

## Dirac-Foldy term and the electromagnetic polarizability of the neutron

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We reconsider the Dirac-Foldy contribution  $\mu^2/m$  to the neutron electric polarizability. Using a Dirac equation approach to neutron-nucleus scattering, we review the definitions of Compton continuum ( $\bar{\alpha}$ ), classical static ( $\alpha_E^n$ ), and Schrödinger ( $\alpha_{\text{Sch}}$ ) polarizabilities and discuss in some detail their relationship. The latter  $\alpha_{\text{Sch}}$  is the value of the neutron electric polarizability as obtained from an analysis using the Schrödinger equation. We find in particular  $\alpha_{\text{Sch}} = \bar{\alpha} - \mu^2/m$ , where  $\mu$  is the magnitude of the magnetic moment of a neutron of mass  $m$ . However, we argue that the static polarizability  $\alpha_E^n$  is correctly defined in the rest frame of the particle, leading to the conclusion that twice the Dirac-Foldy contribution should be added to  $\alpha_{\text{Sch}}$  to obtain the static polarizability  $\alpha_E^n$ . [S0556-2813(97)00101-5]

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### I. INTRODUCTION

The electromagnetic polarizabilities of the nucleon continue to attract interest because of their importance for the understanding of the substructure of the nucleon. The proton and neutron form an isospin doublet with (presumably) similar substructure, and so it is expected that comparing experimental polarizabilities of the two would lead to more insight into this substructure [1,2]. This comparison should take into account the different *definitions* of electromagnetic polarizabilities actually used in the measurements on the proton and the neutron. The electric and magnetic polarizabilities of the proton can be defined [3,4] and measured [5] via Compton scattering at relatively low energies because of the interference of the Rayleigh amplitude (from the polarizabilities) with the Thomson amplitude (from the charged proton). Because the neutron is neutral, there is no such interference and the cross section for elastic Compton scattering is much smaller. Furthermore, the data must come from a neutron bound in a nucleus, say, a deuteron [6], and it is a challenge to interpret it in terms of neutron polarizabilities [7,8]. An alternative would be to determine neutron polarizabilities via quasifree Compton scattering, but the first experiment could only obtain an upper limit for the electric polarizability [9]. It is expected to be redone at SAL with a considerable reduction in the statistical error [10]. The best determination of the electric polarizability of the neutron is obtained, at present, from low-energy neutron-atom scattering [11]. The intense electric field near the surface of the nucleus  $^{208}\text{Pb}$  induces a dipole moment in the neutron which makes a tiny but extractable contribution to the scattering amplitude. The electric polarizability of the neutron is defined as the coefficient

of the  $r^{-4}$  nonrelativistic potential acting between these two systems.

We wish to reexamine the relationship between the definitions of neutron electric polarizability in use, noting that the Compton scattering definition is manifestly relativistic and the neutron-atom scattering definition is not. Furthermore, the Compton scattering definition actually used to extract the electric polarizability of the proton (soon to be extended to the neutron) does not employ a Hamiltonian or a wave equation and the neutron-atom scattering definition is in the context of the Schrödinger equation with its implied Hamiltonian. The common meeting ground of these seemingly disparate definitions in current use is given by the relationship of each one to the classical definition of  $\alpha_E^n$  as the coefficient of  $\mathbf{E}^2$  in  $V^{\text{pol}} = -\frac{1}{2}\alpha_E^n \mathbf{E}^2 \approx -Q^2 \alpha_E^n / (2r^4)$ , where  $V^{\text{pol}}$  represents the interaction of a neutral particle *at rest* with the Coulomb field  $\mathbf{E} \sim Q\hat{r}/r^2$  of an infinitely heavy charged system [12]. In the following we establish these relationships. That is, we (1) remind the reader of the definition of  $\alpha_s$  extracted by experimentalists from the spin-averaged Compton cross section and the definition of the more intuitive Compton polarizability  $\bar{\alpha}$  of, for example, chiral perturbation theory calculations (only the latter  $\bar{\alpha}$  corresponds to a true “deformation” effect on the nucleon); (2) quote the classical limit  $\alpha_E^n$  of the Compton polarizability of a neutral particle,  $\alpha_E^n = \bar{\alpha} + \mu^2/m = \alpha_s + 2\mu^2/m$ , where  $\mu$  is the anomalous (in this case, total) magnetic moment of the neutron [13]; (3) embed the Compton-defined  $\bar{\alpha}$  in a relativistic Dirac description of neutron-atom scattering to establish the nonrelativistic classical limit  $\alpha_E^n = \bar{\alpha} + \mu^2/m$ , where the static polarizability  $\alpha_E^n$  is the coefficient of  $\mathbf{E}^2$  in the neutron’s rest frame; with the *correct* rest frame wave equation this result is identical with the classical limit of the Compton result of (2); (4) assert that the rest frame of a neutron in an external electric field is defined by a vanishing value of the velocity operator, as confirmed by experimental measurements of the Aharonov-Casher effect; (5) note that Schmiedmayer *et al.* [11] and others [14] use the Schrödinger equa-

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tion in order to analyze low-energy neutron-atom scattering experiments; the coefficient of  $\mathbf{E}^2$  in this equation is then  $\alpha_{\text{Sch}}$  and was considered the electric polarizability of the neutron from those experiments; and (6) show that the  $\alpha_{\text{Sch}}$  of Schmiedmayer *et al.* [11] and others [14] is neither the Compton-defined  $\bar{\alpha}$  nor the static  $\alpha_E^n$ , but  $\alpha_{\text{Sch}} = \alpha_E^n - 2\mu^2/m$ .

We conclude from this chain of arguments that twice the Dirac-Foldy contribution  $\mu^2/m$  should be added to  $\alpha_{\text{Sch}}$  to obtain the static polarizability  $\alpha_E^n$  from the existing analysis of neutron-atom scattering experiments. That is the message of our paper.

The details of the discussion of Compton-defined polarizabilities are given in Sec. II, and our Dirac equation discussion of the electromagnetic aspects of neutron-atom scattering is given in Sec. III.

## II. POLARIZABILITIES IN COMPTON SCATTERING

Already in the earliest experimental studies of low-energy Compton scattering from the proton [15] it was realized that the ‘‘polarizabilities’’ entering into the Rayleigh amplitude had two contributions. That is, the external electromagnetic fields both deform the particle and act upon the static distribution of the electric charge and the magnetic moment. Thus we read in Ref. [15] that ‘‘the term ‘polarizability’ used here is not equivalent to the one normally employed for neutral particles.’’ The situation is made more difficult by the fact that the nucleon is a spin-1/2 particle with an anomalous magnetic moment and is described by the Dirac equation. The Compton scattering matrix for a spin-1/2 particle is the sum of six Lorentz-invariant quantum field amplitudes which are free of kinematic singularities and constraints [3]. These six amplitudes each contain single-nucleon pole terms (a structureless Dirac nucleon with charge  $Z$  and magnetic moment  $\mu$  and on-shell vertices). For each amplitude the remainder is called a continuum contribution and is now free of both kinematic singularities and dynamical singularities (from the nucleon poles). If one thinks of polarizabilities as a ‘‘deformation’’ effect on the structure of the nucleon, they would seem to be most naturally defined in terms of the latter continuum contributions. However, there is a freedom in the definition of Compton polarizabilities of spin-1/2 particles due to the fact that the entire Compton matrix is not measured. Instead present experiments measure the spin-averaged cross section which corresponds to only the spin-independent part of the Compton matrix. Bernabéu and Tarrach [16] (BT) note that the nucleon pole contributions to the amplitudes of the complete spin- $\frac{1}{2}$  Compton scattering matrix generate both pole *and* continuum contributions to the spin-averaged amplitudes actually measured as a differential cross section. The most common choice (labeled  $\alpha_s$  by BT and used in this paper as well) includes in the ‘‘polarizability’’ terms from the magnetic moment of the structureless Dirac particle. For this choice the differential cross section takes the form

$$\left(\frac{d\sigma}{d\Omega}\right)_L = \left(\frac{d\sigma}{d\Omega}\right)_{\text{poles}}^{\text{proton}} - \frac{\alpha}{m_p} \omega^2 \left[1 - \frac{3\omega}{m_p}(1-z)\right] \times [(1+z^2)\alpha_s^p + 2z\beta_s^p] + \mathcal{O}(\omega^4), \quad (1)$$

valid through the first three moments of the photon laboratory energy  $\omega$ , where  $z = \cos\theta_L$  [3,13]. The Born terms of the invariant amplitudes go into the Thomson cross section for a pointlike particle with mass  $m$  and charge  $Z$  in its rest frame and the (actually used) Powell cross section of Eq. (1), also for a pointlike particle, but one which includes an anomalous magnetic moment [17]. Equation (1) or its extension to higher energy is used to extract  $\alpha_s^p$  from proton Compton scattering data [5].

Now we return to the classical definition of  $\alpha_E^n$  as the coefficient of an  $\mathbf{E}^2$  or  $r^{-4}$  term in a nonrelativistic wave equation. The concept of a potential as it applies to the interaction of two systems in relativistic quantum field theory and the computation of such van der Waals potentials due to induced dipoles (when the systems are far apart) have been discussed extensively by Feinberg and Sucher [18]. The electromagnetic forces between charged and/or neutral systems are due to the exchange of photons and can be calculated with the aid of dispersion relations from the relativistic Compton amplitudes of photons scattering from the system. Specifically, the potential is to be defined iteratively in such a way that when used in a specified two-body (Dirac) wave equation in the c.m. system it will reproduce, up to a given order, the field-theory amplitude associated with one-photon- and two-photon-exchange graphs. The potential then can be reduced to the Schrödinger form and its long-ranged part compared with the nonrelativistic polarizability potential. Thus there is a clear line of connection between the electric Compton polarizability and the classical electric polarizability which does not depend upon an intuitively appealing but theoretically uncertain mixture of relativistic and nonrelativistic concepts [19].

This program of connecting classical polarizability with the low-energy Compton scattering parameters has been carried out by Feinberg and Sucher for a variety of systems (two spinless and uncharged particles, one neutral spinless and one charged spin- $\frac{1}{2}$  particle [20], etc.), all but the one relevant to our examination of neutron-atom scattering. The long-range potential of these two systems, a very massive charged spin-zero nucleus and a neutral spin- $\frac{1}{2}$  neutron (with an anomalous magnetic moment), has been worked out by Bernabéu and Tarrach [16]. They note that the nucleon pole contributions to the six amplitudes of the complete spin- $\frac{1}{2}$  Compton scattering matrix generate both pole *and* continuum contributions to the spin-averaged amplitudes actually measured as a differential cross section. Thus one can define (in their notation but our units [13]) an  $\alpha_s$  which does include a term with the anomalous magnetic moment  $[-(e\mu Z/m^2 + \mu^2/m)]$  or a  $\bar{\alpha}$  which is given only in terms of the continuum (nonpole) contributions of the spin-averaged amplitudes. The former definition corresponds to the actual analysis of Compton scattering data [5] according to Eq. (1) and the latter is advocated by Bernabéu and Tarrach and used in some theoretical treatments [21]. The latter polarizabilities so defined do not receive any contribution from Born graphs involving the anomalous magnetic moment of the proton and the neutron. The polarizabilities are entirely given in terms of the continuum part of the Compton amplitude. Equivalently  $\bar{\alpha}$  is *defined* to be zero for a point neutral Dirac particle. In the classical limit of a static electric field acting on a

neutral particle of mass  $m$  and magnetic moment  $\mu$  the coefficient of the  $r^{-4}$  potential which survives is given by

$$\alpha_E^n = \bar{\alpha} + \frac{\mu^2}{m} = \alpha_s + \frac{2\mu^2}{m}. \quad (2)$$

It is then this sum which is measured in the scattering of neutrons by heavy nuclei at low energies.

### III. DIRAC EQUATION ANALYSIS OF NEUTRON-ATOM SCATTERING

We see then how a natural definition of the polarizability of a neutral particle  $\bar{\alpha}$  arises in the context of Compton scattering and understand its connection via the Feinberg-Sucher-Bernabéu-Tarrach analysis with the polarizability potential  $V^{\text{pol}} = -\frac{1}{2}\alpha_E^n \mathbf{E}^2$  of a Schrödinger analysis of low-energy neutron scattering. We now establish such a connection again, this time starting from a relativistic Dirac description of the neutron-nucleus scattering. We derive the neutron polarizability as the nonrelativistic limit of a relativistic Dirac Hamiltonian

$$H^D = \beta m + \boldsymbol{\alpha} \cdot \mathbf{p} - i\mu\beta\boldsymbol{\alpha} \cdot \mathbf{E} - \frac{1}{2}\bar{\alpha}\mathbf{E}^2, \quad (3)$$

where the first three terms comprise the standard formula [17,22,23] for a point neutral Dirac particle with an anomalous magnetic moment  $\mu$  in an electric field. As in the BT treatment of Compton scattering,  $\bar{\alpha}$  is that part of the neutron's polarizability that does not contain the nucleon magnetic moment  $\mu$ . Even so, the nonrelativistic reduction of Eq. (3) has a term in  $\mathbf{E}^2$  in addition to the nominal polarizability  $\bar{\alpha}$ :

$$\left( \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{p}}{m} \cdot (\mathbf{E} \times \boldsymbol{\mu}) + \frac{\mu}{2m} (\nabla \cdot \mathbf{E}) + \frac{\mu^2 \mathbf{E}^2}{2m} - \frac{1}{2}\bar{\alpha}\mathbf{E}^2 \right) \psi = E\psi, \quad (4)$$

where we have neglected interaction terms that vanish faster than  $r^{-4}$  at large distance. The second and third terms in Eq. (4) are the Schwinger term arising from the interaction between the (moving) magnetic moment of the neutron and the electric field of the atom, and the Foldy-Darwin scattering from the electric charge distribution of the atom (nucleus + electrons). These terms are taken into account in the nonrelativistic analysis of neutron-atom scattering [24–26]. Then it would seem that the coefficient in the polarizability potential is

$$\alpha_{\text{Sch}} = \bar{\alpha} - \mu^2/m, \quad (5)$$

rather than the Compton-defined  $\bar{\alpha}$ .

This (premature) result could have been anticipated by Foldy's observation that a structureless (point) neutral Dirac particle with an anomalous magnetic moment  $\mu$  in a homogeneous static electric field  $\vec{E}$  is an exactly soluble model [22]. That is,

$$H = \beta m + \boldsymbol{\alpha} \cdot \mathbf{p} - i\mu\beta\boldsymbol{\alpha} \cdot \mathbf{E}. \quad (6)$$

This Hamiltonian can be diagonalized, in the frame where  $\mathbf{p}=0$ , by simply squaring and one finds the energy eigenvalues  $W = \pm \sqrt{m^2 + \mu^2 \mathbf{E}^2} \approx \pm [m + \mu^2 \mathbf{E}^2/2m + \dots]$ . The non-

relativistic limit of this model has a positive coefficient of  $\mathbf{E}^2$  which implies a *negative* polarizability of magnitude  $\mu^2/m$  from this Dirac-Foldy term, just as we find in Eq. (5). Moreover, now we see that the analysis leading to Eq. (5) has been carried out in the frame  $\mathbf{p}=0$ .

But we must be careful to define the polarizability of a particle in that particle's rest frame [12] and the rest frame of a particle is defined by a vanishing value of the velocity operator  $\mathbf{v}$ . The *particle* velocity operator is given as a derivative of the Hamiltonian on the left-hand side of Eq. (4):

$$\mathbf{v} \equiv \frac{\partial H}{\partial \mathbf{p}} = \frac{1}{m} [\mathbf{p} - (\mathbf{E} \times \boldsymbol{\mu})]. \quad (7)$$

That is, the ( $\mathbf{v}=0$ ) frame is *not* the ( $\mathbf{p}=0$ ) frame leading to Eq. (5). From Eq. (7) one can rewrite Eq. (4) in the form familiar from discussions of the Aharonov-Casher effect [27–29]:

$$\left[ \frac{1}{2m} \left( [\mathbf{p} - (\mathbf{E} \times \boldsymbol{\mu})]^2 + \frac{\mu}{2m} (\nabla \cdot \mathbf{E}) - \frac{\mu^2 \mathbf{E}^2}{2m} - \frac{1}{2}\bar{\alpha}\mathbf{E}^2 \right) \right] \psi = E\psi. \quad (8)$$

From this equation one identifies

$$\alpha_E^n = \bar{\alpha} + \mu^2/m \quad (9)$$

to be the coefficient of  $\mathbf{E}^2$  in the particle rest frame ( $\mathbf{v}=0$ ) and  $\alpha_E^n$  is then the static polarizability of the neutron.

This rest frame result is in agreement with the Compton scattering analysis of BT in Eq. (2). In order to avoid any possible misunderstanding, let us emphasize that our discussion of polarizability terms in neutron-nucleus scattering is entirely in the framework of the nonrelativistic limit of the Dirac equation. From that viewpoint, one may argue that it provides an intuitive way of understanding the results of Bernabeu and Tarrach [16] which were obtained from dispersion relations calculations. We do, however, discuss in detail the form of the nonrelativistic wave equation (8) rather than Eq. (4) to be used in conjunction with the BT results.

The observation [30] of the phase shift predicted by Aharonov and Casher [27] for a neutral particle with a magnetic moment (neutron) diffracted around a line of electric charge shows conclusively that Eq. (8) is the correct rest frame equation. For a neutron diffracting around a line charge in a region where  $\nabla \cdot \mathbf{E}=0$ , the Aharonov-Casher phase shift is obtained by evaluating the line integral of  $\mathbf{p} = m\mathbf{v} + (\mathbf{E} \times \boldsymbol{\mu})$  along the path of the diffracted neutrons. (Of course,  $\bar{\alpha}$  could not play any role in this macroscopic experiment, and the fact that the term  $\mu^2 \mathbf{E}^2/2m$  disappears in the Aharonov-Casher geometry is explained in Refs. [28,29].) More recent experiments involving neutral atoms with magnetic moments have measured Aharonov-Casher phase shifts to within a few percent of the theoretically predicted value [31].

The neutron optics experiment, fortified by more exact measurements with atomic systems, demonstrates the Aharonov-Casher insight that velocity is the meaningful relativistic kinematic operator for a neutron in an external electric field. We have used this insight to define the correct static polarizability of a neutral particle with a magnetic moment. From Eqs. (5) and (9) it is clear that the  $\alpha_{\text{Sch}}$  measured

in the experiments of Schmiedmayer *et al.* [11] and others [14] is neither  $\bar{\alpha}$  nor  $\alpha_E^n$ . Indeed from Eqs. (5) and (9) we learn that

$$\alpha_{\text{Sch}} = \alpha_E^n - 2\mu^2/m. \quad (10)$$

Numerically,

$$|\alpha_{\text{Sch}} - \alpha_E^n| = 1.2 \times 10^{-4} \text{ fm}^3 \quad (11)$$

can be compared with

$$\begin{aligned} \alpha_{\text{Sch}}([11]) &= 12 \pm 1.5 \pm 2.0 \times 10^{-4} \text{ fm}^3, \\ \alpha_{\text{Sch}}([14]) &= 0.0 \pm 5 \times 10^{-4} \text{ fm}^3. \end{aligned} \quad (12)$$

This difference is about 10% on the scale of the result of Schmiedmayer *et al.* [11] and quite significant for the central value of the result of Koester *et al.* [14]. Both results came from a Schrödinger equation analysis like Eq. (4). The discrepancy in Eq. (12) perhaps comes from the treatment of individual terms in the electromagnetic interaction of Eq. (4) or from the treatment of the strong interaction between the neutron and the nucleus. In any case, our Dirac equation analysis has nothing to say about the origin of the present experimental discrepancy. We note that these experiments are being repeated [32,33] with an expected experimental error smaller in magnitude than our correction term of Eq. (11).

Finally we note that L'vov [34] obtains (by another argument) a relationship between  $\alpha_E^n$  and  $\bar{\alpha}$  which agrees with Eq. (5) if one equates  $\alpha_E^n$  and  $\alpha_{\text{Sch}}$  as he does. In the neutron rest frame, however, the correct relationship is that of Eq. (10).

In summary, we have reviewed the definition of the electrical polarizability of a neutral spin- $\frac{1}{2}$  particle with a magnetic moment  $\mu$  in the analysis of Compton scattering. We have shown how a Dirac equation analysis of low-energy neutron-atom scattering yields a static polarizability *defined in the rest frame of the neutron*. Our result (10) means that twice the Dirac-Foldy contribution  $\mu^2/m$  should be added to the existing Schrödinger values to obtain the static polarizability of the neutron.

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