the (very small) inelasticities due to bremsstrahlung have been obtained from the tabulated values of Ref. [21] via simple interpolations. The pure hadronic phase shifts (in the S_{31}, P_{33}, and P_{31} channels) are obtained from the corresponding K-matrix elements via Eq. (3) and are subsequently corrected, thus leading to the (so-called) nuclear phase shifts. The nuclear phase shifts are then used to construct the (partial-wave) nuclear amplitudes (the Coulomb phase shifts are also taken into account). Finally, the spin-non-flip and the spin-flip amplitudes are obtained (after the pure Coulomb contributions are added), from which the differential cross section can be determined (e.g., see Appendix A in Ref. [18]).

III. THE ANALYSIS

We have used the standard MINUIT routines of the CERN library. As a first step, we fitted for our seven parameters (three parameters in the s-wave part and two per p-wave channel) to the experimental data of Ref. [5] which dominated the low-energy πN database until the early eighties. The scattering length a_3, thus extracted, is compatible with the Karlsruhe results [3,4]: to be specific, we obtain $a_3=-0.091\pm0.005\ m_\pi^{-1}$ (by using all the data of Ref. [5])

[1]The values of the small phase shifts in the d and f channels have been fixed from Ref. [4].

TABLE I. Recent low-energy measurements of the differential cross section for the elastic $\pi^+ p$ reaction. T_π (in MeV) denotes the pion lab kinetic energy and θ (in degrees) stands for the c.m. scattering angle. The normalization uncertainties Δz (in %) are also quoted. In order to treat all data sets on the same basis, the individual contributions to the normalization uncertainty in the cross-section measurements of Ref. [12] were summed quadratically [23] (the linear combination leads to a normalization uncertainty of 6.5% [12]).

Experiment	T_π	θ	Δz	Ref.
FRANK83 (LAMPF)	29.4–89.6	47.0–154.0	3.7–20.3	[9]
BRACK86 (TRIUMF)	66.8–97.9	89.6–159.7	1.2–1.5	[10]
BRACK88 (TRIUMF)	66.8	101.4–147.1	2.1	[11]
WIEDNER89 (PSI)	54.3	9.6–33.3	3.0	[12]
BRACK90 (TRIUMF)	30.0–66.8	47.6–147.0	2.2–3.6	[13]
BRACK95 (TRIUMF)	87.1, 98.1	36.4–95.2	2.2, 2.0	[14]
JORAM95 (PSI)	32.2–68.6	11.8–132.4	3.3–4.4	[15]

and $a_3 = -0.097 \pm 0.005\ m_\pi^{-1}$ (after excluding the discrepant 67.4 MeV measurements[2]); the a_3 values of Refs. [3] ($-0.101 \pm 0.004\ m_\pi^{-1}$) and [4] ($-0.100\ m_\pi^{-1}$) agree very well with our result.

Using the same technique, we then fitted to the recent differential cross-section measurements [9–15]. In this work, only refereed articles have been taken into account; the data of Ref. [22] have been excluded because of their low signal-to-noise ratio. Some information about the experiments, comprising our database, is displayed in Table I. A discussion on the experiments is provided in Appendix A.

In order to examine the sensitivity of our results to the different treatment of the statistical and systematic uncertainties of the measurements, four methods have been used to analyze the data. The minimization function is defined as

$$\chi^2 = \frac{\sum_{i=1}^{n} w_i [(y_i - y_i^{\text{exp}})/\sigma_i]^2}{\sum_{i=1}^{n} w_i / n}, \tag{7}$$

where n is the total number of the measurements, y_i and y_i^{exp} are (respectively) the estimated and measured values of the differential cross section for the ith entry (the former being calculated from the fitted values of the phase shifts in the S_{31}, P_{33}, and P_{31} channels) and w_i is a weight assigned to the ith measurement. The weights w_i are assumed to be the same for all entries in method I. In method II, the weights w_i are set equal to

$$w_i = \frac{\sigma_{Si}^2}{\sigma_{Si}^2 + \sigma_{Ri}^2}, \tag{8}$$

where σ_{Si} and σ_{Ri} denote, respectively, the systematic and the statistical (random) uncertainties of the ith entry; in this method, the contribution to the χ^2 from experiments with small systematic errors is reduced [24] (it could be argued that the systematic errors, assigned to the measurements, are not very reliable). In method III, the role played by the two uncertainties is reverted. In methods I–III, the uncertainties σ_i, appearing in Eq. (7), are defined by the relation

$$\sigma_i = \sqrt{\sigma_{Si}^2 + \sigma_{Ri}^2}. \tag{9}$$

The experimental data have also been analyzed after all systematic uncertainties were neglected (method IV); however, the results of the fits with method IV were not taken into account wherever averaging is performed in the present work.

IV. RESULTS AND DISCUSSION

Our results for the scattering length a_3 are shown in Table II (first block), along with the χ^2 values of the fits. The χ^2 values, obtained with the first three methods, are reasonable; in method IV, part of the error is neglected, this omission leading to the increase in the χ^2 value. The χ^2 value with method II is slightly lower; this is an indication that the normalization uncertainties, assigned to the differential cross-section measurements, may not be very reliable. The values of the scattering length a_3 are very much compatible among themselves and disagree with the corresponding ones of Refs. [3] and [4].

In order to investigate the stability of these results, we introduced a χ^2 criterion applying to the contribution of each individual data set (series of differential cross-section measurements performed at fixed energy); data sets with high χ^2 contribution per entry (i.e., above the average plus one standard deviation in the overall χ^2 distribution) were removed. This cut eliminates four of the data sets (out of 20) corresponding to 20% of the entries (for a discussion on the excluded measurements, see Appendix A). The χ^2/NDF values (NDF is the number of degrees of freedom), obtained after this cut is imposed, are very close to unity for methods I–III (see Table II, second block). This fact leads to two conclusions. (1) The bulk of the recent $\pi^+ p$ data is self-consistent. (2) The expansions of the K-matrix elements, assumed in the present analysis, describe the interaction sufficiently well. Our a_3 values are insensitive to this cut (see Table II).

The average values of our seven parameters are given in Table III (averages over methods I–III, with and without the imposition of the χ^2 criterion). No statistically significant changes in the parameter values were observed for the different methods of analysis or after applying the χ^2 criterion.

The phase shifts $\delta(S_{31})$, $\delta(P_{33})$, and $\delta(P_{31})$ are shown in Fig. 1 as functions of ϵ. The solid curves represent our solution; they are based on the results of the fits to the measurements of Refs. [9–15] (average over methods I–III, with and without the imposition of the χ^2 criterion). The dotted curves indicate the extent of the one-standard-deviation uncertainty estimated with a Monte Carlo simulation, the results of the fits (parameter values, errors, and the full correlation matrix) having been taken into account. The correlation coefficients, corresponding to different channels, are negligibly small in all cases. With the exception of a small deviation (relative change) in the P_{33} channel above $\epsilon \sim 50$ MeV, the p-wave part of the interaction is compatible with the Karlsruhe analyses. Radical changes are necessary

[2]The 67.4 MeV measurements of Ref. [5] have also been excluded in the Karlsruhe analyses.

TABLE II. The χ^2 values and the scattering length a_3 (in m_π^{-1}) obtained with methods I–IV, with and without the imposition of the χ^2 criterion (see text); NDF is the number of degrees of freedom. The average a_3 (methods I–III, with and without the imposition of the χ^2 criterion) is also given. The values, obtained by the Karlsruhe analyses on the basis of the old measurements (Refs. [3] and [4]), are also shown. The uncertainty in our a_3 values includes the corresponding scale factor $\sqrt{\chi^2/\mathrm{NDF}}$.

Method	χ^2/NDF	a_3
I	426.7/(266−7)∼1.65	−0.0765±0.0034
II	398.3/(266−7)∼1.54	−0.0751±0.0034
III	448.4/(266−7)∼1.73	−0.0775±0.0033
IV	919.0/(266−7)∼3.55	−0.0737±0.0033
I, χ^2 criterion	196.4/(212−7)∼0.96	−0.0779±0.0028
II, χ^2 criterion	165.1/(212−7)∼0.81	−0.0770±0.0027
III, χ^2 criterion	221.8/(212−7)∼1.08	−0.0786±0.0028
IV, χ^2 criterion	443.8/(212−7)∼2.16	−0.0749±0.0028
Average		−0.0771±0.0033
Fixed-t dispersion relations [3]		−0.101±0.004
Partial-wave relations [4]		−0.100

in the S_{31} channel (already presaged by the discrepancy in the a_3 results shown in Table II); the differences in the corresponding phase shift are of the order of about one degree in the energy region considered in this analysis. No statistically significant changes in the description of the (s- and p-wave) phase shifts were observed for the different methods of analysis or after applying the χ^2 criterion.

The values of the scattering length a_3, obtained in the present work (with and without the imposition of the χ^2 criterion), are shown in Fig. 2, along with the corresponding value of Ref. [4] and the ones extracted from the measurements of Ref. [5] with our method I[3]; the imposition of the χ^2 criterion removes the 67.4 MeV data set of Ref. [5]. The similar energy extent of the differential cross-section measurements of Ref. [5] and those of Refs. [9–15] does not leave room for an attribution of the differences to the method in which the electromagnetic effects are treated. Therefore, the discrepancy can only represent the extent of the inconsistency between the measurements of Ref. [5] and the ones of Refs. [9–15].

In Ref. [4], the real part of the partial-wave amplitude $F_{0^+}^{3/2}(\epsilon)$, corresponding to the S_{31} channel, is expanded in powers of k^2:

$$\mathrm{Re}F_{0^+}^{3/2}(\epsilon)=\alpha_{0^+}^{3/2}+\beta_{0^+}^{3/2}k^2+\cdots. \tag{10}$$

The parameter $\alpha_{0^+}^{3/2}$ is our scattering length a_3. $\beta_{0^+}^{3/2}$ is the s-wave effective-range parameter and is related to the parameters in Eq. (4) according to the formula

$$\beta_{0^+}^{3/2}=-a^2\frac{2m_\pi a+b}{2m_\pi}. \tag{11}$$

Using the results of our fits, $\beta_{0^+}^{3/2}=-0.042\pm0.009\ m_\pi^{-3}$; the result of Ref. [4] is $\beta_{0^+}^{3/2}=-0.052\ m_\pi^{-3}$.

[3]The statistical and systematic uncertainties have not been given separately in Ref. [5].

The p-wave scattering volumes, obtained in the present work (average over methods I–III, with and without the imposition of the χ^2 criterion), are equal to $a_{33}=0.205\pm0.004\ m_\pi^{-3}$ and $a_{31}=-0.046\pm0.007\ m_\pi^{-3}$; the corresponding values of Ref. [4] (the uncertainties being taken from Ref. [3]) are $0.214\pm0.002\ m_\pi^{-3}$ and $-0.044\pm0.002\ m_\pi^{-3}$.

Finally, it is worth mentioning that the effective-range expansion is often criticized. The objections are related to the existence of poles in the u and the t channel below (and close to) threshold, the presence of which may affect the (convergence of the) expansions and lead to a distortion of the scattering-length/scattering-volume values. To our opinion, the correctness of our results is justified on the basis of the following reasoning. (a) Fits with the πN interaction model of Ref. [18] to the data considered in the present analysis have led to results identical to the ones obtained herein for the scattering length a_3, scattering volumes, and phase shifts [25]. This πN model is the most general one for applications in the low-energy domain and represents a valid and complementary alternative to the traditionally used scheme of dispersion analyses. It is evident that, with this model, the contributions from the singularities in the unphysical region are explicitly taken into account. The agreement between the results, obtained herein, and the ones, ob-

TABLE III. The average values of our parameters corresponding to methods I–III and being obtained with and without the imposition of the χ^2 criterion. The parameters in the same spin-parity channel are correlated. The parameter a is the scattering length a_3.

S_{31} channel	a (GeV^{-1})	−0.553±0.024
	b	14.7±4.1
	c (GeV^{-1})	−63±48
P_{33} channel	d (GeV^{-2})	9.70±0.39
	e (GeV^{-3})	−31.9±7.1
P_{31} channel	d (GeV^{-2})	−4.70±0.67
	e (GeV^{-3})	20±12

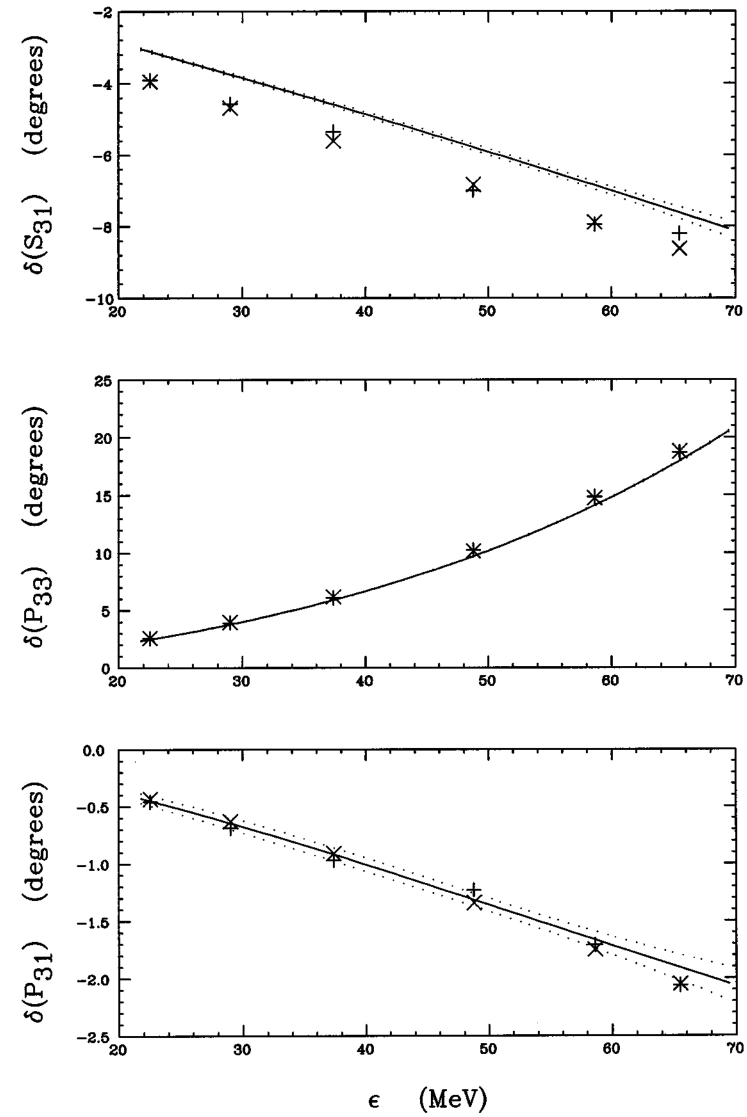

FIG. 1. The phase shifts $\delta(S_{31})$, $\delta(P_{33})$, and $\delta(P_{31})$ as functions of the pion c.m. kinetic energy ϵ. The solid curves represent the results of the fits to the measurements of Refs. [9–15] (methods I–III, with and without the imposition of the χ^2 criterion) and their extent in energy corresponds to the energy interval of the differential cross-section measurements considered in the present analysis. The dotted curves indicate the extent of the one-standard-deviation uncertainty. The data shown correspond to KH80 [3] (plus signs) and KA85 [4] (crosses).

tained with this model, implies that the singularities are distant enough so that the effective-range expansion is a safe approximation. (b) The s- and p-wave K-matrix elements, corresponding to isospin $I=3/2$ and obtained with the πN model of Ref. [18] from low-energy elastic-scattering data [25], are shown in Fig. 3. The fits to these values with first-, second-, and third-order polynomials (in ϵ), the results of which are also shown in Fig. 3, demonstrate that the second-order terms improve the description of the interaction significantly and the contributions of terms above the second order are indeed negligible. This point serves as an important additional verification of the validity of our choice concerning the expansions of the K-matrix elements given in Eqs. (4) and (5). (c) In order to investigate whether the experimental data themselves suggest the inclusion of higher-order terms (in ϵ) in the right-hand side of Eq. (4), fits with all four methods (with and without the imposition of the χ^2 criterion) were performed after including the ϵ^3 term in the expansion of $K_{S31}^{-1}(\epsilon)$. The inclusion of this term does not lead to any improvement; the χ^2 values become negligibly smaller, yet the ratio χ^2/NDF (slightly) increases. For the energy region considered in the present analysis, there is absolutely no sensitivity of the experimental data to any higher-order terms (which is also evident from Fig. 3). (d) The fit to the data of Ref. [5] using the same expansions for the K-matrix elements yields results which agree with the Karlsruhe analyses, thus identifying the source of the discrepancies: inconsistency between the measurements of Ref. [5] and the ones of Refs. [9–15].

The measurements of Bussey *et al.* [6] have not been included in this analysis. Their π^+p data actually consist of only three entries in the energy region considered herein and have been taken close (i.e., at 94.5 MeV) to the highest energy allowed in our work. Including these measurements in our small old database (that is, on top of the measurements of Ref. [5]), which consists of 70 entries and extends down to 20.8 MeV, cannot lead to changes in the corresponding a_3 values (i.e., the ones denoted by the diamonds in Fig. 2).

As far as the ‘‘partial-total’’ or integral π^+p cross sections are concerned, the results of the present analysis are about 10% below the measurements recently reported in the πN Newsletter [26]. Although it is stated therein that these values are now in nearly final form, we would not like to draw conclusions before the corresponding articles appear in a refereed journal. Our predictions for the total nuclear

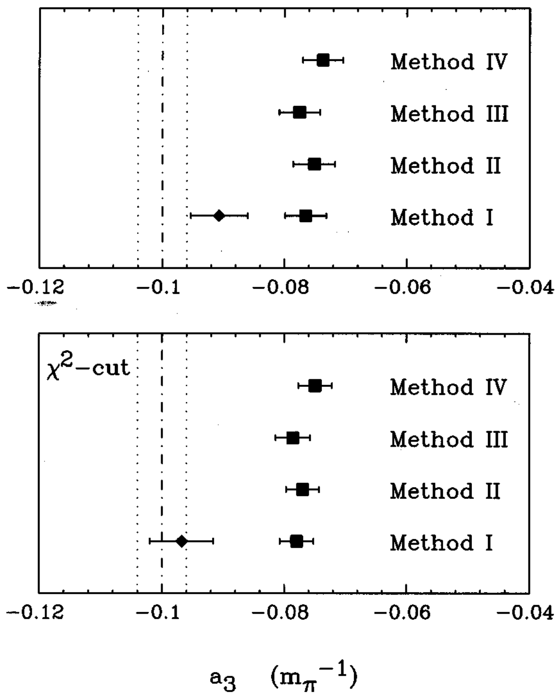

FIG. 2. The values of the scattering length a_3 obtained in the present work (with and without the imposition of the χ^2 criterion). The vertical dash-dotted line corresponds to the a_3 value of Ref. [4], the uncertainty of which is indicated by the dotted lines and has been taken from Ref. [3]. The a_3 values, extracted from the data of Ref. [5] with method I (diamonds), are also shown.

π^+p cross section [see Eqs. (13) and (14) in the first of Refs. [21]] also lie about 10% below the measurements of Refs. [27]; since the lowest energy, considered in these experiments, was above 70 MeV, the inclusion of the corresponding six entries in our database could not have led to changes in our a_3 values.

Measurements of the π^+p analyzing power at 98 MeV have been reported in Ref. [28]. Our predictions, based on the present analysis, lie below the experimental values by one standard deviation approximately. The results of the present work reproduce superbly the recent measurements of the π^+p analyzing power at 68.34 MeV [29].

It is of great interest to compare the present result for the a_3 with the values of the πN scattering lengths, recently obtained from measurements of the $1s$ energy-level shift and width in pionic hydrogen [16] and deuterium [17], in the context of the isospin-symmetry breaking in the strong interactions. If isospin symmetry holds, then the πN processes can be described by two numbers at the πN threshold, e.g., the isoscalar (b_0) and the isovector (b_1) scattering lengths. The π^+p process provides (and is described by) the combination b_0+b_1 ($=a_3$), the π^-p process yields the combination b_0-b_1 and the SCX is directly related to the isovector scattering length b_1. Additionally, the π^-d scattering length leads to the determination of the isoscalar πN scattering length b_0 after the application of a (large) correction term [30] which has been assumed independent of b_1 [31]. In the case that isospin is a good symmetry in the πN system, the various solutions will have a common intersection on the (b_1,b_0) plot [32]. All these measurements are concisely contained in Fig. 4, from which we conclude that, at the present moment, the various experimental results are not incompatible with isospin-symmetry conservation.[4] It is also evident from Fig. 4 that the KA85 solution (represented by the rectangle) is in contradiction with two of the bands shown. As a final remark concerning the interpretation of Fig. 4, we feel that three issues are of crucial importance. (1) To know whether the electromagnetic corrections, applied to the $\pi^\pm p$ scattering data, are reliable and compatible with the corresponding corrections applied at threshold. A redetermination of the whole problem had actually been proposed in the past (e.g., see Ref. [34]) and plans for a new evaluation are under way [35]. (2) Concerning the extraction of the isoscalar πN scattering length b_0 from the measured $1s$ energy-level shift in pionic deuterium, two points are very much worth additional investigation: the electromagnetic corrections and the association of the π^-d scattering length and b_0. (3) The value of b_1, extracted from the measurement of the width of the $1s$ state in pionic hydrogen, introduces the largest uncertainty to the problem. At present, the evaluation of an improved experiment, together with an investigation of the systematic uncertainties (which are due to the Doppler shift), is in progress [36]. It is evident from Fig. 4 that the b_1 value plays a decisive role on the question of the isospin-symmetry conservation in the πN interaction.

Let us finally comment on other recent πN analyses.

(a) Arndt and collaborators published the results of their analysis of πN data using fixed-t dispersion relations [37]. Their database comprises the measurements in all three reaction channels and extends to energies of 2 GeV. The authors varied the πNN coupling constant $g_{\pi NN}$ and the isoscalar scattering length b_0 (denoted as $a^{(+)}$ in their paper) seeking for the minimization of a χ^2 function; several fits were performed with b_0 fixed at three values. However, their interval of b_0 variation is incompatible with the recent pionic-deuterium result [17]. Although it is clearly stated in their paper that the results are not very sensitive to the b_0 value, one does not possess sufficient information to judge whether one can obtain an equally good description of the data for other values of b_0, e.g., for the value extracted on the basis of the measurement of Ref. [17]. In their analysis, the authors obtain an ''optimal'' solution corresponding to $b_0\sim-0.010\ m_\pi^{-1}$ and $b_1\sim-0.088\ m_\pi^{-1}$ (it is not clear what should be used as uncertainty in the values of the scattering lengths of Ref. [37]).

(b) Siegel and Gibbs [38] extracted a b_1 value from low-energy SCX data using a coupled-channel approach with nonlocal potentials. Their value ($3b_1=-0.290\pm0.008\ m_\pi^{-1}$) is compatible with the one recently extracted from the width of the $1s$ state in pionic hydrogen [16] and its uncertainty is smaller. However, the authors have assumed a particular form for the (real part of the) πN scattering amplitude in order to perform the extrapolation to the πN threshold (from the energy corresponding to the experimental data); the p-wave component of their amplitude is considered to be

[4]Notice that a large isospin-breaking effect has been reported [33] around $T_\pi\sim40$ MeV.

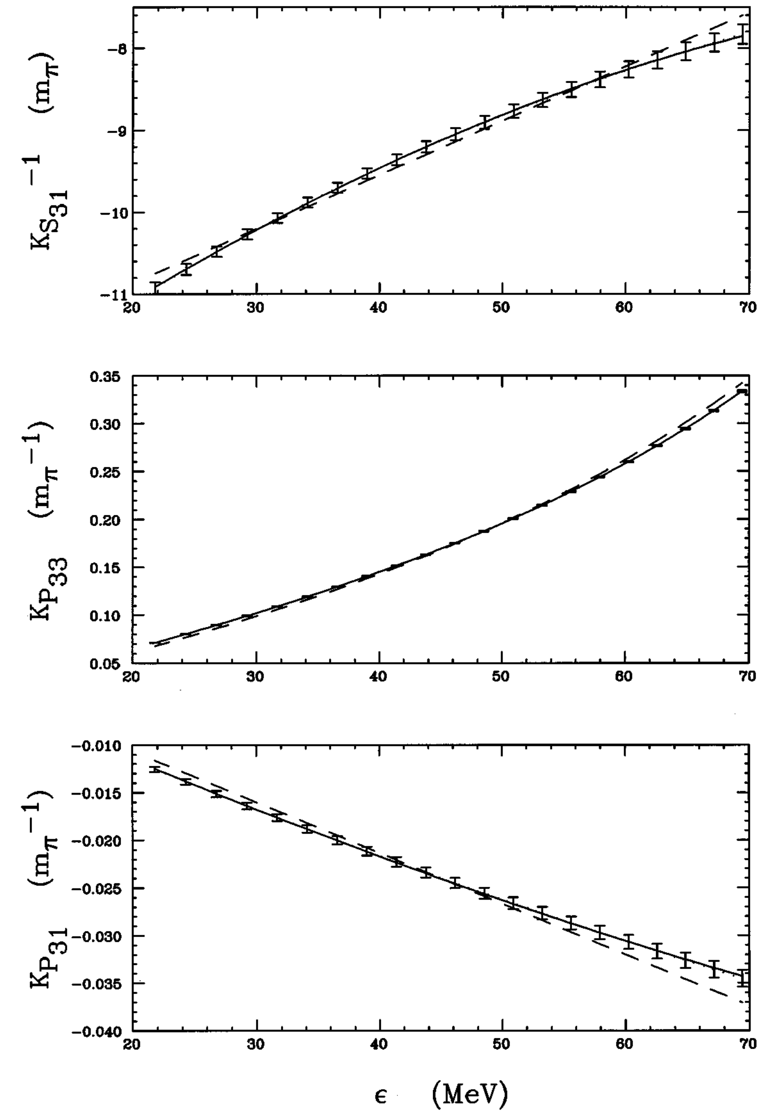

FIG. 3. The energy dependence of the s- and p-wave K-matrix elements corresponding to isospin $I=3/2$ and being obtained with the πN model of Ref. [18] from low-energy elastic-scattering data [25]. The results of the fits to these values with first-, second-, and third-order polynomials (in ϵ) are also shown; the second-order polynomials represent the expansions considered in the present analysis. The results of the fits with up to linear terms in ϵ are denoted by the dashed lines, the ones with up to quadratic terms [i.e., Eqs. (4) and (5)] by the solid lines and the ones with the cubic terms included [on top of the terms shown in Eqs. (4) and (5)] by the dotted lines; the dotted lines are hardly discernible, almost coinciding with the solid lines at all energies. Notice that the resonant term of Eq. (6) has been included in the P_{33} channel.

proportional to $k^2\cos(\theta)$, θ denoting the c.m. scattering angle. In the terminology of Ref. [18], this is equivalent to assuming that the coefficient c_1 of the πN scattering amplitude is a constant in the low-energy region (e.g., below 50 MeV). As deduced from Fig. 4 of Ref. [18], this may not be a safe assumption.

(c) Arndt and collaborators [39] produced a new phase-shift solution (SM95) using forward and fixed-t dispersion relations. Their database comprises *both* the old and the new data. Their value for a_3 is $-0.091\ m_\pi^{-1}$; at the moment, it is not possible to attribute a meaningful uncertainty to this value [40]. The SM95 solution (which is accessible via the SAID on-line system) lies in between our results and the Karlsruhe phase-shift values in the S_{31} channel; it agrees very well with our results in the P_{33} and P_{31} channels.

V. SUMMARY

The recent π^+p low-energy differential cross-section measurements have been analyzed. An expansion of the K matrix containing a quadratic term in the pion c.m. kinetic energy (on top of the standard threshold expansion) has been found to describe the S_{31} and P_{31} channels very well; for the description of the resonant P_{33} channel, a Breit-Wigner formula with an energy-dependent width has been added on top of the background term.

The description of the experimental data is satisfactory. The χ^2/NDF values obtained are reasonable; with small exceptions, the recent π^+p data seem to be consistent among themselves.

Our results in the S_{31} channel disagree with those of the Karlsruhe analyses (which have been based on the old database, i.e., on experiments conducted before 1980). We report values of the phase shift $\delta(S_{31})$ lying about one degree above the phase-shift solutions KH80 and KA85. The scattering length a_3, obtained in the present work, is $-0.077\pm0.003\ m_\pi^{-1}$; it is also in disagreement with the Karlsruhe values.

The p waves, obtained in the present work, are compatible with the KH80 and the KA85 solutions below $\epsilon\sim50$ MeV. Above this energy, the tendency in the P_{33} channel is towards slightly smaller values (by about one degree).

Using the same expansion of the K matrix, we have directly fitted to the Bertin *et al.* data which dominated the low-energy database in the pre-meson-factory era. We then agree with the Karlsruhe values for the scattering length

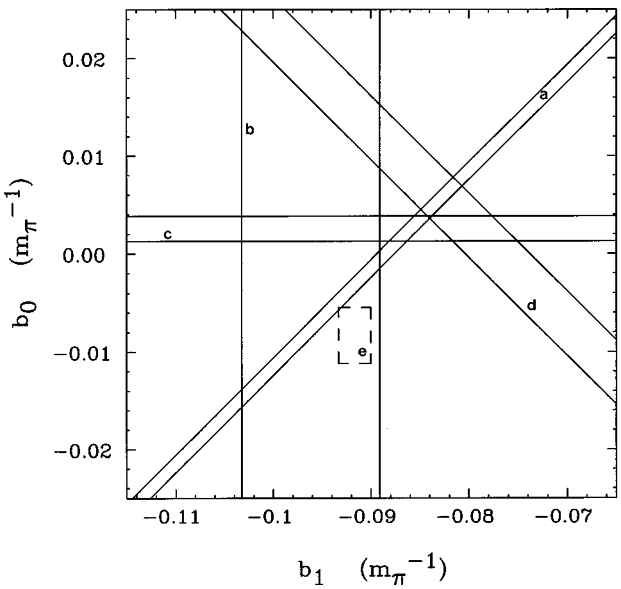

FIG. 4. The various constraints on a (b_1,b_0) plot. The bands correspond to (a) the 1s energy-level shift in pionic hydrogen [16], (b) the width of the 1s state in pionic hydrogen [16], (c) the 1s energy-level shift in pionic deuterium [17] assuming a constant correction factor [30,31], (d) the value of the a_3 extracted in the present analysis, and (e) the KA85 solution [4] (the uncertainties have been taken from Ref. [3]).

a_3. Therefore, the aforementioned discrepancies can only manifest differences between the old and the new scattering data; they cannot be attributed to the method of the analysis.

A comparison of the present result for the a_3 with the values of the πN scattering lengths, recently obtained from measurements of the 1s energy-level shift and width in pionic hydrogen and deuterium, leads to the conclusion that, within the present (experimental and theoretical) accuracies, the various experimental results are not incompatible with the isospin symmetry of the strong interaction.

Note added in proof: The measurements of Ref. [42] are in very good agreement with the results reported herein.

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APPENDIX

Given here are some additional details about the recent π^+p database of the differential cross-section measurements. We feel that such a discussion might be useful as far as future plans, relating to experimental activities, are concerned. In part (a) of the Appendix, we investigate the internal consistency of the π^+p measurements taken at each meson factory. In part (b), we deal with the question of compatibility of the π^+p measurements taken at different meson factories. In parts (c) and (d), we discuss some special aspects concerning the analysis of the data, e.g., commenting on the data sets which are removed after the imposition of our χ^2 criterion, in relation to parts (a) and (b). In part (e), each of the π^+p data sets is compared with the solution describing the bulk of the data. Finally, in part (f), we investigate effects of the absolute normalization of the individual data sets.

a. Internal consistency of the π^+p differential cross-section measurements. Separate fits were performed to the measurements obtained at the three meson factories [9–15]; the data were analyzed with methods I–III. From this analysis, we drew the following conclusions.

The data taken at LAMPF [9] are internally consistent.

The data taken at PSI (Refs. [12] and [15]) are not internally consistent; the fits to these measurements (85 entries) gives a χ^2/NDF between 1.67 and 2.10 depending on the method of analysis. The source of the problem is the JORAM95 32.7 MeV [15] data.

The data taken at TRIUMF (Refs. [10, 11, 13, and 14]) are internally consistent.

b. Consistency of π^+p differential cross-section measurements taken at different meson factories. We considered the combinations PSI+TRIUMF, LAMPF+TRIUMF, and LAMPF+PSI. We came to the following conclusions.

The worst results correspond to the combination of the PSI and the TRIUMF measurements. The reason is that the simultaneous description of the BRACK90 66.8 MeV [13] and the JORAM95 68.6 MeV data is impossible; these measurements contradict one another in the shape of the angular distribution of the π^+p differential cross section.

The LAMPF and the TRIUMF data are not inconsistent.

The LAMPF and the PSI data become consistent after one removes the JORAM95 32.7 and 68.6 MeV measurements.

c. Combined fits, no χ^2-criterion. As the next step, we decided to attempt the global description of all recent π^+p low-energy measurements. To comply with the standards set by the Particle Data Group (see Ref. [20], p. 9), we excluded the four RITCHIE83 data sets [22].[5] We were rather surprised to find that the description of the data was rather satisfactory (Table II, first block).

d. Combined fits, χ^2 criterion. We then decided to investigate the possibility of a bias (in our results) introduced by some of the measurements (the JORAM95 32.7 MeV data set and the inconsistency of the measurements around 67 MeV being always in mind). To this end, we applied a χ^2 criterion to the contribution of each individual data set, in practice excluding all data sets with a χ^2 contribution (per entry) above the mean plus one standard deviation in the overall χ^2 distribution. To our satisfaction, this simple cut removed the JORAM95 32.7 MeV data set and the two mu-

[5]In order to check whether any of the conclusions, drawn in the present work, are affected by the exclusion of the RITCHIE83 data, fits were performed after these measurements were also included in the database. No statistically significant changes, other than the expected increase of the χ^2 values of the fit, were observed. The values of the important quantities (i.e., the scattering length a_3 and the phase shifts) are practically unaffected by the inclusion of these data.

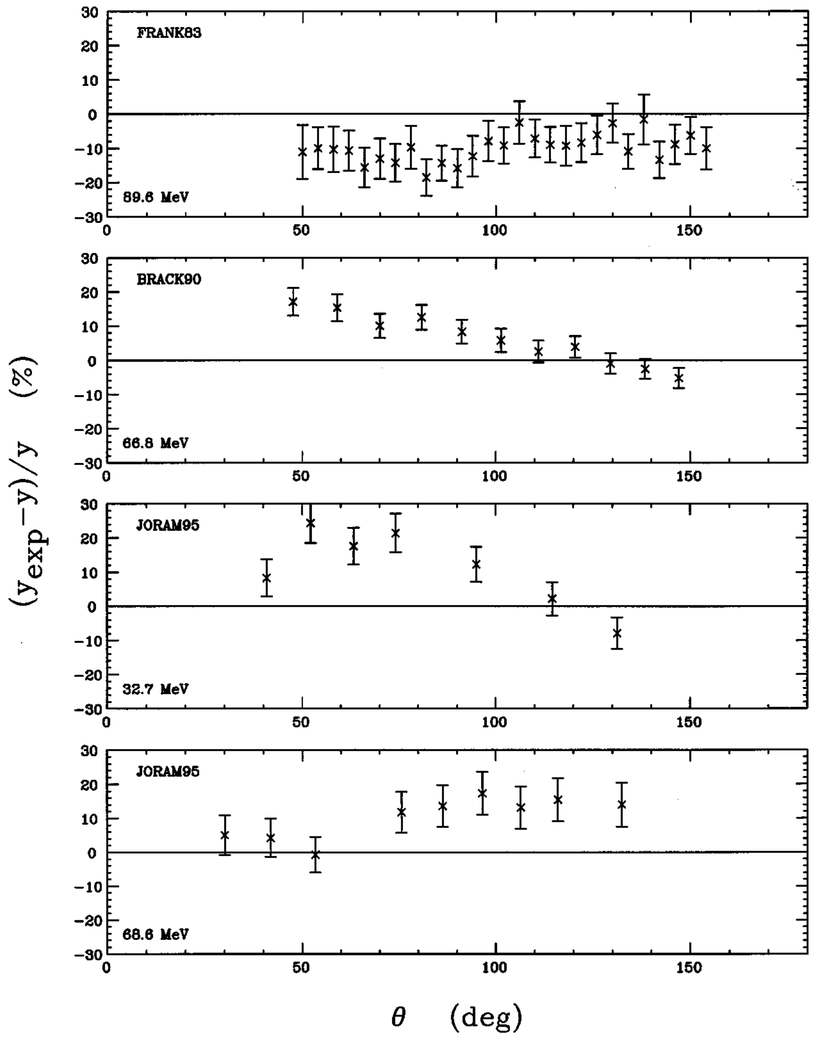

FIG. 5. Comparison between the four data sets, which are excluded after the application of our χ^2 criterion, and the remaining data in the form of relative deviation. θ denotes the c.m. scattering angle. The experimental values are denoted by y_{exp}; y stands for the prediction representing the bulk of the measurements used in the present work. Although the y values correspond to method II, our solutions with the other methods yield very similar results.

tually conflicting measurements around 67 MeV (the cut also removing the FRANK83 89.6 MeV [9] measurements). The description of the remaining data became more than satisfactory (see Table II, second block). Fortunately, none of the conclusions, drawn in the previous stage of the analysis, had to be modified; the values for all our important quantities were found to be insensitive to this cut.

e. Comments on the π^+p measurements. Based on the results of our fits, the following conclusions for the individual π^+p data sets can be drawn.

The RITCHIE83 data lie above our solutions by roughly 10%.

The FRANK83 89.6 MeV measurements lie below our solutions by approximately 10% (see Fig. 5); the other three data sets of Ref. [9] are well reproduced, though the 49.5 and the 69.6 MeV data are not decisive due to their large uncertainty (in the absolute normalization). The FRANK83 low-energy measurement plays an important role in the present analysis. This data set, being the most solid block of measurements close to the πN threshold and spanning between $\theta = 47°$ and 154°, unavoidably exerts a large impact on the determination of the scattering length a_3. In view of the importance of the breaking of isospin symmetry, any experimental activity on the π^+p reaction around (and below) 30 MeV is certainly anticipated with great interest.

The BRACK86 [10] data are very well reproduced; the 66.8 MeV measurements are slightly (one standard deviation) below our solutions.

The BRACK88 [11] data are very well reproduced.

The WIEDNER89 [12] data are well reproduced, yet they do not play a decisive role.

The BRACK90 66.8 MeV data set cannot be reproduced by any means, not even after floating the absolute normalization of these measurements (see Fig. 5); its shape is contradicting the bulk of the π^+p data. The remaining measurements of Ref. [13] are well reproduced in shape, yet they lie slightly above our solutions.

The BRACK95 [14] measurements are very well reproduced.

The JORAM95 68.6 MeV data would have been consistent with our solutions, had they been floated to lower values by roughly 10% (see Fig. 5). The 45.1 MeV data are well reproduced. The 32.7 MeV data have a peculiar shape which is inconsistent with the bulk of the measurements (see Fig. 5). Finally, the data taken in the Coulomb-interference region agree with our solutions, yet they are not decisive in the determination of the important quantities in this article.

f. Effect of the absolute normalization of the individual data sets. Although floating the absolute normalization of an experimental data set is orthogonal to our philosophy, we have eventually decided to perform the analysis with the following minimization function:

$$\chi^2=\sum_{j=1}^{N}\left\{\left(\frac{z_j-1}{\Delta z_j}\right)^2+\sum_{i=1}^{n_j}\left(\frac{y_i-z_j y_i^{\exp}}{\sigma_i}\right)^2\right\}.$$

N denotes the total number of data sets. The number of the differential cross-section measurements in the data set j is denoted by n_j. y_i and $y_i^{\exp}$ are (respectively) the estimated and measured values of the differential cross section for the ith entry of the data set j. The σ_i do not include the normalization uncertainties. The parameter z_j determines the amount at which the particular data set j has to be floated in order to match the bulk of the measurements; for measurements agreeing with the majority of the data, the parameters z_j should be close to unity. The quantity Δz_j is the uncertainty in the absolute normalization of the particular data set j. Evidently, the first term on the right-hand side of the previous equation is the penalty one has to pay for floating the absolute normalization of the measurements. The formula, used here, is similar to the one introduced in Ref. [41].

Two fits have been performed with this minimization function. The first one was carried to all the measurements [9–15]. For the second one, the four data sets described in part (d) of this Appendix were removed. None of the essential values, reported in the present article, change beyond the uncertainties quoted. Our conclusions are also completely unaffected. The χ^2/NDF values obtained are 451.8/$(266-27)\sim 1.89$ (fit to all data) and 267.2/$(212-23)$ ~ 1.41 [the four data sets, described in part (d) of this Appendix, being removed]. The corresponding a_3 values are, respectively, -0.072 ± 0.006 m_π^{-1} and -0.071 ± 0.006 m_π^{-1}.

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