Calculation of the shell model energies for states in a variety of configurations in ^{208}Bi

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The well known experimental levels of ^{208}Bi have been calculated with a generalized intermediate coupling model with Gaussian potential with and without an octupole phonon interaction parameter. The neutron-proton interaction parameters are taken from those obtained in the calculation of 34 experimental levels in ^{210}Bi. The goodness of fit of the calculated energies to the energies of the 36 experimentally assigned states in ^{208}Bi is quite remarkable in spite of the fact that the neutron-proton interaction parameters have been transferred from fitting in ^{210}Bi, a neutron shell away. Finally, the calculated levels of ^{208}Bi are compared with those calculated by a delta force. [S0556-2813(97)04906-6]

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With just one proton more than the 82 proton closed shell, and one neutron less than the 126 neutron closed shell, ^{208}Bi provides an interesting testing ground for shell model calculations on odd-odd spherical nuclei. A considerable amount of experimental data on ^{208}Bi has accumulated [1–4]. The data comprise 8 configurations, and a total of 36 states. A number of other states have been speculatively assigned spin-parities and configurations in ^{208}Bi. However we consider the assignments too tentative to attempt to reproduce them theoretically. In this calculation, we consider only two quasiparticle states, and not four quasiparticle states, a number of which are known.

Because of its position, so close to the double closed shell in ^{208}Pb, a number of calculations of ^{208}Bi have been attempted [5–7]. The phenomenological calculations of Kim and Rasmussen [6] have done the best job of fitting the levels in ^{208}Bi. Using a tensor force and extremely careful fitting of the various force parameters to the $\nu 1g_{9/2}\otimes\pi 0h_{9/2}$ ground state configuration of ^{210}Bi which are very sensitive to the final results, they have calculated the levels in a number of nuclei, including both ^{210}Bi and ^{208}Bi.

On the other hand, the calculations of Kuo [7] used a much larger shell model space than Kim and Rasmussen, and reaction matrix elements which allow no variation in the nucleon-nucleon interaction parameters from the free space values. Thus, their model is a more demanding test of theory. However, Kuo's calculations do not do as well in the prediction of experimentally observed states in ^{208}Bi.

For this reason, we revert to the more phenomenological treatments in this paper. We include tensor force, spin-orbit force, and octupole collectivity in our treatment. Furthermore, we attempt a comparison of the calculated energy levels in ^{208}Bi, using the neutron-proton interaction parameters obtained in a previous calculation of the levels in ^{210}Bi [8], with the experimental levels. In this way there are no adjustable neutron-proton interaction parameters. The only one adjustable parameter originates in the fact that the energies are taken relative to the ground state. A description of the generalized intermediate coupling model (GICM) follows.

In the GICM [8], odd-odd nuclei are assumed to consist of a vibrating even-even core and two outer nucleons (odd proton and odd neutron). Harmonic core vibrations are described phenomenologically with the help of creation and annihilation phonon operators, $b^{\dagger}_{\lambda\mu}$ and $b_{\lambda\mu}$, respectively. In our calculations of ^{208}Bi, only one octupole phonon states are considered.

For a description of one-quasiparticle neutron and proton states, the model of independent quasiparticles is used [9]. In our calculations of ^{208}Bi, neutron states are assumed as hole states and proton states as particle states. Neutron-hole and proton-particle states are taken from odd neighbors, ^{207}Pb and ^{209}Bi, respectively.

We assume [10] that the interaction between the odd nucleon and vibrating core in the first approximation is proportional to $b^{\dagger}_{3\mu}+b_{\overline{3\mu}}$:

$$H_{n\ \mathrm{core}} \quad \text{or} \quad H_{p\ \mathrm{core}}=\xi_3\sqrt{\pi}\frac{\hbar\omega_3}{\sqrt{7}}\sum_{\mu=-3}^{3} Y^{*}_{3\mu}(b^{\dagger}_{3\mu}+b_{\overline{3\mu}}), \tag{1}$$

where ξ_3 is the interaction strength for neutrons or protons, $\hbar\omega_3=2615$ keV is the energy of one octupole phonon state, and $b_{\overline{3\mu}}=(-1)^{3+\mu}b_{3-\mu}$. In our calculations, we assume the same values of ξ_3 for $H_{n\ \mathrm{core}}$ and $H_{p\ \mathrm{core}}$.

The neutron-proton interaction consists of the central part, noncentral tensor part and spin-orbit part [11,12]:

$$\begin{aligned} H_{np}=&V_c(r)(u_0+u_1\boldsymbol{\sigma}_{\mathrm{p}}\cdot\boldsymbol{\sigma}_{\mathrm{n}}+u_2P_M+u_3P_M\boldsymbol{\sigma}_{\mathrm{p}}\cdot\boldsymbol{\sigma}_{\mathrm{n}})\\ &+V_t(r)(u_t+u_{tm}P_M)\left[\frac{1}{r^2}(\boldsymbol{\sigma}_{\mathrm{p}}\cdot\boldsymbol{r})(\boldsymbol{\sigma}_{\mathrm{n}}\cdot\boldsymbol{r})-\frac{1}{3}(\boldsymbol{\sigma}_{\mathrm{p}}\cdot\boldsymbol{\sigma}_{\mathrm{n}})\right]\\ &+V_{ls}(r)(u^s_e+u^s_oP_M)\boldsymbol{l}\cdot\boldsymbol{s}, \end{aligned} \tag{2}$$

TABLE I. Parameters (in MeV) calculated using 34 states in ^{210}Bi, calculated χ^2 values for 36 ^{208} Bi levels, and ground-state neutron-proton interaction strengths E_0 (in MeV). See text for more details.

Parameter	Gauss	Gauss + octupole interaction	Delta	Delta + octupole interaction
u_0	-40.4 ± 2.5	-40.8 ± 0.8	-1.16 ± 0.22	-1.17 ± 0.21
u_1	-2.7 ± 1.1	-2.6 ± 0.9	-0.34 ± 0.10	-0.34 ± 0.10
u_2	-32 ± 6	-31 ± 5		
u_3	-0.5 ± 2.1	-0.5 ± 2.0		
u_t	-73 ± 6	-73 ± 5		
u_{tm}	-108 ± 16	-108 ± 14		
u_e^s	-11 ± 5	-11 ± 5		
u_o^s	35 ± 13	35 ± 12		
ξ_3		0.37 ± 0.11		$0.51+0.17-1.18$
χ^2	52	43	367	414
E_0	0.111	0.111	0	0.002

where r is the distance between neutron and proton, $\boldsymbol{\sigma}_n$ and $\boldsymbol{\sigma}_p$ are Pauli spin matrices, P_M is the space exchange operator, $\boldsymbol{l}$ is the orbital angular momentum of the relative motion of proton and neutron, and $\boldsymbol{s}$ is the total spin of both nucleons. For $V_c(r)$, $V_t(r)$, and $V_{ls}(r)$ in Eq. (2) we use the Gaussian shape

$$V(r)=\exp(-r^2/r_0^2), \tag{3}$$

where $r_0=1.4$ fm, or the delta function

$$V(r)=\delta(r). \tag{4}$$

Using the methods described above for the GICM, we have calculated the levels in ^{210}Bi with and without the interaction between odd nucleons and one octupole phonon, fitting a total of 34 levels [8]. The results are given in Table I. The n-p interaction parameters were then used directly to calculate the 36 known experimental energy levels in ^{208}Bi. We used the harmonic oscillator wave functions for 5 neutron hole states from ^{207}Pb, and 4 proton particle states from ^{209}Bi. The neutron states used for the model space were $(2p_{1/2})^{-1}$ (0 keV), $(1f_{5/2})^{-1}$ (570 keV), $(2p_{3/2})^{-1}$ (898 keV), $(0i_{13/2})^{-1}$ (1633 keV), and $(1f_{7/2})^{-1}$ (2340 keV), while the proton states were $0h_{9/2}$ (0 keV), $1f_{7/2}$ (896 keV), $0i_{13/2}$ (1609 keV), and $1f_{5/2}$ (2826 keV).

Four different calculated values of the 64 levels in ^{208}Bi are given in Table II. The sets E_A and E_B are for the Gaussian shape n-p interaction without and with the interaction between one octupole phonon and odd nucleons; the sets E_C and E_D are for the delta force without and with the interaction between one octupole phonon and odd nucleons.

The experimental energies of the 9^- states $[(\nu 1f_{5/2})^{-1}\otimes\pi 0i_{13/2}$ and $(\nu 0i_{13/2})^{-1}\otimes\pi 1f_{7/2}]$ at 2413 keV and 2475 keV, respectively, are shown with parentheses in Table II. The configurational assignments of the 9^- states are reported for the first time here. When the (p,t) reaction on the 3×10^6 year 271 keV second excited state in ^{210}Bi with $J^\pi=9^-$ and configuration $\nu 1g_{9/2}\otimes\pi 0h_{9/2}$ is carried out, one observes two $l=0$ states with only a small amount of the $l=0$ strength at 2413 and 2475 keV [13]. Thus they each have a small amount of the configuration $[^{206}\mathrm{Pb}]0^+\otimes\nu 1g_{9/2}\otimes\pi 0h_{9/2}$. We propose that the major components of the configurations of these two 9^- states are $(\nu 1f_{5/2})^{-1}\otimes\pi 0i_{13/2}$ and $(\nu 0i_{13/2})^{-1}\otimes\pi 1f_{7/2}$, respectively, in agreement with these calculations.

Figure 1 shows a comparison of the experimental levels of ^{208}Bi with those calculated with the Gaussian shape n-p interaction without the interaction between one octupole phonon and odd nucleons (E_A), and with the delta force without the interaction between one octupole phonon and odd nucleons (E_C); with this interaction, we would obtain an extremely similar figure. 9^- states at 2413 keV and 2475 keV are included and indicated for the Gaussian shape and delta n-p interactions.

Fitting the Gaussian n-p interaction parameters without the interaction between one octupole phonon and odd nucleons to the 36 ^{208}Bi levels (9^- states at 2413 keV and 2475 keV not included) we arrive at $\chi^2=41$ similar to the value 43 obtained using ^{210}Bi parameters with the interaction between one octupole phonon and odd nucleons included. Fitting the Gaussian n-p interaction parameters with the interaction be-

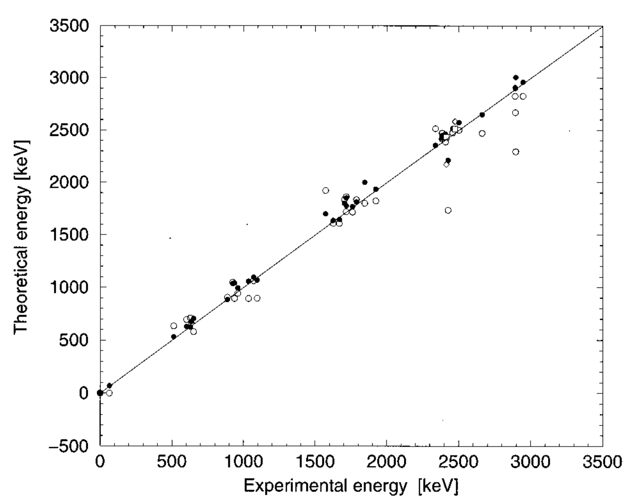

FIG. 1. Plot of the experimental data vs the theoretical calculations without phonon interaction for the Gaussian force (marked by filled circles), for the delta force (marked by open circles). The reported newly assigned 9^- states are marked by squares for the Gaussian force and by diamonds for the delta force.

TABLE II. Comparison of the energies of the experimentally identified states in ^{208}Bi multiplets, $E_{\rm expt}$, to the model results with the Gaussian shape n-p interaction without phonon interaction, $E_{\rm A}$, and with one interacting octupole phonon, $E_{\rm B}$, with the delta force without phonon interaction, $E_{\rm C}$, and with one interacting octupole phonon, $E_{\rm D}$. All energies are in keV. See text for the states in parentheses.

Major configuration	I^π	$E_{\rm expt}$	$E_{\rm A}$	$E_{\rm B}$	$E_{\rm C}$	$E_{\rm D}$	Major configuration	I^π	$E_{\rm expt}$	$E_{\rm A}$	$E_{\rm B}$	$E_{\rm C}$	$E_{\rm D}$
$(\nu 2p_{1/2})^{-1}\otimes\pi 0h_{9/2}$	4^+	63	70	69	0	0		5^+	2384	2423	2376	2472	2384
	5^+	0	0	0	0	0		6^+	2408	2462	2417	2385	2301
$(\nu 1f_{5/2})^{-1}\otimes\pi 0h_{9/2}$	2^+	925	1038	1034	1049	1050		7^+	2339	2355	2309	2513	2428
	3^+	634	674	674	665	666		8^+	2661	2649	2604	2471	2382
	4^+	602	628	627	694	695	$(\nu 1f_{5/2})^{-1}\otimes\pi 1f_{7/2}$	1^+		2239	2204	1915	1864
	5^+	629	623	623	710	712		2^+		1714	1683	1652	1602
	6^+	511	531	531	635	636		3^+		1769	1739	1704	1652
	7^+	651	705	706	581	581		4^+		1553	1525	1570	1518
$(\nu 2p_{3/2})^{-1}\otimes\pi 0h_{9/2}$	3^+	1070	1098	1095	1064	1066		5^+		1727	1696	1792	1738
	4^+	959	995	993	942	942		6^+		1679	1654	1652	1599
	5^+	887	884	884	905	905	$(\nu 1f_{5/2})^{-1}\otimes\pi 0i_{13/2}$	4^-		2530	2500	2362	2313
	6^+	1095	1068	1066	898	898		5^-		2159	2133	2314	2262
$(\nu 2p_{1/2})^{-1}\otimes\pi 1f_{7/2}$	3^+	937	1041	1015	896	844		6^-		2256	2227	2227	2175
	4^+	1034	1056	1028	896	844		7^-		2228	2199	2195	2144
$(\nu 2p_{1/2})^{-1}\otimes\pi 0i_{13/2}$	6^-	1626	1637	1608	1609	1553		8^-		2162	2136	2182	2134
	7^-	1668	1645	1617	1609	1553		9^-	(2413)	2433	2396	2171	2120
$(\nu 0i_{13/2})^{-1}\otimes\pi 0h_{9/2}$	2^-	2894	3004	2967	2294	2267	$(\nu 2p_{3/2})^{-1}\otimes\pi 1f_{7/2}$	2^+		2234	2204	2223	2167
	3^-	1921	1935	1917	1822	1792		3^+		1944	1915	1955	1902
	4^-	1844	2001	1982	1799	1770		4^+		1894	1864	1832	1780
	5^-	1704	1799	1781	1835	1805		5^+		2017	1988	1803	1750
	6^-	1716	1852	1834	1720	1691	$(\nu 2p_{3/2})^{-1}\otimes\pi 0i_{13/2}$	5^-		2785	2754	2482	2433
	7^-	1716	1778	1759	1863	1833		6^-		2504	2470	2432	2374
	8^-	1760	1769	1753	1718	1688		7^-		2542	2510	2469	2411
	9^-	1787	1814	1796	1834	1803		8^-		2664	2626	2503	2446
	10^-	1571	1699	1683	1924	1894	$(\nu 0i_{13/2})^{-1}\otimes\pi 1f_{7/2}$	3^-		3158	3113	3069	3002
	11^-	2427	2211	2194	1734	1704		4^-		2664	2625	2617	2546
$(\nu 2p_{1/2})^{-1}\otimes\pi 1f_{5/2}$	2^+	2945	2959	2951	2826	2828		5^-		2651	2608	2746	2671
	3^+	2890	2908	2905	2826	2828		6^-		2626	2586	2678	2606
$(\nu 1f_{7/2})^{-1}\otimes\pi 0h_{9/2}$	1^+	2892	2903	2860	2669	2593		7^-		2562	2520	2667	2591
	2^+	2501	2571	2529	2500	2419		8^-		2515	2480	2649	2568
	3^+	2458	2516	2471	2475	2390		9^-	(2475)	2512	2473	2582	2500
	4^+	2384	2448	2404	2418	2334		10^-		2726	2679	2512	2421

tween one octupole phonon and odd nucleons does not improve the quality of the fit very much. In this case $\chi^2=37$.

Finally, the 36 ^{208}Bi levels have been fitted with a delta force interaction. Interestingly the interaction between one octupole phonon and odd nucleons plays no role, since we get $\xi_3=0$ as a result of the fit. The other two parameters were $u_0=-1.14$ MeV and $u_1=0.22$ MeV and χ^2 changed to 280. This is a very small improvement with respect to the values obtained without fitting and still approximately 6 times higher than the values obtained for the Gaussian n-p interaction without fitting (see Table I).

In the calculations of χ^2 an error of 10 keV for all experimental energies is assumed.

The experimental value of the strength of the neutron-proton interaction in the ground state, E_0, can be calculated from the binding energies:

$$E_0(^{208}\text{Bi})=B(^{209}\text{Bi})+B(^{207}\text{Pb})-B(^{208}\text{Bi})-B(^{208}\text{Pb}) \tag{5}$$

and is determined to be (from the Wapstra 1993 mass evaluation) $E_0(^{208}\text{Bi})_{\rm expt}=(0.09\pm0.01)$ MeV. The results of our model are presented in Table I. The Gaussian n-p interaction gives values close to the experimental value.

The agreement between experimental and calculated levels for ^{208}Bi in Fig. 1 and Table II for the Gaussian n-p interaction are quite remarkable. Since the neutron-proton and nucleon-phonon interaction parameters of Table I have been taken over from ^{210}Bi, there is in fact no adjustable neutron-proton and nucleon-phonon interaction parameter in the calculation. The only one adjustable parameter comes from fact that the model energies are taken relative to the ground state energy. This agreement occurs in spite of the fact that the neutron shell in ^{210}Bi is different from the neutron shell of ^{208}Bi.

Furthermore, it is clear from the χ^2 values (see Table I) that the addition of the octupole interaction parameter ξ_3 increases the goodness of calculation for the Gaussian type n-p interaction.

It should also be noted that the fit with the Gaussian type n-p interaction is far more successful in reproducing the experimental ^{208}Bi levels than the delta type n-p interaction. In fact, even when the delta force is parameterized by fitting directly the ^{208}Bi levels, the goodness of calculation, as indicated by the χ^2 values, is better with the Gaussian force field without adjustable parameters. Perhaps one of the reasons for the successful calculation of the ^{208}Bi levels is that the change from $A=210$ to 208 is relatively small.

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