Sudakov form factor in a massive vector field theory

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The leading-logarithm approximation for the Sudakov form factor is examined in a theory containing massive fermion and massive neutral vector meson fields. In the on-shell case, where there is only one mass scale (the meson mass), the Sudakov form factor in this model agrees with the result in QED. In the off-shell case, however, with two different mass scales (the meson mass and the off-shell mass of the fermion), the Sudakov form factor differs from the QED result. [S0556-2813(97)01805-0]

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At large spacelike momentum transfer, the leading contributions to the form factor of a fermion coupled to a vector boson exponentiate, yielding the so-called Sudakov form factor. The application of the factorization process that causes the Sudakov form factor in QED and QCD produces the well-known electron and quark electromagnetic form factors. In the derivations of these form factors, an infinitesimal boson mass is introduced, when needed, to avoid infrared divergences. Here we examine the calculation of the Sudakov form factor in a theory containing a massive fermion and a massive neutral vector field.

Quantum theories with massive vector fields are important in the development of quantum hadrodynamics (QHD) [1–3]. Proposed as a way to describe nucleons and the nuclear force at intermediate energies, QHD has been successful in predicting many nuclear properties at the mean-field level. However, some calculations beyond this level have failed, because of the large size of vacuum-polarization loops. It was recognized [4,5] that the size of the vacuum contributions could be reduced by including vertex corrections at the bare nucleon-meson vertices. Several calculations involving vector and scalar meson fields have been performed testing this proposition [6–12], and the results have been encouraging; however, despite these results, the Sudakov form factor in a *massive* vector theory has not been adequately discussed.

In his original work in QED, Sudakov [13] showed that, in the leading-logarithm approximation, the sum of contributions to the electromagnetic form factor at large momentum transfer q exponentiates, and the resulting exponential is highly damped. To avoid infrared divergences that arise from a zero photon mass, Sudakov gave the electrons off-shell momenta p_a and p_b, and found that for $-q^2 \gg |p_a^2|,|p_b^2| \gg m_e^2$, the proper vertex function becomes

$$\overline{u(p_b)}\Lambda^\mu u(p_a) \to \overline{u(p_b)}\gamma^\mu u(p_a) \times \exp\left[-\frac{e^2}{8\pi^2}\ln(-q^2/|p_a^2|) \times \ln(-q^2/|p_b^2|)\right]. \quad (1)$$

Jackiw [14] and Fishbane and Sullivan [15] extended Sudakov's work. They showed that the leading-logarithmic contribution to the electromagnetic form factor of $O(\alpha^{2n})$ in perturbation theory factorizes as $\Gamma^{(n)}=(\Gamma^{(1)})^n/n!$, where $\Gamma^{(1)}$ indicates the lowest-order correction to the bare vertex. In their work, Fishbane and Sullivan gave the photon a small regulator mass ν, and their proof was *independent* of the size of ν. By taking $\nu\approx 0$, they reproduced Sudakov's result for the off-shell vertex, and for the on-shell vertex they found that the vertex function behaves as

$$\overline{u(p_b)}\Lambda^\mu u(p_a) \to \overline{u(p_b)}\gamma^\mu u(p_a) \times \exp\left[-\frac{e^2}{16\pi^2}\ln^2(-q^2/\nu^2)\right], \quad (2)$$

with the regulator mass ν being associated (ultimately) with the energy resolution of the detector.

In later work, Collins [16,17] and Sen [18] studied the behavior of the Sudakov form factor in a broader class of quantum field theories. Their combined efforts showed that the sum of nonleading logarithmic contributions to the proper vertex function, in both Abelian and non-Abelian gauge field theories, either cancel or exponentiate, thus leaving the leading-logarithm results intact.

We consider a theory containing a fermion of mass M and a neutral vector meson of mass m_v interacting through a Yukawa coupling of strength g_v. The off-shell vertex function can be written as [15]

$$\begin{aligned}\Lambda^\mu(p_a,p_b,q) &= \Lambda_+(p_b)(\gamma^\mu\mathcal{F}_1+i\sigma^{\mu\nu}q_\nu\mathcal{F}_2+q^\mu\mathcal{F}_3)\Lambda_+(p_a)\\ &+\Lambda_-(p_b)(\gamma^\mu\mathcal{F}_4+i\sigma^{\mu\nu}q_\nu\mathcal{F}_5+q^\mu\mathcal{F}_6)\Lambda_-(p_a)\\ &+\Lambda_+(p_b)(\gamma^\mu\mathcal{F}_7+i\sigma^{\mu\nu}q_\nu\mathcal{F}_8+q^\mu\mathcal{F}_9)\Lambda_-(p_a)\\ &+\Lambda_-(p_b)(\gamma^\mu\mathcal{F}_{10}+i\sigma^{\mu\nu}q_\nu\mathcal{F}_{11}+q^\mu\mathcal{F}_{12})\Lambda_+(p_a),\end{aligned} \quad (3)$$

where $\mathcal{F}_i\equiv\mathcal{F}_i(q^2,p_a^2,p_b^2)$ are the off-shell form factors, and $\Lambda_\pm(p)=\frac{1}{2}(1\pm \not{p}/M)$ are projection operators.

When the external baryon momenta are on shell ($\not{p}_a=\not{p}_b=M$), the vertex function depends only on the momentum transfer $q^\mu\equiv p_b^\mu-p_a^\mu$, and the only surviving functions are $\mathcal{F}_1 \equiv F_1(q^2)$ and $\mathcal{F}_2 \equiv F_2(q^2)$. The general form for the vertex function then reduces to the well-known on-shell expression [19]

$$\Lambda^\mu(p_a,p_b,q) \Rightarrow \Lambda^\mu(q) = \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} q_\nu F_2(q^2). \tag{4}$$

In general, the anomalous ($i=2,5,8,11$) and longitudinal ($i=3,6,9,12$) form factors in Eq. (3) are suppressed with respect to the charge form factors by a power of q^2; hence, at large $|q|$, the vertex is dominated by the charge form factors. In particular, at nth order in g_v^2,

$$\lim_{|q^2|\to\infty} [\Lambda^\mu(p_a,p_b,q)]^n \approx \gamma^\mu \mathcal{F}_1^{(n)}(q^2,p_a^2,p_b^2). \tag{5}$$

Since $\mathcal{F}_1^{(n)}(q^2,p_a^2,p_b^2) \to [\mathcal{F}_1^{(1)}(q^2,p_a^2,p_b^2)]^n/n!$ in the leading-logarithm approximation (for any value of m_v), the vertex function can be written as

$$\lim_{|q^2|\to\infty} \Lambda^\mu(p_a,p_b,q) = \gamma^\mu f(p_a^2,p_b^2)\exp[\mathcal{F}_1^{(1)}(q^2,p_a^2,p_b^2)]. \tag{6}$$

We are concerned here with the behavior of the Sudakov factor $\mathcal{F}_1^{(1)}$ in the exponent, and we will not deal with the function $f(p_a^2,p_b^2)$. In what follows, we use the dimensionless Euclidean momenta $Q^2=-q^2/M^2$, $m_a^2=-p_a^2/M^2$, and $m_b^2=-p_b^2/M^2$, and also the variable $\mu=m_v/M$.

When the baryon momenta are on shell, the lowest-order vertex correction is

$$F_1^{(1)}(Q^2) = -\frac{g_v^2}{16\pi^2}\int_0^1 du\left(\frac{2[2(1-u)-u^2+Q^2(1-u/2)^2]}{QS(u)}\ln\left[\frac{S(u)+uQ/2}{S(u)-uQ/2}\right] - 2u - \frac{2u[2(1-u)-u^2]}{u^2+\mu(1-u)} + \frac{2S(u)}{Q}\ln\left[\frac{S(u)+uQ/2}{S(u)-uQ/2}\right]\right), \tag{7}$$

where

$$S(u) \equiv \left(u^2 + \frac{u^2Q^2}{4} + \mu^2(1-u)\right)^{1/2}, \tag{8}$$

with $Q \equiv \sqrt{Q^2}$ and μ defined above.

At large Q, the term proportional to Q^2 dominates the integrand, and to extract the leading behavior [proportional to $\ln^2(Q^2)$], one need consider only [7]

$$F_1^{(1)}(Q^2) \approx -\frac{g_v^2}{16\pi^2}\int_0^1 du \frac{2Q}{S(u)} \ln\left(\frac{S(u)+uQ/2}{S(u)-uQ/2}\right). \tag{9}$$

For finite μ and $Q \gg \mu \approx 1$, $S(u)$ behaves linearly in u as long as $u \gg 2\mu/Q$. In this asymptotic limit, the logarithm in the integrand behaves as

$$\ln\left(\frac{S(u)+uQ/2}{S(u)-uQ/2}\right) = \ln\left(\frac{u^2Q^2}{u^2+\mu^2(1-u)}\right) + O(1/Q^2), \tag{10}$$

and the vertex correction becomes, to logarithmic accuracy,

$$F_1^{(1)}(Q^2) \approx -\frac{g_v^2}{16\pi^2}\int_{2\mu/Q}^1 \frac{du}{u} 4\ln\left(\frac{u^2Q^2}{u^2+\mu^2(1-u)}\right) \tag{11}$$

$$\approx -\frac{g_v^2}{16\pi^2}\left(\ln^2(Q^2) - \ln^2(4\mu^2) - \int_{2\mu/Q}^1 \frac{du}{u} 4\ln[\mu^2(1-u)+u^2]\right). \tag{12}$$

The remaining integral generates only one power of $\ln(Q^2)$; hence, in the leading-logarithm approximation, the lowest-order, on-shell vertex correction can be written as

$$F_1^{(1)}(Q^2) \approx -\frac{g_v^2}{16\pi^2}\ln^2(Q^2/\mu^2) = -\frac{g_v^2}{16\pi^2}\ln^2(-q^2/m_v^2), \tag{13}$$

which reproduces the on-shell form factor in QED derived by Fishbane and Sullivan [Eq. (2)].

When the baryon momenta are off shell, the lowest-order correction can be written as

$$\mathcal{F}_1^{(1)}(Q^2,m_a^2,m_b^2) = -\frac{g_v^2}{16\pi^2}\int_0^1 du\left([2(1-u)-u^2+Q^2(1-u/2)^2]I_0(u) + cI_2(u) - 2u\frac{2(1-u)-u^2}{u^2+\mu^2(1-u)} + L(u) - 2u\ln[u^2+\mu^2(1-u)]\right), \tag{14}$$

where

$$I_0(u)\equiv \frac{1}{\sqrt{b^2-4ac}}\ln\left(\frac{-cu^2+u\sqrt{b^2-4ac}+a}{-cu^2-u\sqrt{b^2-4ac}+a}\right), \tag{15}$$

$$cI_2(u)\equiv \frac{b^2-2ac}{2c\sqrt{b^2-4ac}}\ln\left(\frac{-cu^2+u\sqrt{b^2-4ac}+a}{-cu^2-u\sqrt{b^2-4ac}+a}\right)-\frac{b}{2c}\ln\left(\frac{a+bu+cu^2}{a-bu+cu^2}\right)+2u, \tag{16}$$

$$L(u)\equiv u\ln[(a+bu+cu^2)(a-bu+cu^2)]+\frac{b}{2c}\ln\left(\frac{a+bu+cu^2}{a-bu+cu^2}\right)-4u$$
$$-\frac{\sqrt{b^2-4ac}}{2c}\ln\left(\frac{-cu^2+u\sqrt{b^2-4ac}+a}{-cu^2-u\sqrt{b^2-4ac}+a}\right), \tag{17}$$

with

$$a\equiv[\mu^2+(m_a^2+m_b^2)u/2](1-u)+u+u^2Q^2/4,$$

$$b\equiv(m_a^2-m_b^2)(1-u)/2,$$

$$c\equiv-Q^2/4.$$

As in the preceding discussion, only the first term in the integrand can produce a factor of $\ln^2(Q^2)$, since only this term contains an extra power of Q^2 in the numerator. For $Q \gg m_a, m_b, 1$, the factor

$$\mathcal{S}(u)\equiv\sqrt{b^2-4ac}=Q\sqrt{a+b^2/Q^2}\approx\frac{Q}{2}[u^2Q^2+4u+4(1-u)\mu^2+2(1-u)u(m_a^2+m_b^2)]^{1/2} \tag{18}$$

behaves linearly in u when

$$u^2Q^2>4[1-\mu^2+(m_a^2+m_b^2)/2]u+4\mu^2,$$

which can be realized in two different ways:

$$u>4[1-\mu^2+(m_a^2+m_b^2)/2]/Q^2>2\mu/Q$$

or

$$u>2\mu/Q>4[1-\mu^2+(m_a^2+m_b^2)/2]/Q^2.$$

Denoting $\epsilon_1\equiv 2\mu/Q$ and $\epsilon_2\equiv 4[1-\mu^2+(m_a^2+m_b^2)/2]/Q^2$, we observe that in the asymptotic limit $Q\to\infty$, $\epsilon_2>\epsilon_1$ is sensible only for $\mu=0$ (the QED case); otherwise, this inequality will always be violated for large enough Q. In contrast, $\epsilon_2<\epsilon_1$ is sensible only for $\mu\neq 0$. Thus the analysis separates into the massive and massless cases. To logarithmic accuracy, we can set the lower integration limit in Eq. (14) to a cutoff ϵ and study them in turn. In this asymptotic limit, the vertex correction becomes [one must retain the b^2/Q^2 term in Eq. (18)]

$$\mathcal{F}_1^{(1)}(Q^2,m_a^2,m_b^2)\approx-\frac{g_v^2}{16\pi^2}\int_\epsilon^1\frac{du}{u}2\ln\left(\frac{(u^2Q^2)^2}{[u+(\mu^2+m_a^2u)(1-u)][u+(\mu^2+m_b^2u)(1-u)]}\right)$$
$$\approx-\frac{g_v^2}{16\pi^2}[\ln^2(Q^2)-\ln^2(\epsilon^2Q^2)-R(\mu^2,m_a^2,Q^2)-R(\mu^2,m_b^2,Q^2)], \tag{19}$$

with

$$R(\mu^2,m^2,Q^2)\equiv 2\int_\epsilon^1\frac{du}{u}\ln[u+(\mu^2+m^2u)(1-u)]. \tag{20}$$

For finite μ, $R(\mu^2,m^2,Q^2)$ contains only one power of $\ln(Q^2)$, but for $\mu=0$, $R(0,m^2,Q^2)\approx-\ln^2(Q^2)$.

For $\epsilon=\epsilon_1$ [$\epsilon_1>\epsilon_2$ or roughly for $Q\mu\gg(m_a^2+m_b^2)$]

$$\ln^2(Q^2)-\ln^2(\epsilon_1^2Q^2)=\ln^2(Q^2)-\ln^2[Q^2(4\mu^2/Q^2)]$$
$$=\ln^2(Q^2)-\ln^2(4\mu^2), \tag{21}$$

and $R(\mu^2,m^2,Q^2)$ does not contribute to the leading order; hence, the lowest-order, off-shell vertex function

$$\mathcal{F}_1^{(1)}(Q^2,m_a^2,m_b^2) \approx -\frac{g_v^2}{16\pi^2}\ln^2(Q^2) \tag{22}$$

yields a Sudakov factor *half the size of the QED result* [Eq. (1)].

For $\epsilon=\epsilon_2$ [$\epsilon_1<\epsilon_2$ or $\mu\to 0$] the term

$$\ln^2(Q^2)-\ln^2(\epsilon_2^2Q^2)=4\ln(Q^2)\ln[4+2(m_a^2+m_b^2)] - 4\ln^2[4+2(m_a^2+m_b^2)] \tag{23}$$

contains no double logarithm $\ln^2(Q^2)$. One finds, however, that

$$R(0,m^2,Q^2)=2\int_{\epsilon_2}^{1}\frac{du}{u}\ln\{u[1+m^2(1-u)]\} = -\ln^2(Q^2)+O[\ln(Q^2)], \tag{24}$$

and thus, at the leading order, the vertex function

$$\mathcal{F}_1^{(1)}(Q^2,m_a^2,m_b^2) \approx -\frac{g_v^2}{8\pi^2}\ln^2(Q^2) \tag{25}$$

reproduces the QED result [Eq. (1)].

We determined the on-shell and off-shell Sudakov form factors in a theory containing massive fermion and vector meson fields. In the on-shell case where the meson mass provides the only mass scale, the form factor in the massive theory agrees with the QED form factor, and the meson mass can be identified with the infrared regulator used in QED. In the off-shell case with two mass scales, we found two different results. For vanishing vector meson mass, the QED form factor is reproduced, as expected. For finite vector meson mass, however, the form factor displays a double logarithm behavior with a Sudakov factor *half the size* of the QED result.

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