

## Ground-state spectrum of light-quark mesons

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A confining, Goldstone theorem preserving, separable ansatz for the ladder kernel of the two-body Bethe-Salpeter equation is constructed from phenomenologically efficacious  $u$ ,  $d$ , and  $s$  dressed-quark propagators. The simplicity of the approach is its merit. It provides a good description of the ground-state flavor-octet pseudoscalar, vector, and axial-vector meson spectrum facilitates an exploration of the relative importance of various components of the two-body Bethe-Salpeter amplitudes, showing that subleading Dirac components are quantitatively important in the flavor-octet pseudoscalar meson channels, and allows a scrutiny of the domain of applicability of ladder truncation studies. A color-antitriplet diquark spectrum is obtained. The shortcomings of separable *Ansätze* and the ladder kernel are highlighted. [S0556-2813(97)04405-1]

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### I. INTRODUCTION

The spectroscopy of light-quark mesons is made interesting because of the role played by dynamical chiral symmetry breaking, the natural scale of which is commensurate with other scales in this sector. It also explores quark and gluon confinement because most vector and axial-vector mesons have masses that are more than twice as large as typical constituent-quark masses. Covariant, constituent-quark potential models are a useful tool in the study of this problem [1].

A salient feature of the strong interaction spectrum is the fact that  $m_\rho^2 - m_\pi^2 \approx 30m_\pi^2$ , which may be compared with the vector-pseudovector splitting  $m_{a_1}^2 - m_\rho^2 \approx 1.7m_\rho^2$ . Furthermore, the pion mass must vanish in the chiral limit; i.e., when the current-quark mass vanishes, whereas  $m_{a_1}^2 \rightarrow m_\rho^2$ . (We note that a vanishing current-quark mass does not entail a vanishing of the constituent-quark mass.) These observations are an indication that the Goldstone-boson character of the pion is a particular and crucial feature of the strong-interaction spectrum. That these features are difficult to capture in potential models is well illustrated in Refs. [1].

An efficacious framework for studying meson spectroscopy is provided by the QCD Dyson-Schwinger equations [DSE's] [2], which include the "QCD gap equation" (quark DSE), that has proven useful in the study of quark confinement and dynamical chiral symmetry breaking, and the covariant, two-body bound-state Bethe-Salpeter equations [BSE's]. With one exception, Ref. [3], all spectroscopic studies to date have employed the rainbow-ladder truncation of the quark DSE and two-body BSE, which is defined as follows. Rainbow approximation specifies that the dressed quark-gluon vertex in the quark DSE is replaced by the bare vertex:  $\Gamma_\mu^a(k, p) \equiv \gamma_\mu \lambda^a/2$ , where  $\{\lambda^a\}_{a=1}^8$  are the color Gell-Mann matrices. This equation is then solved with a given model form of the dressed-gluon propagator,  $D_{\mu\nu}(k)$ , to yield a dressed-quark propagator,  $S(p) = 1/[i\gamma \cdot p A(p^2)$

$+ B(p^2)]$ . The kernel of the ladder approximation to the two-body BSE is then the customary ladder kernel but with  $D_{\mu\nu}(k)$  and  $S(p)$  employed in place of free-particle propagators.

It has been shown [4] that for any  $D_{\mu\nu}(k)$  that leads to the dynamical generation of a fermion mass in the chiral limit, i.e., to dynamical chiral symmetry breaking, the flavor-octet pseudoscalar meson BSE necessarily admits a  $P^2 = 0$  bound-state solution ( $P_\mu$  is the total momentum of the dressed-quark, antiquark system). No fine tuning is necessary to ensure this outcome and one thus has a natural understanding of the pion as both a Goldstone boson and a bound state of a strongly dressed quark and antiquark. This outcome is the result of an equivalence, in the chiral limit, between the quark DSE and the flavor-octet, pseudoscalar meson BSE. This equivalence is an intrinsic feature of the DSE's, which persists in more sophisticated truncation schemes [3,5].

The most extensive and phenomenologically successful spectroscopic studies in the rainbow-ladder framework are those of Ref. [6], in which the quark DSE is solved numerically for spacelike  $p^2$  using a model gluon propagator. In Landau gauge the behavior of the gluon propagator is constrained by perturbation theory for  $k^2 > 1 - 2 \text{ GeV}^2$  [7] and one models the infrared behavior, which is presently unknown. Such studies have the ability to unify many observables via the few parameters that characterize the behavior of the model dressed-gluon propagator in the infrared.

In this approach, solving the meson BSE's is complicated by the fact that these equations sample the dressed-quark propagator off the spacelike- $p^2$  axis. In Ref. [6] this difficulty was circumvented by employing a derivative expansion of the dressed-quark propagator functions,  $A(p^2)$  and  $B(p^2)$ , and estimating the error introduced thereby. This, however, obscures the discussion and exploration of the role of quark and gluon confinement, a sufficient condition for which is the absence of a Lehmann representation for the dressed-quark and dressed-gluon propagators. The problem becomes more acute in studies of scattering observables.

An algebraic parametrization of a confining dressed-quark propagator, based on numerical solutions of model quark-DSE's, has been used successfully in studies of a large range of mesonic scattering observables; for example:  $f_\pi$ ,  $r_\pi$ , the  $\pi$ - $\pi$  scattering lengths and the pion electromagnetic form factor [8];  $f_K$  and the charged and neutral kaon electromagnetic form factors [9]; the anomalous  $\gamma\pi \rightarrow \gamma$  [10] and  $\gamma\pi \rightarrow \pi\pi$  transition form factors [11]. In these studies the dressed-gluon propagator is only specified implicitly insofar as the model dressed-quark propagator can be used as a constraint on its form via the quark DSE.

It would be useful to make this connection explicit. However, given a dressed-quark propagator it is not possible, in principle, to invert the quark DSE and extract a dressed-gluon propagator; one reason being that the quark DSE involves the dressed-quark-gluon vertex, which depends implicitly on both the dressed-quark and dressed-gluon propagators. In the peculiar case of the rainbow truncation this particular difficulty, at least, is eliminated.

It is known that the rainbow truncation is only quantitatively and qualitatively reliable in Landau gauge [12], which means that it is inappropriate to infer a connection between a given phenomenologically constrained model dressed-gluon propagator and a solution of the gluon DSE, such as those obtained in Refs. [7,13], in any other covariant gauge. The quark DSE is, in general, a pair of coupled, nonlinear integral equations for  $A(p^2)$  and  $B(p^2)$ . In Landau gauge, the kernel in the equation for  $A(p^2)$  is sufficiently complicated, even in rainbow truncation, that it is not possible to invert the equation without introducing kinematic singularities. An explicit connection between the dressed-quark propagator and a confining dressed-gluon propagator via the inversion of the quark DSE is therefore not possible.

A goal of this study, and another [14], is to explore the extent to which pion and kaon observables, as embodied in the model dressed-quark propagators employed in Refs. [8–11], constrain the properties of all light-quark mesons. As we have described, it is not possible to explicitly construct a dressed-gluon propagator from these dressed-quark propagators. However, one may adopt a purely phenomenological approach in order to construct a simple, confining *Ansatz* for the *kernel* of the two-body BSE's that is constrained by the pion and kaon scattering observables. This facilitates the present exploration of the extent to which a confining, Goldstone theorem preserving, BSE approach can generate the ground-state spectrum of light-quark mesons. It also allows one to easily identify those channels in which a ladder truncation of the BSE is applicable and those in which it is inadequate, and to explore the extent to which observable properties are influenced by subleading Dirac components in the Bethe-Salpeter amplitude, e.g., the influence of the pseudovector,  $\gamma_5 \gamma \cdot P$ , piece of the pion Bethe-Salpeter amplitude on the pion mass and decay constant. Such terms have been neglected in almost all studies undertaken to date. A similar situation holds for the calculation of hadronic coupling constants and associated form factors for processes such as  $\rho \rightarrow \pi\pi$  [15,16] and  $\rho \rightarrow \gamma\pi$  [17]. Semiphenomenological  $\bar{q}q$  Bethe-Salpeter amplitudes for the leading Dirac covariant are currently used to facilitate the necessary integrations. The amplitudes provided here have a significant amount of dynamical justification and yet are simple enough

to allow a more realistic study of hadron couplings. We also take this opportunity to explore diquark correlations.

It is with this goal in mind that, in Sec. II, we construct a constrained, confining, flavor-dependent, separable *Ansatz* for the dressed-ladder kernel of the two-body BSE's. Light-quark meson solutions are discussed in Sec. III. Details of the derivation of the explicit form taken by the ladder Bethe-Salpeter equation with the separable kernel *Ansatz*, and with retention of the most general Dirac covariant structure allowed here, are collected into Appendix A. Specific aspects of the solution amplitudes are presented in Appendix B. In Sec. IV we consider the effective masses of color-antitriplet quark-quark (diquark) correlations, which are bound in dressed-ladder truncation. This is a defect of the truncation, which is due to the fact that there are no repulsive terms in the kernel at this level of truncation. It is a peculiarity; repulsive terms appear at every higher order, with ‘‘order’’ referring to the number of explicit dressed-gluon propagators in the kernel, and these eliminate the diquark bound states [3]. Diquark correlations are nevertheless of contemporary interest because a number of studies of the nucleon Faddeev equation have proceeded under the assumption that the quark-quark  $\mathcal{T}$  matrix can be represented as a sum of simple diquark-pole terms, and that the mass splittings are such that only the lowest mass poles need be retained in solving the reduced two-body equation that results [18,19]. Details of the diquark solutions are discussed in Appendices B3 and B4. We summarize and conclude in Sec. V.

## II. SEPARABLE ANSATZ FOR THE BETHE-SALPETER KERNEL

The Dyson-Schwinger equation for the Euclidean-space dressed-quark propagator (Schwinger function),

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}, \quad (1)$$

can be written as

$$S^{-1}(p) = i\gamma \cdot p + m + \int \frac{d^4 k}{(2\pi)^4} g^2 D_{\mu\nu}[(p-k)^2] \times \gamma_\mu \frac{\lambda^a}{2} S(k) \Gamma_\nu^a(k, p), \quad (2)$$

where  $m$  is the (bare) current-quark mass. The Euclidean Dirac matrices satisfy the algebra  $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ , where  $\delta_{\mu\nu}$  is the Kronecker delta, and  $a \cdot b \equiv \sum_{i=1}^4 a_i b_i$ . In Eq. (2),  $D_{\mu\nu}(k)$  is the dressed-gluon propagator and  $\Gamma_\mu(k, p)$  is the dressed quark-gluon vertex.

The homogeneous BSE for a quark-antiquark bound state is

$$\Gamma^{rs}(k; P) = \int \frac{d^4 q}{(2\pi)^4} K^{rs;tu}(q, p; P) \{S_{f_1}(q + \xi P) \times \Gamma(q; P) S_{f_2}[q - (1 - \xi)P]\}^{tu}, \quad (3)$$

where  $P$  is the center-of-mass momentum; the  $f_1$ -flavor-quark carries momentum  $p_{f_1} = q + \xi P$  and the

$\bar{f}_2$ -flavor-antiquark momentum is  $p_{\bar{f}_2} = -q + (1 - \xi)P$ ; and  $K(q, p; P)$  is the quark-antiquark scattering kernel.

The ladder approximation is defined by

$$K^{rs;tu}(q, p; P) \equiv -g^2 D_{\mu\nu}(p - q) I_{r_F t_F} \left( \frac{\lambda^a}{2} \right)_{r_C t_C} \times (\gamma_\mu)_{r_D t_D} I_{u_F s_F} \left( \frac{\lambda^a}{2} \right)_{u_C s_C} (\gamma_\nu)_{u_D s_D}, \quad (4)$$

where  $\{F, C, D\}$  indicate flavor, color, and Dirac indices.

In rainbow approximation, i.e., using

$$\Gamma_\mu^a(k, p) = \gamma_\mu \frac{\lambda^a}{2} \quad (5)$$

in Eq. (2), then, if the dressed-quark propagator is known, Eq. (2) can be used to constrain an *Ansatz* for the ladder approximation to the kernel of Eq. (3).

Herein, solely because of its inherent simplicity, we employ a model of the form

$$g^2 D_{\mu\nu}(p - k) = \delta_{\mu\nu} \Delta(p - k). \quad (6)$$

In being proportional to  $\delta_{\mu\nu}$ , this has the *appearance* of a Feynman gauge propagator. The appearance is misleading, however. A fundamental Slavnov-Taylor identity in QCD entails that the longitudinal piece of the dressed-gluon propagator must be independent of interactions. Equation (6) has the transverse and longitudinal components dressed in exactly the same way. A propagator of this form could only arise if the gauge parameter was chosen so as to completely cancel the transverse interaction contributions, i.e., if the gauge parameter dependent, longitudinal piece of the gluon propagator is interaction dependent. Therefore Eq. (6) can only provide a model effective potential:  $\Delta(p - k)$ , defined in this way, cannot in principle be related to solutions obtained in studies of the gluon DSE, such as Refs. [7,13]. We will describe Eq. (6) as *Feynman-like* gauge.

One can write, without loss of generality,

$$\Delta(p - k) = \sum_{n=0}^{\infty} \Delta_n(p^2, k^2) p^n k^n \frac{1}{2^n} U_n(\hat{p} \cdot \hat{k}), \quad (7)$$

where  $\hat{p}$  is the unit-magnitude direction vector for  $p$  and  $\{U_n(x)\}_{n=0}^{\infty}$  are the complete set of orthonormal Tschebyshev functions, which satisfy

$$\frac{2}{\pi} \int_{-1}^1 dx \sqrt{1-x^2} U_i(x) U_j(x) = \delta_{ij}. \quad (8)$$

Translational invariance is preserved if all contributing Tschebyshev moments are retained.

The quark DSE, Eq. (2), represents two coupled, nonlinear integral equations for  $A(s)$  and  $B(s)$ , where  $s = p^2$ . In rainbow approximation, Eq. (5), and using Eq. (6), these equations are

$$p^2 A(p^2) = p^2 + \frac{8}{3} \int \frac{d^4 k}{(2\pi)^4} \Delta((p-k)^2) p \cdot k \times \frac{A(k^2)}{k^2 A(k^2)^2 + B(k^2)^2}, \quad (9)$$

$$B(p^2) = m + \frac{16}{3} \int \frac{d^4 k}{(2\pi)^4} \Delta((p-k)^2) \frac{B(k^2)}{k^2 A(k^2)^2 + B(k^2)^2}. \quad (10)$$

The simplicity inherent in Feynman-like gauge is obvious.

Introducing the Tschebyshev expansion for  $\Delta(p - k)$ , Eq. (7), these equations become

$$A(s) = 1 + \frac{1}{24\pi^2} \int_0^\infty dt t^2 \Delta_1(s, t) \sigma_V(t), \quad (11)$$

$$B(s) = m + \frac{1}{3\pi^2} \int_0^\infty dt t \Delta_0(s, t) \sigma_S(t), \quad (12)$$

from which one observes that, in rainbow approximation and in Feynman-like gauge, the quark DSE is only sensitive to the zeroth and first Tschebyshev moments of  $\Delta(p - k)$ . Hence, translational invariance of the kernel of the quark DSE is not lost as long as the zeroth and first Tschebyshev moments are retained.

A constrained kernel can now be obtained by employing a rank- $N$ , separable approximation for the Tschebyshev moments:

$$\Delta_n(s, t) = \sum_{i=1}^N F_n^i(s) F_n^i(t). \quad (13)$$

The simplest such approximation is rank-1, which is considered herein, i.e., one writes

$$F_0^1(s) \equiv G(s) = \frac{1}{b} [B(s) - m],$$

$$F_1^1(s) \equiv F(s) = \frac{1}{a} [A(s) - 1], \quad (14)$$

where  $a$  and  $b$  are fixed constants, which are to be determined, and  $A(s)$  and  $B(s)$  are the functions that appear in the quark propagator. As will be seen below, this particular choice for  $F_0^1$  and  $F_1^1$  is sufficient to ensure that Goldstone's theorem is preserved.

Substituting Eqs. (14) via Eq. (13) into Eqs. (11) and (12) one finds that this latter pair of equations [i.e., the quark DSE] is solved if, and only if,

$$a^2 = \frac{1}{24\pi^2} \int_0^\infty dt t^2 [A(t) - 1] \sigma_V(t), \quad (15)$$

$$b^2 = \frac{1}{3\pi^2} \int_0^\infty dt t [B(t) - m] \sigma_S(t). \quad (16)$$

One now has a rank-1, separable *Ansatz* for the ladder kernel of the BSE for like quarks, which is completely determined by the propagator of that quark, i.e., Eq. (4) with

$$\begin{aligned}
g^2 D_{\mu\nu}(p-k) &= \delta_{\mu\nu} \Delta(p-k) \\
&= \delta_{\mu\nu} [G(p^2) G(k^2) + p \cdot k F(p^2) F(k^2)].
\end{aligned} \tag{17}$$

With  $u$  and  $d$  quarks treated as indistinguishable, except for their electric charge, Eq. (17) can be used in the study of the BSE for  $\pi$ ,  $\omega$ , and  $\rho$  mesons, for example.

A simple generalization of this *Ansatz* to mesonlike bound states with arbitrary flavor content is obtained via the identification

$$\begin{aligned}
&S_{f_1}(k+\xi P) \Delta(p-k) S_{f_2}(k-(1-\xi)P) \\
&\equiv S_{f_1}(k+\xi P) \left\{ \frac{1}{2} [G_{f_1}(p^2) G_{f_2}(k^2) + G_{f_1}(k^2) G_{f_2}(p^2)] \right. \\
&\quad \left. + p \cdot k \frac{1}{2} [F_{f_1}(p^2) F_{f_2}(k^2) + F_{f_1}(k^2) F_{f_2}(p^2)] \right\} \\
&\times S_{f_2}[k-(1-\xi)P],
\end{aligned} \tag{18}$$

wherever it appears in the kernel of a given BSE. This can be used in the study of the BSE for  $K$  and  $K^*$  mesons, for example.

We observe that, once the propagators for quarks of flavors  $f_1$  and  $f_2$  are known, Eq. (18) provides a constrained, separable *Ansatz* for the *ladder kernel* of the Bethe-Salpeter equation. If the dressed-quark propagators have no Lehmann representation then this kernel is free of quark and gluon production thresholds and may therefore be described as confining. As remarked above, this *Ansatz* for the kernel is *not* equivalent to an *Ansatz* for the gluon propagator and it is inappropriate to infer comparisons with solutions obtained in studies of the gluon DSE, such as Refs. [7,13]. Such comparisons can only be made when one employs a gauge-fixing procedure that does not violate the relevant Slavnov-Taylor identity; for example, Ref. [20], which employs Landau gauge and is not separable. We note that any attempt to construct a constrained, separable *Ansatz* in other than Feynman-like gauge will introduce kinematic singularities in the analog of Eq. (18).

The BSE is solved in the rest frame by setting  $P=(0,0,0,iM)$  in Eq. (3) and details of this for the case of identical quarks ( $f_1=f_2$ ) are given in Appendix A. The generalization to mesonlike bound states with arbitrary flavor content is straightforward using Eq. (18). In general, the ladder truncation of the BSE reduces to a finite matrix equation that admits solutions for discrete values of the meson mass,  $M$ .

### A. Meson decay constant

The canonical normalization of the Bethe-Salpeter amplitude  $\Gamma$  is given by [21]

$$\begin{aligned}
2P_\mu &= N_c \int \frac{d^4 k}{(2\pi)^4} (\text{tr}_D \{ \bar{\Gamma}(k, -P) \partial_\mu^P S_{f_1}(k+\xi P) \Gamma(k, P) \\
&\quad \times S_{f_2}[k-(1-\xi)P] \} + \text{tr}_D \{ \bar{\Gamma}(k, -P) \\
&\quad \times S_{f_1}(k+\xi P) \Gamma(k, P) \partial_\mu^P S_{f_2}[k-(1-\xi)P] \} ),
\end{aligned} \tag{19}$$

where  $\bar{\Gamma}(k, P)^T = C^{-1} \Gamma(-k, P) C$  defines the corresponding antimeson amplitude. In ladder approximation the kernel of the Bethe-Salpeter equation is independent of the center-of-mass momentum  $P$ , hence there is no contribution of the type  $\partial K / \partial P$  to the normalization.

The pseudoscalar meson decay constant,  $f_P$ , is defined by

$$\langle 0 | \bar{\Psi}(0) \gamma_\mu \gamma_5 \frac{\Lambda^P}{2} \Psi(0) | \Phi(P) \rangle = P_\mu f_P, \tag{20}$$

where  $|\Phi(P)\rangle$  is the pseudoscalar meson state vector,  $\Lambda^P$  are matrices acting in flavor space, and  $\Psi$  is a color triplet and flavor multiplet of Dirac spinors. For the  $K^-$  meson, for example, the relevant flavor matrix is, with  $\{\lambda_{ij}\}_{i=1}^8$  the Gell-Mann matrices,

$$\Lambda^{K^-} = \frac{1}{\sqrt{2}} (\lambda_4 + i\lambda_5) = \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{21}$$

which gives  $\langle 0 | \bar{\Psi}_u(0) \gamma_\mu \gamma_5 \Psi_s(0) | \Phi_{K^-}(P) \rangle = \sqrt{2} P_\mu f_{K^-}$ . Thus, the decay constants for the pseudoscalar meson solutions to the BSE given in Eq. (3) are defined by

$$\sqrt{2} P_\mu f_M = \langle 0 | \bar{\Psi}_{f_2}(0) \gamma_\mu \gamma_5 \Psi_{f_1}(0) | \Phi_M(P) \rangle. \tag{22}$$

To obtain an expression in terms of the Bethe-Salpeter amplitude, we note that the unamputated BS wave function

$$\chi(p, P) = S_{f_1}(p+\xi P) \Gamma(p, P) S_{f_2}[p-(1-\xi)P], \tag{23}$$

can be expressed as

$$\begin{aligned}
(2\pi)^4 \delta^4(p-q) \chi(p, P) \\
= \int d^4 x d^4 y e^{-iP \cdot [(\xi x + (1-\xi)y)]} e^{-i(q \cdot x - p \cdot y)} \\
\times \langle 0 | \Psi_{f_1}(x) \bar{\Psi}_{f_2}(y) | \Phi(P) \rangle.
\end{aligned} \tag{24}$$

(For a color singlet bound state,  $\chi(p; P)$  is diagonal in color space.) Multiplying both sides by  $\gamma_5 \gamma \cdot P$ , taking the matrix trace throughout, evaluating the integrals over  $p$  and  $q$ , and using Eq. (22) one obtains

$$\begin{aligned}
P^2 f_M &= \frac{N_c}{\sqrt{2}} \int \frac{d^4 p}{(2\pi)^4} \text{tr}_D \{ \gamma_5 \gamma \cdot P S_{f_1}(p+\xi P) \Gamma(p, P) \\
&\quad \times S_{f_2}[p-(1-\xi)P] \},
\end{aligned} \tag{25}$$

which provides the relation between the Bethe-Salpeter amplitude and the canonically defined meson decay constant. In this equation  $\Gamma$  is normalized according to Eq. (19).

### B. Dressed quark propagators

The separable *Ansatz* is completely defined once the quark propagators are specified. Following Ref. [9], the scalar and vector parts of the quark propagators are defined in terms of dimensionless functions:

$$\sigma_V^f(s) = \frac{1}{2D} \bar{\sigma}_V^f(x), \quad \sigma_S^f(s) = \frac{1}{\sqrt{2D}} \bar{\sigma}_S^f(x), \quad (26)$$

with  $s=p^2$ ,  $x=s/(2D)$ ,  $D$  is a mass-scale parameter, and where ( $\Lambda=10^{-4}$ ):

$$\begin{aligned} \bar{\sigma}_S^f(x) = & \frac{\bar{m}_f}{x + \bar{m}_f^2} (1 - e^{-2(x + \bar{m}_f^2)}) \\ & + \frac{1 - e^{-b_1^f x}}{b_1^f x} \frac{1 - e^{-b_3^f x}}{b_3^f x} \left( b_0^f + b_2^f \frac{1 - e^{-\Lambda x}}{\Lambda x} \right), \end{aligned} \quad (27)$$

and

$$\bar{\sigma}_V^f(x) = \frac{2(x + \bar{m}_f^2) - 1 + e^{-2(x + \bar{m}_f^2)}}{2(x + \bar{m}_f^2)^2}. \quad (28)$$

Here  $\bar{m}_f = m_f/\sqrt{2D}$ . In this work the  $u$  and  $d$  quarks are considered to be identical, except for their electric charge.

The dressed-quark propagator described by Eqs. (27) and (28) is an entire function in the finite complex  $p^2$  plane and may therefore be interpreted as describing a confined particle [2]. The  $\sim e^{-x}$  form that ensures this is suggested by the algebraic solution of the model DSE studied in Ref. [22], which employed a confining model dressed-gluon propagator and dressed quark-gluon vertex. Furthermore, the behavior of Eqs. (27) and (28) on the spacelike- $p^2$  axis is such that, neglecting  $\ln[p^2]$  corrections associated with the anomalous dimension of the dressed-quark propagator in QCD, which are quantitatively unimportant herein, asymptotic freedom is manifest. In Eq. (27) the term  $\sim 1/x^2$  allows for the representation of dynamical chiral symmetry breaking and the  $\sim m/x$  term represents explicit chiral symmetry breaking.

In Ref. [9] the five parameters  $\{m_u, b_0^u, \dots, b_3^u\}$  in Eqs. (27) and (28) were varied in order to determine whether this model form could provide a good description of the pion observables:  $f_\pi$ ;  $m_\pi$ ;  $\langle \bar{q}q \rangle$ ;  $r_\pi$ ; the  $\pi$ - $\pi$  scattering lengths and partial wave amplitudes; and the electromagnetic pion form factor. A very good fit was found with the  $u$ -quark parameter values listed in Eq. (29):

	$u$ quark	$s$ quark
$\bar{m}_f$	0.00897	0.224
$b_0^f$	0.131	0.105
$b_1^f$	2.90	2.90
$b_2^f$	0.603	0.740
$b_3^f$	0.185	0.185

The scale is set with  $D=0.160 \text{ GeV}^2$ . This same model also provides a good description of the  $\gamma^* \pi \rightarrow \gamma$  [10] and  $\gamma \pi^* \rightarrow \pi \pi$  [11] transition form factors.

Dyson-Schwinger equation studies [23] indicate that while it is a good approximation to represent the  $u$  and  $d$  quarks by the same propagator, this is not true for the  $s$  quark. For example; contemporary theoretical studies suggest that  $2m_s/(m_u + m_d) \sim 17-25$  [24] and  $\langle \bar{s}s \rangle \sim 0.5-0.8 \langle \bar{u}u \rangle$  [25], which is a nonperturbative difference.

In Ref. [9], with this in mind, the model forms in Eqs. (27) and (28) were employed in a study of the kaon observables:  $f_K$ ;  $\langle \bar{s}s \rangle$ ;  $r_{K^0}$ ;  $r_{K^\pm}$ ; and the electromagnetic form factors of the charged and neutral kaon. The sensitivity of these observables to  $\bar{m}_s$  and  $\langle \bar{s}s \rangle$  was too weak for an independent determination and therefore  $\bar{m}_s = 25\bar{m}_u$  and  $b_0^s = 0.8b_0^u$ , which ensures  $\langle \bar{s}s \rangle = 0.8\langle \bar{u}u \rangle$ , were chosen for consistency with other theoretical estimates. The parameter  $b_2^s$  was allowed to vary to provide a minimal residual difference between the  $u/d$ - and  $s$ -quark propagators and a very good fit to the kaon observables was obtained with the value listed in Eq. (29).

To complete the specification of the constrained separable approximation to the kernel of the Bethe-Salpeter equation, the quantities  $a$  and  $b$  in Eqs. (15) and (16) must be determined. However, using Eqs. (27) and (28) neither  $a$  nor  $b$  is finite. Equations (15) and (16) only yield finite values if the large spacelike- $x$  behavior of  $\bar{\sigma}_V$  and  $\bar{\sigma}_S$  is such that

$$\bar{\sigma}_V(x) = \frac{1}{x + \bar{m}^2} + O\left(\frac{1}{x^{2+\delta}}\right), \quad \bar{\sigma}_S(x) = \frac{\bar{m}}{x + \bar{m}^2} + O\left(\frac{1}{x^{2+\delta}}\right), \quad (30)$$

for any  $\delta > 0$ . Dynamical chiral symmetry breaking in QCD entails that at large  $x$  (up to corrections  $\sim [\ln x]^{-\gamma}$ ,  $\gamma < 1$ )  $\bar{\sigma}_S(x) = \bar{m}/(x + \bar{m}^2) + O(x^{-2})$  and hence no quark propagator that properly incorporates the momentum dependence at large  $x$  due to dynamical chiral symmetry breaking will yield finite values of  $a$  and  $b$ . (This behavior is tied to the necessary divergence of the quark condensates in QCD; necessary because condensates are related to two-point Schwinger functions evaluated at zero relative Euclidean spatial separation.)

To complete the specification of the constrained, separable *Ansatz* one must therefore incorporate an ultraviolet regularization in the propagator:

$$\begin{aligned} \bar{\sigma}_S^f \text{Reg}(x) = & \frac{\hat{m}_f}{x + \hat{m}_f^2} (1 - e^{-2(x + \hat{m}_f^2)}) \\ & + \frac{1 - e^{-b_1^f x}}{b_1^f x} \frac{1 - e^{-b_3^f x}}{b_3^f x} \\ & \times \left( b_0^f + b_2^f \frac{1 - e^{-b_4 x}}{b_4 x} \right) \frac{1 - e^{-(\epsilon_S^f x)^2}}{(\epsilon_S^f x)^2} \end{aligned} \quad (31)$$

$$\bar{\sigma}_V^f \text{Reg}(x) = \frac{2(x + \hat{m}_f^2) - e^{-\epsilon_V^2(x + \hat{m}_f^2)^2} + e^{-2(x + \hat{m}_f^2)}}{2(x + \hat{m}_f^2)^2}, \quad (32)$$

which introduces three new parameters:  $\epsilon_V$ ,  $\epsilon_S^u$ ,  $\epsilon_S^s$ , that are not determined by the studies of Ref. [9]. The parameter  $\epsilon_V=0.1$  is chosen so as to ensure that  $\bar{\sigma}_V^f \text{Reg}$  are numerically good approximations to  $\bar{\sigma}_V^f$  on the domain  $0 < x < 3$ ; our results are not sensitive to the domain  $x > 3$ . It is not varied but we have established that our results are insensitive to it, i.e., that changes can be absorbed into a change in  $\epsilon_S^f$ . The regu-

TABLE I. Values of the fitting parameters,  $\epsilon_S^{f_i}$  and  $\hat{m}_{f_i}$ , used in constructing the constrained, separable ansatz; and the values  $a_{f_i}$  and  $b_{f_i}$ , defined in Eqs. (15) and (16), calculated using them. The parameters  $\epsilon_S^u$  and  $\hat{m}_{u/d}$  are chosen so as to fit  $m_\pi = 137.5$  MeV and  $f_\pi = 92.4$  MeV; the parameters  $\epsilon_S^s$  and  $\hat{m}_s$  so as to fit  $m_K = 493.6$  MeV and  $f_K = 113$  MeV. The values of  $\hat{m}_{f_i}$  listed here correspond to  $m_{u/d} = 4.59$  MeV and  $m_s = 112$  MeV. See Eq. (34) and associated text for further details.

$f_i$	$\epsilon_S^{f_i}$	$\hat{m}_{f_i}$	$a_{f_i}^{\text{calc}}$ GeV <sup>2</sup>	$b_{f_i}^{\text{calc}}$ GeV <sup>2</sup>
$u/d$	0.482	0.00811	0.0413	0.0281
$s$	0.580	0.198	0.0385	0.0426

larization parameters modify the large- $p^2$  behavior of the propagator, which entails that the light-quark mass values must be refit ( $\bar{m}_q \rightarrow \hat{m}_q$ ).

Equations (14)–(16), (18), (29), (31), and (32) completely specify the constrained, confining, separable ansatz for the ladder kernel of the Bethe-Salpeter equation. The numerical studies proceed by varying the four parameters  $\hat{m}_f$  and  $\epsilon_S^f$  in order to fit  $f_{\pi/K}$  and  $m_{\pi/K}$  and then predicting the ground-state spectrum of octet mesons. Diquark systems are also studied.

### III. MESONS

The Bethe-Salpeter equation considered for a bound state of a quark of flavor  $f_1$  and an antiquark of flavor  $\bar{f}_2$  is

$$\Gamma(p, P) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu S_{f_1}(q + \xi P) \times \Gamma(q, P) S_{f_2}[q - (1-\xi)P] \gamma_\mu. \quad (33)$$

In this equation  $\Delta(p-q)$  is obtained from Eq. (18) with  $S_{f_i}$  obtained from Eqs. (31) and (32) using the parameters in Eq. (29) and Table I. This equation is solved in each channel as an eigenvalue problem of the form  $K\Gamma = \lambda(P^2)\Gamma$ , with the bound-state mass identified from  $\lambda(P^2 = -M^2) = 1$ .

#### A. Scalar and pseudoscalar mesons

##### 1. $f_1 = u/d = f_2$

In this case the requirement of charge conjugation invariance for the neutral mesons entails  $\xi = 1/2$ . The form of the charge parity  $C = \pm$ ,  $f_1 = u/d = f_2$  Bethe-Salpeter amplitudes,  $\Gamma_C$ , obtained as solutions at the mass-shell point  $P^2 = -M^2$  are given in Eqs. (B1)–(B4).

TABLE II. Calculated meson masses compared with experimental values [24], when known. The column labeled with the superscript “Dom” means that the quantity was calculated using only the leading Dirac amplitude, e.g.,  $\Gamma_\pi(p, P) \propto i\gamma_5 G_u(p^2)$  for the pseudoscalar; “unbound” means that in ladder approximation our constrained, separable Ansatz does not yield a stable bound state in the channel under consideration.

	$m_M^{\text{calc}}$ GeV	$m_M^{\text{calc Dom}}$ GeV	Expt.
$\pi(0^{-+})$	0.139 (fit)	0.116	$\pi^\pm(140), \pi^0(135)$
$f_0/a_0(0^{++})$	0.715	0.743	$f_0(980)/a_0(982)$
$0^{+-}$	1.082	1.092	Not seen
$0^{--}$	1.319	1.299	Not seen
$K$	0.494 (fit)	0.412	$K^\pm(494), K^0(498)$
$K_0^*$	Unbound	Unbound	$K_0^*(1430)$
$\eta(\theta_P = 5^\circ)$	0.549	0.472	$\eta(547)$
$\eta(\theta_P = 0^\circ)$	0.513	0.441	
$\omega/\rho$	0.736	0.755	$\omega(782)/\rho(770)$
$a_1/f_1$	1.34	1.37	$a_1(1260)/f_1(1285)$
$K^*$	0.854	0.866	$K^*(892)$
$K_1$	1.39	1.39	$K_1(1270), K_1(1400)$
$\phi(\bar{s}s \ 1^-)$	0.950	0.957	$\phi(1020)$
$\bar{s}s \ 1^+$	1.60	1.60	$f_1(1510)$

These equations expose a shortcoming of separable Ansatz: the pseudoscalar and pseudovector pieces of the pseudoscalar Bethe-Salpeter amplitude are characterized by the same function, which is not the case in general.

The calculated eigenvectors are given in Eq. (B6) and the bound-state masses in Table II.

The separable Ansatz for the kernel of the Bethe-Salpeter equation yields  $m_{0^{++}} = 0$  when  $\bar{m}_{u/d} = 0$ . This is a necessary consequence of the equivalence between the flavor-octet pseudoscalar BSE and the quark DSE in this chiral limit [4], which is preserved in the approach described herein and discussed in detail in Ref. [3].

One might be tempted to conclude from Eq. (B6) that for  $C = +$  states the leading Dirac component of the amplitude dominates; i.e., the pure  $\gamma_5$  component dominates for the  $0^{-+}$  state and the pure  $I_D$  component for the  $0^{++}$  state. Indeed, it is an often used approximation to neglect subleading Dirac components of the Bethe-Salpeter amplitude in ground-state studies using the Bethe-Salpeter equation. Considering Tables II and III one observes that while this is a good approximation for the heavy  $0^{++}$  state, it represents an erroneous conclusion for the light  $0^{-+}$  state, for which the subleading, axial-vector component provides 17% of the mass and 39% of the decay constant. This feature is also seen in Ref. [3].

TABLE III. Calculated weak decay constants compared with experimental values [24]. The superscript “Dom” has the same meaning as in Table II.

	$f_M^{\text{calc}}$ GeV	$f_M^{\text{calc Dom}}$ GeV	Expt.
$\pi$	0.0924 (fit)	0.056	$\pi^+(0.0924)$
$K^\pm$	0.113 (fit)	0.76	$K^+(0.113)$
$\eta(\theta = 5^\circ)$	0.114	0.086	$0.094 \pm 0.007$ or $0.091 \pm 0.006$
$\eta(\theta = 0^\circ)$	0.111	0.082	

Table II shows that the separable ansatz for the BSE kernel yields a large  $m_{0^{++}} - m_{0^{-+}}$  splitting without fine tuning, thus reproducing this characteristic feature of the strong interaction spectrum. The  $0^{++}$  state can be identified with the  $a_0(980)$  meson. The discrepancy between the calculated and observed masses is consistent with the contention that this state involves a considerable  $\bar{K}-K$  admixture, which can be represented as a contribution to the Bethe-Salpeter kernel but is absent in ladder approximation.

No  $J^{PC}=0^{+-}$  states have been observed in the strong interaction spectrum. However, in general, as observed in Ref. [27], the Bethe-Salpeter equation admits solutions of this type. The amplitudes for such solutions characteristically differ from their  $C=+$  counterparts by the factor  $p \cdot P$  which is odd under charge conjugation. Such states have no analog in quantum mechanics since, for equal-mass constituent particles on shell,  $p \cdot P=0$ .

We expect that such exotic state solutions of the full Bethe-Salpeter equation would be heavy, certainly above the mass of the  $a_1$  meson solution, or unbound. However, the ladder approximation is known to be dynamically inadequate for the  $0^{++}$  states (it is too attractive), and a similar inadequacy for scalar  $C=-$  states would not be surprising. One may identify this as the reason why it is not usual for the  $C=-$  states to be investigated or reported in Bethe-Salpeter studies. For completeness, and a fuller elucidation of the shortcomings of the ladder truncation and the present separable *Ansatz*, in Table II we present masses obtained for the  $0^{--}$  and  $0^{+-}$  states.

One observes that  $m_{0^{--}} \sim 10 m_{0^{-+}}$  and  $m_{0^{+-}} \sim 2 m_{0^{++}}$ , which indicates that with this simple *Ansatz*, the reversal in  $C$  parity introduces a significant repulsive effect. It is, however, too weak and the calculated values are not large enough to be consistent with the observed strong interaction spectrum.

The methods of Ref. [3] indicate that the next order contributions to the kernel [ $\mathcal{O}(g^4)$  in a quark-gluon skeleton graph expansion] for the scalar channel are all predominantly repulsive. This is to be contrasted with the flavor-octet pseudoscalar and vector channels where there are both repulsive and attractive corrections that have a significant amount of cancellation. This highlights the fact that progress in understanding the complex issues of scalar states with  $C=+$  in the Bethe-Salpeter approach certainly requires studies beyond the ladder truncation, including, for example, possibly important meson-loop contributions to the open decay channels. It is our opinion that the same is true in the study of the associated  $C=-$ , exotic states.

## 2. $f_1=u/d, f_2=s$

The Bethe-Salpeter equation for  $\bar{u}-s$  states is Eq. (33) with  $f_1=u/d, f_2=s$ . Consider first the pseudoscalar (kaon) channel. A value of  $\xi$  is determined by ensuring that the electric charge of the  $K^0$  is zero in impulse approximation [9]. The value obtained in Ref. [9] with empirical Bethe-Salpeter amplitudes is  $\xi=0.49$  ( $\approx 0.5$ ), while the present work requires  $\xi=0.56$  ( $\approx 0.5$ ). There is only a weak sensitivity of masses to changes in  $\xi$  of this magnitude throughout this work.

The solution amplitude is given in Eq. (B7) with the calculated eigenvector given in Eq. (B8) and the mass in Table II. We note that there is no charge parity ( $C$ ), symmetry for bound states of distinguishable quarks.

The leading, pseudoscalar Dirac amplitudes ( $\lambda_{1u}, \lambda_{1s}$ ) again appear to be dominant for the kaon, however, as for the pion, the subleading, axial-vector amplitudes ( $\lambda_{3u}, \lambda_{3s}$ ) contribute significantly to the mass (17%) and decay constant (33%).

Each type of covariant in the kaon solution is weighted by two amplitudes that describe the internal momentum dependence in terms of functions that relate to the dressed propagators of the  $u/d$  and  $s$  quarks. These are found to have approximately equal influence in the solution. For example, from Eq. (B8) and Table I one calculates that  $\lambda_1/b_u=9.4 \text{ GeV}^{-1}$  and  $\lambda_2/b_s=9.2 \text{ GeV}^{-1}$ . This means that, using the constrained, separable *Ansatz*, the kaon Bethe-Salpeter amplitude for the pseudoscalar covariant is an approximately even mixture of the  $u$ - and  $s$ -quark mass functions.

It is clear from Table I that

$$\frac{2 \hat{m}_s}{\hat{m}_u + \hat{m}_d} = 24.4, \quad (34)$$

which is essentially the same as the ratio obtained from the mass values in Eq. (29) and is in the range (17–25) suggested by other theoretical analyses [24]. With  $D=0.160 \text{ GeV}^2$ ,  $\hat{m}_{u/d}=0.00811$  corresponds to  $m_{u/d}=4.6 \text{ MeV}$  and  $\hat{m}_s=0.198$  corresponds to  $m_s=112 \text{ MeV}$ . These values *should not* be compared directly with values of  $m_{u/d,s}^{\mu^2=1 \text{ GeV}^2}$  quoted by other authors because the regularization of the vacuum condensates employed herein, via the parameters  $\epsilon_V$  and  $\epsilon_S^f$  in Eqs. (31) and (32), is unconventional and enters through the quantities  $a$  and  $b$  in Eqs. (15) and (16). The ratio, Eq. (34), is likely to be less sensitive to this difference and therefore provides a meaningful point of comparison.

The dressed-quark propagators we employ are confining, with the dressed-quark mass being a function of  $p^2$ ,  $M(p^2)$ , such that there is no dressed-quark mass pole, i.e., no solution of the equation  $p^2[A^f(p^2)]^2 + [B^f(p^2)]^2 = 0$ . A simple estimate of the value of the mass function that is most important in calculations of meson observables is obtained from the solution of  $-p^2[A^f(p^2)]^2 + [B^f(p^2)]^2 = 0$ , which might be called the Euclidean constituent-quark mass,  $M_E^f$ . With the parameter values used herein,  $M_E^u=315 \text{ MeV}$  and  $M_E^s=397 \text{ MeV}$ .

Our calculations yield no true  $0^+$  eigenstate with a mass less than  $2 \text{ GeV}$ , which we consider to be the upper limit for the present approach. The condition for an eigenstate was closest to being satisfied at a mass of  $1.18 \text{ GeV}$ . This is again consistent with a large  $m_{0^{++}} - m_{0^{-+}}$  splitting without fine tuning. This  $0^+$  state might be identified with the  $K_0^*(1430)$ . Such an identification would suggest that this state, like the  $a_0(980)$ , has a sizeable coupling to other channels, which contribute to its mass, i.e., that the ladder kernel is inadequate to properly describe this channel.

## 3. $\eta$ meson

Ladder approximation is inadequate to properly study the  $\eta$ - $\eta'$  complex. A minimal extension that can dynamically

couple the flavor octet and singlet channels is the inclusion of timelike-gluon exchange diagrams. This is not considered here.

Instead we study

$$\begin{aligned} \Gamma_\eta(p, P) = & -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \left[ \frac{1}{3} (\cos\theta_P - \sqrt{2} \sin\theta_P)^2 \right. \\ & \times \Delta_u(p-q) \gamma_\mu S_u(q + \frac{1}{2}P) \Gamma_\eta(q, P) \\ & \times S_u(q - \frac{1}{2}P) \gamma_\mu \\ & + \frac{1}{3} (\sqrt{2} \cos\theta_P + \sin\theta_P)^2 \Delta_s(p-q) \\ & \left. \times \gamma_\mu S_s(q + \frac{1}{2}P) \Gamma_\eta(q, P) S_s(q - \frac{1}{2}P) \gamma_\mu \right], \end{aligned} \quad (35)$$

with

$$\Delta_f(p-q) = G_f(p^2) G_f(q^2) + p \cdot q F_f(p^2) F_f(q^2). \quad (36)$$

Equation (35) is the projected Bethe-Salpeter equation for the meson whose flavor structure is

$$F_\eta = \lambda^8 \cos\theta_P - \lambda^0 \sin\theta_P, \quad (37)$$

with  $\lambda^0 = \sqrt{2/3} \text{diag}(1, 1, 1)$  and  $\theta_P$  an octet-singlet mixing angle. The exact kernel of the Bethe-Salpeter equation would lead to a prediction for  $\theta_P$ .

With the kernel considered herein,  $\theta_P$  is treated as an external parameter on which the mass and other properties of the  $\eta$  meson depend. For example, in this case the expressions for the normalization of the Bethe-Salpeter amplitude, Eq. (19), and the decay constant, Eq. (25), are  $\theta_P$  dependent. The modified forms are given in Eqs. (B9) and (B10), respectively.

The form of the positive charge parity solution of Eq. (35) is given in Eq. (B11). The calculated mass, decay constant, and eigenvector, at a number of values of  $\theta_P$ , are given in Eq. (B12). The experimental values of the mass and decay constant are given in Table II.

In this case the subleading Dirac amplitudes contribute  $\sim 14\%$  to the mass and  $\sim 26\%$  to the decay constant.

The constrained separable *Ansatz* favors a small positive value for the mixing angle,  $\theta_P$ . This can be compared with  $\theta_P = -10^\circ$  estimated in Ref. [24]. As remarked therein, however, there are large uncertainties in this value.

The  $\eta'$  meson can be studied via the projection of the Bethe-Salpeter equation orthogonal to that in Eq. (35), which is obtained from this equation under  $\theta_P \rightarrow \theta_P - \pi/2$ . As remarked above, one expects timelike-gluon exchange, forbidden in the flavor-octet channel, to be important in this mainly singlet channel. The results in Eq. (B12), which one might compare with the experimental values of  $M_{\eta'}^{\text{expt}} = 958 \text{ MeV}$  and  $f_{\eta'} = 89.1 \pm 5$  or  $77.8 \pm 5 \text{ MeV}$ , may be interpreted as a guide to the importance of such contributions in this channel and emphasize the necessity to go beyond ladder approximation for the  $\eta'$  state.

## B. Vector and axial-vector mesons

The ladder approximation to the Bethe-Salpeter equation for vector and axial vector mesons is

$$\begin{aligned} \Gamma_\nu(p, P) = & -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu S_{f_1}(q + \xi P) \\ & \times \Gamma_\nu(q, P) S_{f_2}[q - (1 - \xi)P] \gamma_\mu, \end{aligned} \quad (38)$$

which is identical to Eq. (33) except that the Bethe-Salpeter amplitude carries a Lorentz index. On-shell vector and axial-vector bound states are transverse:

$$P_\nu \Gamma_\nu(p, P) = 0, \quad (39)$$

which constrains the general form of the Bethe-Salpeter amplitude.

### 1. $f_1 = u/d = f_2$

The most general form of the vector meson Bethe-Salpeter amplitude when using the separable ansatz is given in Eq. (B14) and that for the axial-vector meson is given in Eq. (B16). These equations expose another shortcoming of separable *Ansätze*: the vector and axial-vector meson Bethe-Salpeter amplitudes are characterized by the same functions as the pseudoscalar mesons, which is not true in general. In Ref. [6], for example, the vector meson amplitudes were found to be much narrower in momentum space.

The ladder approximation does not distinguish between  $I=0$  and  $I=1$ , hence the vector channel corresponds to both the  $\omega$  and  $\rho$  mesons. Similarly, the axial-vector channel corresponds to the  $f_1$  and  $a_1$  mesons.

The calculated mass for these states is presented in Table II and the eigenvectors in Eq. (B17). With pion and kaon physics used to fix the parameters of the quark propagators as described in Sec. II, these results are predictions. The subleading Dirac amplitudes contribute very little to the  $J=1$  meson masses.

The relevant experimental value to compare the vector meson with is  $M_\omega^{\text{expt}} = 782 \text{ MeV}$ , since it is known that pion loop dressing will lower the  $\rho$ -meson mass while having little effect on the  $\omega$  meson [15]. A recent study of this effect [16] yields  $M_\omega - M_\rho = 21.0 \text{ MeV}$ .

These results in the  $u-d$  sector indicate that the  $u/d$  quark propagator parameters, previously set by pion physics, have produced a separable BSE kernel that captures the dominant physics for the ground-state vector and axial vector channels.

### 2. $f_1 = u/d, f_2 = s$

The general form of the Bethe-Salpeter amplitude for the  $u\bar{s}$  meson, which corresponds to the  $J^P = 1^- K^{*+}$  meson, is given in Eq. (B18). We choose  $\xi$  so as to ensure the neutrality of the  $K^{*0}$  meson, which produces  $\xi = 0.49 \approx 0.5$ .

The form of the amplitude for  $J^P = 1^+$ , which is a nearly equal mix of  $K_1(1270)$  and  $K_1(1400)$ , is simply  $\gamma_5$  times this. The corresponding choice for  $\xi$  yields  $\xi = 0.50$ . The calculated masses are listed in Table II and the eigenvectors in Eq. (B19).

The subleading Dirac amplitudes contribute little to the masses.



### 3. $f_1 = s = f_2$

The  $J^P = 1^{--} \bar{s}s$  state, identified with the  $\phi$  meson, is computed in exactly the same manner as the  $\omega/\rho$  meson except for the replacement of the  $u$ -quark propagator with that for the  $s$  quark. The Bethe-Salpeter amplitude for the  $\phi$  and for the  $1^{++}$  state, identified with the  $f_1(1510)$  meson, are described in Appendix II B 3.

The calculated masses are listed in Table II and the eigenvectors in Eq. (B20). The subleading Dirac amplitudes are again unimportant.

### 4. $J=1$ summary

We observe that the  $J=1$  meson spectrum is satisfactorily reproduced. These higher-mass states explore a larger domain in the complex quark-momentum plane than do the pion and kaon, which are used to constrain the separable ansatz for the ladder kernel. This is an indication that a successful description of a subset of hadronic observables can translate into a uniformly good description of a broad range of phenomena, which is a feature that underlies many applications of this framework and emphasizes the utility of studies such as that of Ref. [20].

## IV. DIQUARK CORRELATIONS

The derivation of the homogeneous Bethe-Salpeter equation from the inhomogeneous equation for the two-body  $\mathcal{T}$  matrix proceeds under the assumption that there exists a bound-state pole in the channel under consideration. In QCD, one expects that confinement ensures the absence of such poles in the quark-quark  $\mathcal{T}$  matrix and hence that there are no solutions to the homogeneous Bethe-Salpeter equation in any color-antitriplet quark-quark (diquark) channel. This is supported by the studies of Ref. [3], which indicate, however, that one must proceed beyond ladder approximation to obtain this result. In ladder approximation one finds bound-state, diquark solutions. This is a defect of the truncation.

Studies of the nucleon as a bound state of three dressed quarks using the covariant Fadde'ev equation have been undertaken [18]. The appearance of the pole in the ladder approximation to the homogeneous, quark-quark Bethe-Salpeter equation was used therein to simplify the three-body problem, i.e., to re-express it as an effective two-body, quark-diquark problem. This technique can also be said to underlie the study of Ref. [19]. Presently, the only justification for this *Ansatz* is the simplicity it introduces into the problem.

Accepting this approach for the present it is then important to identify those diquark correlations that contribute significantly to a given three-body bound state. As a guide one might assume that those diquarks whose mass is greater than that of the three-body bound state under consideration would contribute little to the three-body ground-state mass. Such studies of the “ $u/d$ -diquark spectrum” have been reported in Refs. [26,28]. Herein we extend these studies to  $SU_f(3)$ .

The ladder approximation to the homogeneous Bethe-Salpeter equation for a diquark correlation involving quarks of flavor  $f_1$  and  $f_2$  is

TABLE IV. Calculated diquark effective masses. The superscript “Dom” has the same meaning as in Table II.

$f_1$	$f_2$	$J^P$	$M$ GeV	$M^{\text{Dom}}$ GeV
$u/d$	$u/d$	$0^+$	0.737	0.653
$u/d$	$u/d$	$0^-$	1.50	1.52
$u/d$	$s$	$0^+$	0.882	0.786
$u/d$	$s$	$0^-$	Unbound	Unbound
$u/d$	$u/d$	$1^+$	0.949	0.958
$u/d$	$u/d$	$1^-$	1.47	1.48
$u/d$	$s$	$1^+$	1.05	1.05
$u/d$	$s$	$1^-$	1.53	1.53
$s$	$s$	$1^+$	1.13	1.13
$s$	$s$	$1^-$	1.64	1.64

$$\Gamma_{\mathfrak{z}}(p, P) = - \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu \frac{\lambda^a}{2} S_{f_1}(q + \xi P) \Gamma_{\mathfrak{z}}(q, P) \times \{S_{f_2}[-q + (1-\xi)P]\}^T \left( \gamma_\mu \frac{\lambda^a}{2} \right)^T, \quad (40)$$

where  $T$  denotes matrix transpose. The study of such correlations is simplified if one defines

$$\Gamma_{\mathfrak{z}}^C(p, P) \equiv \Gamma_{\mathfrak{z}}(p, P) C, \quad (41)$$

where  $C = \gamma_2 \gamma_4$  is the charge conjugation matrix. It follows from Eq. (40) that this auxiliary amplitude satisfies

$$\Gamma_{\mathfrak{z}}^C(p, P) = - \frac{2}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu \times S_{f_1}(q + \xi P) \Gamma_{\mathfrak{z}}^C(q, P) S_{f_2}[q - (1-\xi)P] \gamma_\mu. \quad (42)$$

It is immediately obvious that Eq. (42) is identical to Eq. (33) but for a reduction in the (purely attractive) coupling strength:  $4/3 \rightarrow 2/3$ . This observation in Ref. [28] entailed the result that the mass of the scalar ( $u-d$ ) diquark is greater than  $m_\pi$ ; and that of the vector ( $u-u$ ), ( $u-d$ ), and ( $d-d$ ) correlations is greater than the mass of the  $a_1(1280)$  meson. (This result is true in an arbitrary covariant gauge and independent of the form of the gluon propagator. However, it is peculiar to ladder approximation. As discussed in Ref. [3], any other truncation of the kernel of the Bethe-Salpeter equation introduces repulsive terms that eliminate the diquark pole.)

The masses produced in various diquark channels by the present ansatz for the ladder kernel are listed in Table IV. Further details and a brief discussion of these results and the associated Bethe-Salpeter amplitudes are provided in Appendices B3 and B4.

## V. SUMMARY AND CONCLUSIONS

We have constructed a crude, confining, separable *Ansatz* for the ladder kernel of the two-body Bethe-Salpeter equation (BSE) from the phenomenologically efficacious  $u/d$  and  $s$  dressed-quark propagators of Ref. [9]. We have emphasized that no connection can be made between this crude

kernel and the solution of the Dyson-Schwinger equation for the dressed-gluon propagator.

A very good description of the ground-state,  $SU_f(3)$ , flavor-octet pseudoscalar, vector and axial-vector meson spectrum was obtained. However, scalar mesons ( $J^{PC}=0^{++}$ ),  $0^{+-}$  exotics and the pseudoscalar  $\eta-\eta'$  complex were poorly described. We argued that this is a defect of ladder truncation that can be understood as the result of neglecting higher-order terms in the kernel. The present crude construction therefore provides a useful demonstration of the reliability and extent of applicability of the rainbow-ladder truncation of the quark-DSE/meson-BSE complex.

We found that in the flavor-octet pseudoscalar meson channel the subleading Dirac components of the Bethe-Salpeter amplitude, i.e., those terms whose Dirac matrix structure is more than just  $\gamma_5$ , provide quantitatively important contributions to the mass ( $\sim 15\%$  effects) and weak decay constant ( $\sim 35\%$  effects). These terms are unimportant in the vector and axial-vector meson channels.

We saw that separable *Ansätze* have a number of shortcomings. In the pseudoscalar channel one finds that the  $\gamma_5$  and  $\gamma_5 \gamma \cdot P$  components of the meson Bethe-Salpeter amplitude are characterized by the same function,  $B(p^2)$ , which is not true in general. One also finds that the dominant components in the Bethe-Salpeter amplitudes of the vector and axial-vector mesons are characterized by the same functions that characterize these components of the pseudoscalar mesons,  $B(p^2)$ . More sophisticated studies indicate that the vector meson amplitudes are narrower in momentum space. This indicates that the amplitudes we have obtained should be used with caution in the calculation of, for example, meson-meson scattering processes.

The shortcomings notwithstanding, there are areas of study in hadronic physics for which the presently provided Bethe-Salpeter amplitudes have significantly greater dynamical justification than that of currently used approximations. For example, hadronic coupling constants such as  $g_{\rho\pi\pi}$  [16] and  $g_{\gamma\pi\rho}$  [17] have been reproduced from the  $\bar{q}q$  structure of the mesons in terms of a single dominant Dirac covariant if the amplitude is allowed some phenomenological freedom. A more realistic treatment is facilitated by the present work.

The ladder kernel also has the defect that it is purely attractive in both the color-singlet  $\bar{q}-q$  and color-antitriplet  $q-q$  channels. This entails that it yields bound color-antitriplet diquarks. This is a peculiarity of ladder approximation. Measuring ‘‘order’’ by the number of dressed-gluon lines in the Bethe-Salpeter kernel, ladder approximation is the lowest order kernel. Repulsive terms appear at every higher order. It has been shown [3] that in the flavor-octet pseudoscalar and vector meson channels, these repulsive terms are cancelled by attractive terms of the same order. This explains why ladder approximation is phenomenologically successful in these channels. In the color-antitriplet diquark channel the algebra of  $SU_c(3)$  entails that the repulsive terms are stronger; they are not completely cancelled and eliminate the diquark bound states [3].

The artificial diquark spectrum we obtain is nevertheless of contemporary interest because there have been a number of studies of the covariant, three-body Fadde’ev equation that use the existence of diquark poles in the quark-quark  $\mathcal{T}$  matrix to reduce this problem to a two-body, quark-diquark

bound-state problem. Our constrained, separable *Ansatz* indicates that such studies of the baryon should include  $SU_f(3)$  scalar and pseudovector diquarks, since these are low in mass, but can neglect pseudoscalar and vector diquarks.

The diquark results might be used in the following way. The study of Ref. [3] suggests that, even though color-antitriplet states are not bound, one may associate an inverse correlation length,  $M$ , with each channel; the ‘‘bound-state mass’’ providing an estimate of this. One might then construct a ‘‘pseudopole’’ representation of the quark-quark  $\mathcal{T}$  matrix (for example:  $\sim \sum_n a_n \{1 - \exp(-[P^2 + M_n^2])\} / [P^2 + M_n^2]$ ), which would not entail asymptotic (unconfined) diquark states but would provide for a simplification of the covariant, three-body Fadde’ev equation.

Finally, this study shows that in order to directly connect hadron phenomena with the dressed-gluon propagator,  $D_{\mu\nu}(k)$ , one must start with a form of  $D_{\mu\nu}(k)$ , as in Ref. [20]. Other approaches, while they may provide a useful phenomenology, efficacious in that it correlates many observables via few parameters, can only loosely constrain  $D_{\mu\nu}(k)$  and hence the nature of the quark-quark interaction in the infrared.

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## APPENDIX A: BETHE-SALPETER EQUATION FOR EQUAL MASS QUARKS

Here we present details of the solution of the ladder approximation to the Bethe-Salpeter equation in the case of equal mass quarks ( $f_1=f_2$ ) using a separable *Ansatz* for the kernel.

We begin with Eq. (3) subject to Eqs. (4) and (6) with  $f_1=f_2$  and  $\xi=1/2$ :

$$\begin{aligned} \Gamma(p, P) = & -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu S(q + \tfrac{1}{2}P) \Gamma(q, P) \\ & \times S(q - \tfrac{1}{2}P) \gamma_\mu. \end{aligned} \quad (A1)$$

The general form of the scalar and pseudoscalar meson amplitudes is [27]

$$\begin{aligned} \Gamma^{\text{scalar}}(q, P) = & g_I(q^2, P^2, q \cdot P) I + [g_P(q^2, P^2, q \cdot P) P_\mu \\ & + g_u(q^2, P^2, q \cdot P) u_\mu(q)] i \gamma_\mu, \end{aligned} \quad (A2)$$

and

$$\Gamma^{\text{pseud}}(q, P) = g_5(q^2, P^2, q \cdot P) \gamma_5 + [g_{P5}(q^2, P^2, q \cdot P) P_\mu + g_{u5}(q^2, P^2, q \cdot P) u_\mu(q)] i \gamma_\mu \gamma_5, \quad (\text{A3})$$

where

$$u_\mu(p, \hat{P}) = \frac{p_\mu + p \cdot \hat{P} \hat{P}_\mu}{p^2 + (p \cdot \hat{P})^2}, \quad |u(p)| = \sqrt{u(p, \hat{P})^2},$$

$$u_\mu(p) = \frac{u_\mu(p, \hat{P})}{|u(p)|}, \quad (\text{A4})$$

with  $\hat{P}_\mu$  [ $\hat{P}^2 = -1$ ] the direction vector associated with  $P_\mu$ .

In Feynman-like gauge it follows from the Fierz identity that there is no piece proportional to  $[\gamma \cdot P, \gamma \cdot q]$ . For mesons which are even (odd) under charge conjugation,  $g_I$ ,  $g_u$ ,  $g_5$ , and  $g_{P5}$  are even (odd) functions and  $g_P$  and  $g_{u5}$  odd (even) functions of  $q \cdot P$ .

Defining  $k_\mu = q_\mu + \frac{1}{2} P_\mu$  and  $l_\mu = q_\mu - \frac{1}{2} P_\mu$  one has

$$P \cdot u(q) = 0, \quad k \cdot u(q) = l \cdot u(q) = 1,$$

$$k \cdot u(p) = l \cdot u(p) = q \cdot u(p). \quad (\text{A5})$$

Multiplying Eq. (A2) by  $I$ ,  $\gamma \cdot P$  or  $\gamma \cdot u(p)$  and taking traces one projects out a set of coupled integral equations for the scalar meson amplitudes. Defining

$$f(p, P) = g_I(p, P), \quad W(p, P) = i M g_P(p, P),$$

$$U(p, P) = |u(p)| g_{u5}(p, P), \quad (\text{A6})$$

where  $i M = \sqrt{P^2}$ , these equations take the following simple form:

$$f(p) = \frac{16}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p - q) \times [T_{ff} f(q) + T_{fW} W(q) + T_{fU} U(q)],$$

$$W(p) = \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p - q) \times [T_{Wf} f(q) + T_{WW} W(q) + T_{WU} U(q)],$$

$$U(p) = \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p - q) \hat{u}(p, \hat{P}) \cdot \hat{u}(q, \hat{P}) \times [T_{Uf} f(q) + T_{UW} W(q) + T_{UU} U(q)], \quad (\text{A7})$$

where

$$T_{ff}^{\text{scalar}} = k \cdot l |\sigma_V|^2 - |\sigma_S|^2,$$

$$T_{WW}^{\text{scalar}} = (k \cdot l + 2k \cdot \hat{P} l \cdot \hat{P}) |\sigma_V|^2 + |\sigma_S|^2,$$

$$T_{UU}^{\text{scalar}} = \left( k \cdot l - \frac{2}{u(q, \hat{P})^2} \right) |\sigma_V|^2 + |\sigma_S|^2 \quad (\text{A8})$$

$$T_{fW}^{\text{scalar}} = T_{Wf}^{\text{scalar}} = M \Im(\sigma_V^* \sigma_S) + 2i q \cdot \hat{P} \Re(\sigma_V^* \sigma_S),$$

$$T_{fU}^{\text{scalar}} = T_{Uf}^{\text{scalar}} = - \frac{2}{|u(q)|} \Re(\sigma_V^* \sigma_S),$$

$$T_{WU}^{\text{scalar}} = T_{UW}^{\text{scalar}} = 2i \frac{q \cdot \hat{P}}{u(q)} |\sigma_V|^2, \quad (\text{A9})$$

with

$$\sigma_V^* \equiv \sigma_V(k^2), \quad \sigma_V \equiv \sigma_V(l^2), \quad (\text{A10})$$

and similarly for  $\sigma_S$ . In these equations

$$|\sigma_V|^2 \equiv \sigma_V \sigma_V^*, \quad \Re(\sigma_V^* \sigma_S) \equiv \frac{1}{2} (\sigma_V^* \sigma_S + \sigma_V \sigma_S^*),$$

$$\Im(\sigma_V^* \sigma_S) \equiv \frac{1}{2i} (\sigma_V^* \sigma_S - \sigma_V \sigma_S^*). \quad (\text{A11})$$

In terms of the functions

$$f(p, P) = i g_5(p, P), \quad W(p, P) = i M g_{P5}(p, P),$$

$$U(p, P) = |u(p)| g_{u5}(p, P), \quad (\text{A12})$$

the equations for the pseudoscalar states have the form in Eq. (A7) but with the  $T$ 's replaced by

$$T_{ff}^{\text{pseud}} = k \cdot l |\sigma_V|^2 + |\sigma_S|^2,$$

$$T_{WW}^{\text{pseud}} = (k \cdot l + 2k \cdot \hat{P} l \cdot \hat{P}) |\sigma_V|^2 - |\sigma_S|^2,$$

$$T_{UU}^{\text{pseud}} = \left( k \cdot l - \frac{2}{u(q, \hat{P})^2} \right) |\sigma_V|^2 - |\sigma_S|^2, \quad (\text{A13})$$

and

$$T_{fW}^{\text{pseud}} = -T_{Wf}^{\text{pseud}} = -M \Re(\sigma_V^* \sigma_S) + 2i q \cdot \hat{P} \Im(\sigma_V^* \sigma_S),$$

$$T_{fU}^{\text{pseud}} = -T_{Uf}^{\text{pseud}} = - \frac{2}{|u(q)|} \Im(\sigma_V^* \sigma_S),$$

$$T_{WU}^{\text{pseud}} = T_{UW}^{\text{pseud}} = 2i \frac{q \cdot \hat{P}}{u(q)} |\sigma_V|^2. \quad (\text{A14})$$

### 1. Separable Ansatz

The form of the gluon propagator  $\Delta(p - q)$  appearing in Eq. (A7) is not yet specified. Introducing the separable form in Eq. (17) and taking into account the symmetry properties of the functions  $f$ ,  $W$  and  $U$  under  $p \cdot \hat{P} \rightarrow -p \cdot \hat{P}$ , which follow from charge conjugation symmetry, one obtains the following sets of integral equations for the scalar mesons:

*a. Scalar,  $C = +$  mesons*

$$\begin{aligned}
 f(p) &= \frac{16}{3} \int \frac{d^4 q}{(2\pi)^4} G(p^2) G(q^2) \\
 &\quad \times [T_{ff}f(q) + T_{fW}W(q) + T_{fU}U(q)], \\
 W(p) &= \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} p \cdot q F(p^2) F(q^2) \\
 &\quad \times [T_{Wf}f(q) + T_{WW}W(q) + T_{WU}U(q)], \\
 U(p) &= \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} p \cdot q F(p^2) F(q^2) \hat{u}(p, \hat{P}) \cdot \hat{u}(q, \hat{P}) \\
 &\quad \times [T_{Uf}f(q) + T_{UW}W(q) + T_{UU}U(q)]. \quad (A15)
 \end{aligned}$$

*b. Scalar,  $C = -$  mesons*

$$\begin{aligned}
 f(p) &= \frac{16}{3} \int \frac{d^4 q}{(2\pi)^4} p \cdot q F(p^2) F(q^2) \\
 &\quad \times [T_{ff}f(q) + T_{fW}W(q)], \\
 W(p) &= \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} G(p^2) G(q^2) \\
 &\quad \times [T_{Wf}f(q) + T_{WW}W(q)], \\
 U(p) &= 0 \quad (A16)
 \end{aligned}$$

The  $T_{ff}, T_{fW}, \dots$  are given by Eqs. (A8), (A9). There are similar equations for the pseudoscalar mesons.

**2. Form of the solution using the constrained separable Ansatz**

The separable form of the propagator causes the solutions of these equations to be proportional to the functions  $G$  and  $F$ . For the scalar,  $C = +$  meson, for instance, one finds that the solution is of the form

$$\begin{aligned}
 f(p) &= \lambda_f G(p^2), \\
 W(p) &= -i \lambda_W p \cdot \hat{P} F(p^2), \\
 U(p) &= \lambda_U \frac{1}{|u(p)|} F(p^2). \quad (A17)
 \end{aligned}$$

Substituting the above Ansatz into Eqs. (A15) yields a simple matrix equation of the form

$$\begin{pmatrix} \lambda_f \\ \lambda_W \\ \lambda_U \end{pmatrix} = K(M) \begin{pmatrix} \lambda_f \\ \lambda_W \\ \lambda_U \end{pmatrix}, \quad (A18)$$

where  $K(M)$  is a  $3 \times 3$  matrix whose elements are two-dimensional integrals that are completely determined once  $\sigma_V$  and  $\sigma_S$  are specified. This equation is then solved by adjusting the meson mass  $M$  until one of the eigenvalues of  $K$  equals one. This procedure can be implemented by intro-

ducing an eigenvalue,  $\mu(M)$ , on the left-hand side of Eq. (A18); solving for  $\mu(M)$  and the eigenvector at each value of  $M$ ; and repeating the process until one finds  $M$  such that  $\mu(M) = 1$ . At this point one also has the Bethe-Salpeter amplitude for the bound state, which is characterized by the multiplet  $\{\lambda_f, \lambda_W, \lambda_U\}$ .

Bound states of unequal mass quarks [see Eqs. (18) and Sec. III A 2, for example], are not characterized by a charge-conjugation quantum number,  $C$ . In this case the functions  $f$ ,  $W$ , and  $U$  are complex and Eq. (A17) generalizes to forms such as

$$\begin{aligned}
 f(p) &= \lambda_{1u} G_u(p^2) + \lambda_{1s} G_s(p^2) \\
 &\quad + q \cdot \hat{P} [\lambda_{2u} F_u(p^2) + \lambda_{2s} F_s(p^2)] + \dots \quad (A19)
 \end{aligned}$$

In this case the analog of the matrix  $K(M)$  in Eq. (A18) is, in general, a  $12 \times 12$  matrix, which reduces to a  $10 \times 10$  matrix when residual symmetry under  $C$  is taken into account.

**APPENDIX B: BETHE-SALPETER AMPLITUDES IN SEPARABLE APPROXIMATION**

**1. Scalar and pseudoscalar mesons**

*a.  $f_1 = u/d = f_2$*

The most general form for the solutions of Eq. (33) in the scalar and pseudoscalar channels are

$$\Gamma_+^{\text{pseud}}(p, P) = G_u(p^2) [\lambda_f I_D - i \lambda_W \gamma \cdot \hat{P}] i \gamma_5, \quad (B1)$$

$$\begin{aligned}
 \Gamma_+^{\text{scalar}}(p, P) &= G_u(p^2) \lambda_f I_D + i F_u(p^2) \\
 &\quad \times \left[ -\lambda_W p \cdot \hat{P} \gamma \cdot \hat{P} + \lambda_U \frac{1}{|u(p)|} \gamma \cdot \hat{u}(p) \right], \quad (B2)
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_-^{\text{pseud}}(p, P) &= F_u(p^2) \left[ i \lambda_f p \cdot \hat{P} + \lambda_W p \cdot \hat{P} \gamma \cdot \hat{P} \right. \\
 &\quad \left. - \lambda_U \frac{1}{|u(p)|} \gamma \cdot \hat{u}(p) \right] i \gamma_5, \quad (B3)
 \end{aligned}$$

$$\Gamma_-^{\text{scalar}}(p, P) = i F_u(p^2) \lambda_f p \cdot \hat{P} I_D - G_u(p^2) \lambda_W \gamma \cdot \hat{P}, \quad (B4)$$

where  $\hat{P}_\mu$  [ $\hat{P}^2 = -1$ ] is the direction vector associated with  $P_\mu$ , and  $u_\mu(p, \hat{P})$  is defined in Eq. (A4). We note the covariants involving  $\gamma \cdot \hat{u}(p)$  may be brought to a more familiar form through use of the identity

$$\frac{1}{|u(p)|} \gamma \cdot \hat{u}(p) = \gamma \cdot p + p \cdot \hat{P} \gamma \cdot \hat{P}. \quad (B5)$$

Solving Eq. (33) yields the following  $C = \pm$ , pseudoscalar and scalar eigenvectors:

$J^{PC}$	$0^{-+}$	$0^{++}$	$0^{--}$	$0^{+-}$	which are normalized in accordance with Eq. (19).
$\lambda_f$	0.61	0.67	-0.52	0.11	
$\lambda_W$	-0.045	-0.0075	0.084	0.26	(B6)
$\lambda_U$	0.0	-0.050	0.024	0.0	

$b. f_1=u/d, f_2=s$

Solutions for the pseudoscalar Bethe-Salpeter amplitudes for the  $\bar{u}$ - $s$  mesons are of the form

$$\Gamma^{\text{pseud}}(p, P) = \left\{ \lambda_{1u} G_u(p^2) + \lambda_{1s} G_s(p^2) - p \cdot \hat{P} [\lambda_{2u} F_u(p^2) + \lambda_{2s} F_s(p^2)] - [\lambda_{3u} G_u(p^2) + \lambda_{3s} G_s(p^2) - p \cdot \hat{P} [\lambda_{4u} F_u(p^2) + \lambda_{4s} F_s(p^2)]] i \gamma \cdot \hat{P} - i [\lambda_{5u} F_u(p^2) + \lambda_{5s} F_s(p^2)] \frac{1}{|u(p)|} \gamma \cdot \hat{u}(p) \right\} i \gamma_5. \quad (\text{B7})$$

The scalar amplitude has the same form but with  $i \gamma_5 \rightarrow I_D$ . The pseudoscalar amplitude given here corresponds to the  $K^-$ . The  $K^+$  amplitude is obtained by making the replacement  $p_\mu \rightarrow -p_\mu$ .

The calculated pseudoscalar eigenvector is

$$\begin{array}{cccccccccccc} J^P & \lambda_{1u} & \lambda_{1s} & \lambda_{2u} & \lambda_{2s} & \lambda_{3u} & \lambda_{3s} & \lambda_{4u} & \lambda_{4s} & \lambda_{5u} & \lambda_{5s} \\ 0^- & 263 & 390 & -1.3 & -3.3 & -60 & -97 & 2.9 & 7.0 & 2.8 & 6.7, \end{array} \quad (\text{B8})$$

where each of the components is to be multiplied by  $10^{-3}$  and the normalization is in accordance with Eq. (19). No true scalar solution is found.

### c. $\eta$ meson

The normalization condition for the  $\eta_{\theta_P}$  meson is

$$\begin{aligned} 2P_\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \{ & \frac{1}{3} (\cos \theta_P - \sqrt{2} \sin \theta_P)^2 [\text{tr}(\Gamma_\eta(k, -P) \partial_\mu^P S_u(k + \frac{1}{2}P) \Gamma_\eta(k, P) S_u(k - \frac{1}{2}P)) + \text{tr}(\Gamma_\eta(k, -P) S_u(k + \frac{1}{2}P) \\ & \times \Gamma_\eta(k, P) \partial_\mu^P S_u(k - \frac{1}{2}P))] + \frac{1}{3} (\sqrt{2} \cos \theta_P + \sin \theta_P)^2 [\text{tr}(\Gamma_\eta(k, -P) \partial_\mu^P S_s(k + \frac{1}{2}P) \Gamma_\eta(k, P) S_s(k - \frac{1}{2}P)) \\ & + \text{tr}(\Gamma_\eta(k, -P) S_s(k + \frac{1}{2}P) \Gamma_\eta(k, P) \partial_\mu^P S_s(k - \frac{1}{2}P))] \}. \end{aligned} \quad (\text{B9})$$

The formula for the decay constant of the  $\eta_{\theta_P}$  meson is

$$\begin{aligned} P^2 f_\eta = \frac{N_c}{\sqrt{2}} \int \frac{d^4 k}{(2\pi)^4} \{ & \frac{1}{3} (\cos \theta_P - \sqrt{2} \sin \theta_P)^2 \text{tr}[\gamma \cdot P \gamma_5 S_u(p + \frac{1}{2}P) \Gamma_\eta(p, P) S_u(p - \frac{1}{2}P)] \\ & + \frac{1}{3} (\sqrt{2} \cos \theta_P + \sin \theta_P)^2 \text{tr}[\gamma \cdot P \gamma_5 S_s(p + \frac{1}{2}P) \Gamma_\eta(p, P) S_s(p - \frac{1}{2}P)] \}. \end{aligned} \quad (\text{B10})$$

The positive charge-parity solution of Eq. (35) has the form

$$\Gamma_\eta(q, P) = [\lambda_{fu} G_u(q^2) + \lambda_{fs} G_s(q^2) - [\lambda_{Wu} G_u(q^2) + \lambda_{Ws} G_s(q^2)] i \gamma \cdot \hat{P}] i \gamma_5. \quad (\text{B11})$$

Solving Eq. (35) leads to the following values of the mass, decay constant, and eigenvector at the listed values of  $\theta_P$ :

$\theta_P$	$5^\circ$	$0^\circ$	$-5^\circ$	$-10^\circ$	$-90^\circ$	$-95^\circ$
$M_{\eta_{\theta_P}}$	0.549	0.513	0.475	0.436	0.357	0.399
$f_{\eta_{\theta_P}}$	0.114	0.111	0.108	0.105	0.100	0.102
$\lambda_{fu}$	0.18	0.23	0.28	0.33	0.43	0.38
$\lambda_{fs}$	0.47	0.41	0.35	0.29	0.19	0.25
$\lambda_{Wu}$	-0.041	-0.051	-0.059	-0.066	-0.073	-0.071
$\lambda_{Ws}$	-0.11	-0.092	-0.074	-0.058	-0.032	-0.045.

(B12)

The masses and decay constants are given in GeV. The last two columns correspond to the  $\eta'$  flavor projection.

## 2. Vector and axial-vector mesons

$$a. f_1 = u/d = f_2$$

The on-shell constraint of Eq. (39) entails that in constructing the general form of the vector meson Bethe-Salpeter amplitude we can work with the following transverse Euclidean covariants:

$$p_\nu^T \gamma_\nu^T, p_\nu^T \gamma_\nu^T \gamma \cdot p, p_\nu^T \gamma \cdot p, p_\nu^T \gamma \cdot p p \cdot p, \gamma_5 \epsilon_{\mu\nu\lambda\rho} \gamma_\mu p_\lambda p_\rho. \quad (\text{B13})$$

The general amplitude is a linear combination of these covariants weighted by invariant amplitudes  $\mathcal{F}_i(p^2, P^2, p \cdot P)$ . Employing the separable *Ansatz*, Eq. (17), the Bethe-Salpeter equation, Eq. (38), cannot support contributions to  $\Gamma_\nu(p, P)$  that are bilinear in  $p$ . Hence, at the mass shell, the produced vector meson Bethe-Salpeter amplitude is

$$\begin{aligned} \Gamma_\nu^T(p, P) = & p_\nu^T F_u(p^2) \hat{\lambda}_1 + i \gamma_\nu^T G_u(p^2) \hat{\lambda}_2 \\ & + i \gamma_5 \epsilon_{\mu\nu\lambda\rho} \gamma_\mu p_\lambda \hat{P}_\rho F_u(p^2) \hat{\lambda}_3. \end{aligned} \quad (\text{B14})$$

For the axial-vector meson the transverse Euclidean covariants are

$$p_\nu^T \gamma_5 p \cdot P, \gamma_5 \gamma_\nu^T, p_\nu^T \gamma_5 \gamma \cdot p, p_\nu^T \gamma_5 \gamma \cdot p p \cdot P, \epsilon_{\mu\nu\lambda\rho} \gamma_\mu p_\lambda p_\rho. \quad (\text{B15})$$

Again the terms bilinear in  $p$  do not contribute and, using the separable *Ansatz*, the produced axial-vector Bethe-Salpeter amplitude is

$$\Gamma_{5\nu}^T(p, P) = i \gamma_5 \gamma_\nu^T G_u(p^2) \hat{\lambda}_1 + i \epsilon_{\mu\nu\lambda\rho} \gamma_\mu p_\lambda \hat{P}_\rho F_u(p^2) \hat{\lambda}_2. \quad (\text{B16})$$

As above, the Bethe-Salpeter equation is a matrix eigenvalue problem. We obtain the solutions

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$J^P$		$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{\lambda}_4$	$\hat{\lambda}_5$	
$1^-$	$u$	0.020	-0.12	$-4.5 \times 10^{-4}$	$2.0 \times 10^{-4}$	0.014	
	$s$	0.046	-0.21	$-6.5 \times 10^{-4}$	$3.4 \times 10^{-4}$	0.026	
$1^+$	$u$	0.16	$-5.4 \times 10^{-3}$	$-6.7 \times 10^{-3}$	$-1.8 \times 10^{-3}$	$6.8 \times 10^{-4}$	
	$s$	0.34	$4.0 \times 10^{-3}$	$-1.5 \times 10^{-2}$	$9.9 \times 10^{-3}$	$1.9 \times 10^{-3}$	(B19)

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$$c. f_1 = s = f_2$$

The Bethe-Salpeter amplitude for the  $J^{PC} = 1^{--} \bar{s}s$  state  $[\phi]$  has the same form as Eq. (B14) for the  $\rho/\omega$  but with  $G_u \rightarrow G_s$  and  $F_u \rightarrow F_s$ . That for the  $J^{PC} = 1^{++}$  state  $[f_1(1510)]$  is related in a similar way to Eq. (B16) for the  $a_1/f_1$ . The calculated eigenvectors are

$$\begin{aligned} J^{PC} \quad \hat{\lambda}_1 \quad \hat{\lambda}_2 \quad \hat{\lambda}_3 \\ 1^{--} \quad 0.049 \quad -0.35 \quad 0.030 \\ 1^{++} \quad 0.0044 \quad -0.18 \quad 0.0. \end{aligned} \quad (\text{B20})$$

$$\begin{aligned} J^{PC} \quad \hat{\lambda}_1 \quad \hat{\lambda}_2 \quad \hat{\lambda}_3 \\ 1^{--} \quad 0.075 \quad -0.33 \quad 0.049 \\ 1^{++} \quad 0.056 \quad -0.28 \quad 0.0 \end{aligned} \quad (\text{B17})$$

corresponding to the  $\rho/\omega$  and  $a_1/f_1$  channels, respectively. The Bethe-Salpeter amplitudes are normalized according to the vector and axial-vector generalizations of Eq. (19).

$$b. f_1 = u/d, f_2 = s$$

Consider the  $J^P = 1^- K^{*+}$  meson. As for the  $\omega$  meson, there are five transverse covariants, which can be taken to be those in Eq. (B13) except that the explicit factor of  $p \cdot P$  is no longer necessary because  $u\bar{s}$  states are not eigenstates of the charge conjugation operator,  $C$ . The general amplitude is a linear combination of these covariants weighted by invariant amplitudes  $\mathcal{F}_i(p^2, P^2, p \cdot P)$  where odd powers of  $p \cdot P$  are allowed for the same reason. The separable *Ansatz* does not support contributions bilinear in  $p$  and hence, at the mass shell, the produced  $K^{*+}$  amplitude has the form

$$\begin{aligned} \Gamma_T^\nu(p, P) = & p_\nu^T [F_u(p^2) \hat{\lambda}_{1u} + F_s(p^2) \hat{\lambda}_{1s}] + i \gamma_\nu^T [G_u(p^2) \hat{\lambda}_{2u} \\ & + G_s(p^2) \hat{\lambda}_{2s}] + i \gamma_\nu^T p \cdot \hat{P} [F_u(p^2) \hat{\lambda}_{3u} \\ & + F_s(p^2) \hat{\lambda}_{3s}] + i p_\nu^T \gamma \cdot \hat{P} [F_u(p^2) \hat{\lambda}_{4u} + F_s(p^2) \hat{\lambda}_{4s}] \\ & + i \gamma_5 \epsilon_{\mu\nu\lambda\rho} \gamma_\mu p_\lambda \hat{P}_\rho [F_u(p^2) \hat{\lambda}_{5u} + F_s(p^2) \hat{\lambda}_{5s}]. \end{aligned} \quad (\text{B18})$$

The amplitude for the  $K^{*-}$  meson is obtained by reversing the sign of  $p$ , under which the kernel is invariant. The amplitudes for the  $J^P = 1^+ K_1$  meson states are simply  $\gamma_5$  times the appropriate form of Eq. (B18).

The calculated eigenvectors  $\hat{\lambda}_i$  are

## 3. Scalar and pseudoscalar diquark correlations

$$a. f_1 = u/d = f_2$$

To obtain the  $J^P = 0^+$  diquark solution of the Bethe-Salpeter equation given in Eq. (40) one searches for the  $0^-$  auxiliary amplitude solution of Eq. (42). The latter can be written in the form

$$\Gamma_{\bar{3}}^C(p, P) = G_u(p^2) [\lambda_{\bar{f}}^{\bar{3}} - i \lambda_W^{\bar{3}} \gamma \cdot \hat{P}] i \gamma_5, \quad (\text{B21})$$

which is identical in form to the meson amplitude  $\Gamma_+^{\text{pseud}}$  in

Eq. (B1). The  $0^-$  pseudoscalar diquark solution is described by an auxiliary amplitude identical in form to  $\Gamma_+^{\text{scalar}}$  of Eq. (B2). The calculated masses for the scalar ( $0^+$ ) and pseudo-scalar ( $0^-$ ) diquark correlations are listed in Table IV and the eigenvectors are

$$\begin{array}{cccc} J^P & \lambda_f^{\bar{3}} & \lambda_W^{\bar{3}} & \lambda_U^{\bar{3}} \\ 0^+ & 0.96 & -0.29 & 0.0 \\ 0^- & 0.15 & 0.58 & -0.80. \end{array} \quad (\text{B22})$$

For all diquark eigenvectors we use the normalization  $\sum_i |\lambda_i|^2 = 1$ . We note that the subleading Dirac amplitude contributes 11% to the  $0^+$  diquark mass.

The obtained mass values suggest that the  $0^+$  diquark pole will provide a contribution to the truncated quark-quark  $\mathcal{T}$  matrix that is important in the type of Fadde'ev equation studies of the nucleon described in Sec. IV. The much larger

mass found to be associated with the  $0^-$  diquark correlation suggests that it may be neglected in such studies.

$$b. f_1 = u/d, f_2 = s$$

The homogeneous Bethe-Salpeter equation in the  $u/d$ -s quark-quark channel can be written in the form

$$\begin{aligned} \Gamma_{us}^{\bar{3}C}(p, P) = & -\frac{2}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu S_u(q + \xi P) \\ & \times \Gamma_{us}^{\bar{3}C}(q, P) S_s[q - (1-\xi)P] \gamma_\mu, \end{aligned} \quad (\text{B23})$$

where the momentum partitioning parameter is  $\xi = 0.56 \approx 0.5$ , as for the kaon.

The solution of this equation that corresponds to the  $0^+$  diquark is identical in form to  $\Gamma^{\text{pseud}}$  in Eq. (B7). The calculated mass is listed in Table IV and the eigenvector is

$$\begin{array}{cccccccccccc} J^P & \lambda_{1u} & \lambda_{1s} & \lambda_{2u} & \lambda_{2s} & \lambda_{3u} & \lambda_{3s} & \lambda_{4u} & \lambda_{4s} & \lambda_{5u} & \lambda_{5s} \\ 0^+ & 498 & 802 & -10.4 & -33.2 & -165 & -282 & 5.2 & 14.9 & 4.7 & 11, \end{array} \quad (\text{B24})$$

where each component is to be multiplied by  $10^{-3}$ . The subleading Dirac amplitudes contribute 11% to the  $0^+$  diquark mass. The magnitude of its mass is such that this correlation may be important in the Fadde'ev equation studies of the strange octet baryons. No  $0^-$  solution with a mass less than 2 GeV was found. This is in accord with our finding that in the  $0^+$  meson channel, there was insufficient attraction for a clear bound state.

One observes that the diquark mass splitting  $M_{us} - M_{ud} = 145$  MeV. This may be compared with  $m_\Sigma - m_p \approx 250$  MeV. One might infer from this that Fadde'ev equation studies, such as the ones described above, may yield the correct ordering and level separation of the octet baryons.

#### 4. Vector and axial-vector diquarks

$$a. f_1 = u/d = f_2$$

In ladder approximation the homogeneous Bethe-Salpeter equation for vector and axial-vector color-antitriplet diquark correlations has the same form as Eq. (40) except that the Bethe-Salpeter amplitude carries a Lorentz index. It can be recast into the form of Eq. (42) in the same manner.

For the axial-vector ( $1^+$ ) diquark channel, the auxiliary amplitude  $\Gamma_{5\mu}^{\bar{3}C}(p, P)$  is identical in form to the vector meson amplitude in Eq. (B14). The calculated mass is listed in Table IV. The axial-vector diquark mass is larger than that of the vector meson, in agreement with the argument of Ref. [28]. However, it is comparable to the predicted scalar diquark mass. Hence the  $1^+$  diquark pole is likely to provide a contribution to the truncated quark-quark  $\mathcal{T}$  matrix, within the type of simplified nucleon Fadde'ev equation studies described earlier, that is comparable to that of the scalar di-

quark. In the  $1^-$  channel, the auxiliary amplitude  $\Gamma_\mu^{\bar{3}C}(p, P)$  has the axial-vector meson form given in Eq. (B16). The calculated vector diquark mass is listed in Table IV. It is too massive to be of importance.

The eigenvectors are

$$\begin{array}{cccc} J^P & \hat{\lambda}_1 & \hat{\lambda}_2 & \hat{\lambda}_3 \\ 1^- & 0.12 & -1.46 & 0.0 \\ 1^+ & 0.16 & -0.98 & 0.11. \end{array} \quad (\text{B25})$$

Subleading Dirac amplitudes contribute little in these channels.

$$b. f_1 = u/d, f_2 = s$$

The auxiliary amplitude for the  $1^+$  diquark has the same form as the vector meson amplitude in Eq. (B18), while that for the  $1^-$  diquark is simply  $\gamma_5$  times this. The calculated masses are given in Table IV and the eigenvectors are

$$\begin{array}{cccccc} J^P & \hat{\lambda}_1 & \hat{\lambda}_2 & \hat{\lambda}_3 & \hat{\lambda}_4 & \hat{\lambda}_5 \\ 1^- & u & 168 & -4.7 & -0.029 & -7.0 & 4.0 \\ & s & 445 & 6.3 & 0.21 & -17.0 & 39.6 \\ 1^+ & u & 16.2 & -161 & -0.64 & 0.13 & 12.9 \\ & s & 45.8 & -288 & -1.05 & 0.21 & 26.8, \end{array} \quad (\text{B26})$$

where again each component is to be multiplied by  $10^{-3}$ .

These results suggest that the axial-vector diquark can be important in Fadde'ev equation studies of strange baryons,

whereas the vector diquark can be neglected. Again, subleading Dirac amplitudes contribute little in these channels.

$$c. f_1 = s = f_2$$

The auxiliary amplitude for the  $1^+$  diquark is identical in form to that for the  $\phi$  meson while that for the  $1^-$  diquark has the form of the axial counterpart [ $f_1(1510)$ ], both of which are described in Appendix II.B.3. The calculated masses are given in Table IV and the eigenvectors are

$$\begin{array}{cccc} J^P & \hat{\lambda}_1 & \hat{\lambda}_2 & \hat{\lambda}_3 \\ 1^- & 0.030 & -1.64 & 0.0 \\ 1^+ & 0.090 & -0.99 & 0.061. \end{array} \quad (\text{B27})$$

The low mass of the axial-vector diquark suggests that it can be important in Fadde'ev equation studies of all strangeness carrying baryons, whereas again the vector diquark can be neglected. The subleading Dirac amplitudes are unimportant in these channels.

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