# Generalized seniority scheme in light Sn isotopes

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In search of a possible truncation scheme for shell model calculations, the yrast generalized seniority states are compared with the corresponding shell model states for the case of the Sn isotopes $^{104-112}$Sn. For most of the cases the energies agree within a few hundred keV. For the $0^+(2^+)$ states the overlaps decrease from 97% (93%) in $^{104}$Sn to 91% (78%) in $^{112}$Sn when the coefficients of the pairs in the $S$ and $D$ boson operators are allowed to vary with the number of particles. For constant pairing coefficients throughout the entire isotope range, the overlaps are considerably smaller. It is concluded, with the realistic effective interaction applied here, that a truncation scheme based on seniority zero and two states is inadequate when the number of valence particles gets large and that configurations of higher seniority should be included. [S0556-2813(97)02305-4]

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In recent years the region of light Sn isotopes has been intensively investigated both from experimental and theoretical perspectives. The main goal has been to study the excitation mechanisms around the exotic isotope $^{100}$Sn, the heaviest symmetric doubly magic nucleus recently produced in nuclear fragmentation reactions [1,2].

The simplest approach in analyzing the spectra of light Sn isotopes is to consider $^{100}$Sn as an inert core and to treat only neutron degrees of freedom, using the single-particle orbits of the $N=50-82$ shell as model space, i.e., the orbits $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$. Extensive shell model calculations have been performed along this line [3]. Using a Lanczos iteration method, the states for as many as 12 extra-core neutrons have been calculated. Similar studies have also been done in heavy Sn isotopes [4] and in the $N=82$ isotones [5], where systems with up to 14 valence particles have been studied. On the other hand, a large part of the spectra of light Sn isotopes can be rather well described in terms of selected configurations such as those represented by simple quasiparticle excitations [6]. Therefore, one expects that at least a part of the low-lying states in this region can be approximated by shell-model subspaces with reduced dimensions. One alternative in truncating the shell model space to smaller spaces is offered by the generalized seniority scheme (GSEN) [7]. In the mass region of Sn isotopes, GSEN was applied many years ago [8,9], but for heavier isotopes. Because complete shell model calculations were difficult to perform at that time, the GSEN results were compared with the ones given by the quasiparticle Tamm-Dancoff approximation (QTD) [9]. It was concluded that GSEN and QTD gave similar spectra, with differences which were in general less than 100 keV [9]. Later the admixture of seniority four states into seniority zero and two states was analyzed [10,11]. It was found that for some states the admixture from seniority four states could be as large as 20%.

The aim of the present work is to analyze the accuracy of the GSEN scheme for the case of light Sn isotopes. Here we take advantage of the fact that we can perform complete shell model calculations [3] and thus exactly check the accuracy of the GSEN truncation.

One could have a first indication about the validity of the GSEN scheme by analyzing the experimental binding energies (BE) as a function of the number of neutron pairs $n$. In GSEN this dependence is given by [7]

$$\mathrm{BE}(n)=\mathrm{BE}(^{100}\mathrm{Sn})+nV_0+\frac{n(n-1)}{2\delta}. \tag{1}$$

If we fix the parameters $V_0$ and $\delta$ from $^{106}$Sn and $^{108}$Sn, the binding energies for $A=104$ and 110 would be predicted within 90 and 440 keV, respectively. Considering the large uncertainties for the extrapolated BE of $^{100}$Sn [12], one should take these estimates as orientative only. Nevertheless, they may indicate that the generalized seniority zero state

$$(S^+)^{n/2}|0\rangle, \quad S^+=\sum_j C_j(a_j^+ a_j^+)_{J=0}, \tag{2}$$

could provide a reasonable approximation of the exact shell-model ground state. In Eq. (2) $a_j^+$ denotes the particle creation operator. Two versions of GSEN have been analyzed. In version I the amplitudes $C_j$ which give the distribution of the pairs on the various single-particle orbits, are fixed such that the seniority zero state in Eq. (2) reproduces the two-particle shell model state for the ground state of $^{102}$Sn. These values are then used throughout the isotopes from $^{104}$Sn to $^{112}$Sn. Such an approach with the constant pair structure is within the philosophy of the original generalized seniority scheme GSEN [7]. As a simple extension called version II the amplitudes $C_j$, are determined by minimizing the expectation value of the Hamiltonian in the state $(S^+)^{n/2}|0\rangle$, see Eq. (2), for each system separately. This allows the pair structure to change as a function of the number of particles.

The validity of seniority schemes have usually been analyzed with Hamiltonians defined through effective interactions fitted to experimental data. In such cases conclusions

 

TABLE I. Yrast low-lying states for $^{104-112}$Sn. Energies are given in Mev. For the two versions see the text after Eq. (2).

| | $^{104}$Sn | | | $^{106}$Sn | | | $^{108}$Sn | | | $^{110}$Sn | | | $^{112}$Sn | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $J^\pi$ | SM | I | II | SM | I | II | SM | I | II | SM | I | II | SM | I | II |
| $2_1^+$ | 1.45 | 1.51 | 1.53 | 1.42 | 1.56 | 1.54 | 1.57 | 1.80 | 1.64 | 1.63 | 2.17 | 1.71 | 1.65 | 1.65 | 1.72 |
| $4_1^+$ | 1.98 | 2.03 | 2.05 | 2.13 | 2.21 | 2.25 | 2.34 | 2.57 | 2.42 | 2.43 | 3.06 | 2.64 | 2.79 | 2.46 | 2.77 |
| $6_1^+$ | 2.22 | 2.23 | 2.22 | 2.36 | 2.47 | 2.43 | 2.45 | 2.74 | 2.67 | 2.73 | 3.09 | 2.98 | 2.96 | 2.50 | 3.29 |

about the validity of the truncation is affected by the fact that the interaction is renormalized as to include the effects of the truncation, which is just what we want to estimate. Thus in the present calculations we use a microscopically derived effective interaction to describe the Hamiltonian, using the perturbative many-body techniques described in Ref. [13]. In brief, the derivation of the effective interaction is a three-step process. First, one needs a free $NN$ interaction $V$ which is appropriate for nuclear physics at low and intermediate energies. At present, a meson-exchange picture for the potential model seems to offer a viable approach. Among such meson-exchange models one of the most successful ones is the one-boson-exchange model of the Bonn group [14]. As a starting point for our perturbative analysis we use the parameters of the Bonn $B$ potential defined in Table A.1 of Ref. [14]. However, in nuclear many-body calculations the first problem one is confronted with is the fact that the strong repulsive core of the $NN$ potential $V$ is unsuitable for perturbative approaches. This problem is overcome by the next step in our many-body scheme, namely by introducing the reaction matrix $G$. Here we calculate the $G$ matrix using the so-called double-partitioning scheme [13]. The single-particle wave functions were chosen to be harmonic oscillator eigenstates with the oscillator energy $\hbar\Omega = 45A^{-1/3} - 25A^{-2/3} = 8.5$ MeV, for $A = 100$. The last step consists in defining a two-body interaction in terms of the $G$ matrix including all diagrams to third order in perturbation theory and summing so-called folded diagrams to infinite order, see Ref. [13]. The single-particle energies for the orbits $1d_{5/2}$, $0g_{7/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ were fixed as to reproduce the experimental low-lying states of $^{111}$Sn [6].

Another property of the Sn isotopes used to justify the GSEN approximation is the well-known experimental feature of the near constant spacing between the ground state and the first excited $2^+$ states. This indicates that the $2^+$ states may be described as one broken pair upon a ground-state condensate of $0^+$ pairs. Actually one expects a whole group of low-lying excitations to be expressed as generalized seniority two states [7]

$$|J\rangle = D_J^+ (S^+)^{n-1}|0\rangle, \tag{3}$$

where

$$D^+(J) = \sum_{j_1 j_2} X(j_1, j_2; J)(a_{j_1}^+ a_{j_2}^+)_J. \tag{4}$$

In order to investigate these features again two versions of the GSEN are calculated. In version I the amplitudes $X(j_1, j_2; J)$ in the two-particle operators $D^+$ are adjusted to reproduce the corresponding two-particle shell model state in $^{102}$Sn whereas version II is found by diagonalizing the given interaction in the space of all possible seniority two basis states, again for each system separately. In this way one allows the dynamics to build up the intrinsic structure of the $D_J^+$ operators as more pairs are added.

It is worthwhile to stress that the validity of a truncation scheme depends on the effective interaction employed to describe the system. For instance, the validity of Eq. (1) depends on how well the given interaction satisfies the relation [7]

$$\{[H, S^+], S^+\} = \text{const.}(S^+)^2. \tag{5}$$

The results for the excitation energies of the yrast states are shown in Table I. One notices a rather good agreement between the shell-model calculation and the two versions of GSEN for many of the isotopes. Up to the eight-particle case the agreement is reasonably good in both versions, especially in view of the simple model used for the pair states compared to the very large shell basis. As an example, in $^{110}$Sn the number of standard model (SM) basis states for the $2^+$ states is 86 990, which should be compared with 9 in the GSEN calculation. Above eight particles the deviations start to become significant, particularly in version I with fixed pair structures. The version II includes some higher order pair effects by dynamical changes in the $C_j$ and $X(j_1, j_2 : J)$ coefficients, but still deviations are up to 0.5 MeV in the worst cases. However, in conclusion such model calculations as GSEN version I and II show reasonable agreement with the shell model ''experimental data.''

The next piece of information of interest is the properties of the wave functions. These are analyzed through the overlap squared of the generalized seniority states with the exact shell model eigenstate defined by

$$|\langle \text{SM}(n, J=0)|(S^+)^{(n)}|0\rangle|^2$$

and

$$|\langle \text{SM}(n, J)|D_J^+ (S^+)^{(n-1)}|0\rangle|^2. \tag{6}$$

The results are presented in Table II. Compared to the reasonably good agreement between SM and the GSEN versions I and II found for the energies the wave functions show clear deviations. For $^{104}$Sn the differences are between 5 and 10 %, whereas in $^{112}$Sn the differences have increased to $\approx$85% in some cases in GSEN version I. Even the first excited $2^+$ state deviates by $\approx$60% in spite of the fact that this state is well separated from neighboring nonyrast states which could produce mixing. So a fixed pair structure description is not meaningful for the heavy Sn isotopes. Although clear improvement is found in version II, in which

TABLE II. The overlaps square of the generalized seniority wave functions with the corresponding shell model states for different angular momenta. For the two versions see the text after Eq. (2).

| | $^{104}$Sn | | $^{106}$Sn | | $^{108}$Sn | | $^{110}$Sn | | $^{112}$Sn | |
|---|---|---|---|---|---|---|---|---|---|---|
| $J_i^\pi$ | I | II | I | II | I | II | I | II | I | II |
| $0_1^+$ | 0.950 | 0.966 | 0.876 | 0.938 | 0.796 | 0.924 | 0.742 | 0.905 | 0.767 | 0.909 |
| $2_1^+$ | 0.931 | 0.927 | 0.787 | 0.815 | 0.663 | 0.780 | 0.438 | 0.790 | 0.420 | 0.776 |
| $4_1^+$ | 0.906 | 0.906 | 0.798 | 0.821 | 0.482 | 0.743 | 0.236 | 0.764 | 0.173 | 0.680 |
| $6_1^+$ | 0.918 | 0.943 | 0.817 | 0.895 | 0.660 | 0.794 | 0.401 | 0.739 | 0.167 | 0.695 |

the pair structure is allowed to vary with the particle number, the overlaps do show that for many valence nucleons the SM wave functions contain important admixtures beyond the seniority zero and two components of the GSEN scheme. This indicates that a truncation scheme based on seniority zero and two states is inadequate when the number of valence particles gets large. One may expect that the most important additional contributions come from seniority four states, as in the case of heavier Sn isotopes [10,11]. As already pointed out in Refs. [10,11] these admixtures could be rather important when observables such as transition probabilities are calculated.

A complete shell model calculation for the whole chain of isotopes from $^{102}$Sn to $^{130}$S is difficult. The GSEN model has been thought of as a promising approximation. However, the present calculation shows that a model space with pairs of seniority zero and two is too small. Configurations with seniority four and probably six will be necessary for a reasonable description. Such work is in progress.

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