

## Determination of the asymptotic $D$ - to $S$ -state ratio for ${}^3\text{He}$ from the reaction ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$ at $E_{d,\text{lab}}=80\text{--}0$ keV

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The tensor analyzing power  $T_{20}(\theta_{\text{lab}}=90^\circ)$  has been measured to high precision for the reaction  ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$  at  $E_d=80\text{--}0$  keV. Direct capture calculations of  $T_{20}(90^\circ)$  have been performed using asymptotic forms for the bound state  ${}^3\text{He}$  wave function while varying the asymptotic  $D$ - to  $S$ -state ratio  $\eta$ . A best-fit value for  $\eta$  was extracted and found to be  $-0.0399 \pm 0.0091_{-0.0019}^{+0.0012}$ . The model dependence of this result is discussed. [S0556-2813(97)00405-6]

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Understanding the role of the tensor force in the nucleon-nucleon interaction is a central question of nuclear physics. In the case of the three-nucleon system, one observable that probes the tensor force directly is the asymptotic  $D/S$  ratio  $\eta$ . Over the past decade, determination of this observable has been the subject of a multitude of experimental and theoretical investigations from which a consensus is gradually coalescing [1,2]. Various experimental techniques have been utilized to extract  $\eta$ , each involving the measurement of tensor analyzing powers in polarized deuteron-induced reactions. In the work of Vetterli *et al.* [3], the value of  $\eta$  was determined by measuring the tensor analyzing power  $T_{20}$  for the radiative capture reaction  ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$  at  $E_d=19.8$  MeV. In that work the authors concluded that at such high deuteron energies a significant portion of the capture strength occurred in the innermost few fermis of the overlap integral between continuum and bound state wave functions, making the extracted result dependent upon the choice of  ${}^3\text{He}$  (bound state) wave function. The purpose of the present work is to report the results of a new determination of  $\eta$  using the radiative capture of polarized deuterons by protons at very low deuteron lab energies (80–0 keV). We will address the issue of model dependence at these energies in radiative capture-based determinations of  $\eta$  by quantifying, at least roughly, such model dependencies.

In this analysis we employ the direct capture model [4] to describe radiative-capture reactions. In direct capture, the transition between an initial continuum state and a final bound state is treated as a single-step process induced by the electromagnetic interaction. The continuum state is represented by a distorted plane wave that describes two noninteracting pointlike particles, the projectile and the target. The continuum wave function is generated by solving the Schrödinger equation using an optical model potential (and, alternatively, using solely the Coulomb potential). In the present work both realistic wave functions based on Faddeev calculations (see below) and asymptotic wave functions are used to represent the bound state.

We now describe the form of the asymptotic wave functions for the bound state for  ${}^3\text{He}$ . This system is treated as a proton and a deuteron separated by a distance  $r$  and having relative orbital angular momentum  $L$ . We may consider the

${}^3\text{He}$  ground state to be composed of an  $L=0$ , or  $S$ -state, component and an  $L=2$ , or  $D$ -state, component. Solving the Schrödinger equation while including the Coulomb potential gives radial portions of the proton-deuteron wave functions in the asymptotic ( $r \rightarrow \infty$ ) limit of [5]

$$u_L(r) \xrightarrow{r \rightarrow \infty} C_L^C N_W \frac{W_{-\kappa, L+1/2}(2\beta r)}{r}. \quad (1)$$

In this expression  $W_{-\kappa, L+1/2}$  is a Whittaker function [5]. The constants  $\kappa$  and  $\beta$  are the Coulomb parameter and wave number (corresponding to the  $p$ - $d$  separation energy), respectively.  $N_W$  is the zero-range Coulomb asymptotic normalization.  $C_L^C$  is the  $L$ -wave Coulomb asymptotic normalization constant. The asymptotic normalization constants echo the internal dynamics of wave functions through overall normalization [5]. The asymptotic  $D/S$  ratio  $\eta$  is defined to be  $C_D^C/C_S^C$ , and hence is a direct indication of the relative strength of the  $D$ -state component of the bound state wave function. Since the  $D$  state arises from noncentral forces,  $\eta$  provides information about the tensor component of the nuclear interaction.

The primary motivation for using asymptotic forms of the bound state wave functions is that in the limit of large  $r$  the wave function is insensitive to the details of the nuclear interaction and, to a good approximation, is a well-known exponentially decreasing function of the  $p$ - $d$  separation (e.g., a Whittaker function for the  $p + d$  system). If most of the reaction strength occurs in the asymptotic region, i.e., the exponential tail, then this approximation holds and an extraction of  $\eta$  can be viewed as reasonably model independent. The validity of this assumption for the present work is discussed below. Furthermore, it has been shown [6] that the tensor analyzing powers are sensitive to the asymptotic region of the bound state wave function and in particular that  $T_{20}$  scales linearly with  $\eta$ .

With this in mind, we have measured the tensor analyzing power  $T_{20}(\theta_{\text{lab}}=90^\circ)$  for the reaction  ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$  to high precision and compared this with calculations to determine the best-fit value of  $\eta$ . Results for this  $T_{20}$  measurement represent the combination of data recently published [7] with

additional data obtained subsequent to that publication. This report represents a piece of a larger investigation into the dynamics of light nuclei. The research program involves measurements of cross section, vector and tensor analyzing powers, and  $\gamma$ -ray polarizations using proton- and deuteron-induced radiative-capture reactions at beam energies below 100 keV [8].

The present experiment was performed at Triangle Universities Nuclear Laboratory (TUNL). Polarized deuterons from an atomic-beam polarized ion source [9] were accelerated to 80 keV and directed through a Wien filter to orient the spin quantization axis properly. The beam was then stopped in an H<sub>2</sub>O ice target, providing data for deuteron energies from 80 to 0 keV in the lab frame. Outgoing  $\gamma$  rays were detected using two large HPGe detectors with efficiencies of 128 and 145 % relative to 3 $\times$ 3 in. NaI crystals. Energy resolution for both HPGe detectors was 4.2 keV for 5.5 MeV  $\gamma$  rays.

The beam polarization was measured frequently with a low-energy deuteron polarimeter which used the reaction  ${}^2\text{H}(\vec{d}, p){}^3\text{H}$ . The beam was incident on a deuterated titanium foil target while the recoil protons were detected with silicon surface-barrier detectors thick enough to stop the protons. Polarizations were calculated using previously well-determined analyzing powers [10]. Polarization measurements were also performed using a spin-filter polarimeter [11]. Deuteron polarizations were stable throughout the acquisition phase with typical values of  $p_{zz\pm} \approx \pm 0.87$  and an uncertainty of  $\pm 0.04$ .

Data were taken for two polarization states: (1) a state with (theoretical maximum)  $p_{zz} = +1$  and (2) a state with (theoretical maximum)  $p_{zz} = -1$ . The beam fast spin flipped between states at a rate of 10 Hz in order to minimize systematic error from time-dependent changes in the target. The expression for  $T_{20}$  in a given detector in terms of the tensor polarizations  $p_{zz+}$  and  $p_{zz-}$  and counts (normalized to the integrated charge)  $Y_+$  and  $Y_-$  is [12]

$$T_{20} = \sqrt{2} \frac{Y_+ - Y_-}{p_{zz+} Y_- - p_{zz-} Y_+}. \quad (2)$$

Results from this experiment coupled with data from previous runs for measuring  $T_{20}$  [7] yield a final observed value of  $T_{20}(90^\circ) = -0.1051 \pm 0.0108$  where the error is primarily statistical in origin but also includes uncertainty in the beam polarization.

The procedure used to extract  $\eta$  was to calculate  $T_{20}$  for  ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$  at the incident beam energies while varying the choice of  $\eta$  until the calculated value of  $T_{20}$  matched the measured value. The calculation was performed by the radiative-capture program DIRAC, which generates the transition-matrix elements (TME's) connecting the continuum and bound state wave functions by evaluating the overlap integrals numerically. A second program, OBS, generated the corresponding  $T_{20}$  from these TME's using the TME expansions of polarization observables described by Seyler and Weller [14]. The continuum wave functions used in this calculation were distorted plane waves generated using the optical model potential of Guss [13] (see Table I), but neglecting the imaginary surface term. (A comparison with

TABLE I. Optical model potential parameters from Guss [13]. The imaginary surface term, subscripted with "d," is presented for completeness. This term had a negligible effect upon the calculated value of  $\eta$  and was not included in any of the calculations whose results are presented in this report. Well depths are in MeV and lengths are in fermi (fm).

$V_0$	90.73	$W_d$	4.544
$r_0$	1.153	$r_d$	1.104
$a_0$	0.454	$a_d$	1.3
$r_{\text{Coulomb}}$	1.3		

calculations using solely Coulomb distortions is given below.) The calculations were performed for several choices of bound state wave function detailed below. As discussed above, both the  $S$ - and  $D$ -state bound state wave functions contain an overall asymptotic normalization. For each calculation of  $T_{20}$ , the  $S$ -state normalization is chosen in some appropriate manner (see below), while the  $D$ -state normalization is calculated from the  $S$ -state normalization based on the choice for  $\eta$  according to  $\eta = C_D^C/C_S^C$ .

It is important to mention that the DIRAC code can only be used to calculate the electric multipole radiation portion of radiative capture. Unfortunately, at these energies the magnetic multipole  $M1$  contributes  $\sim 30\%$  of the total reaction strength. To account for this, the  $M1$  contribution to the observed value of  $T_{20}$  was removed according to the results of a fit to data from both the  ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$  and  ${}^2\text{H}(\vec{p}, \gamma){}^3\text{He}$  reactions for center of mass energies equivalent to the present experiment. This fit to  $\sigma(\theta)/A_0$ ,  $A_y(\theta)$ ,  $T_{20}(\theta)$ , and  $P_\gamma(\theta)$  (see Fig. 1) gave a result of  $30.2 \pm 7.4\%$   $M1$  strength (doublet and quartet) and  $69.8 \pm 2.6\%$   $E1$  strength, with a  $\chi^2/\nu = 1.57$ . Higher multipoles were found to be negligible. The  $M1$ -removed value of  $T_{20}(90^\circ)$  was found to be  $-0.0742 \pm 0.0176$ . The best-fit process of deter-

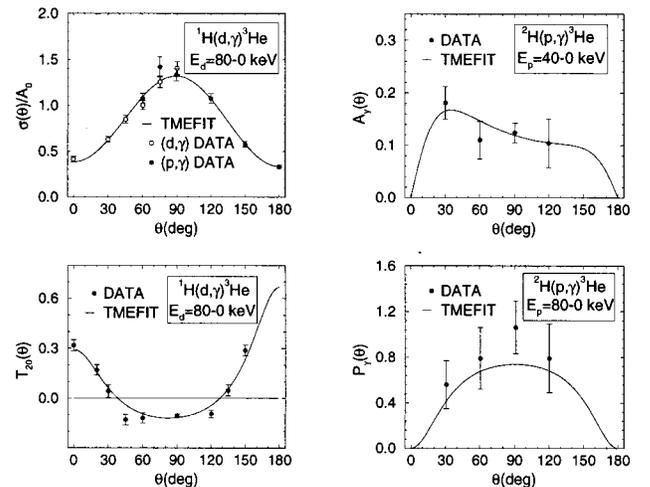


FIG. 1. Results of a simultaneous TME fit to cross section, vector and tensor analyzing power, and  $\gamma$ -ray polarization data for  ${}^1\text{H}(\vec{d}, \gamma){}^3\text{He}$  and  ${}^2\text{H}(\vec{p}, \gamma){}^3\text{He}$  for  $E_{\text{c.m.}} = 27-0$  keV (with the exception of  $\gamma$  ray polarization data, which is for  $E_{\text{c.m.}} = 54-0$  keV). The resultant fit gives 30.2%  $M1$  strength and a  $\chi^2/\nu = 1.57$ .

TABLE II. Extracted values of  $\eta$  that result from varying  $r_{\text{cut}}$  in a purely asymptotic  ${}^3\text{He}$  wave function at  $E_d=19.8$  MeV [15] and at  $E_d=80$  keV (present). At the higher energy,  $\eta$  changes by 0.01 for every change in  $r_{\text{cut}}$  of 0.5 fm. At 80 keV,  $\eta$  changes by roughly 0.001 for every 0.5 fm.

$\eta$ $E_d$ (MeV)	$r_{\text{cut}}$ (fm)					
	1.0	1.5	2.0	2.5	3.0	4.0
19.8	(-0.012)	-0.022	-0.032	-0.042	(-0.052)	
0.080	-0.0353		-0.0379		-0.0402	-0.0428

mining  $\eta$  by means of calculations of  $T_{20}$  is performed with respect to this  $M1$ -removed value of  $T_{20}$ .

The first bound state wave functions used to extract this best-fit  $\eta$  were the purely asymptotic (Whittaker function) forms introduced above. The parameter  $C_S^C$  was chosen to match current experimental measurements [1], while the parameters  $\kappa$  and  $\beta$  were calculated using the observed value for the  $p$ - $d$  separation energy. A limitation of this choice of wave function, however, is that these functions diverge at  $p$ - $d$  separation distance of zero and consequently overpredict the contribution of the innermost few fermis to the capture integral. To counter this overprediction we follow the procedure employed by Vetterli *et al.* of introducing a cutoff distance  $r_{\text{cut}}$  below which the wave function is simply truncated. The best-fit  $\eta$  results for various values of  $r_{\text{cut}}$  are listed in Table II. We see that  $\eta$  changes only slightly ( $\sim 0.002$ – $0.003$ ) for each change of 1 fermi (fm) in  $r_{\text{cut}}$  for the present reaction. Also listed in Table II are results from the analysis of Vetterli [3] at the much higher beam energy of 19.8 MeV. Those results show a dependency upon  $r_{\text{cut}}$  that is ten times greater than for the low-energy calculation. Clearly, at low energies  $\eta$  is only slightly sensitive to the internal dynamics of the bound state wave functions and hence is only slightly model dependent.

To produce a final value for  $\eta$  we constructed a  ${}^3\text{He}$  wave function  $\psi_{\text{matched}}$  which consists of the Whittaker asymptotic form from 3 (4) fm outward for the  $S$ -state ( $D$ -state) wave function and a ‘‘realistic’’ wave function for zero to 3 (4) fm (see Fig. 2). The choice for the innermost 3 (4) fm was a wave function produced by Lehman in 1984 [15] that is a two-body projection of a full three-body Faddeev calculation by Gibson and Lehman (1984) [17]. This wave function was chosen because it matches the experimental binding energy for the  $d + p$  system. The  $S$ -state asymptotic normalization was chosen to be the currently accepted value from experiment  $(C_S^C)^2 = 3.24$  [1]. Extracting  $\eta$  using  $\psi_{\text{matched}}$ , we obtained  $\eta = -0.0399 \pm 0.0091$ .

Since any choice of  ${}^3\text{He}$  wave function is subject to controversy, this result must be taken with a measure of caution. Consequently, we have made an effort to determine rough limits on the model dependence of this extraction of  $\eta$  by constructing two additional ‘‘matched’’ wave functions (see Fig. 2). For the first, we simply multiplied the ‘‘realistic’’ region from  $\psi_{\text{matched}}$  by the arbitrary value of  $2/3$  and matched it with the asymptotic form at about five fermis. This wave function we denote  $\psi_{\text{matched}*2/3}$ . In the same manner we produced a wavefunction  $\psi_{\text{matched}*3/2}$  where the ‘‘realistic’’ region was multiplied by the arbitrary value  $3/2$ . While the technique is admittedly simplistic, it nevertheless

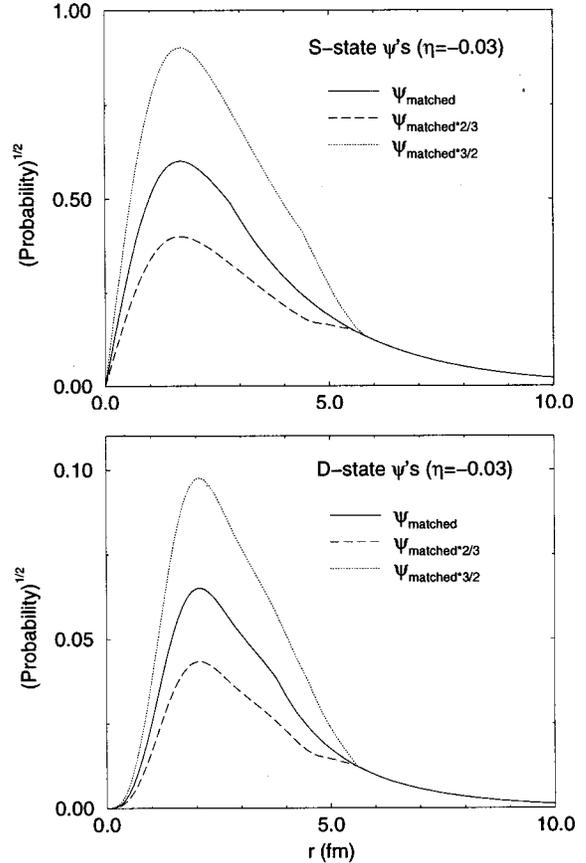


FIG. 2. ‘‘Matched’’ wave functions consisting of a two-body projection of a wave function produced by a three-body Faddeev calculation connected at  $r=3$ – $5$  fm with the asymptotic (Whittaker function) wave function for  $p + d$ .  $\psi_{\text{matched}*2/3}$  and  $\psi_{\text{matched}*3/2}$  were constructed from  $\psi_{\text{matched}}$  by simply multiplying the inner region by  $2/3$  and  $3/2$ , respectively.

achieves the desired aim of creating wave functions with different probability distributions.  $\psi_{\text{matched}*2/3}$  is representative of those wave functions with less strength in the internal region and more in the exponential tail, while the opposite case holds for  $\psi_{\text{matched}*3/2}$ . Resulting values of  $\eta$  for the matched  $\psi$ 's are given in Table III. The rough bounds on the model dependence due to large changes in the internal wave functions are then  ${}^{+0.0012}_{-0.0019}$ . Comparing these with the statistical error on the extraction of  $\eta$  from  $\psi_{\text{matched}}$ ,  $\pm 0.0091$ , we see that the uncertainty due to model dependence is less than the experimental uncertainty.

One final issue which must be addressed is the choice of

TABLE III. Extracted  $\eta$  values for the various ‘‘matched’’ wave functions. The variation in  $\eta$  due to wave functions with significantly different probability distributions is seen to be much smaller than the experimental uncertainty.

$\psi$	$\eta$
$\psi_{\text{matched}*3/2}$	$-0.0387 \pm 0.0087$
$\psi_{\text{matched}}$	$-0.0399 \pm 0.0091$
$\psi_{\text{matched}*2/3}$	$-0.0418 \pm 0.0096$

potential used to generate the continuum wave functions. The extracted values of  $\eta$  given above for the present work all result from using the optical model potential (neglecting the imaginary surface term) of Guss [13] described in Table I. To study  $\eta$ 's sensitivity to the choice of potential we performed additional calculations using solely the Coulomb potential. The final extracted value for  $\eta$  using  $\psi_{\text{matched}}$  and just the Coulomb potential was  $-0.0408 \pm 0.0093$ , or a difference of  $-0.0009$  from the result using the optical model potential. This result indicates that the scattering state wave function in the capture reaction at the energy of this work is not affected by the nuclear potential. Although the optical potential used here does not contain a tensor interaction, three-body continuum Faddeev calculations [16] have shown that the tensor analyzing powers are insensitive to tensor force effects in the continuum. Note, however, that the tensor force is included in the three-body Faddeev calculation [17] used to generate the bound  $S$ - and  $D$ -state wave functions.

In summary, we have performed a high precision measurement of the tensor analyzing power  $T_{20}(90^\circ)$  for the reaction  $^1\text{H}(\vec{d}, \gamma)^3\text{He}$  and extracted a best-fit value for the asymptotic  $D/S$  ratio  $\eta$  using purely asymptotic (Whittaker function) forms for the  $^3\text{He}$  wave functions in a direct capture calculation. We have shown this extracted value for  $\eta$  to be relatively insensitive to the internal dynamics of the choice of bound state  $^3\text{He}$  wave function at low beam energies. We therefore conclude that the radiative-capture method for determination of  $\eta$  at very low beam energies is not subject to the strong model dependencies observed at higher deuteron energies. To extract a final value for  $\eta$  we

have constructed a physically reasonable bound state wave function. The final result of the best-fit process using this wave function was  $\eta = -0.0399 \pm 0.0091_{-0.0019}^{+0.0012}$ . The uncertainty  $\pm 0.0091$  arises from the statistical error of the ( $M1$  removed)  $T_{20}$  measurement. The second uncertainty represents rough upper and lower limits due to the model dependency of the choice of wave functions as described above.

The present measurement agrees well with recent experimental and theoretical determinations of  $\eta$ . Representative of recent experimental work, Ayer *et al.* [2], using a distorted-wave Born approximation extraction of  $\eta$  from ( $\vec{d}, ^3\text{He}$ ) reactions, found  $\eta = -0.0386 \pm 0.0046 \pm 0.0012$ . On the theoretical side, Friar *et al.* [18] performed full three-body Faddeev calculations including Coulomb effects using diverse models for the two- and three-body  $NN$  forces and found  $\eta = -0.0430 \pm 0.001$ . The present measurement agrees within error with both these results (and others—see the summary in Ayer *et al.* [2]), and demonstrates the utility of nuclear physics at very low energies for probing important features of the nuclear interaction.

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