Equation of state of stellar nuclear matter in the temperature-dependent extended Thomas-Fermi formalism

M. Onsi,* H. Przysiezniak, and J. M. Pearson
Laboratoire de Physique Nucléaire, Département de Physique, Université de Montréal, Montréal, Québec, Canada H3C 3J7
(Received 27 September 1996)

Using the full temperature-dependent extended Thomas-Fermi formalism, we calculate the equation of state of inhomogeneous nuclear matter for a force that has been fitted to essentially all nuclear masses, and has the correct properties of neutron matter. Both droplet and bubble phases are considered, under the conditions appropriate to a collapsing star. We examine the relevance of the choice of effective mass to the temperature. [S0556-2813(97)01506-9]

PACS number(s): 21.65.+f, 21.30.Fe, 21.60.−n, 26.60.+c

I. INTRODUCTION

We are involved in a program to develop a microscopic theory of nuclear systems applicable to the wide variety of phenomena encountered at subnuclear and nuclear densities during and after stellar collapse, and in particular to describe all these phenomena, as far as possible, in terms of a single, universal, effective interaction. The main achievement so far has been the development for the first time of a mass formula based entirely on microscopic forces, the ETFSI-1 mass formula [1–5]. The astrophysical interest of such a mass formula lies in the fact that the *r* process of nucleosynthesis depends crucially on the binding energies of nuclei that are so neutron-rich that there is no hope of being able to measure them in the laboratory. It is thus of the greatest importance to be able to make reliable extrapolations of masses away from the known region, relatively close to the stability line, out towards the neutron-drip line.

The ETFSI method is essentially a high-speed approximation to the Hartree-Fock (HF) method, with a macroscopic part given by the extended Thomas-Fermi (ETF) method [6,7], and shell corrections calculated by the so-called Strutinsky-integral (SI) method [1,5]. Pairing is handled in the BCS approximation with a δ-function force. Although this is really a microscopic-macroscopic mass formula, there is a much greater coherence between the two parts than is the case with mass formulas based on the drop(let) model, since the same Skyrme force underlies both parts. In fact, it has been shown [1,2] that the ETFSI method is equivalent to the HF method in the sense that when the two methods fit the same *form* of Skyrme force to the mass data they give essentially the same extrapolation. This presumably accounts for the fact that with just eight parameters the underlying force of the ETFSI-1 mass formula SkSC4 fits the 1492 mass data for $A \geqslant 36$ with an rms error of only 0.736 MeV [4].

However, the force SkSC4 fails to reproduce the energy-density curve of neutron matter, as calculated by realistic nucleon-nucleon forces [8]. This has serious implications for stellar collapse, and to respond to this problem we recently [9] devised a new force, SkSC6, which gives a much improved fit to neutron matter, without a significant deterioration in the quality of the fit to the mass data, the rms error increasing to 0.794 MeV (see also Ref. [3]). This force is thus very well suited to the determination of the equation of state (EOS) of stellar nuclear matter, and the present paper is devoted essentially to this calculation.

Let us recall, therefore, some of the relevant aspects of stellar collapse. Once the iron core of a massive star ($\mathcal{M} \geqslant 8\mathcal{M}_{\odot}$) starts to collapse (see, for example, the review of Bethe [10]), nuclei that are stable under normal terrestrial conditions will begin to capture electrons, forming thereby nuclei that are highly neutron-rich and yet nevertheless stable under the conditions of rising density. At the same time, neutrons will leak out as the temperature rises, forming effectively a vapor in which the nuclei are suspended, like water droplets in a fog. Eventually, as the nuclei come closer and closer together, bridges will form between them, and the entire core of the star will resemble a foam, or Swiss cheese, the holes being filled with a neutron vapor. Finally, this so-called bubble phase goes over into the homogeneous phase of nuclear matter when the density reaches about 60% the density of ordinary nuclei, which is around $\rho \simeq 0.16$ fm$^{-3} = 2.6 \times 10^{14}$ g cm^{-3}.

The lowest density that we consider is about $\rho = 0.01$ fm^{-3}, at which point nuclear matter is still in the droplet phase. However, neutrino trapping will already be well established at this density (it sets in at around $\rho = 10^{-4}$ fm^{-3}), and no further electron capture occurs, with the result that the electron concentration per nucleon Y_e will thereafter be constant; we assume $Y_e = 0.33$, while for the constant neutrino fraction we take $Y_\nu = 0.07$ per nucleon. Under these conditions the collapse will be essentially adiabatic, and thus isentropic, insofar as thermodynamic equilibrium is maintained; for the constant value of the entropy per nucleon during this stage of the collapse we shall take the fairly typical value of $s = 1.0$ (with temperature measured in energy units, entropy is dimensionless).

Of crucial importance for the collapse is the adiabatic index, defined by

$$\Gamma = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_{S, Y_e}, \tag{1.1}$$

*Present address: Faculty of Natural and Applied Sciences, Notre Dame University, Zouk Mosbeh, Lebanon.

with the derivative evaluated under the conditions of constant entropy and leptonic fractions that we assume to hold. The stability of a star depends on the value of Γ in the core being larger than $\frac{4}{3}$, a value that corresponds to ultrarelativistic leptons. Because of nuclear attractions the adiabatic index can fall below this critical value, and when this happens collapse will begin. However, as nuclear densities are approached in the core Γ will rise above $\frac{4}{3}$ again, with the result that the collapse will come rapidly to a halt, and be reversed into a bounce that may lead to a supernova explosion.

Since the EOS of the homogeneous phase of nuclear matter, which prevails at densities greater than about 0.10 fm^{-3}, has already been calculated for force SkSC6 [9], we concentrate in this paper on the inhomogeneous phases found at lower densities, describing these phases by the model of the Wigner-Seitz cell. For the calculation of the nuclear-energy density corresponding to this force it would be inappropriate to use the usual temperature-dependent Thomas-Fermi method (see, for example, Ref. [11]). Rather, since the mass fit that determined this force involved the fourth-order extended Thomas-Fermi method, we calculate the energy density in stellar nuclear matter by using the temperature-dependent extended Thomas-Fermi (TETF) formalism of Bartel, Brack, and Durand (BBD) [12]. We will, of course, miss the shell corrections that are given by the ETFSI method, but these are known [13,14] to be washed out completely for temperatures above 3 or 4 MeV, and are thus negligible for the temperatures that we deal with in this paper.

Now despite its excellent fit to nuclear masses and the neutron gas, there remains a difficulty with this new force SkSC6. As with all the Skyrme-type forces that we have developed for the ETFSI method, the effective nucleon mass M^* has been set equal to the real nucleon mass M. For the calculation of the masses of all but light nuclei, and fission barriers, this is indeed a good approximation, since it leads to the observed density of single-particle (s.p.) states in the vicinity of the Fermi surface being well reproduced in ordinary mean-field calculations (HF or ETFSI) [15]. However, all nuclear-matter calculations with realistic forces (see, for example, Refs. [8,16]) indicate that at the equilibrium density ρ_0 the value of M^*/M lies between 0.6 and 0.8. This is confirmed experimentally, both by the deepest s.p. states of light nuclei [17], and by giant multipole resonances [18,19]. Actually, there is no contradiction, since it has been shown that one can obtain reasonable s.p. level densities in the vicinity of the Fermi surface of finite nuclei with realistic values of M^*, i.e., $M^*/M \simeq 0.6 - 0.8$, provided one takes into account the coupling between particle modes and surface-vibration modes [20]. Such calculations are, however, complicated, and if one sets $M^* \simeq M$ one may regard this as a semiempirical value of M^* that permits considerable phenomenological success with straightforward mean-field calculations.

Thus the force SkSC6 is surely adequate for treating the isolated nuclei that prevail in a star right up to the moment where collapse begins. On the other hand, it is clear that in the final stage, where the nuclear matter is homogeneous, the effective mass $M^* = M$ of force SkSC6 will be inappropriate. A much better force from this standpoint is the one of Rayet, Arnould, Tondeur, and Paulus (RATP) [21], for which $M^*/M = 0.67$ in nuclear matter at the density ρ_0, even though its symmetry coefficient J and incompressibility K_v are very similar to those of force SkSC6 (see Table II of Ref. [9]). However, because of its effective mass the force RATP is not very suitable for the masses of finite nuclei, since there will be significant errors for open-shell nuclei, despite the good fit to doubly magic nuclei.

We showed in Ref. [9] that the difference in M^* between the two forces has a significant impact on the equation of state (EOS) of homogeneous nuclear matter, since the lower the value of M^* the higher will be the temperature for a given entropy and density (see also Ref. [22]). Thus in the present paper all the calculations that we perform with force SkSC6 are repeated with force RATP, in order to examine the extent to which the problem of the effective mass is relevant to the EOS during the inhomogeneous stages of lower density that precede the transition to the homogeneous phase. Exactly the same calculational procedure is adopted for the two forces, and in particular RATP is calculated with the TETF method, even though the HF method was used for the fit to the masses of the doubly magic nuclei; the resulting error is unlikely to be significant.

We describe our calculatational methods, including the model of the Wigner-Seitz cell in Sec. II, while in Sec. III we present and discuss our results. In the Appendix we summarize for convenience the main results of the TETF formalism; we show in particular how we deal with certain limitations of the present form of this formalism [12].

II. MODEL AND CALCULATION

The matter comprising the core of a collapsing star consists of a mixture of neutrons, protons, electrons, and neutrinos (other particles may put in an appearance at supernuclear densities, but this is of no concern here). This mixture is rigorously neutral on the macroscopic scale, so that the fraction of nucleons that are protons will be exactly equal to the number Y_e of electrons per nucleon.

Whatever the density ρ and temperature T of this system it is always possible, using the formalism of Ref. [9], for example, to calculate the energy per nucleon e and the entropy per nucleon s on the assumption that the system is homogeneous. However, equilibrium for given values of ρ and T corresponds to the Helmholtz free energy per nucleon $f = e - Ts$ being minimized. Now for densities close to ρ_0, or larger, thermal equilibrium will indeed be characterized by the homogeneous configuration, while it is equally clear that for terrestrial temperatures and pressures the equilibrium configuration will be quite different, consisting of ^{56}Fe atoms bound in a solid with an appropriate lattice configuration (of course, the time required for this equilibrium to be reached under terrestrial conditions is much greater than the age of the universe).

However, since we are interested only in densities larger than $\rho = 0.01\ \mathrm{fm}^{-3}$, i.e., $1.6 \times 10^{13}\ \mathrm{g\ cm}^{-3}$, we can assume that the electrons, like the trapped neutrinos, form a homogeneous gas [see, for example, Eq. (2) of Ref. [23]]. Then, following Sec. 3 of Ref. [9], we have for the kinetic-energy density of the electrons

$$\mathcal{E}_e=\frac{1}{4\pi^2}\frac{\mu_{0e}^4}{(\hbar c)^3}\left(1+\frac{2}{3}\frac{\pi^2T^2}{\mu_{0e}^2}\right), \tag{2.1}$$

where we have introduced the electron Fermi energy

$$\mu_{0e}=\hbar c(3\pi^2Y_e\rho)^{1/3}. \tag{2.2}$$

This approximation, which assumes the electrons to be completely relativistic, i.e., neglects their mass entirely, but includes the lowest-order departure from total degeneracy, is adequate over the density and temperature range considered here. To this same order the electron entropy density is

$$\mathcal{S}_e=\frac{1}{3}\frac{\mu_{0e}^2}{(\hbar c)^3}T. \tag{2.3}$$

The corresponding expressions for the neutrinos are

$$\mathcal{E}_\nu=\frac{1}{8\pi^2}\frac{\mu_{0\nu}^4}{(\hbar c)^3}\left(1+\frac{2}{3}\frac{\pi^2T^2}{\mu_{0\nu}^2}\right), \tag{2.4}$$

where

$$\mu_{0\nu}=\hbar c(6\pi^2Y_\nu\rho)^{1/3} \tag{2.5}$$

and

$$\mathcal{S}_\nu=\frac{1}{6}\frac{\mu_{0\nu}^2}{(\hbar c)^3}T. \tag{2.6}$$

Model of the Wigner-Seitz Cell. For the nucleons we adopt the model of the spherical Wigner-Seitz cell, according to which all the nucleons are grouped into identical spherically symmetrical clusters of radius R_c. (Relaxing the constraint of spherical symmetry shows that the droplet-bubble transition becomes smeared out over a finite density interval [23], but the overall effect on the EOS does not seem to be very great; moreover, the transition between the bubble and homogeneous phases remains intact.) The average neutron and proton densities over such a cell must thus each be equal to the corresponding macroscopic densities $\overline{\rho_n}$ and $\overline{\rho_p}$, given by

$$\overline{\rho_n}=\overline{\rho}(1-Y_e) \tag{2.7a}$$

and

$$\overline{\rho_p}=\overline{\rho}\,\overline{Y}_e, \tag{2.7b}$$

where $\overline{\rho}$ is the total macroscopic density (written simply as ρ in the foregoing). The neutron and proton density distribution functions within the cell $\rho_n(r)$ and $\rho_p(r)$ are then constrained by

$$\frac{3}{R_c^3}\int_0^{R_c}\rho_q(r)r^2dr=\overline{\rho_q}, \tag{2.8}$$

where q denotes n or p, as the case may be. The total number of nucleons of each type in the cell is

$$N_q=\frac{4\pi}{3}R_c^3\overline{\rho_q}. \tag{2.9}$$

For the neutron and proton density distribution functions we follow Ref. [11], adopting the form

$$\rho_q(r)=\rho_{Bq}+\frac{\rho_{0q}}{1+\exp[(r-C_q)/a_q]}, \tag{2.10}$$

in which ρ_{Bq} is a constant background term on which is superimposed a Fermi-function radial variation. Positive values of the constant ρ_{0q} in this latter term imply that the density decreases from the center of the cell to its surface, and thus correspond to the droplet phase found at the lower end of the density range; likewise negative values correspond to the bubble phase.

The two constants ρ_{Bq} are not independent of the other parameters, but rather are fixed by

$$\rho_{Bq}=\overline{\rho_q}-\frac{3}{R_c^3}I_q\rho_{0q}, \tag{2.11}$$

where

$$I_q=\int_0^{R_c}\frac{1}{1+\exp[(r-C_q)/a_q]}r^2dr. \tag{2.12}$$

The Wigner-Seitz cell is thus characterized by seven parameters, ρ_{0q}, C_q, a_q $(q=n,p)$, and R_c. However, these parameters are not completely independent but must satisfy the constraint that the densities $\rho_q(r)$ be positive at all points in the cell; we return to this problem below. The presence of R_c as a free parameter means that the number of nucleons N_q of each type in the cell is likewise not fixed.

A slight difficulty with the parametrization of Eq. (2.10) is that the density gradients will not vanish identically on the cell surface, which means that the smooth background prevailing between droplets or bubbles is not represented exactly. Moreover, the vanishing of these gradients is formally necessary for the validity of some of the fourth-order terms in the TETF formalism of the Appendix. But these gradients are in practice very small, and no problem is expected. More serious is the argument [24] that the Fermi-like parametrization may not allow a self-consistent calculation to be approximated with an accuracy that is sufficient in all cases. However, it is probably appropriate in the case of force SkSC6, since precisely this parametrization was adopted for finite nuclei in the mass fit that determined this force. As for force RATP, since our interest in it is confined to a comparison with force SkSC6 (in the context of the effective-mass problem), it seems altogether reasonable to use the same parametrization for the two forces.

For a given set of the distribution parameters appearing in Eq. (2.10) the specifically nuclear free-energy and entropy densities $\mathcal{F}_{\rm nuc}$ and $\mathcal{S}_{\rm nuc}=\mathcal{S}_n+\mathcal{S}_p$, respectively, can be determined in the TETF formalism by using the results of the Appendix; see in particular Eqs. (A24) and (A26). The total free-energy density in the cell is then

$$\mathcal{F}=\mathcal{F}_{\rm nuc}+\mathcal{E}_e+\mathcal{E}_\nu-T(\mathcal{S}_e+\mathcal{S}_\nu)+\mathcal{E}_c, \tag{2.13}$$

in which the last term is the total Coulomb energy density, direct plus exchange, given by

$$\mathcal{E}_c = 2\pi e^2(\rho_p - \rho_e)\left\{ \int_0^r \rho_p(r')\left(\frac{r'^2}{r} - r'\right)dr' + \rho_e \frac{r^2}{6}\right\} - \frac{3e^2}{4}\left(\frac{3}{\pi}\right)^{1/3}(\rho_p^{4/3} + \rho_e^{4/3}), \quad (2.14)$$

where $\rho_e = \overline{\rho_p}$ is the electron density. The total entropy density (we need this separately and independently of the free energy, in order to be able to calculate adiabats) is likewise

$$\mathcal{S} = \mathcal{S}_{\rm nuc} + \mathcal{S}_e + \mathcal{S}_\nu . \quad (2.15)$$

Because the average proton density over the cell is equal to the macroscopic proton density $\overline{\rho_p}$, which in turn is equal to the density of the homogeneous electron gas ρ_e, it follows that the cells will be electrically neutral. Since furthermore the cells are assumed to be spherically symmetrical there will be no interaction between them, and it will be sufficient to consider just one cell. That is, the average free energy and entropy per nucleon in the entire system will be given by the corresponding quantities averaged over just one cell:

$$f = 4\pi \int_0^{R_c} \mathcal{F}(r) r^2 dr/(N_n + N_p) \quad (2.16a)$$

and

$$s = 4\pi \int_0^{R_c} \mathcal{S}(r) r^2 dr/(N_n + N_p). \quad (2.16b)$$

Calculational Details. Using the model described in the foregoing, and with $Y_e = 0.33$ and $Y_\nu = 0.07$, we calculate inhomogeneous nuclear matter for each of the two forces SkSC6 and RATP at temperatures of $T = 2.7$, 4.0, 6.0, and 7.0 MeV, and for densities over the range 0.0075 (0.0025) 0.095 fm^{-3}. At each of these points the free energy f per nucleon is minimized with respect to the seven Wigner-Seitz parameters ρ_{0q}, C_q, a_q $(q = n,p)$, and R_c, subject to the constraint that each of the two densities ρ_q remains non-negative everywhere in the cell. (Above 0.085 fm^{-3} we also have to calculate the homogeneous configuration, using the methods of Ref. [9], in order to establish the position of the transition between the bubble and the homogeneous phases.)

It is easy to show that this constraint of non-negative densities is equivalent to requiring that ρ_{0q} be bounded by

$$\rho_{0q}^+ > \rho_{0q} > \rho_{0q}^- , \quad (2.17)$$

where

$$\rho_{0q}^\pm = \frac{\overline{\rho_q}}{(3/R_c^3)I_q - g_q^\pm}, \quad (2.18)$$

in which

$$g_q^+ = \frac{1}{1 + \exp[(R_c - C_q)/a_q]} \quad (2.19a)$$

and

$$g_q^- = \frac{1}{1 + \exp(-C_q/a_q)}. \quad (2.19b)$$

It suffices then to define a new variable θ_q by

$$\rho_{0q} = \frac{1}{2}\{\rho_{0q}^+(1 + \tanh\theta_q) + \rho_{0q}^-(1 - \tanh\theta_q)\}, \quad (2.20)$$

and take this rather than ρ_{0q} in the set of variational parameters; for all real values of θ_q the densities $\rho_q(r)$ will be non-negative throughout the cell.

Once f has been minimized at a given point the entropy per nucleon s is calculated at the corresponding equilibrium configuration: this is required for the subsequent construction of adiabats. With all the points for a given temperature calculated, the corresponding pressures are obtained by numerical differentiation, according to

$$P = \rho^2 \left(\frac{\partial f}{\partial \rho}\right)_{T, Y_e, Y_\nu} . \quad (2.21)$$

For this and the subsequent numerical differentiations we use polynomial smoothing (subroutine SAVGOL of Ref. [25]).

At this point we have at our disposal the pressure P and the entropy per nucleon s as a function of the density for the four temperatures. Using then cubic interpolation (subroutine POLINT [25]) we can determine for each density first the temperature T and then the pressure corresponding to $s = 1.0$. One further numerical differentiation then gives us the adiabatic index Γ according to Eq. (1.1); particular attention has to be paid in this last differentiation to an adequate smoothing of the pressures, especially in the region of phase transitions.

III. RESULTS AND DISCUSSION

Considering both forces SkSC6 and RATP, we construct adiabats corresponding to $s = 1.0$ for our inhomogeneous nuclear matter, as described above. In Figs. 1, 2, and 3, respectively, we show as functions of density the temperature T, the pressure P, and the adiabatic index Γ. The droplet-bubble and bubble-homogeneous phase transitions are clearly seen in all three of these figures, especially the last; they occur at very nearly the same densities for the two forces.

As far as P and Γ are concerned it is clear that only in the homogeneous phase is there any significant difference between the two forces. Thus it would be sufficient for these two quantities to use SkSC6 right through from the beginning of the collapse in the subdrip region of isolated nuclei up to the bubble-homogeneous transition, after which RATP, with its more appropriate effective mass, would be better. On the other hand, we see from Fig. 1 that even in the inhomogeneous phases the temperature is considerably higher for RATP than for SkSC6. Thus, while one could certainly use SkSC6 ($M^*/M = 1.0$) in the subdrip region of isolated nuclei, and RATP ($M^*/M = 0.67$) in the homogeneous region, neither force would be appropriate in the intermediate region if one is interested in the temperature runs.

A quite similar conclusion can be found in the work of Barranco *et al.* [26], who used the forces SkM* [27] and T6

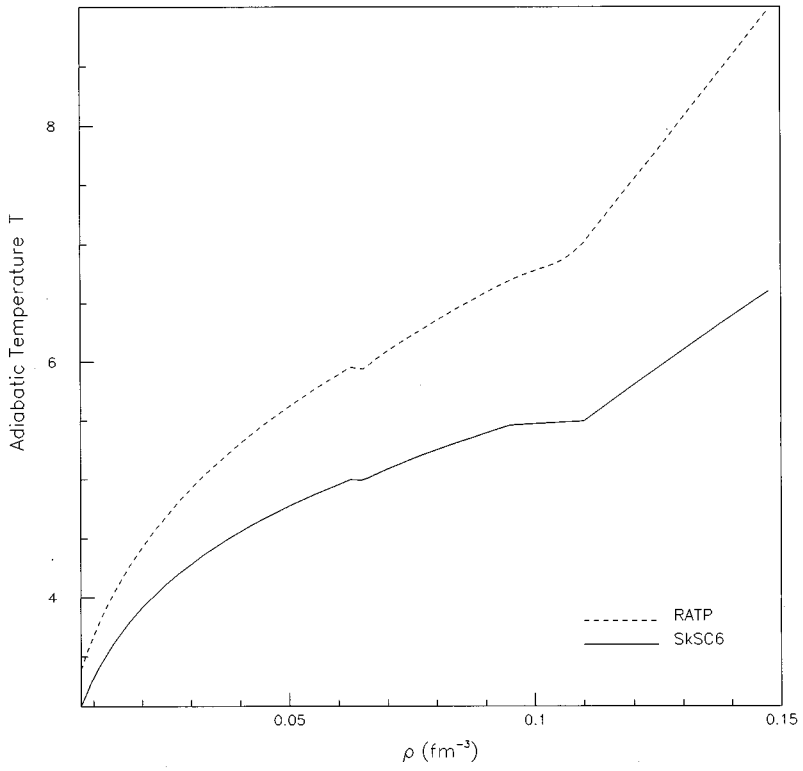

FIG. 1. Adiabatic temperature as a function of density ($s = 1.0$).

[28], for which $M^*/M = 0.79$ and 1.0, respectively. (Their results for these two forces are qualitatively similar to ours for RATP and SkSC6, respectively, although the slightly different values that we have taken for Y_e and Y_ν prevent a quantitative comparison.) However, the evaluation of the role of the effective mass is somewhat obscured in their calculations by virtue of the fact that the incompressibilities of their two forces are rather different (217 MeV for SkM*, as compared to 236 MeV for T6), whereas in our calculations they are very similar (235.4 MeV for SkSC6, 239.6 MeV for RATP). Likewise, it is not altogether clear in Ref. [26] whether the differences between the temperature runs found

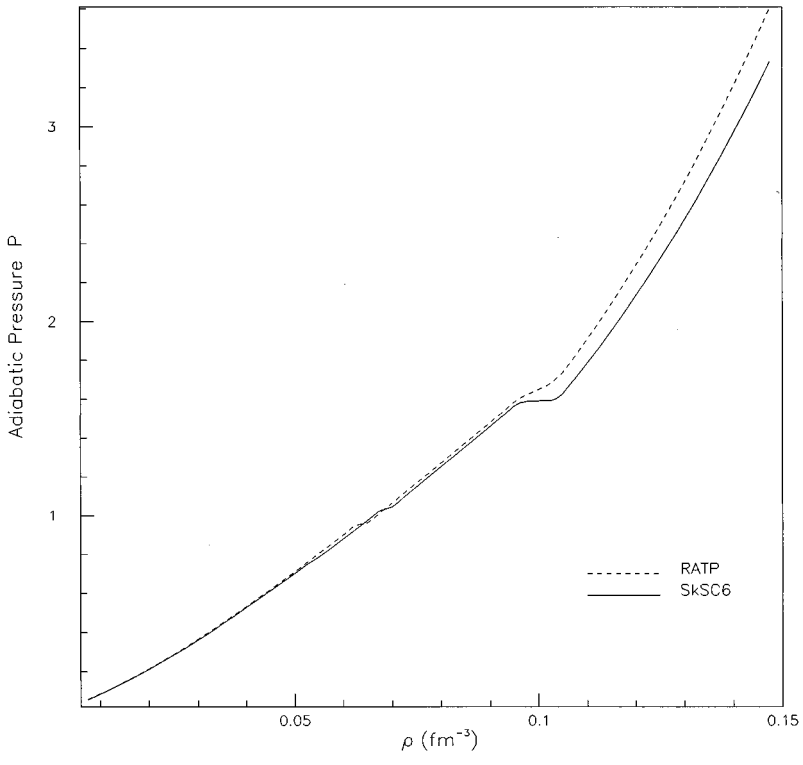

FIG. 2. Adiabatic pressure as a function of density ($s = 1.0$).

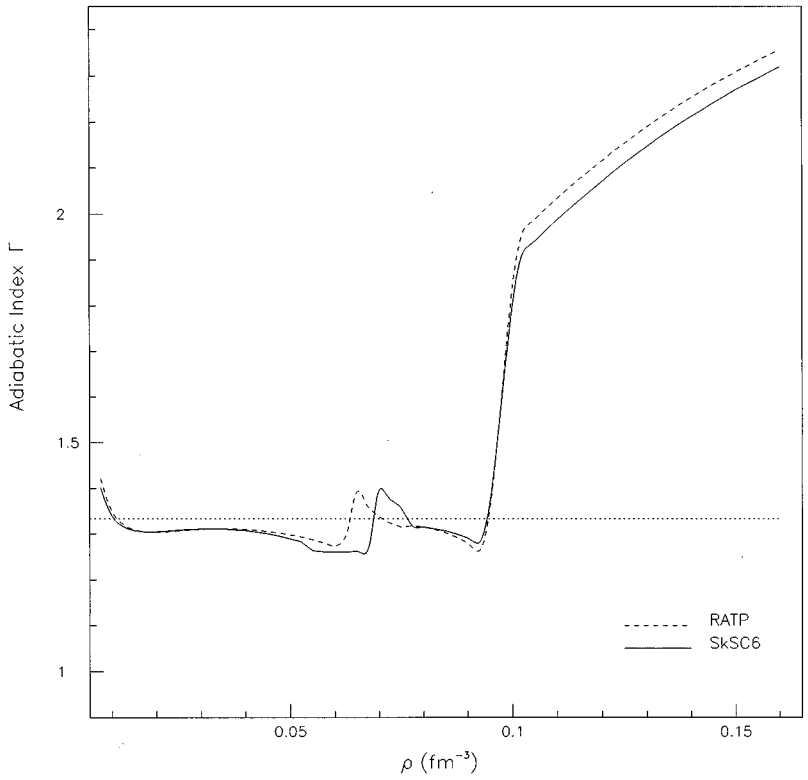

FIG. 3. Adiabatic index Γ as a function of density ($s = 1.0$).

for the two forces reflects the difference between the effective masses or between the incompressibilities. We believe that our calculations show beyond any doubt that the effective mass is quite irrelevant to the pressure and adiabatic index in the inhomogeneous phases, but of considerable importance for the temperature runs.

In any case, we have achieved the main goal of this paper, which was to make a TETF calculation of the EOS of the inhomogeneous phases of stellar nuclear matter using a force (SkSC6) that has been fitted to essentially all the mass data, and has the correct neutron-matter properties. Even though the results are not notably different from what has already been obtained with less precisely fitted forces having the same symmetry coefficient J and effective mass M^* [26], this is an essential step in our overall project of calculating with a single force all the nuclear phenomena occurring at nuclear and subnuclear densities during stellar collapse. Unfortunately, it is clear that with our force SkSC6 we have succeeded only partially in realizing this project, since for densities greater than 0.01 fm^{-3} there emerge the ambiguities associated with the effective mass that we have reported here. Actually, for densities greater than 0.1 fm^{-3}, i.e., for homogeneous nuclear matter, one knows that force RATP is better than force SkSC6, so there is no *practical* problem, even though it would have been more satisfying if we could have found a single universal force. (RATP cannot serve as such a force, on account of its poor fit to the masses of open-shell nuclei.) But over the density range of 0.01 – 0.1 fm^{-3} there is a serious practical problem: how does one interpolate between the SkSC6 and RATP estimates of the temperature, given that there is no clear indication as to the best value of M^* in this density range?

We need some mechanism that will permit a continuous variation of M^*/M between the realistic value of around 0.7 in homogeneous nuclear matter at the density ρ_0 to exactly 1 at zero density, in such a way that the average value over the volume of an isolated finite nucleus is close to 1, the semiempirical value. This cannot be realized with any of the conventional forms of Skyrme force, as in Eq. (A1), since in such forces the variation of M^* with ρ is strictly linear; see, for example Eq. (A3). However, if a t_4 term is introduced, i.e., a term depending simultaneously on ρ and p, then this variation can become nonmonotonic [30], and it is possible to construct a force that has M^*/M close to 0.7 for homogeneous nuclear matter at the density ρ_0, while the value averaged over the nuclear volume is around 1, with the result that there is an excellent fit to the s.p. levels of finite nuclei close to the Fermi surface. But promising as this force might appear as a candidate for a universal effective interaction, it has to be dismissed, since calculations with realistic nucleon-nucleon forces on homogeneous nuclear matter [8] show that M^* varies monotonically with density.

Thus it appears that to obtain the required behavior of M^* within the Skyrme framework one would have to introduce an explicit dependence on the density gradient [31], which means going beyond the limits of a pure force and invoking rather an energy-density formalism. However, even with this device it seems to be impossible to obtain the required behavior of M^* simultaneously with the correct surface properties [32].

In principle, of course, one could resolve this problem quite unambiguously by taking a force with a realistic M^*/M of around 0.7, and performing all calculations with the coupling between particle and surface-vibration modes explicitly included, as in Ref. [20]. But since the basic mass-formula fit itself would have to be undertaken in this way, such a project can scarcely be envisaged at the present time.

Thus if accurate temperatures are required in the density range 0.01 – 0.1 fm^{-3} the most practical solution would probably be to interpolate graphically in some way between the common value of T found for the two forces at 0.01 fm^{-3} and the RATP value found at the bubble-homogeneous transition.

ACKNOWLEDGMENTS

J. M. Lattimer is thanked for very helpful communications. We thank the Computer Center of the American University of Beirut and the Services Informatiques of the Université de Montréal for the support of their computational facilities. The generous help of S. Cortas and B. Kirdi in Beirut and of B. Lorazo in Montreal is gratefully acknowledged. This work was supported in part by NSERC of Canada.

APPENDIX: THE TEMPERATURE-DEPENDENT ETF METHOD

The specifically nuclear terms in Eq. (2.13) for the Helmholtz free-energy density and in Eq. (2.15) for the entropy density, but only these terms, depend on the parameters of the Skyrme-type force being considered, SkSC6 or RATP in our case. These forces have the general form

$$
\begin{aligned}
v_{ij} &= t_0(1+x_0P_\sigma)\,\delta(\mathbf{r}_{ij}) + t_1(1+x_1P_\sigma)\frac{1}{2\hbar^2}\{p_{ij}^2\delta(\mathbf{r}_{ij}) + \text{H.c.}\} \\
&+ t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\mathbf{p}_{ij}\cdot\delta(\mathbf{r}_{ij})\mathbf{p}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma) \\
&\times\{a(\rho_{q_i}+\rho_{q_j})^\alpha + b\rho^\alpha\}\,\delta(\mathbf{r}_{ij}) + \frac{i}{\hbar^2}W_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j).\mathbf{p}_{ij} \\
&\times\delta(\mathbf{r}_{ij})\mathbf{p}_{ij}\,,
\end{aligned}
\tag{A1}
$$

with the parameters for SkSC6 being given in Ref. [9], and those for RATP in Ref. [21]. The parameters a and b appearing in the density-dependent term are usually taken as 0 and 1, respectively, and this is indeed the choice made for RATP. However, for the SkSC forces used in the ETFSI project the choice $a=1$, $b=0$ was made. The latter choice is more physically reasonable, since it implies that the density dependence of the effective interaction between two protons, for example, depends only on the proton density (see Ref. [1] for a fuller discussion). Note that we have no need for the pairing forces that usually accompany these Skyrme forces, since we neglect pairing correlations.

We have now for the nuclear-energy density at any point

$$
\mathcal{E}_{\text{nuc}} = \sum_q \frac{\hbar^2}{2M_q^*}\tau_q + \mathcal{U}_0 + \mathcal{U}_w + \mathcal{U}_{JJ}\,, \tag{A2}
$$

where the effective mass M_q^* is given by

$$
\begin{aligned}
\frac{\hbar^2}{2M_q^*} &= \frac{\hbar^2}{2M_q} + \frac{1}{4}\left\{t_1\left(1+\frac{1}{2}x_1\right) + t_2\left(1+\frac{1}{2}x_2\right)\right\}\rho \\
&+ \frac{1}{4}\left\{t_2\left(\frac{1}{2}+x_2\right) - t_1\left(\frac{1}{2}+x_1\right)\right\}\rho_q\,,
\end{aligned}
\tag{A3}
$$

and the different potential-energy terms are

$$
\begin{aligned}
\mathcal{U}_0 &= \frac{1}{2}t_0\left\{\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(\frac{1}{2}+x_0\right)\sum_q \rho_q^2\right\} \\
&+ \frac{1}{16}\left\{3t_1\left(1+\frac{1}{2}x_1\right) - t_2\left(1+\frac{1}{2}x_2\right)\right\}(\nabla\rho)^2 \\
&- \frac{1}{16}\left\{3t_1\left(\frac{1}{2}+x_1\right) + t_2\left(\frac{1}{2}+x_2\right)\right\}\sum_q (\nabla\rho_q)^2 \\
&+ \frac{1}{6}at_3\left\{\left(1+\frac{1}{2}x_3\right)\rho^\alpha\rho_n\rho_p \right. \\
&\left. + \frac{1}{16}(1-x_3)\sum_q (2\rho_q)^{\alpha+2}\right\} \\
&+ \frac{1}{12}bt_3\left\{\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(\frac{1}{2}+x_3\right)\sum_q \rho_q^2\right\}\rho^\alpha,
\end{aligned}
\tag{A4a}
$$

$$
\mathcal{U}_w = \frac{1}{2}W_0\left\{\boldsymbol{J}_n\cdot\boldsymbol{\nabla}\rho_p + \boldsymbol{J}_p\cdot\boldsymbol{\nabla}\rho_n + 2\sum_q \boldsymbol{J}_q\cdot\boldsymbol{\nabla}\rho_q\right\}, \tag{A4b}
$$

and

$$
\mathcal{U}_{JJ} = -\frac{1}{16}(t_1x_1+t_2x_2)\boldsymbol{J}^2 + \frac{1}{16}(t_1-t_2)\sum_q \boldsymbol{J}_q^2\,. \tag{A4c}
$$

The term $_{JJ}$ is usually neglected, but it is retained in the definition of the RATP and SkSC forces, so we must do likewise. However, in doing so we shall make some approximations, and in particular we shall neglect its contribution to the spin-orbit field, which we thus write as

$$
\boldsymbol{W}_q = \frac{1}{2}W_0\boldsymbol{\nabla}(\rho+\rho_q). \tag{A5}
$$

It then follows that

$$
\mathcal{U}_w = \sum_q \boldsymbol{W}_q\cdot\boldsymbol{J}_q\,, \tag{A6}
$$

which allows us to rewrite Eq. (A2) as

$$
\mathcal{E}_{\text{nuc}} = \sum_q \left(\frac{\hbar^2}{2M_q}f_q\tau_q + \boldsymbol{W}_q\cdot\boldsymbol{J}_q\right) + \mathcal{U}_0 + \mathcal{U}_{JJ}\,, \tag{A7}
$$

where

$$
f_q = \frac{M_q}{M_q^*}. \tag{A8}
$$

Concentrating on the first term of Eq. (A7), we make use of the various ETF expansions of τ_q and $\mathbf{J}_q$ to obtain

$$
\frac{\hbar^2}{2M_q}f_q\tau_q + \boldsymbol{W}_q\cdot\boldsymbol{J}_q = \frac{\hbar^2}{2M_q}\{\Lambda_{0q}(T) + \Lambda_{wq}(T)\}, \tag{A9}
$$

with

$$\Lambda_{0q}(T)=\Lambda_{0q}^{TF}(T)+\Lambda_{0q}^{(2)}(T)+\Lambda_{0q}^{(4a)}(T)+\Lambda_{0q}^{(4b)}(T). \tag{A10}$$

For $T=0$ we have

$$\Lambda_{0q}^{TF}(0)=\frac{3}{5}(3\pi^2)^{2/3}f_q\rho_q^{5/3}, \tag{A11a}$$

$$\Lambda_{0q}^{(2)}(0)=\frac{1}{3}f_q\nabla^2\rho_q+\frac{1}{36}f_q\frac{(\nabla\rho_q)^2}{\rho_q}+\frac{1}{6}\nabla f_q\cdot\nabla\rho_q -\frac{1}{12}\rho_q\frac{(\nabla f_q)^2}{f_q}+\frac{1}{6}\rho_q\nabla^2 f_q, \tag{A11b}$$

$$\Lambda_{0q}^{(4a)}(0)=\frac{1}{(3\pi^2)^{2/3}}f_q\rho_q^{1/3}\left\{\frac{1}{270}\left(\frac{\nabla^2\rho_q}{\rho_q}\right)^2 -\frac{1}{240}\frac{\nabla^2\rho_q}{\rho_q}\left(\frac{\nabla\rho_q}{\rho_q}\right)^2+\frac{1}{810}\left(\frac{\nabla\rho_q}{\rho_q}\right)^4\right\}, \tag{A11c}$$

and

$$\begin{aligned}\Lambda_{0q}^{(4b)}(0)=\frac{1}{(3\pi^2)^{2/3}}\rho_q^{1/3}\Bigg\{&-\frac{1}{240}\frac{(\nabla^2 f_q)^2}{f_q}+\frac{1}{120}\frac{(\nabla^2 f_q)(\nabla f_q)^2}{f_q^2}-\frac{1}{240}\frac{(\nabla f_q)^4}{f_q^3}\\&+\frac{1}{360}(\nabla^2 f_q)\frac{\nabla f_q\cdot\nabla\rho_q}{f_q\rho_q}-\frac{1}{360}(\nabla^2\rho_q)\frac{(\nabla f_q)^2}{f_q\rho_q}\\&-\frac{7}{2160}(\nabla^2 f_q)\left(\frac{\nabla\rho_q}{\rho_q}\right)^2+\frac{1}{540}\left(\frac{\nabla\rho_q}{\rho_q}\right)^2\frac{(\nabla f_q)^2}{f_q}\\&+\frac{7}{2160}\frac{(\nabla\rho_q\cdot\nabla f_q)^2}{f_q\rho_q^2}-\frac{11}{3240}\left(\frac{\nabla\rho_q}{\rho_q}\right)^2\frac{\nabla\rho_q\cdot\nabla f_q}{\rho_q}\\&+\frac{7}{1080}(\nabla^2\rho_q)\frac{(\nabla\rho_q\cdot\nabla f_q)}{\rho_q^2}+\frac{1}{180}(\nabla^2 f_q)\frac{\nabla^2\rho_q}{\rho_q}\Bigg\}\end{aligned} \tag{A11d}$$

[the separation of the fourth-order terms here into $\Lambda_{0q}^{(4a)}(0)$ and $\Lambda_{0q}^{(4b)}(0)$ is such that the latter vanishes for $f_q=1$]. Also at $T=0$ we have

$$\Lambda_{wq}(0)=\Lambda_{wq}^{(2)}(0)+\Lambda_{wq}^{(4)}(0), \tag{A12}$$

in which

$$\Lambda_{wq}^{(2)}=-2\frac{\rho_q}{f_q}S_q^2 \tag{A13a}$$

and

$$\begin{aligned}\Lambda_{wq}^{(4)}(0)=\frac{1}{(3\pi^2)^{2/3}}\frac{\rho_q^{1/3}}{f_q}\Bigg\{&\frac{1}{2}(\nabla\cdot S_q)^2+\frac{1}{24}\boldsymbol{S}_q^2\frac{\nabla^2\rho_q}{\rho_q}\\&+\frac{1}{12}(\nabla\cdot S_q)\left(\frac{\boldsymbol{S}_q\cdot\nabla\rho_q}{\rho_q}\right)-\frac{1}{36}\boldsymbol{S}_q^2\left(\frac{\nabla\rho_q}{\rho_q}\right)^2+\frac{\boldsymbol{S}_q^4}{f_q^2}\\&-\frac{3}{4}(\nabla\cdot S_q)\left(\frac{\boldsymbol{S}_q\cdot\nabla f_q}{f_q}\right)\\&+\frac{1}{8}\boldsymbol{S}_q^2\frac{\nabla^2 f_q}{f_q}+\frac{1}{4}\left(\frac{\boldsymbol{S}_q\cdot\nabla f_q}{f_q}\right)^2\Bigg\},\end{aligned} \tag{A13b}$$

where

$$\boldsymbol{S}_q=\frac{M_q}{\hbar^2}\boldsymbol{W}_q. \tag{A14}$$

The second-order terms here come from Eqs. (3.20) and (3.21) of Brack, Guet, and Håkansson (BGH) [27], while the fourth-order terms come from Eqs. (A4) and (A5) of the same paper. The BGH results are based in turn on the expansions of τ_q and $\boldsymbol{J}_q$, given by Eqs. (3.20) and (3.21) of Ref. [7], respectively. (Actually, it should be noted that the fourth-order expressions are valid only on integrating over the whole of space, or more generally, over a region on the surface of which the density gradients vanish.)

We turn now to the case of $T>0$, basing all the following expressions on BGH [27] and BBD [12]. Up to second order we have

$$\Lambda_{0q}^{TF}(T)=\frac{1}{2\pi^2}\frac{1}{f_q^{3/2}}\left(\frac{2M_qT}{\hbar^2}\right)^{5/2}I_{3/2}(\eta_q), \tag{A15a}$$

$$\begin{aligned}\Lambda_{0q}^{(2)}(T)=&f_q\gamma_q\frac{(\nabla\rho_q)^2}{\rho_q}+\left(\frac{9}{4}\gamma_q-\frac{7}{48}\right)\rho_q\frac{(\nabla f_q)^2}{f_q}\\&+\frac{1}{6}(\rho_q\nabla^2 f_q-f_q\nabla^2\rho_q)\\&+\left(3\gamma_q-\frac{5}{12}\right)\nabla\rho_q\cdot\nabla f_q,\end{aligned} \tag{A15b}$$

and

$$\Lambda_{wq}^{(2)}(T)=\Lambda_{wq}^{(2)}(0), \tag{A15c}$$

where we have introduced the Fermi integral

$$I_\sigma(\eta)=\int_0^\infty\frac{x^\sigma}{1+\exp(x-\eta)}dx, \tag{A16}$$

computed as in Ref. [29], and η_q is given by

$$\rho_q(\boldsymbol{r})=\frac{1}{2\pi^2}\left(\frac{2M_qT}{\hbar^2 f_q}\right)^{3/2}I_{1/2}(\eta_q). \tag{A17}$$

Also

$$\gamma_q=\zeta_q-\nu_q, \tag{A18}$$

in which

$$\zeta_q=-\frac{I_{1/2}(\eta_q)I_{-(3/2)}(\eta_q)}{12I_{-(1/2)}^2(\eta_q)} \tag{A19a}$$

and

$$\nu_q = -\frac{3}{2}\zeta_q + 36\zeta_q^2 - \frac{3}{8}\frac{I_{1/2}^2(\eta_q) I_{-(5/2)}(\eta_q)}{I_{-(1/2)}^3(\eta_q)}. \quad \text{(A19b)}$$

The fourth-order terms for $T>0$ are given by BBD only for $f_q=1$ and $\boldsymbol{W}_q=0$:

$$\Lambda_{0q}^{(4)}(T, f_q=1, \boldsymbol{W}_q=0) = \frac{\hbar^2}{2M_q}\frac{1}{T}\left\{(\theta_1+\chi_1)\frac{(\boldsymbol{\nabla}^2\rho_q)^2}{\rho_q} + (\theta_2+\chi_2)\frac{\boldsymbol{\nabla}^2\rho_q(\boldsymbol{\nabla}\rho_q)^2}{\rho_q^2} + (\theta_3+\chi_3)\frac{(\boldsymbol{\nabla}\rho_q)^4}{\rho_q^3}\right\}, \quad \text{(A20)}$$

where the functions θ_i and χ_i are as in Appendix B of BBD. Since

$$\lim_{T\to 0}\Lambda_{0q}^{(4)}(T, f_q=1, \boldsymbol{W}_q=0) = \frac{\Lambda_{0q}^{(4a)}(0)}{f_q}, \quad \text{(A21)}$$

we approximate the total fourth-order term for $T>0$ by

$$\Lambda_{0q}^{(4)}(T) \simeq \Lambda_{0q}^{(4)}(T, f_q=1, \boldsymbol{W}_q=0) + \frac{f_q-1}{f_q}\Lambda_{0q}^{(4a)}(0) + \Lambda_{0q}^{(4b)}(0) \quad \text{(A22a)}$$

and

$$\Lambda_{wq}^{(4)}(T) \simeq \Lambda_{wq}^{(4)}(0). \quad \text{(A22b)}$$

With the last two terms in Eq. (A22a) and the right-hand side of Eq. (A22b) vanishing if $f_q=1$ and $\boldsymbol{W}_q=0$, we see that these two equations are exact for all T if $f_q=1$ and $\boldsymbol{W}_q=0$, and at $T=0$ for arbitrary f_q and $\boldsymbol{W}_q$. This is the best that one can do in the present state of the theory. Altogether, then, in Eq. (A9) we write

$$\Lambda_{0q}(T) \simeq \Lambda_{0q}^{(TF)}(T) + \Lambda_{0q}^{(2)}(T) + \Lambda_{0q}^{(4)}(T, f_q=1, \boldsymbol{W}_q=0) + \frac{f_q-1}{f_q}\Lambda_{0q}^{(4a)}(0) + \Lambda_{0q}^{(4b)}(0) \quad \text{(A23a)}$$

and

$$\Lambda_{wq}(T) = \Lambda_{wq}^{(2)}(0) + \Lambda_{wq}^{(4)}(0). \quad \text{(A23b)}$$

Likewise for the total nuclear entropy density we have

$$\mathcal{S}_{\rm nuc} = \sum_q \{\sigma_q^{(TF)} + \sigma_q^{(2)} + \sigma_q^{(4)}(f_q=1, \boldsymbol{W}_q=0)\}, \quad \text{(A24)}$$

where

$$\sigma_q^{(TF)} = \frac{5}{6\pi^2}\left(\frac{2M_qT}{\hbar^2 f_q}\right)^{3/2} I_{3/2}(\eta_q) - \eta_q\rho_q, \quad \text{(A25a)}$$

$$\sigma_q^{(2)} = -\frac{\hbar^2}{2M_q}\frac{f_q\nu_q}{T}\left\{\frac{(\boldsymbol{\nabla}\rho_q)^2}{\rho_q} + \frac{9}{4}\rho_q\left(\frac{\boldsymbol{\nabla}f_q}{f_q}\right)^2 + \frac{3}{f_q}\boldsymbol{\nabla}\rho_q\cdot\boldsymbol{\nabla}f_q\right\}, \quad \text{(A25b)}$$

and

$$\sigma_q^{(4)} = \left(\frac{\hbar^2}{2M_q}\right)^2\frac{1}{T^2}\left\{\chi_1\frac{(\boldsymbol{\nabla}^2\rho_q)^2}{\rho_q} + \chi_2\frac{\boldsymbol{\nabla}^2\rho_q(\boldsymbol{\nabla}\rho_q)^2}{\rho_q^2} + \chi_3\frac{(\boldsymbol{\nabla}\rho_q)^4}{\rho_q^3}\right\}; \quad \text{(A25c)}$$

the fourth-order term is valid only for $f_q=1$ and $\boldsymbol{W}_q=0$.

For the nuclear Helmholtz free-energy density $\mathcal{F}_{\rm nuc}=\mathcal{E}_{\rm nuc}-T\mathcal{S}_{\rm nuc}$ we can now write, to within the approximations of the foregoing,

$$\mathcal{F}_{\rm nuc} = \sum_q\left(\frac{\hbar^2}{2M_q}\left\{\Gamma_{0q}^{(TF)} + \Gamma_{0q}^{(2)} + \Gamma_{0q}^{(4)}(f_q=1, \boldsymbol{W}_q=0) + \frac{f_q-1}{f_q}\Lambda_{0q}^{(4a)}(0) + \Lambda_{0q}^{(4b)}(0) + \Lambda_{wq}(0)\right\}\right) + \mathfrak{v}_0 + \mathfrak{v}_{JJ}, \quad \text{(A26)}$$

where

$$\Gamma_{0q}^{(TF)} = -\frac{1}{3\pi^2}\frac{1}{f_q^{3/2}}\left(\frac{2M_qT}{\hbar^2}\right)^{5/2} I_{3/2}(\eta_q) + \frac{2M_qT}{\hbar^2}\eta_q\rho_q, \quad \text{(A27a)}$$

$$\Gamma_{0q}^{(2)} = \zeta_q f_q\frac{(\boldsymbol{\nabla}\rho_q)^2}{\rho_q} + \left(\frac{9}{4}\zeta_q - \frac{7}{48}\right)\rho_q\frac{(\boldsymbol{\nabla}f_q)^2}{f_q} + \frac{1}{6}(\rho_q\boldsymbol{\nabla}^2 f_q - f_q\boldsymbol{\nabla}^2\rho_q) + \left(3\zeta_q - \frac{5}{12}\right)\boldsymbol{\nabla}\rho_q\cdot\boldsymbol{\nabla}f_q, \quad \text{(A27b)}$$

and

$$\Gamma_{0q}^{(4)}(f_q=1, \boldsymbol{W}_q=0) = \frac{\hbar^2}{2M_q}\frac{1}{T}\left\{\theta_1\frac{(\boldsymbol{\nabla}^2\rho_q)^2}{\rho_q} + \theta_2\frac{\boldsymbol{\nabla}^2\rho_q(\boldsymbol{\nabla}\rho_q)^2}{\rho_q^2} + \theta_3\frac{(\boldsymbol{\nabla}\rho_q)^4}{\rho_q^3}\right\}. \quad \text{(A27c)}$$

Finally, for the $_{JJ}$ term, given by Eq. (A4c), since it is small we truncate the semiclassical expansion of $\boldsymbol{J}_q$ at the second order,

$$\boldsymbol{J}_q = -\frac{2M_q}{\hbar^2}\frac{1}{f_q}\rho_q\boldsymbol{W}_q. \quad \text{(A28)}$$

[1] A. K. Dutta, J.-P. Arcoragi, J. M. Pearson, R. Behrman, and F. Tondeur, Nucl. Phys. **A458**, 77 (1986).
[2] F. Tondeur, A. K. Dutta, J. M. Pearson, and R. Behrman, Nucl. Phys. **A470**, 93 (1987).
[3] J. M. Pearson, Y. Aboussir, A. K. Dutta, R. C. Nayak, M. Farine, and F. Tondeur, Nucl. Phys. **A528**, 1 (1991).
[4] Y. Aboussir, J. M. Pearson, A. K. Dutta, and F. Tondeur, Nucl. Phys. **A549**, 155 (1992).
[5] Y. Aboussir, J. M. Pearson, A. K. Dutta, and F. Tondeur, At. Data Nucl. Data Tables **61**, 127 (1995).
[6] B. Grammaticos and A. Voros, Ann. Phys. (N.Y.) **123**, 359 (1979).
[7] B. Grammaticos and A. Voros, Ann. Phys. (N.Y.) **129**, 153 (1980).
[8] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981).
[9] M. Onsi, H. Przysiezniak, and J. M. Pearson, Phys. Rev. C **50**, 460 (1994).
[10] H. A. Bethe, Rev. Mod. Phys. **62**, 801 (1990).
[11] M. Barranco and J.-R. Buchler, Phys. Rev. C **24**, 1191 (1981).
[12] J. Bartel, M. Brack, and M. Durand, Nucl. Phys. **A445**, 263 (1985).
[13] M. Brack and Ph. Quentin, Phys. Lett. **52B**, 159 (1974).
[14] P. Bonche and D. Vautherin, Nucl. Phys. **A372**, 496 (1981).
[15] M. Barranco and J. Treiner, Nucl. Phys. **A351**, 269 (1981).
[16] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).
[17] A. N. James, P. T. Andrews, P. Kirby, and B. G. Lowe, Nucl. Phys. **A138**, 145 (1969).
[18] O. Bohigas, A. M. Lane, and J. Martorell, Phys. Rep. **51**, 267 (1979).
[19] H. Krivine, J. Treiner, and O. Bohigas, Nucl. Phys. **A336**, 155 (1980).
[20] V. Bernard and Nguyen Van Giai, Nucl. Phys. **A348**, 75 (1980).
[21] M. Rayet, M. Arnould, F. Tondeur, and G. Paulus, Astron. Astrophys. **116**, 183 (1982).
[22] X. Vinas, M. Barranco, J. Treiner, and S. Stringari, Astron. Astrophys. **182**, L34 (1987).
[23] C. J. Pethick and D. G. Ravenhall, Annu. Rev. Nucl. Sci. **45**, 429 (1995).
[24] E. Suraud, Astron. Astrophys. **143**, 108 (1985).
[25] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in Fortran* (Cambridge University Press, Cambridge, 1992).
[26] M. Barranco, F. Garcias, M. Pi, and E. Suraud, *Nuclear Astrophysics*, edited by M. Lozano, M. I. Gallardo, and J. M. Arias (Springer-Verlag, Berlin, 1989) p. 265.
[27] M. Brack, C. Guet, and H. -B. Håkansson, Phys. Rep. **123**, 275 (1985).
[28] F. Tondeur, M. Brack, M. Farine, and J. M. Pearson, Nucl. Phys. **A420**, 297 (1984).
[29] M. Onsi, A. M. Chaara, and J. M. Pearson, Z. Phys. A **348**, 255 (1995).
[30] M. Farine, J. M. Pearson, and F. Tondeur, Nucl. Phys. **A615,** 135 (1997).
[31] M. Prakash, J. Wambach, and Z. Y. Ma, Phys. Lett. **128B**, 141 (1983).
[32] J. M. Lattimer (private communication).