COMMENTS

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Comment on ''Intermittency and correlations in 200 GeV/nucleon S+S and S+Au collisions''

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The W80 Collaboration [Phys. Rev. C **50**, 1048 (1994)] has presented an analysis of normalized factorial moments (NFM) of multiplicity distributions in central ^{32}S+S and ^{32}S+Au collisions at 200 GeV/nucleon. They claim to observe no intermittency signal, as characterized by a variation of the NFM with rapidity interval, beyond that produced by folding the FRITIOF event generator with a detailed model of their detector. In this Comment, it is pointed out that the difference in magnitude between the measured NFM and the detector simulation, which was not considered by WA80, also contains important information which can be used to extract the true NFM. In particular, F_2 is extracted and compared to other experiments. [S0556-2813(97)05205-9]

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The study of non-Poisson fluctuations of charged particle multiplicity distributions in small pseudorapidity intervals $\delta\eta\leqslant 1$ by WA80 [1] and many other experiments has been heavily influenced by the utilization of normalized factorial moments [2,3] which are unity for a Poisson distribution. The normalized factorial moment with the clearest interpretation is

$$F_2=\frac{\langle n(n-1)\rangle}{\langle n\rangle^2}=\frac{\langle n^2\rangle-\langle n\rangle}{\langle n\rangle^2}=\frac{\sigma^2+\langle n\rangle^2-\langle n\rangle}{\langle n\rangle^2}$$
$$=1+\frac{\sigma^2}{\mu^2}-\frac{1}{\mu}, \tag{1}$$

where $\mu\equiv\langle n\rangle$ is the mean and $\sigma\equiv\sqrt{\langle n^2\rangle-\langle n\rangle^2}$ is the standard deviation. A mechanism, dubbed intermittency, was proposed [2], which would be indicated by a power-law increase of multiplicity distribution moments over pseudorapidity bins as the bin size is reduced:

$$F_q(\delta\eta)\propto(\delta\eta)^{-\phi_q}. \tag{2}$$

Many experiments applied the formalism to their data, leading to the observation [3] of the predicted power law behavior in the region $1\geqslant\delta\eta\geqslant 0.1$. However, the observation of tantalizing power laws tended to obscure the fact that multiplicity distributions were well known to be non-Poisson because of short-range rapidity correlations in multiparticle production [4,5].

The q-fold normalized factorial moments for intermittency analyses are nothing other than integrals of the q-particle short-range rapidity correlation functions on the interval $\delta\eta$. For instance, the $q=2$ moment measures the weighted average of the normalized two-particle correlation function:

$$F_2(\delta\eta)-1=K_2(\delta\eta)=\frac{\int^{\delta\eta}dy_1dy_2\rho_1(y_1)\rho_1(y_2)R(y_1,y_2)}{\int^{\delta\eta}dy_1dy_2\rho_1(y_1)\rho_1(y_2)}, \tag{3}$$

where K_2 is a normalized factorial cumulant [6]. It turns out that all the higher order $F_q(\delta\eta)$ moments are dominated by $K_2(\delta\eta)$. The normalized two-particle correlation function, $R(y_1,y_2)$, from two-particle short-range rapidity analyses is typically parametrized as an exponential [5]:

$$R(y_1,y_2)=\frac{C_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)}=\frac{\rho_2(y_1,y_2)}{\rho_1(y_1)\rho_1(y_2)}-1$$
$$=R(0,0)e^{-|y_1-y_2|/\xi}, \tag{4}$$

where $\rho_q(y_1,\ldots,y_q)$ are the q-particle inclusive rapidity densities and ξ is the two-particle short-range rapidity correlation length. The relationship between the intermittency formalism and the two-particle correlation becomes clear when $\rho_1(y)=dn/dy$ is constant on the interval $(0\leqslant y_1,y_2\leqslant\delta\eta)$, in which case

$$K_2(\delta\eta)=F_2(\delta\eta)-1=R(0,0)\left\{2\,\frac{(x-1+e^{-x})}{x^2}\right\}, \tag{5}$$

where the quantity in braces is a function, denoted $G(x)$, of the scaled variable $x\equiv\delta\eta/\xi$. In heavy ion collisions, the correlation strength is very weak, and the correlation length is very short [7], $R(0,0)\sim 1\%$ and $\xi\sim 0.1$, which explains the small values of $K_2=F_2-1$ and the continuous change of $F_2(\delta\eta)$ in the range $1\geqslant\delta\eta\geqslant 0.1$: $F_2(\delta\eta)$ appears to be a

TABLE I. WA80 Data and Monte Carlo results within the acceptance ($\delta\eta=0.45$, $\delta\phi=220°$).

Reaction	$\langle n\rangle$ data (MC)	σ data (MC)	K_2 data (MC)	K_2^T data − MC
Central $^{32}S+S$	13.2 (12.9)	4.37 (3.98)	0.0338 (0.0177)	0.0161
Central $^{32}S+Au$	31.7 (29.7)	7.67 (6.09)	0.0270 (0.0084)	0.0186

power law, but actually follows Eq. (5)—the logarithmic derivative $\phi_2=-R(0,0)xG'(x)$ appears to be a constant $\phi_2\sim-0.2R(0,0)$, because $xG'(x)$ is slowly varying for $0.5\leqslant x\leqslant 10$.

The explanation of the ''intermittency signal'' in heavy ion collisions by a very short range correlation clarifies why experiments have resorted to studying small volumes in multidimensional phase space to enhance the effect and makes the susceptibility to instrumental effects evident—any short range correlation generated by the detector mimics the effect, e.g., electronic cross-talk, Dalitz and external conversions, etc. Fortunately, these, and other such detector effects that generate false intermittency signals can be very well simulated by the Monte Carlo packages FRITIOF and GEANT as done by WA80 [1]. The issue in this Comment is that the measurement of $F_2(\delta\eta)$ can be corrected when the detector background—''the Monte Carlo''—is known.

In terms of the correlation analysis, the measured F_2 can be related to a two-particle correlation function $R(y_1,y_2)$, which is taken to be the sum of a true effect plus an instrumental effect, $R(y_1,y_2)=R^{\mathrm{T}}(y_1,y_2)+R^{\mathrm{I}}(y_1,y_2)$. It then immediately follows from Eq. (3) that the measured $K_2(\delta\eta)=F_2(\delta\eta)-1$ is just the sum of the integrals of the true plus the instrumental terms, $K_2(\delta\eta)=K_2^{\mathrm{T}}(\delta\eta)+K_2^{\mathrm{I}}(\delta\eta)$; and the true effect $K_2^{\mathrm{T}}(\delta\eta)$ is then simply

$$K_2^{\mathrm{T}}(\delta\eta)=K_2(\delta\eta)-K_2^{\mathrm{I}}(\delta\eta), \tag{6}$$

$$F_2^{\mathrm{T}}(\delta\eta)=F_2(\delta\eta)-K_2^{\mathrm{I}}(\delta\eta), \tag{7}$$

where $K_2^{\mathrm{I}}=\Delta F_2^{\mathrm{I}}\equiv F_2^{\mathrm{MC}}-F_2^{\mathrm{input}}$ is the instrumental background given by the Monte Carlo. The tacit assumption that the detector does not otherwise distort the measurement of the moments is true for the WA80 analysis for large values of $\delta\eta$ (see below).

WA80 [1] has presented $F_q(\delta\eta)$ over a range $0.45\geqslant\delta\eta\geqslant 0.056$ for central S+S and S+Au collisions, triggered on E_T in the largest 14th or 20th percentile. (This Comment is addressed only to the central data in their publication, which deviate only slightly from Poisson, so that $K_2\ll 1$). The data are presented on a plot of $\ln F_2$ versus $-\ln\delta\eta$ to search for the power law of Eq. (2). Apparently, the Monte Carlo background for $F_2(\delta\eta)$ has the same ϕ_2 slope as the measurement, but has an amplitude that is significantly lower. The authors choose to display their data by ''scaling'' up the Monte Carlo background calculation to agree with the data at the largest $\delta\eta=0.45$ ''to emphasize the physically important parameter of the data (the slope ϕ_q) while suppressing the modest difference in the magnitude of F_2 between the Monte Carlo and the data.'' They then find no further difference of $\ln F_2(\delta\eta)$ between the data and the Monte Carlo, which indicates that the ''slope'' is the same.

The main point of this Comment is that the magnitude as well as the slope are important parameters since both are sensitive to the correlation length and strength [see Eq. (5)]. Furthermore, due to the use of the FRITIOF event generator which gives Poisson distributions with $F_q^{\mathrm{input}}=1.000$, the ''scaled'' out factor ($\equiv\ln F_2^{\mathrm{data}}-\ln F_2^{\mathrm{MC}}$) is numerically equal to the true result K_2^{T} corrected for instrumental background [8]. The discussion is somewhat complicated by the severe distortion of the WA80 data at small values of $\delta\eta$, which is clearly described [1] as being caused by the finite two-track resolution for tracks with pseudo-rapidity difference $d\eta\leqslant 0.05$. Fortunately, the data point with the largest $\delta\eta=0.45$ is least affected, so we concentrate only on that data point. These data (multiplicity distribution), shown in Fig. 6(b) of the WA80 paper [1], deviate from the MC: on the upper edge, presumably from the correlation effect under discussion; and on the lower edge, a slight deviation sensitive to the trigger. In Table I, the ''data'' and ''(MC)'' entries for $\langle n\rangle$ and σ are taken directly from Table II of the WA80 paper [1], K_2 is calculated using Eq. (1), and K_2^T using Eq. (6). The error on K_2^T is estimated as ± 0.006–0.008 from Fig. 8 of the paper [1]. It is rewarding to note that the values of K_2^T are almost exactly equal to the ''scaling'' factors for both the reactions as quoted by WA80 (on the caption of Fig. 8).

The values of K_2^T represent the averages of the normalized two-particle correlation function $R(y_1,y_2)$ on the $\delta\eta=0.45$ interval, which are $\sim$ 1%, as asserted above. The comparison of these properly corrected results, K_2^T, to other experiments is most easily done by utilizing the NA35 measurements of negative binomial fits to multiplicity distributions in central nucleus-nucleus collisions at the same energy [9]. The negative binomial parameter, $1/k(\delta\eta)=K_2(\delta\eta)$, is a standard measure of the deviation of a distribution from Poisson. For the interval $\delta\eta=0.5$, $\delta\phi=180°$, NA35 quotes the results [9] $K_2=0.0132\pm 0.0027$, for S+S, and $K_2=0.006\pm 0.003$, for S+Au, in rather good agreement with the values (and estimated errors) of K_2^T from Table I. The agreement is equally good with the E802 measurement from central O+Cu collisions at 14.6 GeV/c per nucleon [7], $K_2=0.014\pm 0.002$, for $\delta\eta=0.5$, $\delta\phi=200°$. However, E802 shows, in addition, a clear ''intermittency'' effect in the slope, $\phi_2=0.006\pm 0.002$, completely consistent with the parameters for the correlation analysis they derived from Eq. (5).

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[2] A. Bialas and R. Peschanski, Nucl. Phys. **B273**, 703 (1986); **B308**, 847 (1988).
[3] For an extensive review of this work, see A. Bialas, Nucl. Phys. **A525**, 345c (1991).
[4] A. H. Mueller, Phys. Rev. D **4**, 150 (1971).
[5] Detailed citations are not reasonable in the confines of a Comment. See, for example, M. J. Tannenbaum, Phys. Lett. B **347**, 431 (1995) for more complete references.
[6] The normalized factorial cumulants, K_q, which are zero if there is no direct q particle correlation (Poisson distribution), are just the normalized factorial moments, F_q, with all q-fold combinations of lower order correlations subtracted [4]: $K_2=F_2-1$, $K_3=F_3-(1+3K_2)$, $K_4=F_4-(1+6K_2+3K_2^2+4K_3)$ . . . .
[7] E802 Collaboration, T. Abbott *et al.*, Phys. Lett. B **337**, 254 (1994); Phys. Rev. C **52**, 2663 (1995). The actual results are $\xi=0.18\pm0.05$ and $R(0,0)=0.031\pm0.005$ for central O+Cu collisions.
[8] Since $F_2=1+K_2$ is close to unity because $K_2\sim1\%$, $\ln F_2=F_2-1=K_2$ to an excellent precision. Thus, the ‘‘scaled’’ out factor satisfies the equality, $\ln F_2^{\mathrm{data}}-\ln F_2^{\mathrm{MC}}=(F_2^{\mathrm{data}}-1)-(F_2^{\mathrm{MC}}-1)$, which, for FRITIOF input, is identical to Eq. (6) for the corrected result.
[9] NA35 Collaboration, J. Bächler *et al.*, Z. Phys. C **57**, 541 (1993).