Isospin breaking in the pion-nucleon coupling from QCD sum rules

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We use QCD sum rules for the three-point function of a pseudoscalar and two nucleonic currents in order to estimate the charge dependence of the pion-nucleon coupling constant $g_{NN\pi}$ coming from isospin violation in the strong interaction. The effect can be attributed primarily to the difference of the quark condensates $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$. For the splitting $(g_{pp\pi_0} - g_{nn\pi_0})/g_{NN\pi}$ we obtain an interval of 1.2×10^{-2} to 3.7×10^{-2}, the uncertainties coming mainly from the input parameters. The charged pion nucleon coupling is found to be the average of $g_{pp\pi_0}$ and $g_{nn\pi_0}$. Electromagnetic effects are not included. [S0556-2813(97)01906-7]

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The effect of isospin-violating meson-nucleon couplings has recently seen a strong revival of interest in the investigation of charge symmetry breaking (CSB) phenomena [1–4] (for a comprehensive review see [5], and references therein). On a microscopical level, isospin symmetry is broken by the electromagnetic interaction as well as the mass difference of up and down quarks $m_u \neq m_d$. It is the aim of this paper to examine the difference between the pion-nucleon coupling constants $g_{pp\pi^0}$, $g_{nn\pi^0}$, and $g_{pn\pi^+}$ using the QCD sum rule method, which has been established as a powerful and fruitful technique for describing hadronic phenomena at intermediate energies [6–8]. Here we will only look at effects which arise from isospin breaking in the strong interaction. In the QCD sum rule method this is reflected by $m_u \neq m_d$ as well as by the isospin breaking of the vacuum condensates. Electromagnetic effects are not examined. Our work follows the approach of Refs. [7,9,10] and extends their analysis to the isospin-violating case.

We start from the three-point function of two nucleonic (Ioffe) [11] and one pseudoscalar interpolating currents with the appropriate isospin quantum numbers [7,9,10,12,13], e.g.,

$$A_{NN\pi^i}(p_1, p_2, q)$$

$$= \int d^4x_1 d^4x_2 e^{ip_1 x_1} e^{-ip_2 x_2}$$

$$\times \langle 0 | \mathcal{T} \eta_N(x_1) P_i^{T=1}(0) \overline{\eta_N}(x_2) | 0 \rangle, \tag{1}$$

where i stands for $+$ or 0 and N for proton or neutron, respectively. The expressions for the pseudoscalar isovector currents read

$$P_{i=0}^{T=1}(x) = \bar{u}(x) i \gamma_5 u(x) - \bar{d}(x) i \gamma_5 d(x), \tag{2a}$$

$$P_{i=+}^{T=1}(x) = \sqrt{2} \bar{u}(x) i \gamma_5 d(x), \tag{2b}$$

and those for the Ioffe currents are

$$\eta_p(x) = \epsilon_{abc}\{[u^a(x) \mathcal{C} \gamma_\mu u^b(x)] \gamma_5 \gamma^\mu d^c(x)\}, \tag{3a}$$

$$\eta_n(x) = \epsilon_{abc}\{[d^a(x) \mathcal{C} \gamma_\mu d^b(x)] \gamma_5 \gamma^\mu u^c(x)\}. \tag{3b}$$

The momenta p_1 and p_2 are those of the nucleon, and $q = p_1 - p_2$ that of the pion; $\mathcal{C} = i \gamma_2 \gamma_0$ is the charge conjugation matrix. In the following we will only keep terms up to first order in isospin violation, i.e., $m_d - m_u$.

The phenomenological side of the QCD sum rules for the three-point functions A are obtained by saturating the general expressions for the A's (1) with the corresponding nucleon and pion intermediate states. In order to connect to hadronic observables we have to know the overlap between the pion states and the interpolating fields. The axial Ward identity

$$\partial^\mu A_\mu{}^a = i \bar{q} \gamma_5 \left\{ \mathcal{M}, \frac{\tau^a}{2} \right\} q \tag{4}$$

gives

$$m_0 \langle 0 | \bar{u} i \gamma_5 u - \bar{d} i \gamma_5 d | \pi^0 \rangle = m_{\pi^0}{}^2 f_{\pi^0} + \mathcal{O}[(m_u - m_d)^2] \tag{5a}$$

$$\sqrt{2} m_0 \langle 0 | \bar{u} i \gamma_5 d | \pi^+ \rangle = m_{\pi^+}{}^2 f_{\pi^+} + \mathcal{O}[(m_u - m_d)^2] \tag{5b}$$

with $q = \binom{u}{d}$, $\mathcal{M} = m_0 \mathbb{1} + [(m_u - m_d)/2] \tau^3$ and $m_0 = (m_u + m_d)/2$.

Hereby we have used that

$$(m_u - m_d) \langle 0 | \bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d | \pi^0 \rangle = \mathcal{O}[(m_u - m_d)^2].$$

Furthermore we can set $m_{\pi^0}{}^2 = m_{\pi^+}{}^2 = m_\pi{}^2$ as well as $f_{\pi^0} = f_{\pi^+} = f_\pi$, because the differences between the charged and the neutral quantities are also of $\mathcal{O}[(m_u - m_d)^2]$ [14].

We also need the current algebra relation

$$m_0 \langle \bar{u}u + \bar{d}d \rangle = (-)m_\pi{}^2 f_\pi{}^2 + \mathcal{O}[(m_u - m_d)^2] \tag{6}$$

which follows from Eqs. (5) and the partially conserved axial-vector current (PCAC) relation

$$\partial^\mu A_\mu{}^a = m_\pi{}^2 f_\pi \pi^a. \tag{7}$$

The pion-nucleon couplings are defined through the interactions

$$\mathcal{L}_{pp\pi^0} = g_{pp\pi^0}\bar{p}\,i\,\gamma_5\pi^0 p, \tag{8a}$$

$$\mathcal{L}_{nn\pi^0} = (-)g_{nn\pi^0}\bar{n}\,i\,\gamma_5\pi^0 n, \tag{8b}$$

$$\mathcal{L}_{pn\pi^+} = \sqrt{2}\,g_{pn\pi^+}\bar{n}\,i\,\gamma_5\pi^+ p. \tag{8c}$$

It should be remarked that in our notation all three couplings are positive and have the same value in the isospin conserving limit.

We then obtain the following expressions for the phenomenological sides of the three-point functions, Eqs. (1):

$$A_{pp\pi^0} = i\lambda_p^2 \frac{m_\pi^2 f_\pi}{m_0} \frac{(+)g_{pp\pi^0}}{-q^2+m_\pi^2} \frac{1}{p_1^2-M_p^2}$$

$$\times \frac{1}{p_2^2-M_p^2} 2M_p\gamma_5\slashed{q} + \cdots, \tag{9a}$$

$$A_{nn\pi^0} = i\lambda_n^2 \frac{m_\pi^2 f_\pi}{m_0} \frac{(-)g_{nn\pi^0}}{-q^2+m_\pi^2} \frac{1}{p_1^2-M_n^2}$$

$$\times \frac{1}{p_2^2-M_n^2} 2M_n\gamma_5\slashed{q} + \cdots, \tag{9b}$$

$$A_{pn\pi^+} = i\lambda_p\lambda_n \frac{m_\pi^2 f_\pi}{m_0} \frac{\sqrt{2}g_{pn\pi^+}}{-q^2+m_\pi^2} \frac{1}{p_1^2-M_p^2}$$

$$\times \frac{1}{p_2^2-M_n^2} \frac{M_p+M_n}{2}\gamma_5\slashed{q} + \cdots, \tag{9c}$$

where the λ_N's are the overlaps between the Ioffe currents (2) and the corresponding single nucleon states. The $\cdots$ denote contributions from higher resonance intermediate states and the continuum. We will come back to these contributions later.

By saturating the three-point function Eq. (1) for the neutral current with pseudoscalar isovector intermediate states and deriving Eqs. (9a) and (9b) we have assumed so far that the π^0 mass eigenstate is a pure isovector state. However, due to π-η mixing the correlator in Eq. (1) with the current $P_{i=0}^{T=1}$ will pick up a contribution from the $|\eta\rangle$ state as well.[1] In order to avoid this we have to use a correlator where the pseudoscalar meson current has only overlap with the physical $|\pi\rangle$, i.e., the mass eigenstate and not with the $|\eta\rangle$. As it has been shown in Ref. [14] this is possible in lowest order chiral perturbation theory by using the linear combination of the SU(3) flavor octet pseudoscalar currents

$$P_{a=3} + \theta P_{a=8}, \tag{10}$$

where

$$P_{a=3} = \bar{u}(x)i\gamma_5 u(x) - \bar{d}(x)i\gamma_5 d(x) \equiv P_{i=0}^{T=1}, \tag{11}$$

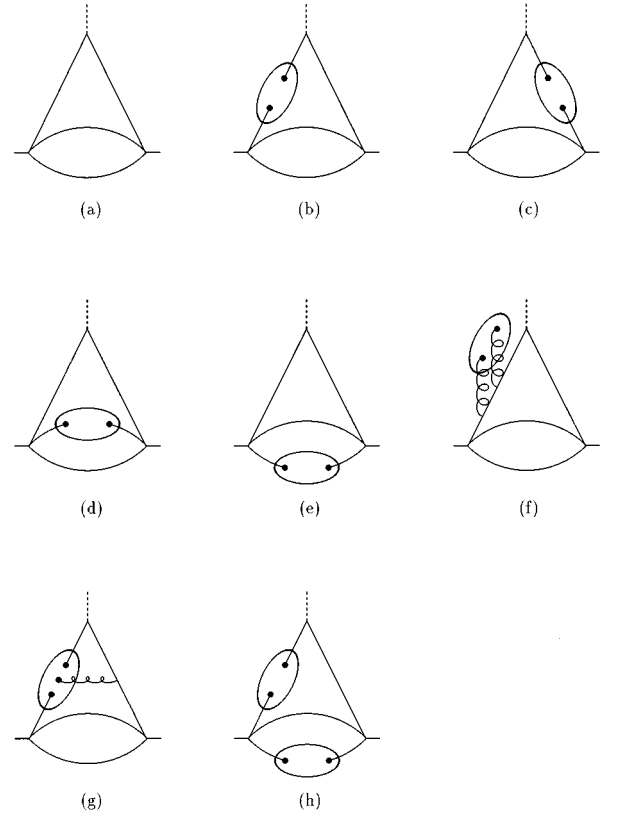[1] We are grateful to K. Maltman for pointing this out to us.

FIG. 1. Diagrams in the OPE.

$$P_{a=8} = \frac{1}{\sqrt{3}}[\bar{u}(x)i\gamma_5 u(x) + \bar{d}(x)i\gamma_5 d(x) - 2\bar{s}(x)i\gamma_5 s(x)]$$

rather than the pure isovector current in the correlator Eq. (1). The θ denotes the π-η mixing angle which defines the mass eigenstates $|\pi\rangle$ and $|\eta\rangle$ in terms of the flavor octet eigenstates $|\pi_{a=3}\rangle$ and $|\pi_{a=8}\rangle$:

$$|\pi\rangle = |\pi_{a=3}\rangle + \theta|\pi_{a=8}\rangle, \tag{12a}$$

$$|\eta\rangle = |\pi_{a=8}\rangle - \theta|\pi_{a=3}\rangle. \tag{12b}$$

It should be noted in this context that there exists actually a whole family of possible choices for interpolating currents involving linear combinations of $P_{a=8}$ and the flavor singlet current $P_{a=0}$, which have no overlap with the η but only with the π. Our choice (10) is the appropriate one if one ignores possible mixing to the SU(3) flavor singlet state, i.e., the η', because in this case the current (10) is the only choice which has no overlap with the flavor singlet state either.

Furthermore it should be noted that we have neglected all higher pseudoscalar, isovector resonances π', π'', ... In other words, we have assumed that pion pole dominance works at spacelike $q^2 \approx -1$ GeV^2, where the three-point function method can be applied [7,9]. We will discuss this point later as well.

The next step is to perform the operator product expansion (OPE) for the three-point functions under consideration. Typical diagrams are shown in Fig. 1. Following Refs.

[7,9,10] we keep only terms which are proportional to $\not{q}\gamma_5$ and have a $1/q^2$ pole. We identify the residua of this pole with one on the phenomenological side, assuming hereby that $|q^2| \gg m_\pi{}^2$, so that the pion mass can be neglected in Eqs. (9). Finally we take $p_1{}^2 = p_2{}^2 = -P^2$ in the equation of the pole residua and perform a Borel transformation with respect to P^2. It should be noted that the OPE side contains, of course, also terms which do not have a $1/q^2$ pole. They will give rise to a form factor, i.e., a q^2 dependence of the pion-nucleon couplings [13], which we do not consider in the present context.

In our case one can easily convince oneself that up to and including order 4, only the diagrams in Figs. 1(b) and 1(c), which contain the quark condensates $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$ contribute. Diagrams containing the gluon condensate $\langle G^2 \rangle$ [Fig. 1(f)] come in at order 6, because from dimensional arguments they are proportional to the current quark masses m_u or m_d, respectively. The mixed condensates $\langle \bar{u}G \cdot \sigma u \rangle$ and $\langle \bar{d}G \cdot \sigma d \rangle$ [Fig. 1(g)] are genuinely of two orders higher than the quark condensates. Four quark condensates [Fig. 1(h)] enter already at order 8. Because reliable values for the isospin breaking of the mixed condensates and the four quark condensates are missing, we prefer to stop the OPE at order 4 and do not take the higher order power corrections into account.

Applying the prescription described above one can easily derive the Borel sum rules for the three-point functions of Eq. (1):

$$(-)\frac{1}{\pi^2}\left\{\left[\frac{5}{6}\langle \bar{u}u \rangle + \frac{1}{6}\langle \bar{d}d \rangle\right] + \frac{\theta}{\sqrt{3}}\left[\frac{5}{6}\langle \bar{u}u \rangle - \frac{1}{6}\langle \bar{d}d \rangle\right]\right\}$$
$$= \lambda_p{}^2 \frac{m_\pi{}^2 f_\pi}{m_0} M_p(+)g_{pp\pi^0}\left(\frac{1}{M^2}\right)^3 e^{-(M_p{}^2/M^2)}, \quad (13a)$$

$$(-)\frac{1}{\pi^2}\left\{(-)\left[\frac{5}{6}\langle \bar{d}d \rangle + \frac{1}{6}\langle \bar{u}u \rangle\right] + \frac{\theta}{\sqrt{3}}\left[\frac{5}{6}\langle \bar{d}d \rangle - \frac{1}{6}\langle \bar{u}u \rangle\right]\right\}$$
$$= \lambda_n{}^2 \frac{m_\pi{}^2 f_\pi}{m_0} M_n(-)g_{nn\pi^0}\left(\frac{1}{M^2}\right)^3 e^{-(M_n{}^2/M^2)}, \quad (13b)$$

and

$$(-)\frac{1}{\pi^2}\left[\frac{1}{2}\langle \bar{u}u \rangle + \frac{1}{2}\langle \bar{d}d \rangle\right]$$
$$= \lambda_p \lambda_n \frac{m_\pi{}^2 f_\pi}{m_0} \frac{M_p + M_n}{2} g_{pn\pi^+}$$
$$\times \left(\frac{1}{M^2}\right)^2 \frac{e^{-(M_p{}^2/M^2)} - e^{-(M_n{}^2/M^2)}}{M_n{}^2 - M_p{}^2}. \quad (14)$$

It should be noted hereby that the strange quark in the current $P_{a=8}$ [Eq. (10)] does not contribute in the OPE up to that order which we are taking into account.

Already at this point we see by taking the difference between Eq. (13a) and Eq. (13b) and comparing with Eq. (14) that up to first order in isospin breaking the charged pion-nucleon coupling is exactly the arithmetic average of the two neutral pion nucleon couplings, i.e., we have

$$g_{pn\pi^+} = \frac{1}{2}[g_{pp\pi^0} + g_{nn\pi^0}]. \quad (15)$$

which is a simple consequence of the u and d quark contents of the three-point functions and valid within the approximations considered.

In order to obtain the splitting between $g_{pp\pi^0}$ and $g_{nn\pi^0}$ we take the sum between Eq. (13a) and Eq. (13b) and divide by either one of them. Expanding again up to first order in isospin breaking, we obtain

$$\left(-\frac{\delta g}{g_{\pi NN}}\right) + \left(-\frac{\delta \lambda_N{}^2}{\lambda_N{}^2}\right) + \left(-\frac{\delta M_N}{M_N}\right) + 2\left(\frac{\delta M_N}{M_N}\right)\left(\frac{M_N{}^2}{M^2}\right)$$
$$= -\frac{2}{3}\gamma + \frac{4}{3}\frac{\theta}{\sqrt{3}}. \quad (16)$$

Here we have used the following notations for the isospin splittings:

$$\delta M_N = M_n - M_p, \delta g = g_{nn\pi^0} - g_{pp\pi^0}, \delta \lambda_N{}^2 = \lambda_n{}^2 - \lambda_p{}^2 \quad (17)$$

and the average values

$$M_N = \frac{1}{2}(M_n + M_p), \quad g_{NN\pi} = \frac{1}{2}(g_{nn\pi^0} + g_{pp\pi^0}),$$

$$\lambda_N{}^2 = \frac{1}{2}(\lambda_n{}^2 + \lambda_p{}^2). \quad (18)$$

Furthermore we have introduced the parameter

$$\gamma = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} - 1 \quad (19)$$

to denote the isospin breaking in the quark condensates and set

$$\langle \bar{q}q \rangle = \frac{1}{2}[\langle \bar{u}u \rangle + \langle \bar{d}d \rangle]. \quad (20)$$

From Eq. (16) we also see that we need to know the value of $\delta \lambda_N{}^2$, i.e., the isospin breaking in the overlaps between the nucleon states and the corresponding interpolating currents. To obtain $\delta \lambda_N{}^2$, we follow Refs. [7,9,10] and use the sum rules for the nucleon two point functions

$$\int d^4x e^{ikx}\langle 0|\mathcal{T}\eta_N(x)\overline{\eta_N}(0)|0\rangle = \not{k}\Pi_1^N(k^2) + \Pi_2^N(k^2), \quad (21)$$

which have been considered in the case of isospin breaking in Refs. [15–17]. We will take the chiral odd sum rule for the amplitudes $\Pi_1(k^2)$ which is known to work better than the chiral even ones for $\Pi_2(k^2)$ [18]. Including again condensates up to order 4 we have [cf. Eqs. (8) and (11) in Ref. [17]]:

$$(2\pi)^4 \frac{\lambda_p{}^2}{4} = e^{M_p{}^2/M^2}\left[\frac{M^6}{8} + \frac{M^2 g_c^2 \langle G^2 \rangle}{32}\right.$$
$$\left. + (2\pi)^2 \frac{M^2}{4} m_d \langle \bar{d}d \rangle\right], \quad (22a)$$

$$(2\pi)^4\frac{\lambda_n{}^2}{4}=e^{M_n{}^2/M^2}\left[\frac{M^6}{8}+\frac{M^2 g_c^2\langle G^2\rangle}{32}\right.$$

$$\left.+(2\pi)^2\frac{M^2}{4}m_u\langle\bar{u}u\rangle\right]. \tag{22b}$$

It should be noted that in this sum rule the gluon condensate $g_c^2\langle G^2\rangle$ enters in the same order as the quark condensate (i.e., order 4) and therefore is taken into account, whereas in the sum rule for the three-point function [Eqs. (13a)–(14)] it enters two orders higher than the quark condensate and was therefore omitted.

We take the difference between Eq. (22a) and Eq. (22b) for p and n and divide by either one of them, giving

$$\left(-\frac{\delta\lambda_N{}^2}{\lambda_N{}^2}\right)=(-2)\left(\frac{\delta M_N}{M_N}\right)\left(\frac{M_N{}^2}{M^2}\right)-(2\pi)^2(m_\pi{}^2 f_\pi{}^2)$$

$$\times\frac{M^2}{M^6+\frac{1}{4}g_c^2\langle G^2\rangle M^2}\left[2\frac{m_d-m_u}{m_d+m_u}+\gamma\right.$$

$$\left.-2\frac{\delta M_N}{M_N}\frac{M_N{}^2}{M^2}\right]. \tag{23}$$

Putting Eq. (23) into Eq. (16) we obtain the final sum rule

$$\left(-\frac{\delta g}{g_{NN\pi}}\right)=-\frac{2}{3}\gamma+\frac{4}{3}\frac{\theta}{\sqrt{3}}+\left(\frac{\delta M_N}{M_N}\right)+(2\pi)^2(m_\pi{}^2 f_\pi{}^2)$$

$$\times\frac{M^2}{M^6+\frac{1}{4}g_c^2\langle G^2\rangle M^2}\left[2\frac{m_d-m_u}{m_d+m_u}+\gamma\right.$$

$$\left.-2\frac{\delta M_N}{M_N}\frac{M_N{}^2}{M^2}\right], \tag{24}$$

where we have used Eq. (6).

For the isospin breaking in the quark masses we use the most recent analysis of current quark mass ratios [19], giving a value of $(m_d-m_u)/(m_d+m_u)=0.29\pm0.05$. As we can see, one of the crucial ingredients in Eq. (24) is the numerical value for the parameter γ. Various analyses concerning this quantity have been performed using different methods: QCD sum rules for scalar and pseudoscalar mesons [8,20–22], QCD sum rule analyses of the the baryon mass splittings [23], and the D and D^* isospin mass differences [24] as well as effective models for QCD incorporating the dynamical breaking of chiral symmetry [16,25]. The range for γ resulting from these analyses is rather large: $0.002<-\gamma<0.010$. This range is also consistent with the result obtained from one-loop chiral perturbation theory assuming reasonable values for the strange quark condensate $\langle\bar{s}s\rangle$ [14].

For the π-η mixing angle θ we take the value obtained in lowest order chiral perturbation theory [14]

$$\theta=\frac{1}{4}\sqrt{3}\frac{m_d-m_u}{m_s-m_0}. \tag{25}$$

Using the numerical values for the quark mass ratios from Ref. [19] we find $\theta=(10\pm0.8)\times10^{-3}$. Next to leading order corrections are typically of the order 30%, e.g., the decay
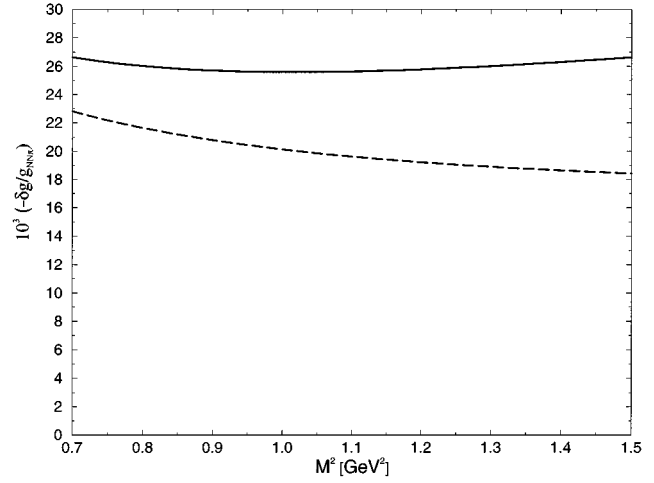

FIG. 2. Dependence of $(g_{pp\pi_0}-g_{nn\pi_0})/g_{NN\pi}$ on the square of the Borel mass M^2. As example we have used the parameters $\gamma=-0.01$, $\delta M_N=2$ MeV, and $(m_d-m_u)/(m_d+m_u)=0.28$. The dashed curve is obtained by omitting the transitions $N\to N^*$. In the full curve these contributions are included.

constants f_π and f_η differ by about 30% if loops are included. It seems therefore appropriate to assign an error of 30% to the contribution coming from π-η mixing, i.e., to the term $(4/3)(\theta/\sqrt{3})$ in Eq. (24). We have already mentioned that in the treatment of the π-η mixing we have ignored the mixing between η and η' as well as π and η'. The treatment of the η' in the current approach is difficult due to the anomaly in the SU(3) singlet pseudoscalar current. The value of the π-η mixing angle θ increases by about 30%, if the η' is included [26].

In order to obtain δM_N, we correct the experimental value for the proton and neutron mass difference by electromagnetic effects, rendering an interval of 1.6 MeV$<\delta M_N<2.4$ MeV [27,31]. For $g_c^2\langle G^2\rangle$ we take the standard value of 0.474 GeV^4, noting that its numerical contribution to Eq. (24) is rather small.

The dashed curve of Fig. 2 shows $[-(\delta g/g_{NN\pi})]$ obtained from Eq. (24) in the Borel window 0.7 GeV$^2<M^2<1.5$ GeV^2 using typical values for the parameters.

Up to now we have saturated the phenomenological side of the sum rules only with the N ground state and have omitted transitions between N and excited N^* states as well as contributions from the pure continuum. As has been shown, e.g., in Refs. [28–30] in a single variable dispersion sum rule, the transitions $N\to N^*$ gives rise to a single pole term $\sim1/(p^2-M_N{}^2)$ in addition to the double pole term of Eq. (9). This single pole term will not be suppressed in the Borel sum rules (24). It is easy to see that the inclusion of this contribution would add a term to the left-hand side (LHS) of Eq. (24) which is of the same general form multiplied by an additional power of M^2, i.e., it can be written as $C(1/M^2)^2 e^{-(M_N^2/M^2)}$. The constant C can be treated as effective parameter which is optimized in order to obtain the best fit to the Borel curve. In the isospin conserving case [7,9,10] it seems to be justified to neglect this contribution due to the fact that the sum rule for $g_{NN\pi}$ saturated only with the ground state is practically independent on the Borel mass

M^2. This indicates that the parameter C is compatible with zero. Furthermore the on shell value for $g_{NN\pi}$ is reproduced rather well in this approach. A recent QCD sum rule analysis for $g_{NN\pi}$ using two-point functions [30] also finds that this transition is very small. However, in our case we are looking at isospin violation, and there could be a small difference of the parameter C for the proton and neutron contributing to the sum rule (24) in the same order of magnitude as $\delta g/g_{NN\pi}$. It is not difficult to take the excited states into account. The LHS of Eq. (24) becomes $[-(\delta g/g_{NN\pi})]+AM^2$, where the unknown parameter A is optimized in the Borel analysis, which means, effectively, by fitting a straight line to the dashed curve of Fig. 2. Doing so results in the full line curve of Fig. 2 as the final Borel curve for $[-(\delta g/g_{NN\pi})]$, which is very stable in the window under consideration.

It should be noted that the M^2 dependence of the Borel curve is practically unaffected by the large uncertainty in the input parameter γ and only depends on the ratio $(m_d-m_u)/(m_d+m_u)$, because the numerical contribution of γ as well as δM_N to the M^2 dependent term in Eq. (24) is very small. The term $-(2/3)\gamma+(4/3)(\theta/\sqrt{3})$ only affects the intersection with the y axis but not the M^2 dependence.

Finally let us look at the effect of a pure continuum starting at a threshold s, which would result in multiplying the RHS of the Eqs. (13a)–(14) with the function $E_1(x)=1-(1+x)e^{-x}$ with $x=s/M^2$. If one assumes that the continuum thresholds for proton s_p and neutron s_n are equal, there is no effect to the isospin breaking sum rule (24). Allowing for a difference of $|\delta s|/s=0.2\%$ (compatible with $\delta M_N/M_N$), with $s_n>s_p$ and using a typical value of $s=2.25$ GeV^2 would give a contribution of $\approx 0.17\%$ to $[-(\delta g/g_{NN\pi})]$ at $M^2=1$ GeV^2. This is noticeably smaller than other errors inherent in the sum rule method.

In order to obtain an estimated error for $[-(\delta g/g_{NN\pi})]$ we calculate the minimum and the maximum values obtained from Eq. (24) after fitting the constant A and using the extreme values for the input parameters $-\gamma$, $(m_d-m_u)/(m_d+m_u)$, δM_N as well as the the contribution from π-η mixing, as discussed above. This gives an interval of

$$17\times 10^{-3} < \left(-\frac{\delta g}{g_{NN\pi}}\right) < 30\times 10^{-3}. \qquad (26)$$

Furthermore, from various other isospin violating sum rule analyses (e.g., Ref. [17]) we know that the next higher order condensate $\langle \bar{q} G\cdot\sigma q\rangle$, which has been omitted here due to the reasons mentioned above, may account for about 25% of the leading term. This means that we can expect an additional uncertainty of this magnitude. This leaves us with a final interval of

$$12\times 10^{-3} < \left(-\frac{\delta g}{g_{NN\pi}}\right) < 37\times 10^{-3}. \qquad (27)$$

The contribution coming from π-η mixing, the term $(4/3)(\theta/\sqrt{3})$ in Eq. (24) amounts to about $(8\pm 2.5)\times 10^{-3}$. As stated above this value would be about 30% larger if η-η' mixing was included. The large uncertainty in the input parameter γ and the lack of phenomenological data do not call for a more detailed investigation at the present stage.

TABLE I. Comparison of $(g_{pp\pi_0}-g_{nn\pi_0})/g_{NN\pi}$ obtained in different approaches including isospin-violation effects from strong interaction. In order to compare the numerical values of Refs. [1,35] with the other results we have used $\delta M_N/M_N=0.002$.

	$(g_{pp\pi_0}-g_{nn\pi_0})/g_{NN\pi}$
This work	$\approx 0.012\cdots 0.037$
Ref. [35]	$\approx 0.010\cdots 0.014$
Ref. [1]	≈ 0.006
Ref. [38]	0.005 ± 0.0018
Ref. [39]	≈ 0.0067

Finally let us compare our result with those of previous studies, which analyze the isospin splitting of the pion nucleon couplings arising from the strong interaction, i.e., essentially the quark mass difference m_d-m_u. It should be noted that direct experimental values are not available. The Nijmegen phase shift analysis for NN and $N\bar{N}$ scattering data [32,33] which is consistent with data from πN scattering [34], but includes electromagnetic effects, finds $(\delta g/g_{NN\pi}) = 0.002$, but with an error of 0.008; thus, there is no evidence for a difference and they also find no evidence for a difference between $g_{pn\pi^+}$ and $g_{NN\pi^0}$ within the statistical errors of their analysis.

From Table I we see that our range for $[-(\delta g/g_{NN\pi})]$ in Eq. (27) is compatible with the values obtained by other authors, both in sign and order of magnitude.

(1) The quark gluon model of Henley and Zhang [35].

(2) The quark pion model of Mitra and Ross [36,5]. This has recently been used by Piekarewicz [1], who obtained a violation of the ''triangle identity'' consistent with the πN data analysis of Ref. [37].

(3) The use of the quark mass difference m_d-m_u and π-η mixing [38].

(4) The chiral bag model [39], which also has our relation [Eq. (15)] for the charged coupling to be valid.

(5) On the other hand, the use of the cloudy bag model [40] leads to $(\delta g/g_{NN\pi})\approx 0.006$, with the opposite sign to our result.

It should be noted that there are electromagnetic corrections, whose direction are unknown. The charge difference we obtain due to the strong interaction would, by itself, lead to a difference on the scattering lengths $|a_{nn}|-|a_{pp}|\approx -0.5\pm 0.2$ fm, smaller than, but in the opposite direction to the observed difference [5]. Of course, there are other effects which play a role, e.g., ρ-ω mixing.

We are aware that using the three-point function is *a priori* less suitable than the two-point function for calculating the pion-nucleon coupling on shell, because it works at spacelike $q^2\approx -1$ GeV^2 and needs the detour of comparing the $1/q^2$ pole residua [7,9,10]. As we have already mentioned earlier this means that one has to assume that π pole dominance can still be applied in this region and higher pseudoscalar resonances are neglected or, in other words we use the PCAC interpolating pseudoscalar field at those values of q^2. A quantitative analysis of the contribution of these higher resonances would require some knowledge about their coupling to the nucleon. There are various indications and consistency checks that the concept proposed in Refs. [7,9,10] is

a reasonable one: First of all in the isospin-violating case one is likely to be less sensitive to the higher resonance contributions due to cancellation which presumably occur if summing over the higher pionic excitations. Furthermore, in the isospin conserving case [7,9,10] the experimental value of $g_{NN\pi}$ is reproduced rather well. The Borel stability in both the isospin conserving [7,9,10] and the isospin-violating case (this work) is a further consistency check, although this of course is only a necessary but not sufficient condition. Finally there is the analysis of the πNN form factor [13] using this approach. The q^2 dependence of $g_{NN\pi}(q^2)$, which contains effectively the higher pionic resonances in the spectral function (1), is consistent with various other approaches using the same interpolating current but working at lower q^2. This indicates that the interpolation between low and high q^2 region is done reasonably well. Despite all these arguments the importance of the higher pseudoscalar resonances remains a matter to be settled and needs further quantitative investigation [42].

On the other hand the use of a two-point function [7,10,41,30] has other, and we believe worse problems in the consideration of isospin violation. The single nucleon pole sum rule, as it has been used in Refs. [7,10,41], suffers principally from the problem that the contribution from the transition $N \rightarrow N^*$ enters exactly in the same form as $g_{NN\pi}$ itself, namely as single pole. After Borel transform one obtains a term $g_{NN\pi}+A$ instead of $g_{NN\pi}+AM^2$ as in case of the double pole sum rule. Hence within the single pole sum rule itself there is *a priori* no way to separate the $N \rightarrow N^*$ contribution A from $g_{NN\pi}$. In Refs. [7,10,41] the $N \rightarrow N^*$ transition has been ignored. In the isospin-conserving case this is *a*

posteriori justified because the numerical value of this term turns out to be small, as it has been discussed above. However, we do not know if this is true in the isospin violating case. The double nucleon pole sum rule [30] would avoid this problem and moreover is able to give a value for the $N \rightarrow N^*$ contribution. Unfortunately as one can see from the analysis in Ref. [30] already in the isospin-conserving case this sum rule seems to be rather sensitive to the condensate input, which is the quark condensate $\langle \bar{q}q \rangle$ and especially the higher order mixed condensate $\langle 0|\bar{q}\widetilde{G}\gamma_\mu q|\pi\rangle$, which has to be included to give a reasonable value for $g_{NN\pi}$. In case of the three-point function the condensate input parameters are much better under control. For these reasons we prefer to work with the three point function sum rule.

To summarize we have calculated the splitting between the pion-nucleon coupling constants $g_{pp\pi^0}$, $g_{nn\pi^0}$, and $g_{pn\pi^+}$ due to isospin breaking in the strong interaction by using the QCD sum rules for the corresponding pion nucleon three-point functions. We have taken OPE diagrams up to order 4 into account. Our result for the splitting in the neutral couplings is $1.2\times10^{-2}<(g_{pp\pi^0}-g_{nn\pi^0})/g_{NN\pi}<3.7\times10^{-2}$. The charged coupling $g_{pn\pi^+}$ is found to be the average of the two neutral ones.

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