Running coupling constants in Walecka model and renormalization-group equations

Wang Zisheng, Ma Zhongyu, and Zhuo Yizhong
China Institute of Atomic Energy, P.O. Box 275(18), Beijing 102413, China

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The effective running coupling constants in the Walecka model are investigated by introducing renormalization-group equations at finite density, where couplings are divided into two parts: density dependence and momentum dependence. The results are applied to one-loop calculations in nuclear matter. [S0556-2813(97)03006-9]

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A model of strongly interacting nucleons and mesons was proposed by Walecka [1] to study the bulk properties of high-density matter. It is quite successful in the mean-field theory and in the relativistic Hartree approximation (RHA) [2]. However, the calculated incompressibility of nuclear matter seems to be too large. Alternatively, a nonlinear scalar field is proposed to fit the incompressibility of the nuclear matter [3–5]. But the self-coupling parameter of the quartic term is negative, which will cause the many-body system to collapse at high density [6,7], while a nonlinear vector field [8] or derivative scalar coupling term [9] obviously leads the model to not being able to be renormalized.

In loop expansion, especially, the contributions of two-loop order to energy density give a too large correction [10], which cannot satisfy the convergent requirement. It is possible for these problems to be attributed to the treatment of the starting point of solving the Walecka model at one-loop order, where the meson fields are replaced by classical expectation values of the ground state at a finite density of the nuclear matter.

In this paper, we try first to consider quantum corrections of meson fields by the effective action formalism and its derivative expansions. Then, the scalar and vector meson fields are rescaled and renormalization-group equations are derived at the finite density. Finally, replacing the scalar and vector couplings by the effective running coupling constants obtained by renormalization-group equations, we investigated the bulk properties of the nuclear matter at one-loop order.

The properties of symmetric nuclear matter can be described in terms of a sigma ϕ and a omega V_μ meson field in the Walecka model [1]. It is convenient to employ an effective action formalism in which the quantum corrections may be directly included in the action [11–14]. Integrating out the fermion fields, one finds

$$S_{\rm eff}=-i{\rm Tr}\,\ln[\gamma_\mu(i\partial^\mu-g_vV^\mu)-(M-g_s\phi)]+\int d^4x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\mu_v^2V_\mu V^\mu\right)+\int d^4x\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}\mu_s^2\phi^2\right),\tag{1}$$

where $F^{\mu\nu}=\partial^\mu V^\nu-\partial^\nu V^\mu$, the trace is performed with respect to space, momentum, and internal variables.

It is noted that in general the fields ϕ and V_μ are x dependent. So that $[p_\mu,\phi]=-i\hbar\partial_\mu\phi\neq 0$ and $[p_\mu,V_\nu]=-i\hbar\partial_\mu V_\nu\neq 0$ because of the quantum corrections. Thus the functional operations formally indicated in Eq. (1) can be performed neither in momentum nor in coordinate spaces. In particular, it is not clear how to manipulate Eq. (1) into the form, $S=\int d^4x\mathcal{L}_{\rm eff}$. Fortunately, an elegant method [15–21] of solving this problem has been developed by expanding $S_{\rm eff}$ in terms of the derivatives of the meson fields,

$$S_{\rm eff}=\int d^4x[-i{\rm tr}\,\ln[\gamma_\mu(i\partial^\mu-g_vV^\mu)-(M-g_s\phi)]+\tfrac{1}{2}Z_s(\phi)\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{4}Z_v(\phi)F_{\mu\nu}F^{\mu\nu}-\tfrac{1}{2}\mu_s^2\phi^2+\tfrac{1}{2}\mu_v^2V_\mu V^\mu],\tag{2}$$

where the trace is only operated with respect to momentum and internal variables. And the effective action is only expanded to second-order derivatives including all divergences because the higher derivative terms are not important at a low-energy case. Z_s and Z_v will be written into two parts of momentum and density dependence, such as

$$Z_s(\phi)=1+\frac{1}{4\pi^2}g_s^2\ln\frac{\Lambda_s^2}{M^{*2}}=1+\frac{1}{2\pi^2}g_s^2\left(\ln\frac{\Lambda_s}{M}-\ln\frac{M^*}{M}\right),\tag{3}$$

$$Z_v(\phi)=1+\frac{1}{12\pi^2}g_v^2\ln\frac{\Lambda_v^2}{M^{*2}}=1+\frac{1}{6\pi^2}g_v^2\left(\ln\frac{\Lambda_s}{M}-\ln\frac{M^*}{M}\right),\tag{4}$$

where $M^*=M-g_s\phi,\Lambda_s$ and Λ_v are cutoff parameters for momentum. Obviously, the second terms in Z_s and Z_v are from the quantum corrections [15,16]. Now, one may renormalize for fields ϕ and V_μ by

$$\phi_R=Z_s^{1/2}(\phi_0)\phi,\quad V_{R\mu}=Z_v^{1/2}(\phi_0)V_\mu,\tag{5}$$

and couplings g_s and g_v by

$$g_{Rs}=Z_s^{-1/2}(\phi_0)g_s,\quad g_{Rv}=Z_v^{-1/2}(\phi_0)g_v,\tag{6}$$

at point of the classical expection value of the meson fields at the finite density of the nuclear matter. Clearly the renormalization point ϕ_0 may be taken at any point in the physical region. Thus the coupling constants in any renormalization scheme should be regarded as a function of the renormalization point. In view of the renormalization group the coupling constants are Λ and M^* dependent and are called the effective running coupling constants [21–25].

In order to describe the change of the effective couplings $\overline{g}_s$ and $\overline{g}_v$ in terms of M^* and Λ_s or Λ_v, the renormalized factors are regarded as the functions of $t_i=\ln(\Lambda_i/M)$ and $T=\ln(M^*/M)$,

$$Z_i=Z_i(g_i,t_i,T)=Z_i(g_i,\tau_i), \quad i=s,v. \tag{7}$$

It is noted that $\Lambda_i>M$ and $M^*\leqslant M$ should be satisfied in the physical region. Therefore, the ranges of t_i and T are $t=[0,\infty]$ and $T=[-\infty,0]$, which can be combined to become $\tau=[-\infty,\infty]$ by setting $t_i=\tau_i$ and $\overline{\Lambda_i}=\Lambda_i$ for $\tau_i>0$ and $T=\tau_i$ and $\overline{\Lambda_i}=M^*$ for $\tau_i\leqslant 0$.

The renormalization-group equations can be derived by differentiating both sides of the trival identity $Z_i^{-1}Z_i=1$ with respect to τ_i and by applying the chain rule [23],

$$\left(\frac{\partial}{\partial\tau_i}-\beta_{\tau_i}\frac{\partial}{\partial g_i}+\gamma_i\right)Z_i=0, \tag{8}$$

where

$$\gamma_i=-\frac{\partial}{\partial\tau_i}\ln Z_i(g_i,\tau_i)\bigg|_{g_i}=M\frac{\partial}{\partial M}\ln Z_i(g_i,\tau_i)\bigg|_{\overline{\Lambda_i},g_i,\tau_i}, \tag{9}$$

$$\beta_s=M\frac{\partial}{\partial M}[g_{Rs}(g_s,\tau_s)]\bigg|_{\overline{\Lambda_s},g_s,\tau_s}=\begin{cases}\dfrac{1}{4\pi^2}g_{Rs}^3, & \tau_s>0,\\[2ex] -\dfrac{1}{4\pi^2}g_{Rs}^3, & \tau_s\leqslant 0.\end{cases} \tag{10}$$

$$\beta_v=M\frac{\partial}{\partial M}[g_{Rv}(g_v,\tau_v)]\bigg|_{\overline{\Lambda_v},g_v,\tau_v}=\begin{cases}\dfrac{1}{12\pi^2}g_{Rv}^3, & \tau_v>0,\\[2ex] -\dfrac{1}{12\pi^2}g_{Rv}^3, & \tau_v\leqslant 0.\end{cases} \tag{11}$$

Equations (10) and (11) imply that the β functions are cutoff independent. The physical coupling constants should satisfy the differential equations [23,25],

$$\frac{d}{d\tau_i}\overline{g}_i(g_i,\tau_i)=\beta_i(\overline{g}_i). \tag{12}$$

For the range of $\tau_i=[0,\infty]$, which is corresponding to the case of zero density in the Dirac sea, Eq. (12) may be solved by using Eqs. (10) and (11). One finds

$$\overline{g^2_{Fs}}=g^2_{Fs}/[1-(1/2\pi^2)g^2_{Fs}\ln(\Lambda_s/M)],$$

$$\overline{g^2_{Fv}}=g^2_{Fv}/[1-(1/6\pi^2)g^2_{Fv}\ln(\Lambda_v/M)], \tag{13}$$

where the normalization conditions $g^2_{Fi}=\overline{g^2_{Fi}}(\tau_i=0,g_i)$ are used. One can see that the Walecka model is not ultraviolet asymptotically free because the running coupling constants increase with the increases of the cutoff parameters. The conclusion is obvious because the Walecka model is not a non-Abelian field theory, which has been obtained in particle physics earlier. And the conditions, such as $g^2_{Fs}<2\pi^2/\ln(\Lambda_s/M)$ and $g^2_{Fv}<6\pi^2/\ln(\Lambda_v/M)$, should be satisfied. In the following, we will set $g_{Fs}=g_{Fv}\simeq 1$, which is phenomenologically used in the strong interaction [26]. It is noted that the contributions of Λ_s and Λ_v to the coupling constants, and energy density (Table I) are very small. This means that the effective cutoff momentum is about $\Lambda\sim M$ [see Eqs. (3), (4), and (13)], which is in agreement with the one, such as $\Lambda\sim 1$ GeV, usually used in the strong interaction.

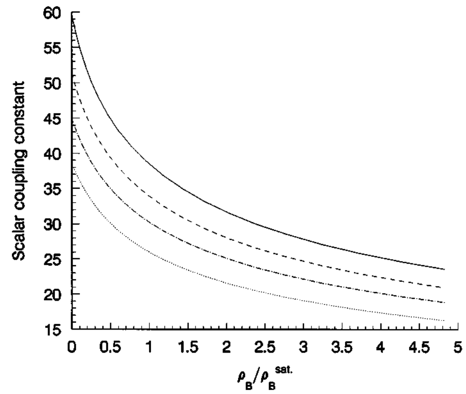

FIG. 1. Scalar coupling $\overline{g^2_{Ds}}$ decrease with density $\rho_B/\rho_B^{\rm sat}$ increasing for the different masses of scalar meson $m_s=550$ MeV (solid line), 500 MeV (dashed one), 458 (dot-dashed), and 400 MeV (dot one).

For the region of $\tau_i=[-\infty,0]$, which is corresponding to the case of the finite density in the Fermi sea, integrating over Eq. (12) and using Eqs. (10) and (11), we find

$$\overline{g^2_{Ds}}=g^2_{Ds}/[1+(1/2\pi^2)g^2_{Ds}\ln(M/M^*)],$$

$$\overline{g^2_{Dv}}=g^2_{Dv}/[1+(1/6\pi^2)g^2_{Dv}\ln(M/M^*)], \tag{14}$$

which mean that the couplings decrease as the baryonic density increases (see Figs. 1 and 2). These are in agreement

TABLE I. Parameters used in calculations for bulk point $k_F=1.42$ fm^{-1}.

m_v (MeV)	m_s (MeV)	g_s^2	g_v^2	M^*/M	K_v^{-1} (MeV)	Λ_s (GeV)	Λ_v (GeV)
783	550	62.9	79.8	0.72	470		
783	550	59.7	43.1	0.83	183		
783	500	51.4	50.1	0.82	190		
783	458	45.0	57.5	0.81	194		
783	400	38.6	75.7	0.78	205		
783	500	47.4	51.1	0.82	192	5	5
783	500	45.0	53.3	0.81	198	20	20
783	500	45.0	53.7	0.81	198	20	15
783	500	45.2	52.1	0.81	196	15	20

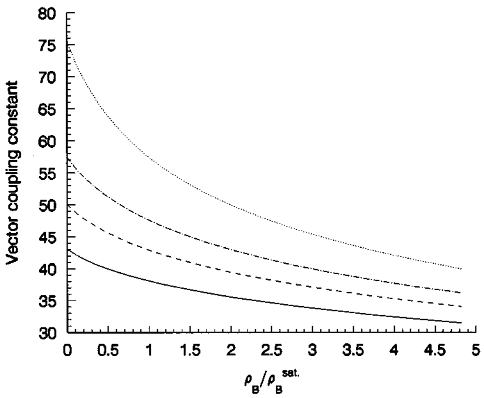

FIG. 2. Vector coupling $\overline{g}^2_{Dv}$ decrease with density $\rho_B/\rho_B^{\text{sat}}$ increasing for the different masses of scalar meson m_s=550 MeV (solid line), 500 MeV (dashed one), 458 (dot-dashed), and 400 MeV (dot one).

with the phenomenological analysis [27,28] in nuclear physics. It is noted that $g^2_{Fi}g^2_{Di}=g^2_i$ and $\overline{g}^2_{Fi}\overline{g}^2_{Di}=\overline{g}^2_i$ are used in Eq. (14).

After all these steps, the original Walecka action is now recast into the form,

$$S_{\text{eff}}=\int d^4x\mathcal{L}_{\text{eff}}=\int d^4x\Big\{-i\text{tr}\ln\{\gamma_\mu[i\partial^\mu-\overline{g}_v(V^\mu_R-W^\mu)]-(M-\overline{g}_s\phi_R)\}-\frac{1}{4}F_{R\mu\nu}F^{\mu\nu}_R+\frac{1}{2}m^2_vV_{R\mu}V^\mu_R+\frac{1}{2}\partial_\mu\phi_R\partial^\mu\phi_R-\frac{1}{2}m^2_s\phi^2_R\Big\},\tag{15}$$

where $\overline{g}_{Rs}=\overline{g}_{Fs}\overline{g}_{Ds}$, $\overline{g}_{Rv}=\overline{g}_{Fv}\overline{g}_{Dv}$ and $m^2_s=\mu^2_s/Z_s$, $m^2_v=\mu^2_v/Z_v$. The effective Lagrangian has the same form as the Walecka model but with the density and momentum dependence of the masses and coupling constants in the dense matter, which is Brown and Rho's conjecture by means of a scale invariance [29].

For the purpose of studying the finite baryonic matter, a chemical potential μ has been introduced in the effective Lagrangian (15). μ is represented by the four-vector $\overline{g}_vW_\mu=(-i\mu,0,0,0)$. Thus, energy density at one-loop order can be obtained by [21,30]

$$\epsilon(M^*,\rho_B)=\frac{\overline{g}^2_v}{2m^2_v}\rho^2_B+\frac{m^2_s}{2\overline{g}^2_s}(M-M^*)^2+\frac{\gamma}{(2\pi)^3}\int_0^{k_F}d^3k\times(\vec{k}^2+M^{*2})^{1/2}-\frac{\gamma}{16\pi^2}\Big[M^{*4}\ln\frac{M^*}{M}+M^3(M-M^*)-\frac{7}{2}M^2(M-M^*)^2-\frac{13}{3}M(M-M^*)^3-\frac{25}{12}(M-M^*)^4\Big],\tag{16}$$

where the spin-isospin degeneracy of the vacuum is γ=4 in the nuclear matter. The effective mass of the nucleon in the nuclear matter can be obtained by minimizing the energy density ϵ with respect to M^*, which yields

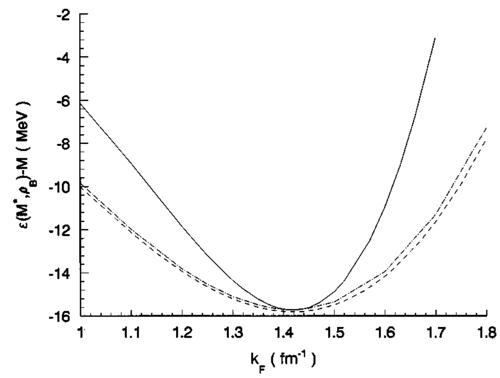

FIG. 3. Nuclear matter saturation curve as the function of the Fermi momentum k_F (fm^{-1}) for the RHA with bare couplings (solid line) and m_s=550 MeV (dashed one) and 400 MeV (dot-dashed line) in this work.

$$\frac{\rho^2_B}{2m^2_v}\frac{\partial\overline{g}^2_v}{\partial M^*}+\frac{m^2_s}{\overline{g}^2_s}(M-M^*)-\frac{m^2_s}{2\overline{g}^4_s}(M-M^*)^2\frac{\partial\overline{g}^2_s}{\partial M^*}+\frac{\gamma}{(2\pi)^3}\int_0^{k_F}d^3kM^*(\vec{k}^2+M^{*2})^{-1/2}-\frac{1}{\pi^2}\Big[M^{*3}\ln\frac{M^*}{M}+M^2(M-M^*)-\frac{5}{2}M(M-M^*)^2+\frac{11}{6}(M-M^*)^3\Big]=0.\tag{17}$$

It is noted that the contribution of the vector meson to the effective mass has been entered as shown by Eq. (17), which seems to arrive at the effects that the nonlinear vector fields had been introduced in the Walecka model [8].

In order to produce the saturation property of the symmetric nuclear matter with the binding energy -15.75 MeV at the saturation point k_F=1.42 fm^{-1}, several sets of parameters are adjusted for different masses of σ meson (Table I).

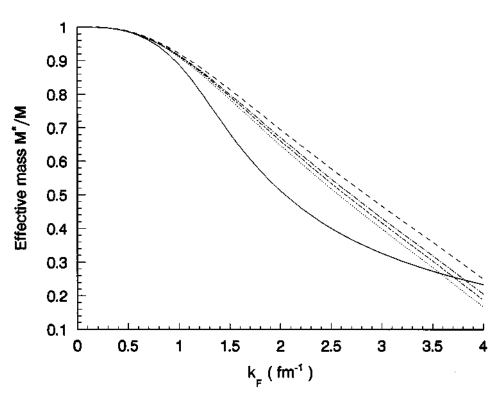

FIG. 4. Effective mass M^*/M of nucleon in nuclear matter as the function of the Fermi momentum k_F (fm^{-1}) for the RHA with bare couplings (solid line) and m_s=550 MeV (dashed one), 500 MeV (dot-dashed), 458 MeV (line-dashed), and 400 MeV (dot one) in this work.

We see in Table I that the vector coupling constant has a large decrease in comparison with the one in the RHA with bare couplings [30] (first row in Table I), which may be a required condition to guarantee convergence because an almost equal contribution of scalar and vector meson to energy density is needed in two-loop calculations [31] (another paper for two loop analysis is needed).

By using the effective running couplings, the saturation curve is much softer than the one with bare couplings [30] (see Fig. 3), which makes the incompressibility of the nuclear matter have a large decrease from 470 to 200 MeV or so. The results are in agreement with the empirical data of Treiner *et al.*, $K_\infty^{-1}=210\pm30$ MeV [32]. The effective mass of the nucleon in the nuclear matter is showed in Fig. 4, where the effective mass at saturation increases from $0.72M$ to 0.80–$0.83M$. The result is simlar to the calculations of the phenomenological Skyrme interaction ($M^*/M=0.78$, $K_v^{-1}=217$ MeV), including the derivative scalar coupling term in the Walecka model ($M^*/M=0.85$, $K_v^{-1}=225$ MeV) [9] and the quark-meson coupling model ($M^*/M=0.85$, $K_v^{-1}=200$ MeV) [33,34]. A larger effective mass is also helpful for the description of the giant quadrupole resonance and level density in the vicinity of the Fermi surface as stressed by Jaminon and Mahaux [35]. It is noted in studies of finite nuclei that the value of the nucleon effective mass at equilibrium seems to be between $0.58M$ and $0.65M$ in order to reproduce spin-orbit splittings in nuclei [36,37]. However, their analysis is only based on the mean-field theory with a nonlinear meson. If one consider vacuum effects, the value should be increased as shown by the change from $0.56M$ to $0.72M$ in the nuclear matter by fitting the binding energy [30,2]. Of course, such an increase should be verified by a concrete calculation.

In this paper, by extending the renormalization-group equations at zero density to the case of the finite density, we find that some problems in the Walecka model may be solved in this way. We have also noted that another possible application in nuclear physics, such as the properties of the meson and coupling constants at finite density, may possibly compare with Brown and Rho's analysis by the way of a scaling effective Lagrangian in a dense medium [29].

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