Pion nucleus potential from scattering data and a test of charge symmetry

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A modified form of the inverse scattering method of Newton and Sabatier is applied to generate local complex potentials for the scattering of $\pi^{\pm}$ on zero spin and zero isospin nuclei. The phase shifts and inelasticity parameters, used as inputs, are extracted from a partial wave analysis of the elastic differential cross-section data by parametrizing the nuclear amplitude. Analytic mapping techniques are used to get a better accuracy in the phase shift analysis. The nuclear parts of the inversion potentials for π^{+} and π^{-} scattering on ^{4}He are compared as a possible test of charge symmetry breaking. The method has been repeated for several incident pion energies below the (3,3) resonance threshold. [S0556-2813(97)02805-7]

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I. INTRODUCTION

A number of methods have been developed over the last several decades to determine the potential between two interacting particles from a knowledge of their scattering data. Rigorous mathematical methods have been constructed. They have the specific advantage of generating the potential directly from the available experimental information, without having to introduce a particular bias in constructing a model potential. In the usual quantum mechanical problem involving strong interaction, one generally introduces a model optical potential containing several free parameters which are optimized to fit the scattering and the bound state data. Such model potentials may not reveal the true information content of the data, as sometimes several sets of free parameters can equally well reproduce good agreement with the experiment. The information contained in the data may thus be masked by the model itself, obscuring the revelation of the true nature of the interaction. It is for this reason that the inverse scattering formalism has drawn a lot of attention in recent years. A potential constructed by inversion is the best phenomenological potential which one can construct, as it embodies a minimum of model dependence. As such it should serve as a guide for a better physical understanding of the interaction between two particles.

The inverse scattering problem is related to the spectral theory of a Sturm-Liouville eigenvalue problem. In connection with the Schrodinger equation, two alternative procedures have been suggested [1]; these correspond to taking either the energy or the angular momentum as the spectral variable. The first method developed by Gelfand and Levitan with its variant form due to Marchenko [2] yields the potential at a fixed angular momentum. The later method, which we follow in this work, is a fixed energy scattering problem, i.e., finding a potential from a set of partial wave phase shifts at a fixed energy. Though for quite some time the problem has been formally solved by Newton [3], Sabatier [1], and others [4], it is only in the last decade that the method has found successful applications [5,6]. May, Munchow, and Scheid [5], in particular, have discussed a modified version of the Newton-Sabatier formalism and have applied it to the derivation of nucleus-nucleus potentials from the corresponding phase shifts derived with the use of some standard optical potential [7].

Our interest in the present work concerns the application of the fixed energy inversion to the determination of pion-nucleus interactions. The motivation comes from a need for reliable pion-nucleus potentials, since the pion-nuclear physics provides an important application of many-body theory to a fairly simple, strongly interacting system. Since the pion is spinless, many of the complications of spin-dependent effects present in the nucleon-nucleus interaction play no role, provided we are careful to close nuclei also with zero spin [8,9]. Pion-nucleus interactions also provide new possibilities for studying charge symmetry-breaking effects due to the existence of two pions of opposite charges. The pion-nucleus scattering data has traditionally been analyzed using optical potentials, such as the Kisslinger potential [10] or some variants of it [11]. However, with such models, potentials as different as the Kisslinger potential and the Laplacian potential can provide equally good fits to the pionic atom data, as well as the pion-nucleus scattering data.

In an earlier work [12] we discussed a modified form of the Newton-Sabatier formalism for the calculation of potential from the phase shifts at a fixed energy and subsequently this was applied to the inversion of phases of elastic π^{-}-^{4}He scattering [13]. The procedure yields a unique solution for the interaction, if it is assumed to be local and is known from a certain radial distance up to infinity. This is indeed the case with the pion-nucleus interaction where the unknown strong interaction potential is of short range and is superimposed over the background Coulombic interaction of infinite range. A somewhat similar method has been applied to the study of the interaction between light heavy ions by May and Scheid [14]. In this paper we extend the earlier work of Ref. [13] to calculate the potential for π^{+} and π^{-} with ^{4}He and ^{12}C at several incident pion energies below 80 MeV for which elastic cross-section data are available in the literature. A very consistent picture which shows several distinct and systematic trends in the shape of the potentials and their dependence on energy has been obtained. The phase shifts are calculated from the differential cross-section data of Ref. [15]. The inversion potentials are compared with the standard Laplacian optical potentials [11]. The potentials for π^{+} and π^{-} with isoscalar nuclei ^{4}He are compared to get an

TABLE I. Phase shifts and inelasticity parameters for π^+-^{4}He scattering.

E_π in MeV		$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$	$l=7$	$l=8$	$l=9$
51	2δ (deg)	−15.0	18.0	2.0	0.2025	0.0216	0.0024	0.00027	0.00002	0.000004	−0.000006
	η	0.9456	0.9260	0.9930	0.9994	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
60	2δ (deg)	−16.0	22.79	3.199	0.3745	0.0459	0.0058	0.00078	0.00009	0.000015	0.000008
	η	0.9037	0.9523	0.9895	0.9988	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
68	2δ (deg)	−17.2	26.8	4.2	0.5463	0.0745	0.0105	0.00155	0.00022	0.00003	−0.000006
	η	0.9423	0.8788	0.9792	0.9973	0.9996	0.9999	0.9999	0.9999	0.9999	0.9999
75	2δ (deg)	−15.8	31	5.6	0.8060	0.1199	0.0185	0.0029	0.00047	0.000079	0.000002
	η	0.9965	0.7777	0.9792	0.9974	0.9996	0.9999	0.9999	0.9999	0.9999	0.9999

estimate of the charge symmetry in pion nucleus interactions.

In Sec. II we discuss the general procedure of calculating potentials for a set of phase shifts at a fixed energy, where both the phase shifts and the potential may be complex. The modifications needed to take into account the presence of the Coulomb force are also discussed. In Sec. III the method is applied to the elastic scattering data of $\pi^\pm$ on ^{4}He and ^{12}C. The salient features of the resulting potentials are presented. The potentials for π^+ and π^- are compared and the status of charge symmetry in pion-nucleus interactions is discussed.

II. THE INVERSION PROCEDURE

The interaction of charged pions with the nucleus contains the Coulomb interaction term $V_C(r)$ which has an infinite range and a short range nuclear potential $V_N(r)$, the only restriction on the unknown part $V_N(r)$ being that it decreases faster than $r^{-3/2}$ for large r [3] and, as we know, this is not a stringent condition in the case of strong interaction potentials. Let us introduce a dimensionless coordinate

$$\rho = kr = \left[\frac{2\mu E}{\hbar^2}\right]^{1/2} r, \tag{1}$$

where μ is the reduced mass of the pion-nucleus system, and E is the center of mass energy. Assuming a spherically symmetric potential $V(r)$ for the pion, the Schrodinger equation for the ℓth partial wave can be written as

$$\rho^2\left[\frac{d^2}{d\rho^2} + 1 - U(\rho)\right]\Phi_\ell(\rho) = \ell(\ell+1)\Phi_\ell(\rho), \tag{2}$$

where

$$U(\rho) = \frac{V(r)}{E}, \quad V(r) = V_N(r) + V_C(r), \tag{3}$$

and

$$\Phi_\ell(\rho) = r\psi_\ell(r), \text{ where } \psi_\ell(r) \text{ is the wave function for the } \ell\text{th partial wave.} \tag{4}$$

Let $\Phi_\ell^0(\rho)$ be the wave function when the nuclear part of the interaction is switched off, so that $\Phi_\ell^0(\rho) = F_\ell(\rho)$, the regular solution of the Coulomb problem.

Outside the range of the nuclear interaction, i.e., for $\rho > \rho_0$ the nuclear part of the potential may be neglected and then the wave function in this region can be expressed as a linear combination of the regular and irregular Coulomb wave functions $F_\ell(\rho)$ and $G_\ell(\rho)$

$$\begin{aligned}\Phi_\ell(\rho) &= A_\ell[\cos\delta_\ell F_\ell(\rho) + \sin\delta_\ell G_\ell(\rho)], \quad \text{for } \rho > \rho_0 \\ &= A_\ell T_\ell(\rho).\end{aligned} \tag{5}$$

The unknown amplitudes A_ℓ are to be determined as discussed below. The nuclear phase shifts δ_ℓ are extracted from the experimental cross-section data after properly subtracting the Coulomb effects [13]. The phase shifts are, in general, complex.

Let us define a kernel

$$K(\rho,\rho') = \sum_{\ell=0}^{\infty} C_\ell \Phi_\ell(\rho)\Phi_\ell^0(\rho'), \tag{6}$$

TABLE II. Phase shifts and inelasticity parameters for π^+-^{12}C scattering.

E_π in MeV		$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$	$l=7$	$l=8$	$l=9$
30	δ (deg)	−4.1	11.5	1.9	0.1463	0.0109	0.00083	0.00006	−0.000002		
	η	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0		
40	δ	−8.4	12.6	3.7	0.368	0.0358	0.00354	0.00036	0.00003	0.000002	−0.000005
	η	0.94	0.96	0.97	0.9965	0.9996	0.9999	0.9999	0.9999	0.9999	0.9999
50	δ	−0.5	23.9	8.2	0.966	0.1132	0.0135	0.00165	0.00019	0.00002	0.000005
	η	0.79	0.7999	0.9499	0.9907	0.9989	0.9998	0.9999	0.9999	0.9999	0.9999

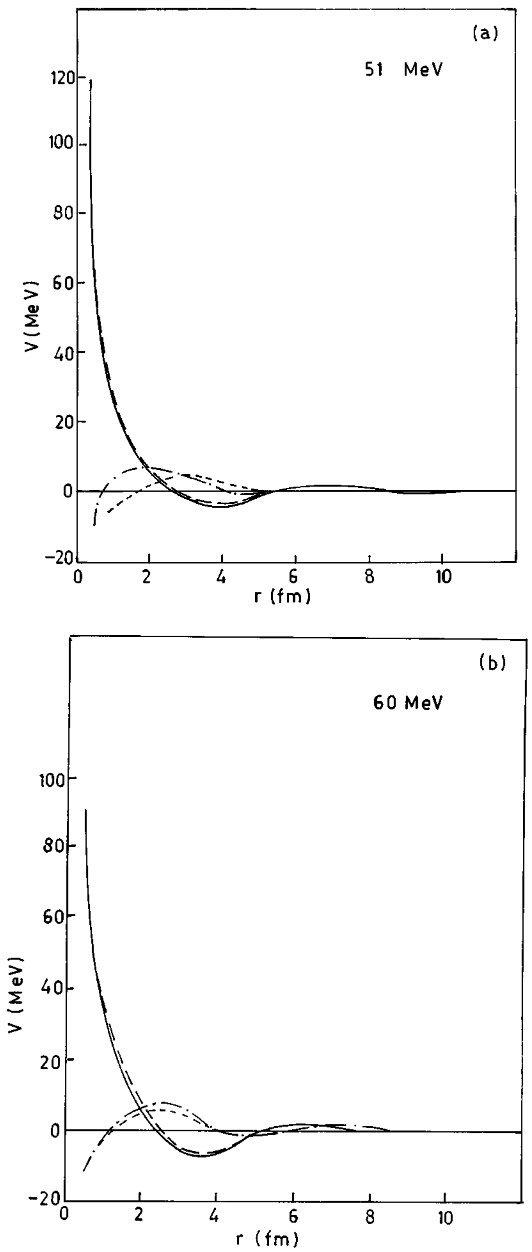

FIG. 1. (a)–(d): Nuclear part of the $\pi^{\pm}$-^{4}He optical potential obtained by inversion. The solid line and the dashed line represent the real part of the inversion potential for π^- and π^+, respectively. The dash-dotted curve and the short-dashed curve are twice the negative of the imaginary part of the inversion potential for π^- and π^+, respectively. Only one of the curves is shown when there is an overlap between the former two or the latter two.

It has been shown by Newton [16] and by Coudray and Coz [17] that, if one defines the potential as

$$U(\rho)=U_C(\rho)-(2/\rho)\,\frac{d}{d\rho}\,[\rho^{-1}K(\rho,\rho)], \tag{7}$$

the kernel $K(\rho,\rho')$ turns out to be the unique solution of the Gelfand-Levitan linear integral equation [18]

$$K(\rho,\rho')=g(\rho,\rho')-\int_0^{\rho}d\rho''(\rho'')^{-2}K(\rho,\rho'')g(\rho'',\rho'), \tag{8}$$

where

$$g(\rho,\rho')=\sum_{\ell=0}^{\infty}C_{\ell}\Phi_{\ell}^{0}(\rho)\phi_{\ell}^{0}(\rho'). \tag{9}$$

The wave function $\Phi_{\ell}(\rho)$ then satisfies the integral equation

$$\Phi_{\ell}(\rho)=\Phi_{\ell}^{0}(\rho)-\int_0^{\rho}d\rho'(\rho')^{-2}K(\rho,\rho')\Phi_{\ell}^{0}(\rho'). \tag{10}$$

Substituting $K(\rho,\rho')$ from Eq. (6) into Eq. (10) above, we get a set of coupled equations

$$\Phi_{\ell}(\rho)=\Phi_{\ell}^{0}(\rho)-\sum_{\ell'=0}^{\infty}C_{\ell'},L_{\ell\ell'}(\rho)\Phi_{\ell'}(\rho), \tag{11}$$

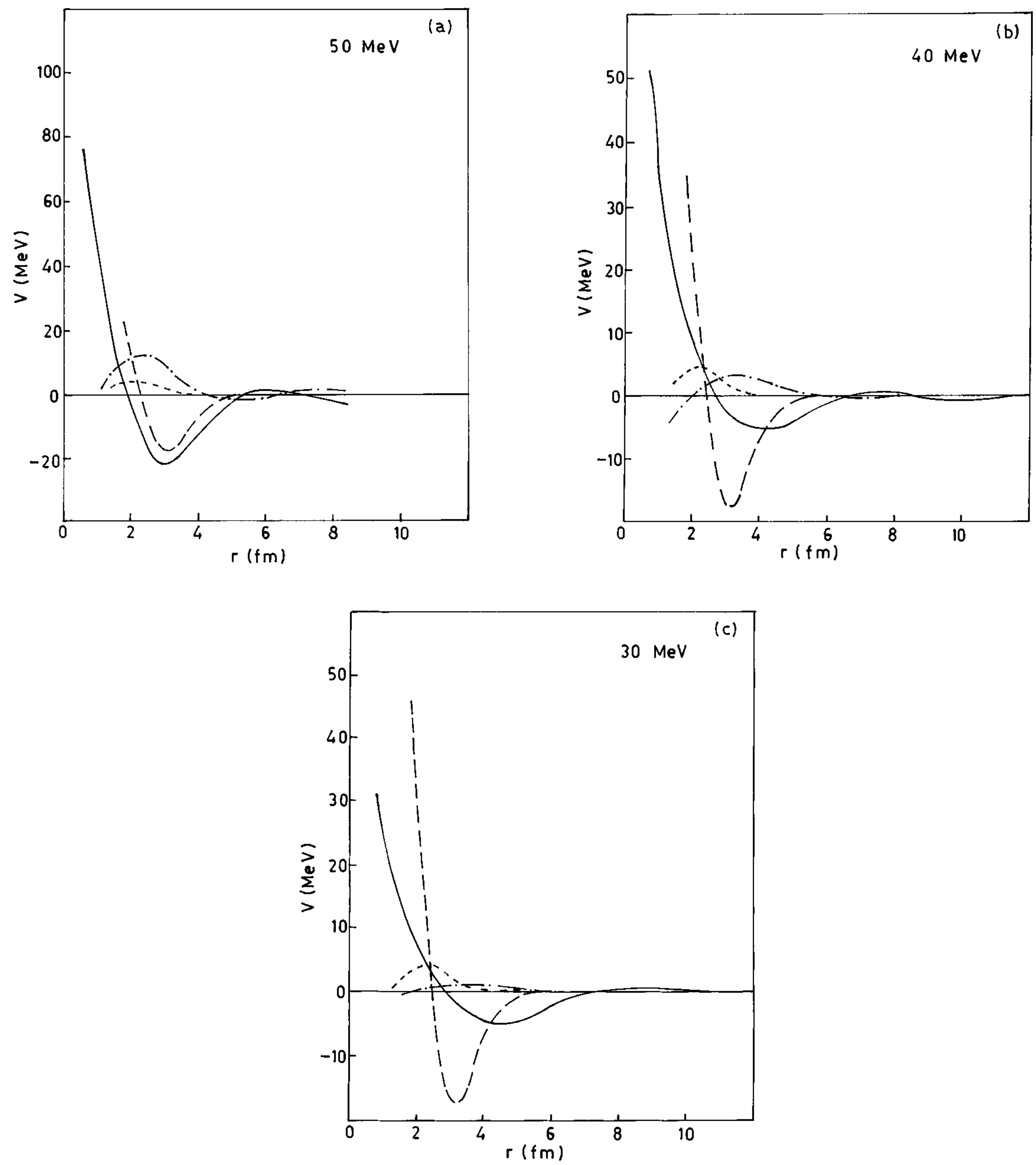

FIG. 2. (a)–(c): π^+-^{12}C optical potential. The solid curve is the real part of the inversion potential. The dashed curve is the real part of the Laplacian potential. The dash-dotted curve is twice the negative of the imaginary part of the inversion potential. The short-dotted curve is twice the negative of the imaginary part of the Laplacian potential.

where the matrix $L_{\ell\ell'}$ is given by

$$L_{\ell\ell'}(\rho)=\int_0^{\rho}\Phi^0_{\ell}(\rho)\Phi^0_{\ell'}(\rho)\rho^{-2}d\rho. \tag{12}$$

Equation (11) can be rewritten as

$$\sum_{\ell'=0}^{\infty}[\delta_{\ell\ell'}T_{\ell'}(\rho)A_{\ell'}+L_{\ell\ell'}(\rho)T_{\ell'}(\rho)b_{\ell'}]=F_{\ell}(\rho), \tag{13}$$

where we have introduced a new set of coefficients $b_{\ell}=C_{\ell}A_{\ell}$. Solving the set of linear equations (13) at two radial distances $\rho=\rho_1,\rho_2$ ($>\rho_0$) provides the unknown coefficients A_{ℓ} and b_{ℓ}, and hence the coefficients C_{ℓ}. From these the kernel $K(\rho,\rho)$ and the nuclear part of the potential are obtained using Eqs. (6) and (7), respectively.

In potential problems the number of significant phase shifts is roughly $L=kr_0$. Normally the phase shifts become negligibly small for partial waves of value somewhat higher than L. In the works of May, Munchow, and Scheid [5] the summation series in Eq. (13) is truncated at the value of $\ell'=L$ for computational reasons, even though theoretically an infinite number of partial wave phase shifts contain all the required information for reproducing the true interaction. In this work, Eq. (13) at more than two values of ρ are considered, which in effect overdetermines the solution. The solutions are then optimized by a standard procedure [19]. However, in that procedure the total number of coefficients C_ℓ calculated from the equivalent of Eq. (13) is only equal to L. As a result, the series for $K(\rho,\rho)$ of Eq. (6) is truncated at the value of $\ell'=L$. Unfortunately, even though the phase shifts for $\ell>L$ may be negligible, the coefficients C_ℓ ($\ell>L$) may not be small. This was earlier demonstrated by Sabatier [20]. Thus the truncation of the series for $K(\rho,\rho)$ at $\ell=L$ will introduce a considerable amount of error in the value for the potential. In an earlier work [12] we presented a prescription for calculating C_ℓ's for $\ell>L$. Including a larger number of coefficients in the series sum for the potential naturally increases the accuracy in the reproduction of the actual potential. A sufficient number of C_ℓ's for $\ell>L$ are included so as to obtain a convergence in the potential with successive C_ℓ's.

III. RESULTS AND DISCUSSIONS

In an earlier work [13] we studied the inversion of the phases of elastic π^--^{4}He scattering below the $\Delta(3,3)$ threshold and discussed the salient features of the resulting potential. In this work we extend the formalism to calculate the potential for $\pi^\pm$-^{4}He and π^+-^{12}C at several incident pion energies. Phase shifts for the first ten partial waves upto the angular momentum state $L=9$ are found from analyses of differential cross-section data using the conformal mapping technique of Cutkowsky and Deo [21]. Partial waves for $\ell>9$ contribute insignificantly to the cross section in the energy region considered in this work. The ambiguities of complex phase shift analysis [22] are taken care of by minimizing the chi-square for a fit to the differential cross-section data. The phase shifts for π^+-^{4}He and π^+-^{12}C are given in Table I and Table II, respectively. Phase shifts for π^--^{4}He have been reported earlier in Ref. [13].

The resulting complex phases are used to calculate the potential with the help of the method outlined in the previous section. Figs. 1(a)–1(d) show the real and imaginary parts of the complex potential for $\pi^\pm$-^{4}He at incident pion energies of 51, 60, 68, and 75 MeV. The potential for π^+-^{12}C at energies of 50, 40, and 30 MeV are shown in Figs. 2(a)–2(c). The inversion potentials are compared with their Laplacian counterparts [11] which are given by a form

$$V(r)=\frac{1}{2E}\left[q(r)-k^2\alpha(r)-\tfrac{1}{2}\nabla^2\alpha(r)\right]. \tag{14}$$

Here k is the center of mass momentum and E the center of mass total energy of the pion. The terms $q(r)$ and $\alpha(r)$ result from the s-wave part and the p-wave part, respectively, of the pion-nucleon interaction. The detailed expressions for these and the values of the parameters occurring in these expressions are given in Ref. [13] and hence are not repeated here. For the evaluation of q and α one needs a knowledge of the nuclear matter density distribution functions for the target nucleus. We assume the same density distributions for both the protons and neutrons. For ^{4}He we have used a simple density distribution function

$$\rho(r)=(2/\pi^{3/2}a^3)\exp(-r^2/a^2), \tag{15}$$

whereas for ^{12}C we have taken

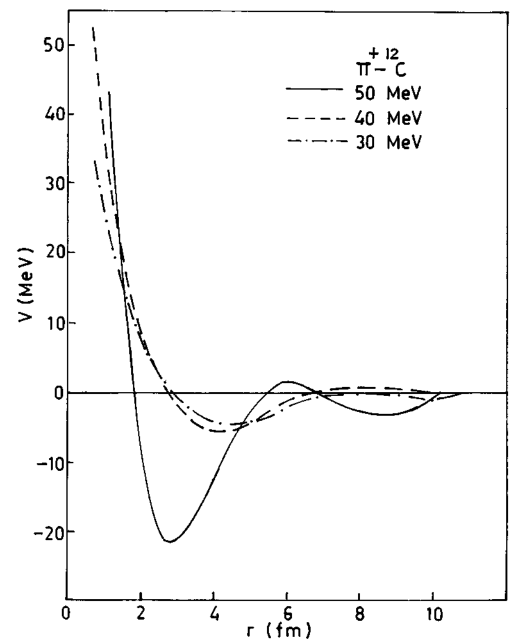

FIG. 3. The real part of inversion potential for π^+-^{12}C at the incident pion energies of 30, 40, and 50 MeV.

$$\rho(r)=(4/\pi^{3/2}a^3)\left[1+\frac{A-4}{6}(r/a)^2\right]\exp[-r^2/a^2]. \tag{16}$$

The constant ''a'' is adjusted to fit the corresponding rms nuclear radius.

In Fig. 2 the phenomenological Laplacian potentials as described above for π^+-^{12}C are shown directly against the calculated inversion potentials for comparison. Such a comparative study for the π^--^{4}He has already been presented in Ref. [13]. As is seen, the inversion formalism yields pion-nucleus potentials which conform to our knowledge about this interaction from earlier phenomenological models. However, the phenomenological potential and the inversion potential differ in their details. A general feature of the pion-nucleus potentials for both ^{4}He and ^{12}C nuclei is that the interaction is attractive at large radii, becoming repulsive for shorter distances. The strength of the attractive part of the real potential, as well as the imaginary potential increase with increasing incident energy of the pion. This conforms to the experimental observation that the inelastic absorption is higher at higher energies. Figure 3 shows the real part of the potential for π^+-^{12}C at three energies 50, 40, and 30 MeV.

Figure 1 shows a direct comparison of the real and imaginary parts of the inversion potentials for π^+ and π^-. Due to the existence of two pions of opposite charges, the pion-nucleus interaction provides unique possibility of studying the charge symmetry breaking in nuclear interactions. The observed difference, if any, between the Coulomb corrected cross sections for π^+ and π^- scattering on nuclei with zero isospin can be interpreted as a manifestation of different masses and widths of $\Delta(3,3)$ isobar states excited in the respective processes. Masterson *et al.* [23] claim to have discovered such a charge symmetry-breaking effect in the elastic scattering of $\pi^\pm$ on d, ^{3}He, and ^{3}H. Their observed estimates of the splittings of $\Delta(3,3)$ states are in suitable agreement with predictions of models [24] which take into account the different quark composition of $\Delta(3,3)$ resonances. Nevertheless the evidence is not yet conclusive: there remains some doubt concerning the conclusion on charge symmetry breaking [25]. These doubts arise from the fact that for any firm conclusion about charge symmetry breaking one must calculate the hadronic amplitude with very high precision. The matter is complicated by the presence of Columb correction, for which doubts still exist regarding the accuracy of the methods in use. Since the model-independent pion-nucleus potentials calculated in this work have taken the Coulomb effect into consideration, they provide us with a suitable tool for the comparison of any violation of charge symmetry. From Figs. 1(a)–1(d) it is noted that there is no significant difference between the hadronic part of the potentials for π^+ and π^-. Even though we have not shown in the graphs the error bars in the potentials due to the uncertainties in the experimental cross section, at the present stage of the experimental accuracy, they are expected to be larger than any slight differences between the two potentials. However, no firm conclusions can be drawn, since the effect of charge symmetry breaking is expected to be very small anyway. Considering the smallness of the effects of the charge symmetry violation, it would have been appropriate to determine accurately the uncertainties in the potentials, so as to be able to specify within what limits charge symmetry is obtained. However, at the present state of the inverse scattering algorithm, there is no specified procedure to estimate the propagation of errors in the phase shifts for different partial waves to the final inversion potentials. Much more work in this direction is needed to get the expected answers from such analyses. It has been shown by Khankhasayev *et al.* [25] that for a mass difference of 6 MeV and a difference in widths of 8 MeV for Δ^{++} and Δ^-, the difference between the hadronic phases for scattering of π^+ and π^- on ^{4}He at 75 MeV amounts to a mere 0.72°. Such a difference in hadronic phases may be detected in phase shift analysis, if the differential cross sections of elastic scattering are measured with 1–2% precision.

In conclusion we can state that the present study gives us the confidence that the inverse scattering theory has reached a stage where it can directly be applied to complex physical systems to provide us with model-independent interactions reliably.

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