Medium-induced parton energy loss in γ+jet events of high-energy heavy-ion collisions

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The effect of medium-induced parton energy loss on jet fragmentation is studied in high-energy heavy-ion collisions. It is shown that an effective jet fragmentation function can be extracted from the inclusive p_T spectrum of charged particles in the opposite direction of a tagged direct photon with a fixed transverse energy. We study the modification of the effective jet fragmentation function due to parton energy loss in AA as compared to pp collisions, including E_T smearing from initial-state radiations for the photon-tagged jets. The effective fragmentation function at $z=p_T/E_T^\gamma\sim 1$ in pA collisions is shown to be sensitive to the additional E_T smearing due to initial multiple parton scatterings whose effect must be subtracted out in AA collisions in order to extract the effective parton energy loss. Jet quenching in deeply inelastic lepton-nucleus scatterings as a measure of the parton energy loss in cold nuclear matter is also discussed. We also comment on the experimental feasibilities of the proposed study at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies and some alternative measurements such as using Z^0 as a tag at the LHC energy. [S0556-2813(97)04805-X]

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I. INTRODUCTION

Hard processes are considered good tools to study ultrarelativistic heavy-ion collisions because they happen early in the reaction processes and thus can probe the early stage of the evolution of a dense system, during which a quark-gluon plasma (QGP) could exist for a short period of time. Among the proposed hard probes, large transverse momentum jets or partons are especially useful because they interact strongly with the medium. For example, an enhanced acoplanarity and energy imbalance of two back-to-back jets [1] due to multiple scatterings, jet quenching due to the medium-induced radiative energy loss of a high-energy parton propagating through a dense medium [2] can provide important information on the properties of the medium and interaction processes that may lead to partial thermalization of the produced parton system. The medium-induced radiative energy loss of a fast parton traversing a dense QCD medium is also interesting by itself because it illustrates the importance of quantum interference effects in QCD. As recent studies have demonstrated [3,4], it is very important to take into account the destructive interference among many different radiation amplitudes induced by multiple scatterings in the calculation of the final radiation spectrum. The so-called Landau-Pomeranchuk-Midgal effect [5] can lead to very interesting, and sometimes nonintuitive results for the radiative energy loss of a fast parton inside a QCD medium. Recently, Baier, Dokshitzer, Mueller, Peigné, and Schiff (BDMPS) showed [4] that the energy loss per unit distance, dE/dx, grows linearly with the total length of the medium L, which in turn can be related to the total transverse momentum broadening squared Δk_T^2 of the parton from multiple scatterings. It turns out that both quantities, dE/dx and Δk_T^2, are related to the parton density of the medium that the parton is traveling through. One can therefore determine the parton density of the produced dense matter by measuring the energy loss of a fast parton in high-energy heavy-ion collisions.

Unlike in the QED case, where one can measure directly the radiative photon spectrum and thus the energy loss of a fast electron, one cannot measure directly the energy loss of a fast parton in QCD. Since a parton is experimentally associated with a jet, a cluster of hadrons in a finite region of the phase space, an identified jet can contain particles both from the fragmentation of the leading parton and from the radiated partons. If we neglect the k_T broadening effect, the total hadronic energy contained in a jet should not change even if the leading parton suffers radiative energy loss. However, significant changes could happen to particle distributions inside the jet, or the fragmentation function and jet profile, due to the induced radiation of the leading parton. Therefore, one can measure the radiative energy loss indirectly via the modification of the jet fragmentation function and jet profile.

A jet fragmentation function is defined as the particle distribution in the fractional energy. In order to measure the fragmentation function one has to first determine the initial energy of the fragmenting parton either through other measured kinematic variables as in e^+e^- and e^-p or calorimetric measurements as in pp and $p\bar{p}$ collisions. However, because of the large value of $dE_T/dyd\phi$ and its fluctuation in high-energy heavy-ion collisions [6], the conventional calorimetric study of a jet cannot determine the jet energy to such an accuracy as required to determine the parton energy loss. In search of an alternative measurement, Wang and Gyulassy [7] proposed that single-particle p_T spectrum can be used to study the effect of parton energy loss, since the suppression of large E_T partons naturally leads to the suppression of large p_T particles. Because the particle spectrum at large p_T is the convolution of jet cross sections and fragmentation functions, particles with a fixed p_T can come from the fragmentation of partons of different initial energies with some average value $\langle E_T^{\rm jet}\rangle$. In this case, the suppression of the p_T spectrum due to parton energy loss is then related to the modification of jet fragmentation function at an averaged $\langle z\rangle = p_T/\langle E_T^{\rm jet}\rangle$. Since $\langle E_T^{\rm jet}\rangle$ is approximately proportional to p_T, by varying p_T one can then study the energy dependence of the modification of jet fragmentation functions at a fixed $\langle z\rangle$ [8].

In order to study the modification of the *whole* jet fragmentation function due to parton energy loss in the full z range, we and Sarcevic [9] proposed to measure the particle p_T distribution in the opposite transverse direction of a tagged direct photon. Since a direct photon in the central rapidity region ($y=0$) is always accompanied by a jet in the opposite transverse direction with roughly equal transverse energy, the p_T distribution of particles in that direction is directly related to the jet fragmentation function with known initial energy, $E_T^{\rm jet}\approx E_T^{\gamma}$. In such $\gamma+$jet events, the background due to particle production from the rest of the system was estimated to be well below the p_T spectrum from jet fragmentation at moderate large p_T. Therefore, one can easily extract the fragmentation function from the experimental data without much statistical errors introduced by the subtraction of the background. By comparing the extracted jet fragmentation function in AA to that in pp collisions, one can then measure the modification of the fragmentation function and determine the parton energy loss.

Because of the complexity of the problem, it will be helpful for us to first discuss all possible relevant processes which might have some effects on the study of parton energy loss in $\gamma+$jet events in high-energy AA collisions. One can order the processes in a chronological order with respect to the hard process of direct photon production.

(1) As the two nuclei approach and pass through each other, the two participating beam partons which later produce the direct photon will suffer initial-state interactions with other oncoming nucleons. The participating beam partons will then suffer radiative energy loss and acquire transverse momentum kicks because of these soft interactions. The initial state interactions will also cause the shadowing of the parton distributions inside the nuclei. These initial-state effects will certainly affect the production rate of direct photons at a given transverse energy E_T^{γ}. However, they will not influence the propagation and fragmentation of the accompanying produced jet parton in events triggered with a direct photon with a fixed E_T^{γ}.

(2) After the hard process in which a direct photon and a jet parton are produced, the jet parton can also scatter from the beam nucleons within a tube of a transverse size at most 1 fm in the rest part of the colliding nuclei which has not passed through. Since the colliding nuclei pass through each within a very short period of time, $t\sim 1$ fm/c (this is the spatial size of wee partons, while the valence partons have a Lorentz contracted size of $R_A m_N^2/2s$), the produced jet parton in central rapidity region will not have time to interact with other beam nucleons outside the tube. Since we are only interested in jet partons in the central rapidity region, these scatterings will not cause the jet partons to lose transverse energy. Rather, together with the initial-state interactions, they will change the final transverse momentum of the jet parton, resulting in an E_T broadening in addition to the E_T smearing caused by initial-state radiations associated with the hard process. The effects of this E_T broadening should also exist and can be studied independently in pA collisions.

(3) In the triggered events, there are many other processes which can also produce hard or semihard partons [10] and thus form a dense medium in the central rapidity region. The photon-tagged jet parton will then interact with these partons during its propagation through the dense medium. We call these interactions final-state interactions. The induced radiative energy loss and transverse momentum broadening, referred to as k_T broadening in this paper, are the focus of our study.

In this paper, we will study in detail the effect of parton energy loss on the jet fragmentation function as extracted from the p_T spectrum in the opposite direction of a triggered direct photon. In particular, we will take into account the E_T smearing of the jet due to initial-state radiations associated with the $\gamma+$jet processes. We will show that the particle spectrum from the jet fragmentation at $p_T\sim E_T^{\gamma}$ is very sensitive to the E_T broadening from initial- and final-state scatterings with beam partons. One can then use our proposed measurement to determine the E_T broadening in pA collisions. This small but finite effect must then be subtracted out when one determines the medium-induced parton energy loss in AA collisions. We will also investigate the sensitivity of the modification of the fragmentation function to the energy and A dependence of the parton energy loss. The change of the profile function in the azimuthal angle due to the k_T broadening of the parton from multiple scatterings inside the medium will also be discussed. Here k_T is the parton transverse momentum with respect to the original jet direction, which can be related to the parton energy loss according to BDMPS study [4]. Finally, we will discuss the experimental feasibility of the proposed study and alternative measurements using Z^0 particles as a tag. We will also discuss how similar measurements can be made in deeply inelastic lepton-nucleus scatterings, from which one can determine the energy loss of a fast parton passing through cold nuclear matter.

II. MODIFIED FRAGMENTATION FUNCTIONS

The fragmentation functions of partons hadronizing in the vacuum have been studied extensively in e^+e^-, ep, and $p\bar{p}$ collisions [11]. These functions describe particle distributions in the fractional energy, $z=E_h/E_{\rm jet}$, in the direction of a jet. Similar to parton distributions inside hadrons, the fragmentation functions are also nonperturbative in nature. However, parton cascades during the early stage of the fragmentation can be described by perturbative QCD. The measured dependence of the fragmentation functions on the momentum scale is shown to satisfy the QCD evolution equations very well. We will use the parametrizations of the most recent analysis [12] in both z and Q^2 dependence for jet fragmentation functions $D^0_{h/a}(z,Q^2)$ to describe the fragmentation of a parton (a) into hadrons (h) in the vacuum.

The fragmentation of a parton inside a medium is different from that in the vacuum, because of its final-state interactions with the medium and the associated radiations. Such interactions and medium-induced radiations will cause the deflection and energy loss of the propagating parton which in effect will modify the fragmentation functions from their corresponding forms in the vacuum. In principle, one could study the modification of jet fragmentation functions in perturbative QCD in which induced radiation of a propagating parton in a medium and Landau-Pomeranchuk-Migdal interference effect can be dynamically taken into account. However, for the purpose of our current study, we can use a phenomenological model to describe the modification of the jet fragmentation function due to an effective energy loss dE/dx of the parton. Such an approach is useful and possibly necessary for experimental studies of the parton energy loss and multiple final-state scatterings.

In this phenomenological model we assume that the size and lifetime of the system is small compared to the hadronization time of a fast parton. In the case of a QGP, a parton cannot hadronize inside the deconfined phase. A fast parton will hadronize outside the system and the fragmentation can be described as in e^+e^- collisions, however, with reduced parton energy. The interaction of a parton a with the medium can be characterized by the mean free path λ_a of parton scatterings, the radiative energy loss per scattering ϵ_a and the transverse momentum broadening squared Δk_T^2. The energy loss per unit distance is thus $dE_a/dx = \epsilon_a/\lambda_a$ which in principle depends on Δk_T^2 and λ_a. We assume that the probability for a parton to scatter n times within distance L is given by a Poisson distribution,

$$P_a(n,L) = \frac{(L/\lambda_a)^n}{n!} e^{-L/\lambda_a}. \tag{1}$$

We also assume that the mean free path of a gluon is half that of a quark, and the energy loss dE/dx is twice that of a quark. The emitted gluons, each carrying energy ϵ_a on the average, are assumed to hadronize also according to the fragmentation function. For simplification, we will neglect the energy fluctuation given by the radiation spectrum for the emitted gluons. We assume the momentum scale in the fragmentation function for the emitted gluons to be set by the minimum scale $Q_0^2 = 2.0\ \mathrm{GeV}^2$. Since the emitted gluons only produce hadrons with very small fractional energy, the final modified fragmentation functions in the moderately large z region are not very sensitive to the actual radiation spectrum and the momentum scale dependence of the fragmentation functions for the emitted gluons. In this paper, we will also neglect possible final-state interactions between hadrons from parton fragmentation and the hadronic environment at the late stage of the evolution of the whole system. However, it is important for future investigations to estimate the influence of pure hadron scatterings on the final observed jet fragmentation functions.

We will consider parton fragmentation in the central rapidity region of high-energy heavy-ion collisions. In this case, we only need to study partons with initial transverse energy E_T and traveling in the transverse direction in a cylindrical system. With the above assumptions, the modified fragmentation functions for a parton traveling a total distance L can be approximated as [9],

$$D_{h/a}(z,L,Q^2) = \frac{1}{C_N^a} \sum_{n=0}^{N} P_a(n,L) \frac{z_n^a}{z} D_{h/a}^0(z_n^a,Q^2) + \langle n_a \rangle \frac{z_a'}{z} D_{h/g}^0(z_a',Q_0^2), \tag{2}$$

where $z_n^a = z/(1 - n\epsilon_a/E_T)$, $z_a' = zE_T/\epsilon_a$, and $C_N^a = \Sigma_{n=0}{}^N P_a(n)$. $D_{h/a}^0(z,Q^2)$ are the jet fragmentation functions in the vacuum which we take the parametrized form in Ref. [12]. We limit the number of inelastic scatterings to $N = E_T/\epsilon_a$ to conserve momentum. One can check that the above modified fragmentation functions satisfy the momentum sum rule by construction, $\Sigma_h \int z D_{h/a}(z,L,Q^2)dz = \Sigma_h \int z D_{h/a}^0(z,Q^2)dz = 1$. For large values of N, the average number of scatterings within a distance L is approximately $\langle n_a \rangle \approx L/\lambda_a$. The first term in the above equation corresponds to the fragmentation of the leading partons with reduced energy $E_T - n\epsilon_a$ after n inelastic scatterings. This term normally dominates for leading particles in the moderate and large z region. Since the fragmentation functions $D_{h/a}^0(z,Q^2)$ generally decrease with z, especially quite rapidly at moderate and large z region, the reduction in energy will lead to the suppression of leading particles or the decrease of the fragmentation functions in this region as compared to the case in vacuum. The second term in the above equation comes from the emitted gluons each having energy ϵ_a on the average. This term is generally significant only in the small z region and it increases the effective fragmentation functions in the small z region, or enhances soft particle production. We should note that our assumptions on the hadronization of the emitted gluons are too schematic to give a quantitative description of the physics involved in that small z region.

For a given parton energy E_T and the total distance L, the above effective fragmentation functions depend on only two parameters, the mean free path λ_a and energy loss per scattering ϵ_a. As demonstrated in Ref. [9], contributions from the leading partons who have suffered at least one inelastic scattering is completely suppressed for z values close to 1. The remaining contribution comes from those partons which escape the system without a single inelastic scattering, with a probability $\exp(-L/\lambda_a)$, which depends on λ_a but is independent of the parton energy E_T and the parton energy loss dE_a/dx. On the other hand, in the intermediate z region, particles from the fragmentation of the leading partons with reduced energy dominates. The suppression of the fragmentation functions is controlled by the total energy loss, $\langle \Delta E_T \rangle = \langle n_a \rangle \epsilon_a = L\,dE_a/dx$, which depends only on dE_a/dx. One, therefore, could determine, in principle, these two parameters, λ_a and dE_a/dx, simultaneously from the measured suppression of the effective fragmentation functions, for fixed E_T and L. However, as we will see in the next section, the complication of not knowing the jet energy precisely will render such arguments unrealistic. In certain cases, one has to resort to a model-dependent global fitting of the modification of the fragmentation functions in order to determine the mean free path and parton energy loss.

III. THE INCLUSIVE FRAGMENTATION FUNCTION OF PHOTON-TAGGED JETS

As we have emphasized in the Introduction, the most important point in the study of parton energy loss through the measurement of the modification of the parton fragmentation functions is the determination of the initial parton or jet energy. However, the direct measurement of a jet energy to the accuracy as required to determine an energy loss of a few GeV is unfeasible due to the large background and its fluctuation in high-energy heavy-ion collisions. To overcome this difficulty, it was proposed [9] that direct photons in heavy-ion collisions can be used to tag the energy of jets which always accompany the direct photons. Because of initial-state radiations associated with the production of a direct photon, the accompanying jet is not always exactly in the opposite direction of the photon and its transverse energy also differs from collision to collision, though the averaged jet energy is well approximated by the energy of the triggered photon. In Ref. [9], the variation of jet energy was not considered in the study of the effective inclusive jet fragmentation function in $\gamma+$jet events and the modification due to parton energy loss. In this paper, however, we would like to explore the effect of E_T smearing due to initial-state radiations. We will see that such a smearing complicates the simple procedure to determine the parton energy loss and the mean free path of final-state scatterings as outlined in the previous study [9].

Let us consider events which have a direct photon with fixed transverse energy E_T^γ in the central rapidity region, $|y_\gamma|\leqslant\Delta y/2$, $\Delta y=1$. Given the jet fragmentation functions $D^0_{h/a}(z)$, with z the fractions of momenta of the jet carried by hadrons, one can calculate the differential p_T distribution of hadrons, averaged over the kinematical region $(\Delta y,\Delta\phi)$, from the fragmentation of a photon-tagged jet in pp collisions,

$$\frac{dN_{pp}^{\gamma-h^\pm}}{dyd^2p_T}\equiv\frac{1}{d\sigma_{pp}^\gamma/dy_\gamma dE_T^\gamma}\,\frac{d\sigma_{pp}^{\gamma-h^\pm}}{dy_\gamma dE_T^\gamma dyd^2p_T}$$

$$\frac{d\sigma_{pp}^{\gamma-h^\pm}}{dy_\gamma dE_T^\gamma dyd^2p_T}=\sum_{a,h}\int dE_T^a dy_a d\phi_a \times\frac{d\sigma_{pp}^{\gamma-a}}{dE_T^\gamma dy_\gamma dE_T^a dy_a d\phi_a}\,\frac{D^0_{h/a}(p_T/E_T^a)}{p_T E_T^a}\times\int_{(\Delta y,\Delta\phi)}\frac{dy}{\Delta y}\,\frac{d\phi}{\Delta\phi}f_0(y_a-y,\phi_a-\phi), \quad (3)$$

where the summation is over jet (a) and hadron (h) species and $f_0(y,\phi)$, assumed to be the same for all hadron species, is the normalized hadron intrinsic profile around the parton axis. If the azimuthal angle of the photon is ϕ_γ and $\bar{\phi}_\gamma=\phi_\gamma+\pi$, the restricted kinematical region for the selected hadrons is defined as $(\Delta y,\Delta\phi)=(|y|\leqslant\Delta y/2, |\phi-\bar{\phi}_\gamma|\leqslant\Delta\phi/2)$. One could also use more complicated geometry, such as a circle with a given radius, for the phase space restriction to define a jet. The inclusive differential cross section for direct photon production is

$$\frac{d\sigma_{pp}^\gamma}{dy_\gamma dE_T^\gamma}=\sum_a\int dE_T^a dy_a d\phi_a\frac{d\sigma_{pp}^{\gamma-a}}{dE_T^\gamma dy_\gamma dE_T^a dy_a d\phi_a}. \quad (4)$$

We now define the E_T-smearing function, $g_{pp}(E_T^a,E_T^\gamma)$, and parton correlation function, $f_{\rm jet}(y_a,\phi_a)$, as

$$g_{pp}(E_T^a,E_T^\gamma)f_{\rm jet}(y_a,\phi_a)\equiv\frac{1}{d\sigma_{pp}^\gamma/dy_\gamma dE_T^\gamma}\,\frac{d\sigma_{pp}^{\gamma-a}}{dE_T^\gamma dy_\gamma dE_T^a dy_a d\phi_a}. \quad (5)$$

In a perturbative calculation to the lowest order in α_s, which was used in our earlier study [9], the E_T-smearing function and parton correlation function are simply two δ functions, $g_{pp}(E_T^a,E_T^\gamma)=\delta(E_T^a-E_T^\gamma)$, $f_{\rm jet}(y_a,\phi_a)\propto\delta(\phi_a-\bar{\phi}_\gamma)$, due to momentum conservation. The intrinsic hadron profile function $f_0(y,\phi)$ in this leading-order calculation should be the measured jet profile. In calculations beyond the leading-order, the photon and jet parton have a finite imbalance in transverse momentum due to the initial-state radiations. The final-state radiations also contribute to the measured jet profile $f(y,\phi)$ which should be the convolution of the parton correlation function $f_{\rm jet}(y_a,\phi_a)$ from perturbative calculations to a given order and the intrinsic hadron profile $f_0(y,\phi)$,

$$f(y,\phi)=\int dy_a d\phi_a f_{\rm jet}(y_a,\phi_a)f_0(y_a-y,\phi_a-\phi). \quad (6)$$

Note that the differential cross section $d\sigma^{\gamma-a}$ and the fragmentation function $D^0_{h/a}(z,Q^2)$ in Eq. (3) both depend on the factorization scheme and the associated scale. So are the E_T-smearing function and parton correlation function in Eq. (5). Unlike the collinear divergences in the next-to-leading-order jet cross sections, which are cancelled via the definition of a jet with a finite size, the collinear divergences in Eq. (3) are subtracted out via the definitions of parton distributions and fragmentation functions. Therefore, the differential cross section $d\sigma^{\gamma-a}$ beyond the leading order in Eq. (3) is not the same as the γ-jet cross section in which a jet is defined via the transverse energy within a finite region in phase space [14]. The two cross sections can be related via some divergence-free physical observables, e.g., total hadronic energy within the acceptance $(\Delta y,\Delta\phi)$, which can be computed from Eq. (3).

With the above definitions of the E_T-smearing function and jet profile function, we can rewrite Eq. (3) as

$$\frac{dN_{pp}^{\gamma-h^\pm}}{dyd^2p_T}=\sum_{a,h}r_a(E_T^\gamma)\int dE_T g_{pp}(E_T,E_T^\gamma)\times\frac{D^0_{h/a}(p_T/E_T)}{p_T E_T}\,\frac{C(\Delta y,\Delta\phi)}{\Delta y\Delta\phi}, \quad (7)$$

where $C(\Delta y,\Delta\phi)=\int_{|y|\leqslant\Delta y/2}dy\int_{|\phi-\bar{\phi}_\gamma|\leqslant\Delta\phi/2}d\phi f(y,\phi-\bar{\phi}_\gamma)$ can be considered as an overall acceptance factor for finding the jet fragments in the given kinematic range.

We will approximate the fractional production cross section $r_a(E_T^\gamma)$ of an a-type jet associated with the direct photon, by the lowest order calculation,

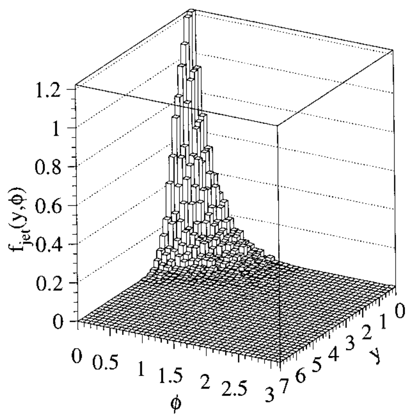

FIG. 1. The normalized parton correlation from HIJING simulations in rapidity y and azimuthal angle ϕ with respect to the opposite direction of a tagged photon with $E_T^\gamma = 10$ GeV in pp collisions at $\sqrt{s} = 200$ GeV.

$$r_a(E_T^\gamma) = \frac{d\sigma_a^\gamma/dy_\gamma dE_T^\gamma}{d\sigma^\gamma/dy_\gamma dE_T^\gamma}; \quad \sigma^\gamma = \sum_a \sigma_a^\gamma, \tag{8}$$

$$\frac{d\sigma_a^\gamma}{dy_\gamma dE_T^\gamma} = \sum_{bc} \int_{x_{b\min}}^1 dx_b f_{b/p}(x_b) f_{c/p}(x_c) \times \frac{2}{\pi} \frac{x_b x_c}{2x_b - x_T e^{y_\gamma}} \frac{d\sigma}{d\hat{t}}(bc \to \gamma + a), \tag{9}$$

where $x_c = x_b x_T e^{-y_\gamma}/(2x_b - x_T e^{y_\gamma})$, $x_{b\min} = x_T e^{y_\gamma}/(2 - x_T e^{-y_\gamma})$, and $x_T = 2E_T/\sqrt{s}$. The parton distributions in a proton, $f_{a/p}(x)$, will be given by the MSRD$-'$ parametrization [17]. In our following calculations for AA collisions, we will use the impact-parameter averaged parton distributions per nucleon in a nucleus (A,Z),

$$f_{a/A}(x) = S_{a/A}(x)\left[\frac{Z}{A} f_{a/p}(x) + \left(1 - \frac{Z}{A}\right) f_{a/n}(x)\right], \tag{10}$$

where $S_{a/A}(x)$ is the parton nuclear shadowing factor which we will take the HIJING parametrization [13]. We have explicitly taken into account the isospin of the nucleus by considering the parton distributions of a neutron which are obtained from that of a proton by isospin symmetry.

To simulate higher order effects, event generators such as PYTHIA [15], which was used in the HIJING program [13], normally use the parton shower model. In this model, one introduces a cutoff μ_0 for the parton virtuality in the chain of parton shower, thus avoiding both infrared and collinear singularities. In addition, initial- and final-state radiations are treated separately and the interference between them is also neglected. Shown in Fig. 1, is the parton correlation function in rapidity y and azimuthal angle ϕ with respect to the opposite direction of a triggered photon with $E_T^\gamma = 10$ GeV in HIJING [13] simulations of pp collisions at $\sqrt{s} = 200$ GeV. In the simulations, the final-state radiations are switched off so that we can study the effect of momentum imbalance due to initial state radiations. We can see that most of the jet partons fall into the kinematic region, $(|y| \leq 1, |\phi - \phi_\gamma| \leq 0.5)$. The jet profile, which is the convolution of the parton correlation function and hadron intrinsic profile around the parton axis, has a similar shape with a slightly larger width according to both our simulations and experimental measurements in high-energy $p\bar{p}$ collisions [16]. For calculations throughout this paper, we will use $\Delta y = 1$ and $\Delta\phi = 2$. We find the acceptance factor defined in Eq. (7), $C(\Delta y, \Delta\phi) \approx 0.5$, independent of both the colliding energy and the photon energy E_T^γ, using HIJING [13] Monte Carlo simulations.

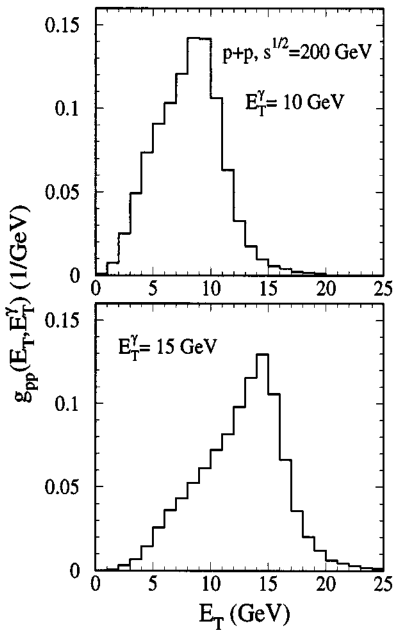

FIG. 2. The E_T-smearing function for the photon-tagged parton jets with $E_T^\gamma = 10$, 15 GeV, from HIJING simulations of pp collisions at $\sqrt{s} = 200$ GeV. The averaged value of E_T is $\langle E_T \rangle = 8.08$, 12.57 GeV, respectively

Since a parton jet in the parton shower model is well defined, one can then calculate the double differential cross section for γ-jet events. Shown in Fig. 2 are the normalized E_T distributions of the jet parton,

$$\frac{1}{N_{\gamma-\text{jet}}} \frac{dN_{\gamma-\text{jet}}}{dE_T}, \tag{11}$$

with given E_T^γ of the tagged direct photon, from HIJING simulations. As we can see that the transverse energy of the jet parton has a wide smearing around E_T^γ due to the initial-state radiations associated with the hard processes. Because of the rapidly decrease with E_T of the direct photon production cross section, the distribution is biased toward smaller E_T than E_T^γ. The average E_T is thus smaller than E_T^γ. In this paper, we will use the jet parton distribution in Eq. (11) to approximate the E_T-smearing function in Eq. (5).

If one triggers a direct photon with a given E_T^γ, one should average over the E_T smearing of the jet in the calculation of particle distributions in the opposite direction of the tagged photon. Such a smearing is important especially for hadrons with p_T comparable or larger than E_T^γ.

If we define the inclusive fragmentation function associated with a direct photon in pp collisions as,

$$D_{pp}^{\gamma}(z)=\sum_{a,h} r_a(E_T^{\gamma})\int dE_T g_{pp}(E_T,E_T^{\gamma})\frac{E_T^{\gamma}}{E_T}D_{h/a}^{0}\left(z\frac{E_T^{\gamma}}{E_T}\right), \tag{12}$$

with $z=p_T/E_T^{\gamma}$ the hadrons' momenta as fractions of the direct photon's transverse energy, we can rewrite the p_T spectrum [Eq. (7)] of hadrons in the opposite direction of a tagged photon as

$$\frac{dN_{pp}^{\gamma-h^{\pm}}}{dy d^2p_T}=\frac{D_{pp}^{\gamma}(p_T/E_T^{\gamma})}{p_T E_T^{\gamma}}\frac{C(\Delta y,\Delta\phi)}{\Delta y\Delta\phi}. \tag{13}$$

Considering parton energy loss in central AA collisions, we model the modified jet fragmentation functions as given by Eq. (2). Including the E_T smearing and averaging over the γ-jet production position in the transverse direction, the inclusive fragmentation function of a photon-tagged jet in central $A+A$ collisions is

$$D_{AA}^{\gamma}(z)=\int\frac{d^2r t_A^2(r)}{T_{AA}(0)}\sum_{a,h} r_a(E_T^{\gamma})\times\int dE_T g_{AA}(E_T,E_T^{\gamma})\frac{E_T^{\gamma}}{E_T}D_{h/a}\left(z\frac{E_T^{\gamma}}{E_T},L\right), \tag{14}$$

where $T_{AA}(0)=\int d^2r t_A^2(r)$ is the overlap function of AA collisions at zero impact parameter. The E_T-smearing function $g_{AA}(E_T,E_T^{\gamma})$ in AA collisions should be different from that in pp collisions due to initial multiple parton scatterings. However, for the moment, we will regard them as the same and postpone the discussion of the difference to the next section. We have assumed that direct photon production rate is proportional to the number of binary nucleon-nucleon collisions. Neglecting expansion in the transverse direction, the total distance a parton produced at (r,ϕ) will travel in the transverse direction is $L(r,\phi)=\sqrt{R_A^2-r^2(1-\cos^2\phi)}-r\cos\phi$. Using the above inclusive fragmentation function in Eq. (13), one can similarly calculate the p_T spectrum of particles in the opposite direction of a tagged photon in AA collisions. Setting the parton energy loss $dE/dx=0$, the above equation should be reduced to the inclusive fragmentation function in pp collisions in Eq. (12) and the corresponding p_T spectrum should also become the same as in pp collisions.

To measure the modification of the inclusive fragmentation function in experiments, one should first select events with a direct photon of energy E_T^{γ}. Then one measures the particle spectrum in the kinematical region $(\Delta y,\Delta\phi)$ in the opposite direction of the tagged photon. After subtracting the background which is essentially the p_T spectrum in ordinary events, one can use Eq. (13) to extract the inclusive jet fragmentation function, $D^{\gamma}(z)$, from the resultant spectrum. One can then compare the extracted inclusive jet fragmentation function in central AA collisions to that in pp or peripheral AA collisions to study the modification due to parton energy loss. Note that the centrality requirements for the signal (γ+jet) and background (ordinary) events should be the same. The overall acceptance factor $C(\Delta y,\Delta\phi)$ in AA collisions remains approximately the same as in pp collisions with small but measurable corrections due to the k_T broadening of the leading parton as we will discuss later.

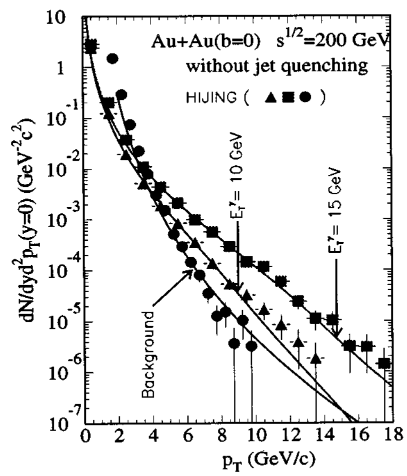

FIG. 3. The differential p_T spectrum of charged particles from the fragmentation of a photon-tagged jet with $E_T^{\gamma}=10$, 15 GeV and the underlying background in central Au+Au collisions at $\sqrt{s}=200$ GeV. The direct photon is restricted to $|y|\leq\Delta y/2=0.5$. Charged particles are limited to the same rapidity range and in the opposite direction of the photon, $|\phi-\phi_{\gamma}-\pi|\leq\Delta\phi/2=1.0$. Solid lines are calculations using Eq. (7) and points are HIJING simulations of 20 K events.

To demonstrate the feasibility of the above prescribed procedure, we show in Fig. 3 the calculated p_T distributions of charged hadrons from the fragmentation of photon-tagged jets with $E_T^{\gamma}=10$, 15 GeV and the underlying background from the rest of a central Au+Au collisions at the RHIC energy. We set $dE/dx=0$ so the effect of parton energy loss is not included yet. The points are HIJING simulations of 20-K events and solid lines for jet fragmentation are numerical results from Eq. (13) with the fragmentation functions given by the parametrization of e^+e^- data [12]. The numerical result (solid line) for the background coming from jet fragmentation in ordinary central events is obtained by the convolution of the jet cross section and fragmentation functions [8]. As we can see, the spectra from jet fragmentation are significantly higher than the background at moderately large transverse momenta. The background in pp collisions is about 1200 (the number of binary nucleon-nucleon collisions) times smaller than in central Au+Au collisions. One can therefore easily extract the inclusive fragmentation function from the experimental data without much statistical errors from the subtraction of the background. This conclusion remains valid even if one includes the parton energy loss in AA collisions because both the background and the particles from jet fragmentation are suppressed by approximately the same amount due to jet quenching [8,9]. As one can expect from Fig. 3, for direct photons with $E_T^{\gamma}<6$ GeV at the RHIC energy, the hadron spectrum from the jet fragmentation is much smaller than the background. In this case, one can no longer accurately extract the effective fragmentation function with finite number of events.

Shown in Fig. 4, are similar calculations for central Pb+Pb collisions at the LHC energy with $E_T^{\gamma}=60$ GeV for

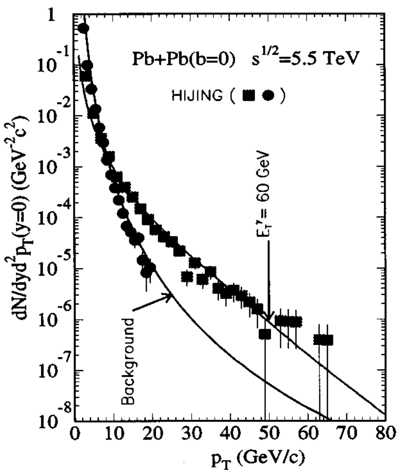

FIG. 4. The same as Fig. 3, except for $E_T^\gamma=60$ GeV at $\sqrt{s}=5.5$ TeV.

the tagged photons. It is clear that the overall background is much larger than at the RHIC energy. Therefore, one needs to trigger on large E_T^γ photons. Our calculation shows that the fragmented hadron spectrum from the photon-tagged jets with $E_T^\gamma=40$ GeV is roughly as large as the background at the LHC energy, from which one can barely extract the effective fragmentation function. For $E_T^\gamma<40$ GeV, the fragmentation spectrum is too small to be extracted.

As a general criterion on the minimum value of E_T^γ in our prescribed procedure in central AA collisions, one should require

$$E_T^\gamma > E_{T\min}^\gamma, \quad T_{AA}(0)\frac{d\sigma_{\rm jet}}{dydE_T}(E_{T\min}^\gamma)=1(\mathrm{GeV}^{-1}). \tag{15}$$

Since large p_T hadrons in the background also come from jet fragmentation, this is to ensure that E_T^γ is large enough such that the average number of jets with $E_T=E_T^\gamma$ in each central collisions is less than 1. Only then, the inclusive fragmentation function of the photon-tagged jet can be extracted with confidence after the subtraction of the background. Shown in Table I, are values of $E_{T\min}^\gamma$ for different central $A+A$ collisions at $\sqrt{s}=200$ GeV. This is consistent with what one can expect from Fig. 3.

IV. EFFECTS OF JET E_T SMEARING

From Figs. 3 and 4, one also notices that there are significant number of particles with p_T larger than E_T^γ, from fragmentation of the photon-tagged jets. This is because of the

TABLE I. The minimum transverse energy of the triggered photon, $E_{T\min}^\gamma$, required in order for the fragmentation function of photon-tagged jets to be reliably extracted in central $A+A$ collisions at $\sqrt{s}=200$ GeV.

A	80	120	160	200
$E_{T\min}^\gamma$ (GeV)	5.0	5.5	6.0	6.4

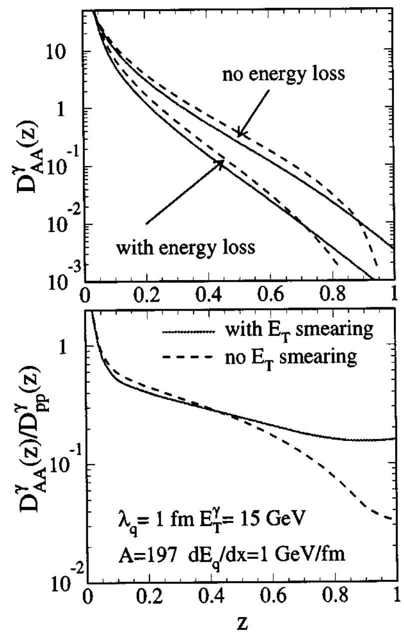

FIG. 5. Upper panel: The inclusive fragmentation functions with (solid lines) and without (dashed lines) E_T smearing for $dE_q/dx=1$ GeV/fm and $dE_q/dx=0$. Lower panel: The ratios of the inclusive fragmentation functions with and without energy loss. The mean free path of a quark is assumed to be $\lambda_q=1$ fm and the triggered photon has $E_T^\gamma=15$ GeV in central Au+Au collisions at $\sqrt{s}=200$ GeV.

E_T smearing of the jet caused by initial-state radiations. To illustrate the effect of the E_T smearing, we plot in Fig. 5 the inclusive fragmentation functions (upper panel) with (solid lines) and without (dashed lines) E_T smearing both for $dE_q/dx=1$ GeV/fm and $dE_q/dx=0$. The lower panel shows the ratios of the inclusive fragmentation functions with and without energy loss. We assume that the mean free path of a quark is $\lambda_q=1$ fm and the triggered photon has $E_T^\gamma=15$ GeV in central Au+Au collisions at $\sqrt{s}=200$ GeV. Notice that we now define z as a hadron's fractional energy of the triggered photon. Because of the E_T smearing of the jet caused by initial-state radiations, hadrons can have p_T larger than E_T^γ. Therefore, the effective inclusive jet fragmentation function does not vanish at $z=p_T/E_T^\gamma>1$. As we can see, the effect of E_T smearing is only significant at large z. In particular at $z\simeq1$, the modified fragmentation function without E_T smearing has contributions only from those partons which escape the system without a single inelastic scattering, thus is controlled only by the mean free path [9]. However, after taking into account the E_T smearing, one also has contributions from the fragmentation of jets with E_T larger than E_T^γ even if the jet has suffered energy loss. Therefore, the modification of the inclusive fragmentation function in this region of z depends on both the mean free path and the parton energy loss. Only at very large $z>2$, the modification factor becomes independent of the energy loss, depending only on the mean free path. However, the production rate becomes also extremely small. For small and intermediate values of z, both the inclusive fragmentation function and the modification due to parton energy loss are not very sensitive to the E_T smearing.

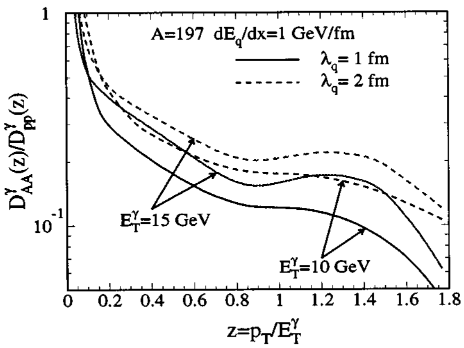

FIG. 6. The modification factor of the photon-tagged inclusive jet fragmentation function in central Au+Au collisions at $\sqrt{s}=200$ GeV for a fixed $dE_q/dx=1$ GeV/fm.

To study the effect of jet E_T smearing in detail, we show in Fig. 6 ratios of the inclusive fragmentation function in central Au+Au collisions with energy loss $dE_q/dx=1$ GeV/fm over the one in pp collisions without energy loss. We shall refer to this ratio as the *modification factor*. The enhancement of soft particle production due to induced emissions is important only at very small fractional energy z. The fragmentation function is suppressed for large and intermediate z due to parton energy loss. For fixed dE/dx and z, the suppression becomes less as E_T^γ or the average $\langle E_T\rangle$ increases. One can notice that there is an interesting structure in the region of $z>0.8$ which is also a consequence of the jet E_T smearing. In this region, contributions from fragmentation of jets with E_T larger than E_T^γ dominates. Since the leading particles are relatively less suppressed for larger E_T with fixed dE/dx and z, the decrease of the modification factor will then saturate at around $z\approx 1$ until the large E_T tail of the E_T smearing for the photon-tagged jet (as shown in Fig. 2) becomes insignificant. Therefore, the structure of the modification factor in the large z region results from the competition between the energy dependence of jet quenching and the falling off of the tail of the jet E_T smearing. The structure becomes more prominent for larger E_T^γ and it should also depend on the energy dependence of the parton energy loss.

Another advantage of studying jet quenching in photon-tagged events is that the results are not sensitive to some of the effects of initial-state multiple interactions, e.g., the nuclear shadowing of parton distributions [7] and energy loss of the beam partons, which can affect the direct photon production rate. However, there is one exception, i.e., the E_T broadening from multiple initial- and final-state scatterings with the beam partons. Such E_T broadening, which also causes the so-called Cronin effect [18] in pA collisions, should also increase the E_T smearing of the photon-tagged jets in both pA and AA collisions. This E_T broadening has also been seen in dijet events of fixed target experiments [19]. Even though the Cronin effect for inclusive cross sections decreases with the colliding energy as indicated by current experiments [20] and theoretical estimates also predict it to be small [10] at the RHIC collider energy and beyond, the small E_T broadening in the photon-tagged events should still have finite effects and one would like to have a handle on it experimentally.

One can directly measure the E_T distributions of the photon-tagged jets in pp and pA collisions and study the possible change, using the conventional calorimetrical study of jets. However, due to small but finite background in pp and pA collisions, it is difficult to measure the calorimetric energy of jets with an accuracy of less than 1 GeV. Here we propose to study the E_T broadening indirectly by measuring the modification factor for the photon-tagged jets in pA collisions, since the inclusive fragmentation function is very sensitive to the E_T smearing as we have demonstrated and the final jet partons in the central rapidity region do not experience transverse energy loss in pA collisions. In particular at $z=p_T/E_T^\gamma\sim 1$, only those jets with $E_T>E_T^\gamma$ contributes. The spectrum in this region is extremely sensitive to the E_T broadening. Therefore, one should be able to measure even very small E_T broadening via the modification factor in pA collisions.

Let us assume that the transverse momentum kick from initial- and final-state scatterings with the beam partons has a Gaussian distribution with a width Δ_{pA}, one can then calculate the effective jet fragmentation function similarly to Eq. (12) but with a modified E_T-smearing function $g_{pA}(E_T,E_T^\gamma)$,

$$D^\gamma_{pA}(z)=\sum_{a,h} r_a(E_T^\gamma)\int dE_T g_{pA}(E_T,E_T^\gamma)\frac{E_T^\gamma}{E_T}D^0_{h/a}\left(z\frac{E_T^\gamma}{E_T}\right). \tag{16}$$

The modified E_T-smearing function can be obtained as the convolution of the E_T-smearing function in pp collisions with a Gaussian distribution,

$$\begin{aligned} g_{pA}(E_T,E_T^\gamma)&=\int dE_T^a\int\frac{d^2p_T}{\pi\Delta_{pA}^2}e^{-p_T^2/\Delta_{pA}^2}g_{pp}(E_T^a,E_T^\gamma)\\ &\quad\times\delta\left(E_T-\sqrt{E_T^{a2}+p_T^2+2p_TE_T^a\cos\phi}\right)\\ &=\int_0^\pi\frac{d\phi}{\pi}\int_0^{E_T^2}\frac{dp_T^2}{\Delta_{pA}^2}g_{pp}(E_T^a,E_T^\gamma)\\ &\quad\times e^{-p_T^2/\Delta_{pA}^2}\frac{E_T}{\sqrt{E_T^2-p_T^2(1-\cos^2\phi)}}, \end{aligned} \tag{17}$$

where $E_T^a=\sqrt{E_T{}^2-p_T^2(1-\cos^2\phi)}-p_T\cos\phi$. For not very large values of Δ_{pA} relative to E_T^γ, the peak of the modified smearing function $g_{pA}(E_T,E_T^\gamma)$ is simply shifted to larger values of E_T as compared to $g_{pp}(E_T,E_T^\gamma)$. One can characterize the E_T shift by

$$\Delta E_T=\int dE_T E_T[g_{pA}(E_T,E_T^\gamma)-g_{pp}(E_T,E_T^\gamma)]. \tag{18}$$

To demonstrate the sensitivity of the effective fragmentation function on the E_T broadening due to multiple parton scatterings, we show in Fig. 7 the modification factor $D^\gamma_{pA}(z)/D^\gamma_{pp}(z)$ in pA collisions for three values of ΔE_T with $E_T^\gamma=10$ and 15 GeV, respectively, at the RHIC energy. It is clear that the modification factor is sensitive to the additional E_T smearing even for very small values of ΔE_T. The shape of the modification factor simply reflects the fact

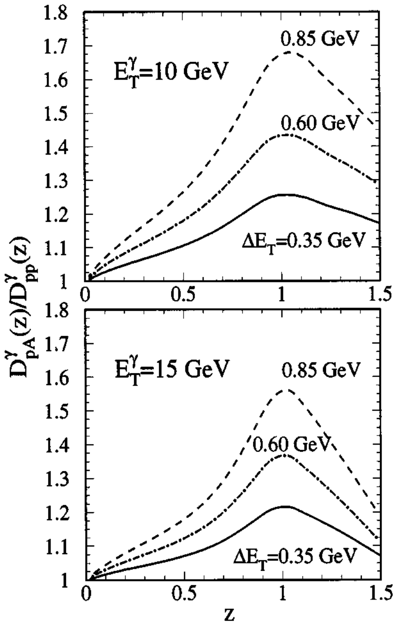

FIG. 7. The modification factor for the photon-tagged jet fragmentation function in pA collisions with $E_T^\gamma = 10$ and 15 GeV at $\sqrt{s} = 200$ GeV, for different values of ΔE_T due to E_T broadening.

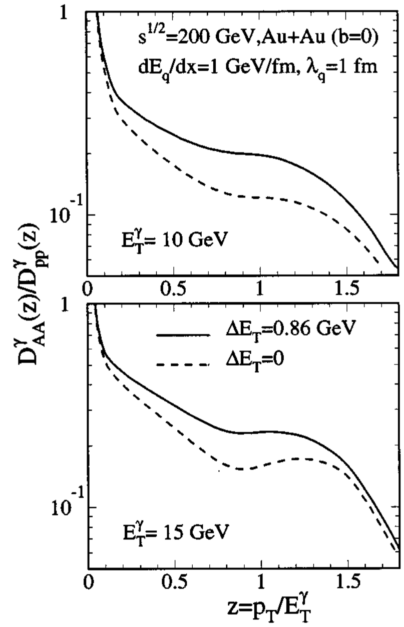

FIG. 8. The modification factor for the inclusive fragmentation function of photon-tagged jets with (solid) and without (dashed) E_T broadening due to initial parton scatterings, in central Au+Au collisions at $\sqrt{s} = 200$ GeV. The parton energy loss is fixed at $dE_q/dx = 1$ GeV/fm and the mean free path $\lambda_q = 1$ fm.

that the smearing function is most modified around the peak $E_T \approx E_T^\gamma$. Comparison between the calculation for $E_T^\gamma = 10$ and 15 GeV for fixed values of ΔE_T shows that the relative effect of multiple scatterings decreases with increasing E_T^γ.

The E_T-smearing function for AA collisions can be modeled the same way as in Eq. (17) for pA collisions, except that Δ_{pA} is replaced by Δ_{AA}. According to the classical random-walk approximation [4,21], Δ^2_{pA} should be proportional to $A^{1/3}$, or the average number of proton-nucleon subcollisions. In such an approximation, $\Delta^2_{AA} = 2\Delta^2_{pA}$. Using this modified E_T-smearing function in Eq. (14), we can calculate the modified effective fragmentation function for the photon-tagged jets in AA collisions, including both the effect of parton energy loss through the dense medium and the additional E_T smearing due to initial- and final-state scatterings with the beam partons. Shown in Fig. 8 are the calculated modification factors with (solid line) and without (dashed line) E_T broadening. It is clear that the E_T broadening has a significant effect on the final modification factor in AA collisions. One therefore has to study pp, pA, and AA collisions systematically and subtract the effect caused by the E_T broadening due to initial multiple scatterings to obtain the modification factor only due to parton energy loss in AA collisions. In the following discussions, we assume that such an effect has already been subtracted out and we only concentrate on the effect of parton energy loss.

V. EXTRACTING PARTON ENERGY LOSS

Given the modification of the inclusive fragmentation function of photon-tagged jets, one, in principle, should be able to extract the parton energy loss and the parton mean free path in our phenomenological model. The optimal case is when the average total energy loss is significant as compared to the initial jet energy, and yet the p_T spectrum from jet fragmentation is still much larger than the underlying background. However, because of the complication of the initial-state radiations, one still cannot determine precisely the energy of the photon-tagged jet in each central AA event. Therefore, the parton energy loss and mean free path cannot be determined independently in a tangible way. As compared to our earlier results [9] where we did not take into account of the E_T smearing of the photon-tagged jets, the modification of the averaged fragmentation function due to energy loss is quite sensitive to the value of the mean free path for $dE_q/dx =$ 1 GeV as shown in Fig. 6.

To study the sensitivity of the modification to the energy loss, we plot in Fig. 9 the modification factor at a fixed value of $z = 0.4$ as functions of dE_q/dx. For small values of dE_q/dx, the suppression factor is more or less independent

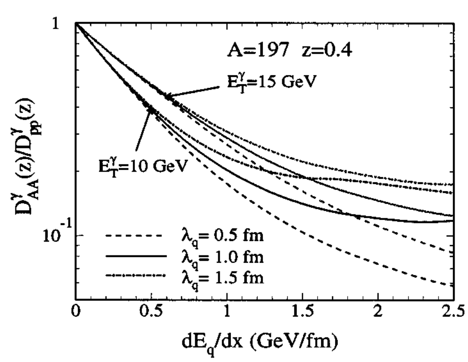

FIG. 9. The modification factors for the inclusive fragmentation function of photon-tagged jets at given $z = p_T/E_T^\gamma = 0.4$ as functions of parton energy loss dE_q/dx with different values of the mean free path λ_q, in central Au+Au collisions at $\sqrt{s} = 200$ GeV.

of the mean free path. This is referred to as the ''soft emission'' scenario in Ref. [9] where the suppression is dominated by the leading parton with an average total energy loss $\langle \Delta E_T^a \rangle = \langle n_a \rangle \epsilon_a = \langle L \rangle dE_a/dx$. The suppression factor should scale with dE_a/dx, depending very weakly on the mean free path. Assuming an exponential form of the fragmentation function $\sim e^{-cz}$ for $z=0.2\sim0.8$, one can show that the suppression factor has a form $(1-\Delta E_T^a/E_T^\gamma)\exp(-cz\Delta E_T^a/E_T^\gamma)$. For large values of $dE_q/dx \geq 1$ GeV/fm, the ratio is sensitive to the mean free path. However, as one can see from Fig. 9, the suppression factor flattens out as dE_q/dx increases, especially for large values of the mean free path λ_a. This can be understood as the ''hard emission'' scenario in which the parton enegy loss per emission $\epsilon_a = \lambda_a dE_a/dx$ is large. In this scenario, z_n after n times emission becomes so small, the $n \neq 0$ contribution is completely suppressed in the modified fragmentation function in Eq. (2). The only contribution is from $n=0$ term, i.e., from the partons which escape the system without any induced radiation. In this case, the suppression factor is controlled by $\exp(-L/\lambda_a)$, a factor independent of dE_a/dx and E_T^γ. One thus needs to measure the suppression factor at smaller values of z or a global fit to determine both the energy loss dE_a/dx and the mean free path from the experimental data.

Recent theoretical studies [4] of parton energy loss in a dense medium of a finite size L indicate that the energy loss per unit distance dE/dx could be proportional to the total distance that the parton has traveled since it is produced,

$$\frac{dE_a}{dx} = \frac{C_a \alpha_s}{8} \frac{\mu^2}{\lambda} L \ln \frac{L}{\lambda} = \frac{C_a \alpha_s}{8} \Delta k_T^2, \tag{19}$$

which was also shown to be proportional to the average transverse momentum broadening squared Δk_T^2, with respect to the direction of the initial parton momentum, where μ is the Debye mass of the medium and λ is the mean free path of the parton, $C_a = 4/3$ for quark and 3 for a gluon. The k_T broadening results from multiple scatterings which also induce the radiative energy loss for the propagating parton.

One way to test this experimentally is to study the modification factor at any given z value for different nucleus-nucleus collisions or for different centralities (impact parameters). Shown in Fig. 10 are the modification factors for the inclusive fragmentation function at $z=0.4$ as functions of $A^{1/3}$. We assume that the radius of the cylindrical system is $R_A = 1.2A^{1/3}$ fm. In one case (dashed lines), we assume a constant energy loss $dE/dx = 0.5$ GeV/fm. The modification factor decreases almost linearly with $A^{1/3}$. In another case (solid lines), we assume $dE_q/dx = 0.2(L/\mathrm{fm})$ GeV/fm. The average transverse distance a parton travels in a cylindrical system with transverse size R_A is $\langle L \rangle = 0.905 R_A$. We choose the coefficient in dE_q/dx such that its average value roughly equals 0.5 GeV/fm for $A=20$. To implement such an energy loss in our model, we assume the energy loss per scattering, for a parton traveling a total distance L, to be $\epsilon_a = \lambda_a 0.2(L/\mathrm{fm})$ GeV. As we can see, the suppression factor for a distance-dependent dE/dx decreases faster than the one with constant dE/dx. Unfortunately, we have not found a unique way to extract the average total energy loss so that one could show that it is proportional to $A^{2/3}$ for the distance-

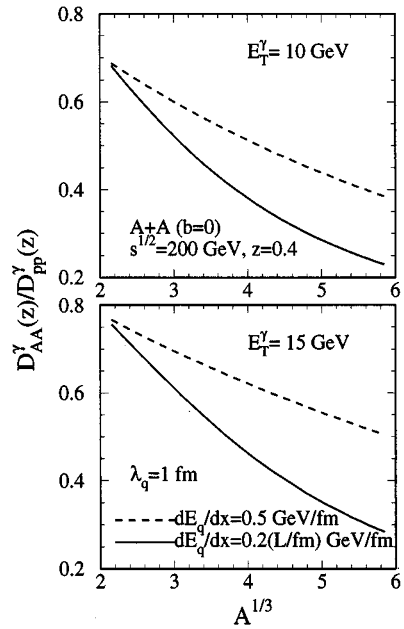

FIG. 10. The modification factors for the inclusive fragmentation function of photon-tagged jets at given $z=p_T/E_T^\gamma=0.4$ as functions of $A^{1/3}$ in central $A+A$ collisions at $\sqrt{s}=200$ GeV. The solid lines are for a distance-dependent parton energy loss dE_q/dx, while the dashed lines are for a constant dE_q/dx.

dependent dE/dx. One possible procedure to determine the A dependence of the energy loss is to first determine dE/dx and λ using a global fit to the measured modification factor for each type of central AA collisions and then find the A dependence of the extracted dE/dx. However, such a procedure and our model depend on the assumption of the size of the dense medium produced in AA collisions.

VI. k_T BROADENING AND JET PROFILE

In our discussions so far, we have assumed that the jet profile in the opposite direction of the tagged photon remains the same in AA as in pp collisions, since we used the same acceptance factor $C(\Delta y, \Delta\phi)$. Such an acceptance factor is determined by the effective jet profile in the opposite direction of the tagged photon. One can imagine that there should be two sources of corrections. One is due to the initial- and final-state multiple parton scatterings with the colliding nucleons. As we have discussed, such multiple scatterings can cause the broadening of the jet E_T smearing. They shall also increase the acoplanarity of the jet with respect to the tagged photon. One can study this effect directly via the effective jet profile in $pA \to \gamma + \mathrm{jet} + X$ processes as in dijet events [19]. Let us assume that such increased acoplanarity can be measured and corrected. The second correction to the effective jet profile comes from multiple scatterings suffered by the leading parton while it propagates inside the dense medium. These multiple scatterings induce radiative energy loss and in the meantime also cause the k_T broadening of the final parton with respect to its original transverse direction, giving rise to an additional acoplanarity. Such a change to the jet profile could affect the acceptance factor, which will be an overall factor to the measured jet fragmentation func-

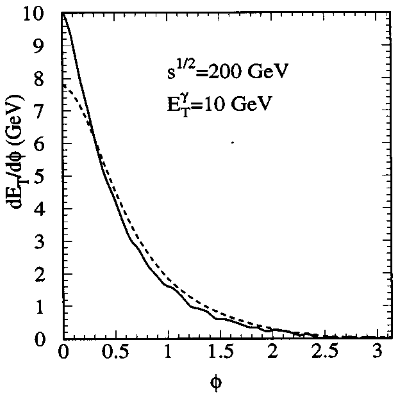

FIG. 11. Jet profile $dE_T/d\phi$ (within $|y|<0.5$) with respect to the opposite direction of the tagged photon. The solid line is the original profile in pp collisions from HIJING simulations, while the dashed line is the modified profile function with $\Delta k_T^2=4$ GeV$^2/c^2$.

tion if we assume the jet profile to be the same for particles with different fractional energies.

Since we only consider jets in the central rapidity region ($y=0$), we assume that the final multiple scatterings will only change the jet profile in the azimuthal direction. We define the jet profile function as $f(\phi)=dE_T/d\phi$. If the initial effective jet profile is $f_0(\phi)$ and the k_T broadening distribution is given by a Gaussian form [4], the final effective jet profile function is then,

$$f(\phi)=\int_0^\infty dk_T^2\frac{1}{\Delta k_T^2}e^{-k_T^2/\Delta k_T^2}f_0(\phi-\phi_{\rm jet}), \tag{20}$$

where $\sin\phi_{\rm jet}=k_T/E_T$ and Δk_T^2 is the average k_T broadening squared.

To demonstrate the effect of the k_T broadening due to final multiple scatterings, we plot in Fig. 11 (solid line) the azimuthal angle distribution of E_T (within $|y|<0.5$) with respect to the opposite direction of the tagged photon with $E_T^\gamma=10$ GeV. We have subtracted the background so that $dE_T/d\phi=0$ at $\phi=\pi$. The profile distribution includes both the intrinsic distribution from jet fragmentation and the effect of initial-state radiations. The acceptance factor is simply the fractional area within the $|\phi|<\Delta\phi/2$ region. The k_T broadening of jets due to multiple scatterings will broaden the profile function. Shown as the dashed line is the profile function from Eq. (20) for $\Delta k_T^2=4$ (GeV/c)^2. It is clear that with a modest value of the k_T broadening, the acceptance factor only changes by a few percent.

Since the change of the effective jet profile function is related to the average k_T broadening, one can combine the measurement with the measured energy loss to verify the relationship between dE/dx and Δk_T^2 as in Eq. (19).

VII. JET QUENCHING IN DEEP INELASTIC LEPTON-NUCLEUS SCATTERINGS

Even though we have so far applied the parton energy loss in Eq. (19) to a fast parton inside a dense matter, the generic form and its derivation is also valid for a parton propagating inside cold nuclear matter or hot hadronic medium. The properties of the medium are manifested in the total transverse momentum kick Δk_T^2. For a hot QGP, Δk_T^2 directly reflects the temperature, while for cold nuclear matter it is related to the gluon density inside a nucleus [4]. If there is a dramatic difference between the transverse momentum broadening or the parton energy loss in QGP and cold nuclear matter, then the measurement of parton energy loss in high-energy heavy-ion collisions can be used as a possible probe of QGP formation. It is thus also important to measure the parton energy loss in cold nuclear matter.

As we have mentioned in the Introduction, initial-state interactions with beam nucleons prior to a hard process can also cause the participating partons to lose energy, thus affecting the final cross section. Among many hard processes, such as the Drell-Yan lepton pair and heavy quarkonium production at large x_F in pA collisions [22], the simplest processes where parton energy loss in cold nuclear matter can be directly measured are probably deeply inelastic lepton-nucleus scatterings. In such processes, one can relate the suppression of the leading hadrons to the attenuation of the quark jet inside the nuclear matter. There are many earlier studies of this problem in the literature [23–26]. In this paper, we would like to revisit this problem within our framework of modified fragmentation functions.

In deeply inelastic ℓA collisions, a quark or antiquark is knocked out of a nucleon by the virtual photon which carries energy ν and virtuality Q. In the rest frame of the nucleus, the photon's energy is transferred to the quark which then will propagate through the rest of the nucleus. If the hadronization time of the quark in the order of $2\nu/\Lambda_{\rm QCD}^2$ is much larger than the nuclear size, most of the leading hadrons from the jet fragmentation are formed outside of the nucleus. We can then attribute the attenuation of the leading hadrons from the quark fragmentation to the energy loss of the propagating quark inside the nuclear matter. Let us assume that the longitudinal position of the nucleon from which the quark is knocked out is $x_\parallel$ in the direction of the virtual photon. Using Eq. (2) for the modified fragmentation functions due to parton energy loss and averaging over the longitudinal and transverse position of the interaction point inside the nucleus, we can obtain the effective quark fragmentation functions in deeply inelastic lepton-nucleus collisions,

$$D_{h/a}^{\ell A}(z,Q^2)=\frac{3}{4}\int_0^{R_A^2}\frac{dx_\perp^2}{R_A^2}\int_{-L_A(x_\perp)}^{L_A(x_\perp)}\frac{dx_\parallel}{R_A}D_{h/a}(z,\Delta L,Q^2), \tag{21}$$

where $L_A(x_\perp)=\sqrt{R_A^2-x_\perp^2}$, $\Delta L=-x_\parallel+L_A(x_\perp)$, and a hard-sphere nuclear distribution is used.

In a parton model, the nuclear structure function is defined as

$$F_2^{\ell A}(x,Q^2)=ZF_2^{\ell p/A}(x,Q^2)+(A-Z)F_2^{\ell n/A}(x,Q^2), \tag{22}$$

$$\frac{1}{x}F_2^{\ell p/A}(x,Q^2)=\frac{4}{9}[2V_{N/A}(x,Q^2)+2\overline{u}_{N/A}(x,Q^2)]+\frac{1}{9}[V_{N/A}(x,Q^2)+2\overline{d}_{N/A}(x,Q^2)+2\overline{s}_{N/A}(x,Q^2)],$$

$$\frac{1}{x}F_2^{\ell n/A}(x,Q^2)=\frac{4}{9}[V_{N/A}(x,Q^2)+2\overline{u}_{N/A}(x,Q^2)]+\frac{1}{9}[2V_{N/A}(x,Q^2)+2\overline{d}_{N/A}(x,Q^2)+2\overline{s}_{N/A}(x,Q^2)],$$

where $V_{N/A}(x,Q^2)$ (normalized to 1) is the effective valence quark distribution, $\overline{q}_{N/A}(x,Q^2)$'s are the effective sea quark distributions per nucleon inside a nucleus, and $x=Q^2/2m_N\nu$. Here we neglect the isospin asymmetry in the sea quark distributions, i.e., $\overline{u}_{N/A}(x,Q^2)=\overline{d}_{N/A}(x,Q^2)$. Because of nuclear effects such as shadowing, the effective parton distributions per nucleon inside a nucleus are different from that inside a nucleon in the vacuum. There are many different mechanisms for the nuclear modification of the parton distributions and they could be different for valence and sea quark distributions [27]. In this paper, we assume that the nuclear modification factors for quark distributions are the same and can be given by the ratio of structure functions,

$$R_{A/D}(x,Q^2)=\frac{V_{N/A}(x,Q^2)}{V_{N/D}(x,Q^2)}=\frac{\overline{q}_{N/A}(x,Q^2)}{\overline{q}_{N/D}(x,Q^2)}=\frac{2F_2^{\ell A}(x,Q^2)}{AF_2^{\ell D}(x,Q^2)}. \tag{23}$$

With parton fragmentation model, one can also define the semi-inclusive structure function associated with production of hadrons with momentum $z\nu$ in the direction of the virtual photon,

$$F_2^{\ell A\to h^\pm}(x,z,Q^2)=ZF_2^{\ell p/A\to h^\pm}(x,z,Q^2)+(A-Z)F_2^{\ell n/A\to h^\pm}(x,z,Q^2),$$

$$\begin{aligned}\frac{1}{x}F_2^{\ell p/A\to h^\pm}(x,z,Q^2)=&\frac{4}{9}[2V_{N/A}(x,Q^2)+2\overline{u}_{N/A}(x,Q^2)][D^{\ell A}_{\pi/V}(z,Q^2)+D^{\ell A}_{K/V}(z,Q^2)]+\frac{1}{9}[V_{N/A}(x,Q^2)+2\overline{d}_{N/A}(x,Q^2)]\\&\times[D^{\ell A}_{\pi/V}(z,Q^2)+D^{\ell A}_{K/S}(z,Q^2)]+\frac{1}{9}2\overline{s}_{N/A}(x,Q^2)[D^{\ell A}_{\pi/S}(z,Q^2)+D^{\ell A}_{K/V}(z,Q^2)],\end{aligned}$$

$$\begin{aligned}\frac{1}{x}F_2^{\ell n/A\to h^\pm}(x,z,Q^2)=&\frac{4}{9}[V_{N/A}(x,Q^2)+2\overline{u}_{N/A}(x,Q^2)][D^{\ell A}_{\pi/V}(z,Q^2)+D^{\ell A}_{K/V}(z,Q^2)]+\frac{1}{9}[2V_{N/A}(x,Q^2)+2\overline{d}_{N/A}(x,Q^2)]\\&\times[D^{\ell A}_{\pi/V}(z,Q^2)+D^{\ell A}_{K/S}(z,Q^2)]+\frac{1}{9}2\overline{s}_{N/A}(x,Q^2)[D^{\ell A}_{\pi/S}(z,Q^2)+D^{\ell A}_{K/V}(z,Q^2)].\end{aligned} \tag{24}$$

In principle, one should also take into account hadron production from the fragmentation of the nuclear remnants. However, one can neglect them for relatively large values of z. In the above equation, we have used the following definitions for the quark fragmentation functions:

$$D_{\pi/V}\equiv D_{\pi/u}=D_{\pi/\overline{u}}=D_{\pi/d}=D_{\pi/\overline{d}},$$

$$D_{\pi/S}\equiv D_{\pi/s}=D_{\pi/\overline{s}},$$

$$D_{K/V}\equiv D_{K/u}=D_{K/\overline{u}}=D_{K/s}=D_{K/\overline{s}},$$

$$D_{K/S}\equiv D_{K/d}=D_{K/\overline{d}}. \tag{25}$$

With the above model assumptions, one can study the quark energy loss by measuring the modification of the jet fragmentation functions via the ratio of the above semi-inclusive structure functions for different nucleus (A and D deuterium) targets,

$$R_{A/D}(z)=\frac{2F_2^{\ell A\to h^\pm}(x,z,Q^2)}{AF_2^{\ell D\to h^\pm}(x,z,Q^2)}. \tag{26}$$

Shown in Fig. 12, is the calculation of the above ratio within our model of the modified fragmentation functions, together with experimental data from E665 [28]. In the experiments, the averaged values of Q^2, ν, and x are different for different values of z. We have taken into account such kinematic effects in our calculation, especially the nuclear modification of the quark distributions. In the experimental measurements, small values of z are correlated to small values of x, where nuclear shadowing of parton distributions is important. This is why the ratio $R_{A/D}(z)$ is smaller than 1 even at small values of z. Within the errors, the data are consistent with our calculation with parton energy loss of $dE/dx=0.5$–1 GeV/fm. It is clear that in order to pin down the quark energy loss inside the nuclear matter, one still needs more accurate measurements in deeply inelastic ℓA collisions. Because of the finite total parton energy loss pos-

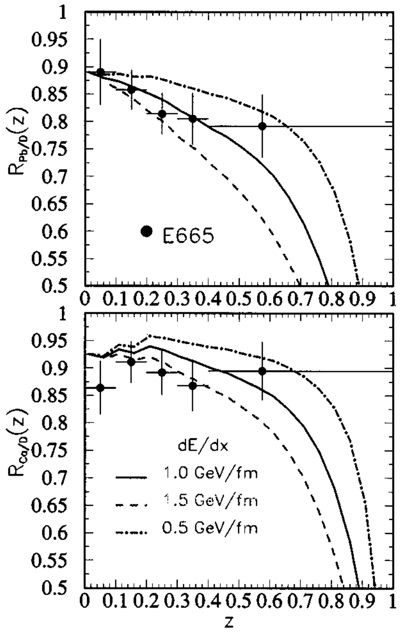

FIG. 12. The suppression factor of charged hadron production in the jet fragmentation region in deeply inelastic ℓA collisions as a function of $z = E_h/\nu$. The lines are calculations using modified fragmentation functions with parton energy loss dE_q/dx. The data are from Ref. [28].

sible inside a finite nucleus, events with small values of ν (energy carried by the struck quark) are more desirable.

VIII. EXPERIMENTAL FEASIBILITIES

To have an estimate of the experimental feasibility of our proposed γ+jet measurement, we list in Table II the number of γ+jet events per year per unit rapidity and unit (GeV) E_T at the RHIC collider energy. We assume a central Au+Au cross section of 125 mb with impact parameters $b<2$ fm. We have taken a luminosity of $\mathcal{L}=2\times10^{26}$ cm^{-2} s^{-1} with 100 operation days per year. The γ+jet cross sections are taken from the compilation by the hard probes collaboration [29]. As we can see, although the number of direct photons with $E_T^\gamma=7$ GeV is large enough, the rate for $E_T^\gamma=15$, 20 GeV is still too small to give any statistically significant measurement of the fragmentation function and its modification in AA collisions. If one can increase the luminosity by a factor of 10, the numbers of events for both $E_T^\gamma=10$ and 15 GeV are significant enough for a reasonable determination of the fragmentation function of the photon-tagged jets.

Given a number of events, one still has to overcome the large background of π^0's to identify the direct photons. Plotted in Figs. 13 and 14, are the production rates of direct photons (solid line) and π^0's (dashed and dot-dashed lines) for central Au+Au collisions at the RHIC and LHC energies. The rate of π^0 production is calculated with the same jet fragmentation functions employed in this paper and convoluted with jet production cross sections [8]. We can see that without jet quenching, π^0 production rate is about 20 times larger than the direct photons at $p_T=10$ GeV/c at $\sqrt{s}=200$ GeV. Fortunately, jet quenching due to parton energy loss can significantly reduce π^0 rate at large p_T as shown by the dot-dashed line. However, one still has to face π^0's about three times higher than the direct photons at $p_T=10$ GeV/c. At larger p_T, the situation improves, but one loses the production rate. Since the isolation cut method normally employed in pp collisions to reduce the background for direct photons does not work any more, the only way one can identify them with high accuracies has to be through the means of improved detectors.

Similarly, we also list in Table III the number of γ+jet

TABLE II. Rate of direct photon production in central Au+Au collisions at $\sqrt{s}=200$ GeV, with luminosity $\mathcal{L}=2\times10^{26}$ cm^{-2} s^{-1} and 100 operation days per year.

E_T^γ (GeV)	7	10	15	20
$(dN^{\gamma-\mathrm{jet}}/dydE_T)$/year	20500	3550	400	70

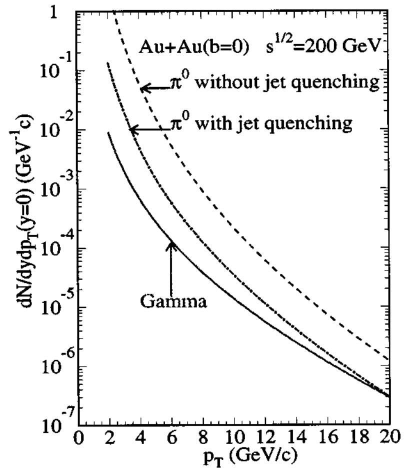

FIG. 13. The spectrum of direct photon production (solid) as compared to π^0 spectrum with (dot-dashed) and without (dashed) parton energy loss ($dE_q/dx=1$ GeV/fm, $\lambda_q=1$ fm) in central Au+Au collisions at $\sqrt{s}=200$ GeV.

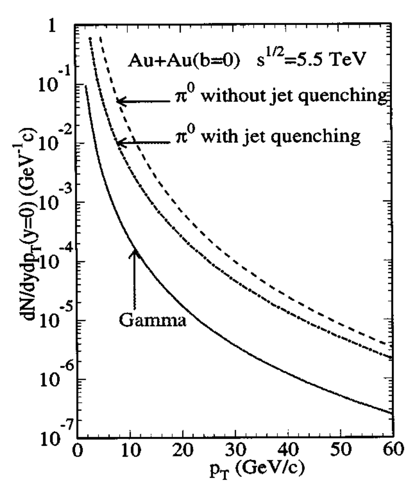

FIG. 14. The same as Fig. 13, except at $\sqrt{s}=5.5$ TeV.

TABLE III. Rate of direct photon production in central Au+Au collisions at $\sqrt{s}=5.5$ TeV, with luminosity $\mathcal{L}=2\times10^{27}$ cm^{-2} s^{-1} and 50 operation days per year.

E_T^γ (GeV)	40	50	60
$(dN^{\gamma-\mathrm{jet}}/dydE_T)$/year	2880	1070	490

TABLE IV. Rate of Z^0 production in central Au+Au collisions at $\sqrt{s}=5.5$ TeV, with luminosity $\mathcal{L}=2\times10^{27}$ cm^{-2} s^{-1} and 50 operation days per year.

$P_T^{Z^0}$ (GeV)	>20	>40	>60
$dN^{Z^0-\mathrm{jet}}/dy$/year	21100	7700	3470

events per year per unit rapidity and unit (GeV) E_T at the LHC energy. We assume a luminosity of $\mathcal{L}=2\times10^{27}$ cm^{-2} s^{-1} with 50 operation days per year for Au+Au collisions. The production rates are reasonably high due to both the high luminosity and collider energy. However, the corresponding background of π^0's is also high (see Fig. 14) which may make the detection of direct photons more difficult.

At the LHC energy, $\sqrt{s}=5.5$ TeV, the production rate for Z^0+jet becomes large even for reasonably large $P_T^{Z^0}$. Listed in Table IV are the number of Z^0+jet events per year per unit rapidity integrated over $P_T^{Z^0}$ with different low cut-off values. The production cross sections are provided by Han based on calculations as described in Ref. [30]. Note that the given Z^0 production rates are integrated ones, thus appearing to be larger than the differential rate of direct photon production at the same transverse momentum. One can detect Z^0 through the dilepton channel which has almost no background in the range of the dilepton invariant mass near M_{Z^0}. One can then apply the same procedure as we have discussed in this paper for direct photon events and measure the modification of the effective jet fragmentation function due to parton energy loss. However, the drawback of using the dilepton channel of Z^0 decay is that the effective number of events via this channel is about 6.7% of the total number of Z^0 events.

IX. CONCLUSIONS AND DISCUSSIONS

In summary, we have studied systematically how one can measure the parton energy loss in $\gamma+$jet events in central high-energy heavy-ion collisions within the framework of a modified fragmentation function model. We have demonstrated that an effective fragmentation function of the photon-tagged jets can be extracted from the inclusive charged hadron spectrum in the opposite direction of the tagged photon. We have estimated the background from large p_T hadron production to be small as compared to hadron production from the jet fragmentation for relatively large values of p_T. We also provided estimates of the lower limits on E_T^γ in central $A+A$ collisions in order for such extraction to be possible. We further show that the effective fragmentation function is sensitive to the parton energy loss possibly experienced by the parton during its propagation through the produced dense matter. The sensitivity is characterized by the so-called modification factor via the comparison of the effective fragmentation function in AA with the one in pp collisions.

We have explicitly taken into account the E_T smearing of the photon-tagged jets for a fixed value of E_T^γ due to initial-state radiations. We also demonstrated that the defined modification factors in pA collisions probe the E_T broadening due to multiple initial- and final-state parton scatterings with the beam nucleons. One should subtract out the effect of such E_T broadening when extracting the parton energy loss from the modification factor in AA collisions. We have also made detailed analysis of the modification factor within our model and studied the sensitivity to different forms of the parton energy loss, e.g., A dependence and the effective jet profile as a function of the azimuthal angle in the transverse plane. We have also applied our model to jet quenching in deeply inelastic lepton-nucleus collisions from which one can extract the parton energy loss inside cold nuclear matter. Finally, we have examined the experimental feasibilities of our proposed study. Our estimates show that in order to have accurate measurements, one needs somewhat increased luminosity about $\mathcal{L}=2\times10^{27}$ cm^{-2} s^{-1} at the RHIC energy. At the LHC energy, one can alternatively use Z^0 as the trigger and study the associated jet fragmentation.

The analysis in this paper has been based on a simplified model of modified fragmentation functions. In particular, we have neglected the fluctuation of the induced radiation according to a given radiation spectrum and we assumed the average energy loss to be independent of the initial parton energy. Since the initial parton energy in the γ+jet events can still fluctuate due to initial state radiations, the final particle spectrum should also be subject to any variation of the energy loss with the initial parton energy. Such sensitivities should be studied in the future. However, one can also address this issue experimentally by changing the energy of the tagged photon.

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[1] D. A. Appel, Phys. Rev. D **33**, 717 (1986); J. P. Blaizot and L. D. McLerran, *ibid.* **34**, 2739 (1986); M. Rammerstorfer and U. Heinz, *ibid.* **41**, 306 (1990); S. Gupta, Phys. Lett. B **347**, 381 (1995).

[2] M. Gyulassy and M. Plümer, Phys. Lett. B **243**, 432 (1990); M. Thoma and M. Gyulassy, Nucl. Phys. **B351**, 491 (1991); M. Plümer, M. Gyulassy, and X. -N. Wang, Nucl. Phys. **A590**, 511c (1995).

[3] M. Gyulassy and X. -N. Wang, Nucl. Phys. **B420**, 583 (1994); X. -N. Wang, M. Gyulassy, and M. Plümer, Phys. Rev. D **51**, 3436 (1995).

[4] R. Baier, Yu. L. Dokshitzer, S. Peigné, and D. Schiff, Phys. Lett. B **345**, 277 (1995); R. Baier, Yu. L. Dokshitzer, A. Mueller, S. Peigné, and D. Schiff, Nucl. Phys. **B478**, 577 (1996); **B483,** 291 (1997); **B484,** 265 (1997).

[5] L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 535 (1953); A. B. Midgal, Phys. Rev. **103**, 1811 (1956).

[6] X.-N. Wang and M. Gyulassy, Proceedings of the Fourth Workshop on Experiments and Detectors for RHIC, 1990, Brookhaven National Laboratory, edited by M. Fatyga and B. Moskowitz, p. 79, BNL-52262.

[7] X. -N. Wang and M. Gyulassy, Phys. Rev. Lett. **68**, 1480 (1992).

[8] X.-N. Wang (unpublished).

[9] X. -N. Wang, Z. Huang, and I. Sarcevic, Phys. Rev. Lett. **77**, 231 (1996).

[10] X.-N. Wang, Phys. Rep. **280,** 287 (1997); X.-N. Wang, in *Quark-Gluon Plasma II*, edited by R. C. Hwa (World Scientific, Singapore, 1995).

[11] P. Mättig, Phys. Rep. **177**, 141 (1989).

[12] J. Binnewies, B. A. Kniehl, and G. Kramer, Z. Phys. C **65**, 471 (1995).

[13] X. -N. Wang and M. Gyulassy, Phys. Rev. D **44**, 3501 (1991); Comput. Phys. Commun. **83**, 307 (1994).

[14] S. D. Ellis, Z. Kunszt, and D. E. Soper, Phys. Rev. Lett. **62**, 726 (1989); Phys. Rev. D **40**, 2188 (1989); Phys. Rev. Lett. **69**, 1496 (1992).

[15] T. Sjöstrand and M. van Zijl, Phys. Rev. D **36**, 2019 (1987); T. Sjöstrand, Comput. Phys. Commun. **39**, 347 (1986); T. Sjöstrand and M. Bengtsson, *ibid.* **43**, 367 (1987).

[16] UA1 Collab., G. Arnison *et al.*, Phys. Lett. B **172**, 461 (1986); C. Albajar *et al.*, Nucl. Phys. **B309**, 405 (1988).

[17] A. D. Martin, W. J. Stirling, and R. G. Roberts, Phys. Lett. B **306**, 145 (1993).

[18] P. Bordalo *et al.*, Phys. Lett. B **193**, 373 (1987).

[19] E609 Collaboration, M. D. Corcoran *et al.*, Phys. Lett. B **259**, 209 (1991).

[20] D. M. Alde *et al.*, Phys. Rev. Lett. **64**, 2479 (1990).

[21] S. Gavin and M. Gyulassy, Phys. Lett. B **214**, 241 (1988); S. Gavin and R. Vogt (unpublished).

[22] S. Gavin and J. Milana, Phys. Rev. Lett. **68**, 1834 (1992).

[23] G. Nilsson, B. Andersson, and G. Gustafson, Phys. Lett. **B83**, 379 (1979).

[24] N. N. Nikolaev, Z. Phys. C **5**, 291 (1980).

[25] A. Bialas and M. Czyzewski, Phys. Lett. B **222**, 132 (1989).

[26] M. Gyulassy and M. Plümer, Nucl. Phys. **B346**, 1 (1990).

[27] K. J. Eskola, Nucl. Phys. **B400**, 240 (1993).

[28] P. Madden, Ph.D. thesis, University of California, San Diego, 1996.

[29] H. Satz and X.-N. Wang, Int. J. Mod. Phys. A **10**, 2881 (1995).

[30] V. Barger, T. Han, J. Ohnemus, and D. Zeppenfeld, Phys. Rev. Lett. **62**, 1971 (1989); Phys. Rev. D **40**, 2888 (1989), and references therein.