## Meson exchange currents in kaon scattering on the lightest nuclei

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(Received 24 September 1996; revised manuscript received 19 November 1996)

The  $K^+$  scattering on the lightest nuclei, d, <sup>3</sup>He, and <sup>4</sup>He is studied in the framework of multiple-scattering theory. Effects from meson exchange currents (MEC) tied to the  $K^+N \rightarrow KN\pi$  reaction are evaluated. We found that at momentum transfers  $Q^2 < 0.5$  (GeV/c)<sup>2</sup> contributions from MEC are much smaller than kaon rescattering corrections. This makes the conventional multiple-scattering picture a reliable tool to study these reactions in this kinematical domain and to extract the  $K^+n$  scattering amplitude from the  $K^+d$  data. At larger transferred momentum MEC can become more relevant. [S0556-2813(97)05206-0]

PACS number(s): 25.10.+s, 24.10.Eq, 25.80 Nv

### I. INTRODUCTION

The  $K^+$ -nucleon interaction is one of the weakest among other strongly interacting systems. Therefore, it is expected that for the description of the kaon-nuclear interaction a simple impulse approximation (IA) should be more reliable. However, recent theoretical [1-7] studies in this framework show systematic discrepancies with the experimental data, in particular, for the ratio of the  $K^+$ -nuclear to  $K^+$ -deuteron total cross sections. This puzzling situation still persists in the description of  $K^+$ -<sup>12</sup>C and  $K^+$ -<sup>40</sup>Ca scattering and most of the theoretical and experimental studies are concentrated on these targets. The reader can find a review of this problem in Ref. [4]. A more recent experimental and theoretical study of  $K^+$  scattering on C (natural) and <sup>6</sup>Li shows agreement between theory and experiment within errors for the elastic cross sections, but discrepancies for the ratio of the total cross sections where systematic errors can be eliminated [8]. On the other hand, the systematic study of the  $K^+$  interaction with the lightest nuclei (deuteron, <sup>3</sup>He, and <sup>4</sup>He) could provide us with new knowledge about the dynamics of the kaonnuclear interaction, which could be applied then to the case of heavier nuclei. Examples of the practical realization of such a scheme were done in the study of pion-nuclear [9-13]and kaon-nuclear [14] scattering within multiple-scattering theory. In the present paper we will extend it by including meson exchange current (MEC) corrections.

From the study of electromagnetic interactions it is well known that at high momentum transfers MEC's play an important role and it is important to find their relevance in  $K^+$  nucleus scattering. In fact, the determination of the  $K^+n$  amplitude from the study of the  $K^+$ -deuteron scattering is tied to the hypothetical strength of the MEC. This point is stressed in Ref. [14] where the uncertainty in the  $K^+$  nucleus cross section from likely effects of the MEC is repeatedly mentioned and kept in mind as a possible source of corrections needed to explain systematic discrepancies of the calculations with the data.

Attempts to calculate MEC effects in  $K^+$ -nuclear scattering were done for nuclear matter [5,6]. Remaining uncertainties in Refs. [5,6] tied to the off-shell extrapolation of the  $K\pi$  amplitude were settled in Ref. [7] using the chiral Lagrangians involving the octet of pseudoscalar mesons and nucleon and  $\Delta$  in the baryon sector. The calculations in Ref. [7] were also done in spin-isospin saturated nuclear matter where some cancellations occur due to the T=0 and J=0 character of the medium. In the present paper we shall evaluate the MEC using nuclear wave functions with detailed spin-isospin structure. As a starting point to construct the MEC operator for kaon-nuclear scattering we shall use the amplitude for the  $K^+N \rightarrow KN\pi$  reaction obtained recently in Ref. [15] in the framework of standard chiral perturbation theory.

Finally let us make one comment about  $K^+d$  scattering. As we mentioned above, this reaction is the main source of information about  $K^+$  interaction with the neutron. From the study of pion-deuteron scattering we know that, for this purpose, the three-body Faddeev approach would be the best theoretical tool which offers the possibility to treat the *NN* interaction and scattering processes within a unified and consistent framework. Fortunately, in the case of  $K^+NN$  system, due to the absence of kaon absorption processes, these channels are uncoupled. This makes the three-body approach closer to the potential multiple-scattering theory. However, this statement would be more accurate if the MEC contributions would be small. In our paper we will show that, indeed, this is the case, providing a justification of the conventional calculations done so far.

The structure of the paper is the following. In Sec. II we consider the formalism for the description of the MEC. Section III presents our results and our conclusions are summarized in Sec. IV.

#### **II. FORMALISM**

The conventional approach in the description of the  $K^+$ -nuclear interaction is the potential multiple-scattering theory. Due to the absence of the absorption channels such an approach is rather precise in the treatment of the  $K^+$ -nucleus dynamics. In the present paper we follow closely the formalism and steps of Ref. [14] where  $K^+$  scattering on He targets was studied using S, P, D, F partial amplitudes for  $K^+N$  scattering from Ref. [16] and including the spin-isospin dependence of the first-order optical potential.

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FIG. 1. A diagram for MEC in kaon-nucleus scattering: (a) pion pole term, (b) contact term.

We calculate this potential in a similar way as it was done for pion scattering on the lightest nuclei [12,13]. Note only that for the deuteron we use realistic wave functions obtained with the Paris *NN* potential [17]. In the description of the <sup>3</sup>He ground state we use the solution of the Faddeev equation with the Reid-soft-core potential [18]. In both cases *S* and *D* components of the nuclear wave function are taken into account. <sup>4</sup>He is described using a charge form factor extracted from electron scattering. The contributions from kaon rescattering are taken into account using the Kerman-McManus-Thaler version of multiple-scattering theory [19]. Within this framework the Lippmann-Schwinger integral equation is solved with a separable form for the off-shell extrapolation of the *KN* amplitude.

The main aim of our paper is the study of contributions from MEC which are not included in the potential multiplescattering theory. Originally it was supposed that the MEC are caused by the  $K^+$  interaction with the nuclear pion cloud, i.e.,  $K^+$  scattering from virtual pions exchanged between two nucleon [see Fig. 1(a)]. The detailed investigations of this mechanism in  $K^+$ -<sup>12</sup>C scattering have been done by Jiang and Koltun [5]. This work was improved in Ref. [6] by the addition of extra terms contributing to the imaginary part of the  $K^+$  self-energy from the  $K^+$  interaction with the pion virtual cloud,  $\delta \Pi_K$ . In addition, the static approximation used to deal with the virtual pion cloud was removed in Ref. [6], resulting in appreciable numerical changes. Uncertainties remained in the real part in Refs. [5,6] tied to the offshell extrapolation of the  $K\pi$  amplitude.

However, it was also mentioned in Ref. [5] that the addition of contact terms from chiral Lagrangians [Fig. 1(b)] should partly cancel the contribution of the real part of  $\delta \Pi_K$ , much as it happens in the  $\pi N \rightarrow \pi \pi N$  reaction [20,21], or in the evaluation of the pion self-energy from the interaction of the pion with the virtual pion cloud [22,23]. A detailed calculation of the real part of  $\delta \Pi_k$  including the pion pole [Fig. 1(a)] and contact term [Fig. 1(b)] was carried out in Ref. [7] using standard chiral Lagrangians for the octet of pseudoscalar mesons. An exact cancellation was found for symmetric nuclear matter, unlike in the case of the pion selfenergy, where the terms cancelled only partially [22,23].

In view of the results obtained in Ref. [7] for the real part of  $\delta \Pi_K$  and the terms involved in the evaluation of the imaginary part of  $\delta \Pi_K$ , one envisages some reduction of the results obtained in Ref. [6], which were already rather small.

However, the cancellation found in Ref. [7] in nuclear



FIG. 2. Diagrammatic representation of the MEC for the four isospin channels in terms of the effective  $NNKK\pi$  vertices  $t_1, \ldots, t_4$ . In diagram (a) the proton line p to the right contributes only in the <sup>3</sup>He case.

matter was for T=0 systems and only because kaons propagate forward in nuclear matter. Away from the forward direction, or if  $T\neq 0$ , this does not occur. It is thus worth investigating in detail whether the effects of these MEC can show up at large angles, as it is usually the case in electromagnetic MEC.

In the present work we shall construct the MEC operator starting from the  $K^+N \rightarrow KN\pi$  amplitude obtained in Ref. [15]. The procedure is similar to the way the MEC operator for photon absorption processes is generated starting from the  $\gamma N \rightarrow N\pi$  amplitude [24], or for pion photoproduction starting from the  $\gamma N \rightarrow N\pi\pi$  amplitude [25]. First, using the results of Ref. [15] we define the effective  $NNKK\pi$  vertex  $t_{\rm eff}$ :

$$t_{\rm eff} = \frac{1}{2} t_{\rm pole} + t_{\rm cont}, \qquad (1)$$

where  $t_{\text{pole}}$  and  $t_{\text{cont}}$  are the pion pole and contact terms. In Fig. 1 they correspond to Fig. 1(a) and 1(b), respectively, without the nucleon line to the right. In a second step we assume that the pion produced is off-shell and we let it be absorbed by a second nucleon. Finally, taking into account all the possible isospin channels we obtain MEC operators in four effective vertices  $(t_1, \ldots, t_4)$ . They are associated with the diagrams depicted in Fig. 2. In the evaluation of the spin-isospin matrix elements from the two-body MEC operator we shall use the method which was suggested in Ref. [25]. It consists of writing explicitly the wave function in terms of spin and isospin up and down states and evaluating the matrix elements of the operators in that base. Then, neglecting the nuclear D states (we expect that due to momentum sharing arguments their contributions are small and in any case they are corrections in corrections) we get for different initial and final nuclear-spin states for the  $K^+d$  scattering

$$A \equiv 2\langle d + |t_{\rm eff}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda}|d+\rangle = -\langle\uparrow|t_{d}|\uparrow\rangle\langle\uparrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle,$$
(2a)  

$$B \equiv 2\langle d0|t_{\rm eff}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda}|d+\rangle$$
$$= -\frac{1}{\sqrt{2}}(\langle\uparrow|t_{d}|\uparrow\rangle\langle\downarrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle+\langle\downarrow|t_{d}|\uparrow\rangle\langle\uparrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle),$$
(2b)  

$$C \equiv 2\langle d - |t_{\rm eff}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda}|d+\rangle = -\langle\downarrow|t_{d}|\uparrow\rangle\langle\downarrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle,$$
(2c)

$$D \equiv 2\langle d + |t_{\text{eff}}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda}|d0\rangle$$
  
$$= -\frac{1}{\sqrt{2}}(\langle\uparrow|t_{d}|\uparrow\rangle\langle\uparrow|\vec{\sigma}\cdot\vec{p}|\downarrow\rangle + \langle\uparrow|t_{d}|\downarrow\rangle\langle\uparrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle), \quad (2d)$$
  
$$E \equiv 2\langle d0|t_{\text{eff}}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda}|d0\rangle$$
  
$$= -A - \frac{1}{2}(\langle\uparrow|t_{d}|\downarrow\rangle\langle\downarrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle + \langle\downarrow|t_{d}|\uparrow\rangle\langle\uparrow|\vec{\sigma}\cdot\vec{p}|\downarrow\rangle),$$

(2e)

for the  $K^{+3}$ He scattering

$$F \equiv 6 \langle {}^{3}\text{He}\uparrow | t_{\text{eff}}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda} | {}^{3}\text{He}\uparrow \rangle$$
  
$$= -\langle \uparrow | 2t_{1} + t_{24}^{(+)} | \uparrow \rangle \langle \uparrow | \vec{\sigma}\cdot\vec{p} | \uparrow \rangle - \langle \uparrow | t_{1} + \sqrt{2}t_{2} | \downarrow \rangle$$
  
$$\times \langle \downarrow | \vec{\sigma}\cdot\vec{p} | \uparrow \rangle - \langle \downarrow | t_{1} + \sqrt{2}t_{4} | \uparrow \rangle \langle \uparrow | \vec{\sigma}\cdot\vec{p} | \downarrow \rangle, \qquad (3a)$$

$$G \equiv 6\langle^{3}\mathrm{He}\downarrow|t_{\mathrm{eff}}(1)\vec{\sigma}_{2}\cdot\vec{p}\,\tau_{2}^{\lambda}|^{3}\mathrm{He}\uparrow\rangle$$
  
=  $-\langle\downarrow|t_{24}^{(-)}|\uparrow\rangle\langle\uparrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle-\langle\downarrow|t_{24}^{(-)}|\downarrow\rangle\langle\downarrow|\vec{\sigma}\cdot\vec{p}|\uparrow\rangle.$  (3b)

In Eqs. (2)–(3)  $|\uparrow\rangle$  and  $|\downarrow\rangle$  are the up- and down-spin states of the single nucleon. The operators  $t_d$  and  $t_{24}^{(\pm)}$  are the following combinations of the effective vertices:

$$t_d = t_1 - t_3 + t_{24}^{(+)}, \quad t_{24}^{(\pm)} = \sqrt{2}(t_2 \pm t_4).$$
 (4)

Some details for the calculation of the matrix elements (2)–(3) are given in the Appendix.

One should note that the diagrams in Figs. 1(a) and 1(b) involve a model for the  $K^+N \rightarrow K^+N\pi$  process based on the pion pole and contact terms. The model of Ref. [15] contains also a term for  $\Delta$  excitation which was proved important in the  $K^+N \rightarrow KN\pi$  reaction in Ref. [26]. This term would lead to the MEC term of Fig. 3. However, in the case of the deuteron this term is zero because the intermediate  $\Delta N$  state cannot have an isospin zero like the deuteron. In the case of <sup>3</sup>He and <sup>4</sup>He, and considering only *S*-wave nuclear states, we also find that the  $\Delta$  excitation term does not contribute to the non-spin-flip amplitude. However, it contributes to the spin-flip amplitude *G* in the case of <sup>3</sup>He, but we find it negligible compared to the MEC related to the chiral terms.



FIG. 3.  $\Delta$  excitation term in the MEC.

The final expression for the MEC amplitude, which has to be added to the first-order kaon-nuclear potential, is given by

$$V_{\text{MEC}}^{fi} = -\frac{E_A(\vec{k})}{4\pi E} \int \frac{d\vec{p}}{(2\pi)^3} \frac{F_A(\vec{Q},\vec{p})}{\vec{p}^2 + m_\pi^2} T_{fi}^{(2)}(\vec{k},\vec{k}',\vec{p})F(\vec{p}) \\ \times F(\vec{Q} - \vec{p})\frac{f_\pi}{m_\pi},$$
(5)

where  $f_{\pi}^2/4\pi = 0.08$  is the  $\pi NN$  coupling constant,  $T_{fi}^{(2)}$  are the spin-isospin matrix elements  $A, \ldots, G$  from Eqs. (2) and (3),  $E_A(\vec{k})$  and  $E = E_K(\vec{k}) + E_A(\vec{k})$  are the kaon and total kaon-nuclear energies in the c.m. frame,  $F(\vec{q})$  is the off-shell form factor:  $F(\vec{q}) = (\Lambda^2 - m_{\pi}^2)/(\Lambda^2 + \vec{q}^2)$  with  $\Lambda = 1300$ MeV,  $\vec{Q} = \vec{k} - \vec{k}'$  is the transferred momentum.

The two-body nuclear form factor  $F_A(\vec{Q}, \vec{p})$  in the case of the deuteron is given by

$$F_{d}(\vec{Q},\vec{p}) = \int u_{S}^{2}(x)j_{0} \left( \left| \vec{p} - \frac{1}{2}\vec{Q} \right| x \right) dx,$$
(6)

where  $u_S(x)$  is the S-wave part of the deuteron wave function taken from Ref. [17]. For <sup>3</sup>He we have

$$F_{^{3}\text{He}}(\vec{Q},\vec{p}) = \int u_{S}^{2}(x,y)j_{0}\left(\left|\vec{p}-\frac{1}{2}\vec{Q}\right|x\right)j_{0}\left(\frac{1}{3}Qy\right)dxdy,$$
(7)

where *x* and *y* are the standard Jacobi coordinates.  $u_S(x,y)$  is the *S*-wave part of the radial wave function for the trinucleon system obtained in Ref. [18] by solving the Faddeev equation. In the case of a simple harmonic oscillator model the two-body form factor can be expressed in the analytical form  $F_{^{3}\text{He}}(\vec{Q},\vec{p}) = e^{-b^2 Q^2/6} e^{-b^2 \vec{p} \cdot (\vec{p} + \vec{Q})/2}$ .

On finishing this section let us make a few comments about the MEC. First, it turns out that in the case of  $K^+d$ scattering in the forward direction the MEC contributions from the contact and pion pole terms cancel each other. This result is consistent with the results of Ref. [7] where this effect was proved for a more general case:  $K^+$  scattering with symmetric nuclear matter. The next comment is connected with the spin-flip transition in kaon scattering on <sup>3</sup>He. The one-body part of the spin-flip amplitude is propor-



FIG. 4. Elastic differential cross sections  $d\sigma/dt$  for  $K^+d$  scattering at lab kaon momenta  $k_{lab}=342$ , 587, and 890 MeV/c. The dotted and dashed curves are the PWIA calculations without and with deuteron *D* state, respectively. The solid curves are the full calculations (with kaon rescattering and Coulomb interaction). Experimental data are from Ref. [27] (o) and Ref. [28] ( $\oplus$ ).

tional to  $\sin\theta_K$ . The same angle dependence was also obtained for the MEC spin-flip amplitude. Note that this amplitude contains only the  $t_2$  and  $t_4$  effective vertices. This mean that there are no contributions from  $\pi^0$ -exchange diagrams.

The last comment concerns the method used here for deriving the nuclear matrix elements of the two-body operator. It is similar to the method used in quark models. On the other hand there is an alternative way based on the algebra of tensor operators. For the MEC we have done calculations with both methods in order to have extra confidence in the results, which were identical in both cases.

### **III. RESULTS AND DISCUSSION**

We begin with the consideration of  $K^+d$  scattering. In the analysis of the experimental data the simple expressions obtained in plane-wave impulse approximation (PWIA), with only a deuteron *S* state, are normally used in order to extract information about the kaon-neutron scattering amplitude. However, this is an approximate expression where the deuteron *D* state and rescattering corrections are neglected. In Fig. 4 we demonstrate the accuracy of such an approximation. In fact, it is good at momentum transfers  $Q^2 = -t < 0.2$  (GeV/c)<sup>2</sup> and it turns out that most of the analyzed experimental points are concentrated in this region. This means that the results which could be obtained using more accurate expressions would lead to small changes in the kaon-neutron amplitude. We come to the same conclusion in the analysis of the total cross section.

Another correction which is not taken into account in the simple PWIA approach is the effect from MEC coming from diagrams depicted in Figs. 1 and 2. We have found (see Fig. 5) that in  $K^+$ -d elastic scattering their contributions are small in comparison with a contribution from the conventional one-body mechanism. For example, due to the cancellation effects found in Ref. [7], there is no MEC contribution in the



FIG. 5. MEC effects in elastic  $K^+d$  scattering at  $k_{\rm lab}=587$  MeV/ c. The dashed and dash-dotted curves are PWIA and PWIA + MEC results. The solid curve is the result of the full calculation. By the dotted curve we denote the MEC contribution alone. Experimental data are from Ref. [27] (o) and Ref. [28] ( $\bullet$ ).

forward direction. At backward angles due to the interference with the one-body (PWIA) amplitudes they reduce the differential cross section in about a factor of 2, but this effect is practically cancelled by contributions from kaon rescattering. The results for the  $K^+$ -<sup>4</sup>He scattering are similar.

In  $K^{+-3}$ He scattering (see Fig. 6), in contrast to the deuteron and <sup>4</sup>He cases, the contribution from MEC in the forward direction is not zero. However, in this region the conventional one-body mechanism dominates. At backward angles the contribution from kaon rescattering is more important. We have also found that in this region it is very important to use realistic Faddeev wave functions instead of the simple harmonic oscillator model. In the case of MEC this essentially enhances the contribution of the non-spin-flip amplitude.



FIG. 6. The same as in Fig. 5, but for elastic  $K^+$ -<sup>3</sup>He scattering calculated using the Faddeev wave function from Ref. [18].

The MEC mechanisms which we are using here correspond to the consideration of the real part of the  $K^+$  selfenergy from the interaction of the kaon with the virtual pion cloud. The corrections from the imaginary part are very small at low energies and were found to provide corrections of 10–20 % at  $K^+$  momenta around 800 MeV/c for <sup>12</sup>C. In the lightest nuclei considered here the effects should be certainly smaller. Also, as indicated above, the consideration of the findings of Ref. [7] would lead to further reductions in the imaginary part of  $\delta \Pi$ , and we should expect the corrections from the imaginary part to be small, as the one found here from the real part, essentially negligible within the accuracy of the present data and the possible one in a near future. On the other hand, the MEC evaluated here would account for the 2p2h excitation diagrams considered in Ref. [7]. In addition one would have contributions from  $ph\Delta h$ components which are of the same order of magnitude as those of the 2p2h (and of opposite sign). In view of the smallness of the contributions obtained from the 2p2h terms we refrain from extending the calculations to account for these corrections which would not change the conclusion drawn here, i.e., the small relevance of the MEC in this problem compared to the one-body and rescattering contributions.

#### **IV. CONCLUSION**

The  $K^+$  scattering on the deuteron is the main source of information about the  $K^+$ -neutron interaction. The basic theoretical approach which is normally used for this purpose is the simple plane-wave impulse approximation which does not take into account contributions from the *D*-state components of the nuclear wave functions, from kaon rescattering and meson exchange currents. In the present paper we have analyzed these ingredients including all of them together. At the same time we have evaluated cross sections for <sup>3</sup>He and <sup>4</sup>He targets using these ingredients.

The MEC operator for the finite nuclei was constructed using as an input the amplitude for the  $K^+N \rightarrow K^+N\pi$  reaction, which was obtained in Ref. [15] in the framework of the standard chiral perturbation theory. We have found that MEC contributions are small at momentum transfers  $Q^2 < 0.5$  (GeV/c)<sup>2</sup>. In this region more important corrections are coming from kaon rescattering, especially in the total cross section at low energies ( $k_{lab} < 500$  MeV). From this result we can conclude that the conventional multiplescattering theory which does not include MEC contributions is a reliable approach for kaon scattering on the lightest nuclei in this kinematical domain.

At larger momentum transfers the MEC can become more relevant and, for instance, for  $K^+$  scattering in the deuteron and <sup>3</sup>He, with kaon momentum around 600 MeV/c and backward angles, they can reduce the cross section in about a factor of 2. However, the cross sections in that region are about four and five orders of magnitude, respectively, smaller than at forward angles. In the case of  $K^+$  scattering on the deuteron we have demonstrated that at  $k_{lab} < 400$ MeV/c and momentum transfers  $Q^2 < 0.2$  (GeV/c)<sup>2</sup>, where most of the analyzed experimental data are concentrated, the corrections from the deuteron D state and kaon rescattering are small. Therefore, the kaon-neutron-scattering amplitude that one would obtain using our more accurate approach would be the same one obtained so far.

## ACKNOWLEDGMENTS

This work is partially supported by the CICYT Contract No. AEN-96-1719. S.S.K. acknowledges financial support from the Ministerio de Educacion y Ciencia during his stay at the University of Valencia and to say many thanks to Juan el Rey whose help allowed us to finish this work.

# APPENDIX

In this appendix we present some details for the calculations of the MEC. First, let us consider expressions for the four effective vertices  $t_1, \ldots, t_4$  which correspond to the diagrams depicted in Fig. 2 without the nucleon line to the right. The vertices  $t_1$  and  $t_2$ , which describe the  $(K^+, K\pi)$ reaction on the proton, were considered in detail in Ref. [15]. The other vertices  $t_3$  and  $t_4$  are new. They describe the  $(K^+, K\pi)$  reaction on the neutron. As we discussed in Sec. II, all these vertices involve the contribution from the pion pole and contact terms. The explicit expressions for them are the following (in the notation of Ref. [15]):

Pion pole terms

$$t_1^p = 2C_1 \frac{\langle p | \gamma^{\mu} \gamma_5 \tau_3 | p \rangle}{q^2 - m_{\pi}} q_{\mu} \{ 2m_K^2 - 2k \cdot k' + p \cdot q + m_{\pi}^2 \},$$
(A1)

$$t_{2}^{p} = 4\sqrt{2}C_{1}\frac{\langle n|\gamma^{\mu}\gamma_{5}\tau_{-}|p\rangle}{q^{2}-m_{\pi}}q_{\mu}\{m_{K}^{2}-p\cdot k-2p\cdot k'-k\cdot k' + m_{\pi}^{2}-p^{2}\},$$
(A2)

$$t_{3}^{p} = 2C_{1} \frac{\langle n | \gamma^{\mu} \gamma_{5} \tau_{3} | n \rangle}{q^{2} - m_{\pi}} q_{\mu} \{ 2m_{K}^{2} - 2k \cdot k' + p \cdot q + m_{\pi}^{2} \},$$
(A3)

$$t_{4}^{p} = 4\sqrt{2}C_{1}\frac{\langle p | \gamma^{\mu}\gamma_{5}\tau_{+} | n \rangle}{q^{2} - m_{\pi}}q_{\mu}\{m_{K}^{2} - q \cdot k - 2q \cdot k' - k \cdot k' + m_{\pi}^{2} - q^{2}\}.$$
(A4)

Contact terms

$$t_{1}^{c} = (C_{1} - C_{2}) \langle p | \gamma^{\mu} \gamma_{5} \tau_{3} | p \rangle \{ k_{\mu}^{\prime} - k_{\mu} - 2p_{\mu} \}, \quad (A5)$$

$$t_{2}^{c} = -\sqrt{2}C_{1}\langle n|\gamma^{\mu}\gamma_{5}\tau_{-}|p\rangle\{p_{\mu}+k_{\mu}'+2k_{\mu}\}, \quad (A6)$$

$$t_{3}^{c} = C_{2} \langle n | \gamma^{\mu} \gamma_{5} \tau_{3} | n \rangle \{ k_{\mu}^{\prime} - k_{\mu} - 2p_{\mu} \}, \qquad (A7)$$

$$t_{4}^{c} = -\sqrt{2}C_{1}\langle p | \gamma^{\mu}\gamma_{5}\tau_{+} | n \rangle \{ p_{\mu} - 2k_{\mu}^{\prime} - k_{\mu} \}, \quad (A8)$$

where  $|p\rangle$  and  $|n\rangle$  are the proton and neutron states, respectively,  $\tau_+$  and  $\tau_-$  are the standard raising and lowering isospin operators ( $\tau_+|n\rangle = |p\rangle$  and  $\tau_-|p\rangle = |n\rangle$ ). In Eqs. (A1)– (A8) the coupling constants  $C_1$  and  $C_2$  are defined as

$$C_1 = \frac{D+F}{2} \frac{1}{12f^3}, \quad C_2 = \frac{D-F}{2} \frac{1}{12f^3}$$
 (A9)

with D = 0.85, F = 0.52, and f = 92.4 MeV. Recall also the convention used for the momentum conservation for the pion pole term:  $\vec{k} = \vec{k}' + \vec{p} + \vec{q}$ .

In order to construct the MEC operator in the nuclear application we use a nonrelativistic expression for the  $\pi NN$  vertex, i.e.,  $\langle N | \gamma^{\mu} \gamma_5 | N \rangle q_{\mu} = -\vec{\sigma} \cdot \vec{q}$  and take into account that pions are off-shell  $(q_0 = p_0 = 0)$ . Finally after some algebra we obtain the following expression for the operator  $t_d$ , defined in Eq. (4), and which describes the MEC in kaon deuteron elastic scattering:

$$t_d = 3C_1 \left[ 2 \frac{m_\pi^2 - \vec{p} \cdot \vec{q} - \vec{Q}^2}{\vec{q}^2 + m_\pi^2} \vec{\sigma} \cdot \vec{q} + \vec{\sigma} \cdot (2\vec{p} + \vec{Q}) \right]. \quad (A10)$$

Using this simple expression the matrix elements  $A, \ldots, E$  from Eqs. (2) can be easily derived. From Eq. (A10) we can also see that in the forward direction, where the momentum transfer is  $\vec{Q} = \vec{k} - \vec{k}' = 0$  and  $\vec{p} = -\vec{q}$ , the pion pole term is canceled by the contact term.

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In the case of kaon scattering on <sup>3</sup>He we have another combination of  $t_1, \ldots, t_4$  vertices [see Eqs. (3)]. They can be derived in a similar way. Here we present only the final result for the non-spin-flip *F* and spin-flip *G* matrix elements from Eq. (3):

$$F = -6C_1 \frac{m_\pi^2 - \vec{p} \cdot \vec{q} - \vec{Q}^2}{\vec{q}^2 + m_\pi^2} \vec{p} \cdot \vec{q} - 2(2C_1 - C_2)(2\vec{p}^2 + \vec{Q} \cdot \vec{p}),$$
(A11)

$$G = 6\sqrt{2}C_1 \left\{ 2\frac{\vec{p} \cdot (\vec{k} + \vec{k}')}{\vec{q}^2 + m_\pi^2} i[\vec{p} \times \vec{q}]_{+1} - i[\vec{p} \times (\vec{k} + \vec{k}')]_{+1} \right\},$$
(A12)

where the product  $i[\vec{A} \times \vec{B}]_{+1} = A_{+1}B_0 - A_0B_{+1}$  is defined in the covariant spherical basis. Note that expressions (A10)–(A11) for the contributions of the pion terms, together with Eq. (5), are symmetrical relative to the permutation  $\vec{p} \leftrightarrow \vec{q}$ .

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