Effect of multiparticle collisions on pion production in relativistic heavy-ion reactions

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In the present work we discuss the effect of N-body processes on pion multiplicity in relativistic heavy-ion reactions. This effect is analyzed in the energy range from the pion threshold up to 2 GeV/nucleon, for several projectile-target systems. The analysis is carried out in the context of intranuclear cascade calculations. It is shown that the inclusion of multibaryonic collisions is a crucial element in the study of the pion production mechanisms, being strongly dependent on the adopted correlation range for the particles involved in the N-body processes. [S0556-2813(97)05305-3]

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I. INTRODUCTION

The study of relativistic heavy-ion collisions at intermediate energies still makes claims for a theoretical approach to the reaction mechanism that includes properly the many-body correlations in the baryonic interacting system. After the phenomenological thermal and hydrodynamic pictures were used to describe some aspects of the nucleus-nucleus reactions [1], models based on transport equations have been largely applied and presented as good candidates to describe many-body correlations [2]. These approaches are well established from the microscopic point of view in terms of a time-dependent Hartree-Fock calculation [3]. However, the basic element in these approaches is the *one-body* phase-space distribution function; *many-body* correlations at short distances cannot be properly incorporated into models based on the Boltzmann-Uehling-Uhlenbeck (BUU) transport equation. The most recent attempt to include many-body effects in these transport schemes has been carried out by Batko *et al.* [4], who have modified the collisional term in the BUU equation by introducing an effective cross section for collisions involving three or more baryons.

The importance of N-body ($N>2$) effects to the description of the heavy-ion reaction mechanism was first pointed out by Kodama *et al.* [5], by using Monte Carlo simulations in the intranuclear cascade model. In that work it was demonstrated that nonbinary particle collisions due to dynamical density fluctuations cannot be neglected. In this context of intranuclear cascade (INC) models, however, the relevance of multibaryonic collisions to particle production has not yet been discussed. This important issue, already discussed in a nonrelativistic BUU-equation approach for kaon production [4], still needs to be examined regarding pion production.

The main purpose of the present work is to study the effect of multiparticle collisions on pion production in the broad energy range from the nucleon-nucleon pion threshold to energies around 2 GeV/nucleon. Although the BUU scheme deals directly with a self-consistent mean field, also including the Pauli-blocking effect in the binary collisional term, we believe that the BUU description of the evolution of the system in terms of the one-body distribution function only, is not appropriate to discuss effects originated in the dynamics of density fluctuations. It seems that an INC calculation, with some improvement in the prescription to mimic the Pauli blocking, continues to be a useful, available tool to provide a reliable estimate for effects which depend on multibaryonic interactions.

We remind the reader that, for low energies near the pion threshold, particle production significantly cools down the formed hot system and can interfere in the occurrence of a possible liquid-vapor phase transition of the nuclear matter [6], discussed since early studies on heavy-ion collisions. Furthermore, it is the available energy to heat the nuclear system that will determine the fragmentation process of the residual nucleus in the final stage of the nuclear reaction. Besides, the subthreshold pion production itself still claims for an analysis focused on multiparticle collisional processes.

For higher energies the interest is to explore the relativistic character of the INC calculation [7]. Current BUU schemes used to discuss the effect of N-body collisions on particle production are nonrelativistic [4]. Consequently, kinematic effects on particle production, as well as the effect of multibaryonic collisions included in these BUU calculations, should be reviewed.

The calculation of the present work follows the general aspects of our previous relativistic INC models [5,7]. In the present study we explore the effect of N-body processes on pion production, and discuss the sensitivity of results to different ways of defining these multiparticle collisions.

This work is organized as follows. In Sec. II we give a brief description of the present INC model, emphasizing the details related to the inclusion of many-body collisions. In Sec. III, we present and discuss our results. Finally, we draw some conclusions and make our final remarks in Sec. IV.

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II. THE *N*-BODY INTRANUCLEAR CASCADE CALCULATION

The inclusion of N-body interactions in the present INC calculation begins by checking the neighborhood of any two colliding particles of a conventional cascade calculation. If other baryons can interfere in the binary collision, the process should rather be considered as an N-body process, with N being the number of the intruders plus the two colliding particles. The fundamental point is how to define the interfering neighborhood. Let us consider two interacting baryons i and j, separated by the distance d_{ij}. The simplest way to decide on whether a generic third particle k is in the geometric interfering neighborhood of the i-j pair or not, is by comparing the effective ranges for the k-i and k-j interactions with the distances r_{ki} and r_{kj}, respectively. Namely, if any of the conditions

$$r_{ki}<\sqrt{\frac{\sigma_{ki}(s)}{\pi}} \quad \text{or} \quad r_{kj}<\sqrt{\frac{\sigma_{kj}(s)}{\pi}} \tag{1}$$

is satisfied, the particle k is considered to be in the interfering neighborhood of the baryons i and j. [The total cross sections for any two particles, $\sigma(s)$, are parametrized as functions of s, the invariant-mass squared.]

In order to better appreciate the effect of the interfering neighborhood on the pion production, one can alternatively restrict this interfering neighborhood. Let us say that particle k interferes in the i-j collision if, in addition to condition (1), one also requires that

$$r_{ki}<d_{ij} \quad \text{or} \quad r_{kj}<d_{ij}. \tag{2}$$

With the additional condition (2), the number of particles involved in the N-body processes results is smaller than in the previous situation. We can say that the criterion which uses only the first condition leads to larger clusters surrounding the interacting i-j pair. Thus, hereafter, we will call this criterion the *large-cluster criterion* (LCC), in contrast to the second one, referred to as *small-cluster criterion* (SCC).

Since neither experimental information for the cross sections of these N-body interactions is available, nor any efficient theoretical approach to calculate it, the simplest way to take into account the effect of the other particles on the i-j collision is by considering a redistribution of energy and momentum among the N particles in the cluster. This energy-momentum redistribution prescription must, of course, preserve strictly the total energy-momentum conservation. As a matter of fact, we have picked out equally probable configurations in the N-body phase space restricted by energy-momentum conservation constraints [5,8]. After this redistribution, the resulting energies and momenta of the baryons i and j are then used as input for the processing of their binary interaction. By using this prescription we are imposing a local thermalization mechanism, similar to the one adopted in Ref. [4] to define an effective cross section for the N-body processes. (This point will be further discussed in Sec. III.)

We list below all the elementary reactions that we have included in the present version of the INC model:

$$B_1B_2 \to B_1B_2, \tag{3}$$

$$NN \rightleftharpoons N\Delta, \tag{4}$$

$$NN \rightleftharpoons NN^*, \tag{5}$$

$$\Delta \rightleftharpoons N\pi, \tag{6}$$

$$N^* \rightleftharpoons N\pi, \tag{7}$$

where B means any baryon, N means a nucleon, and Δ and N^* are the lowest-mass (1232 MeV) and the next higher (1440 MeV) baryonic resonances, respectively. The cross section for the elastic baryon-baryon process, Eq. (3), has been taken from a compilation of experimental data [9].

The direct reactions (4) and (5) are the Δ- and N^*-formation processes, respectively. The cross sections for all possible isospin channels in these reactions, taken from Ref. [10], are given in terms of the parametrization by VerWest and Arndt [11] of the data for nucleon-nucleon single-pion production. The resonance-recombination processes, inverse reactions in Eqs. (4) and (5), were calculated by using the extended detailed balance relation [12]. Although other versions of the extended detailed balance have been used [13], a recent experimental analysis [14] has shown that the version of Wolf *et al.* presents a better performance in reproducing the data for the delta-recombination cross section.

The Δ and N^* resonances may decay into a pion and a nucleon through the direct reactions in Eqs. (6) and (7). The decay time of a resonance is randomly chosen from the characteristic exponential law with a lifetime given by $\tau=\hbar/\Gamma$, where $\Gamma_\Delta=115$ MeV and $\Gamma_{N^*}=200$ MeV. We note that pion absorption effects—inverse reactions in Eqs. (6) and (7)—are explicitly included. Their cross sections have been taken from Ref. [10]. Finally, in our model the pion production may occur only through a resonance decay, once we disregarded the direct s-state pion production.

In conventional versions of the INC model [15–17], the Pauli-blocking effect has been taken into account through prescriptions which adopt some sort of energy cutoff for soft collisions (collisions with relative kinetic energy lower than the local Fermi energy are not processed). The way the Pauli-blocking effect has been considered in our present INC model represents an improvement over the previously adopted treatments; we have used phase-space exclusion volumes attached to each fermion. For every elementary interaction processed, we check whether the exclusion volume associated with the final state of each colliding fermion includes more than $2S+1$ similar fermions, where S is the spin of that fermion. Whenever this happens, the collision is not allowed. The exclusion volume corresponds to a hypercubic cell in phase space, with the spatial size equal to typical nucleon dimension ($\Delta x_\alpha=1.13$ fm), and the momentum size given by the uncertainty relation $\Delta x_\alpha \Delta p_\alpha=h$, with $\alpha=1,2,3$.

III. RESULTS AND DISCUSSION

We now discuss the main results obtained with the present INC model. The frequency of occurrence of N-body processes ($N=2,3,4,5$), as a function of time, is shown in Fig. 1 for central Ca-Ca reactions at incident energy $E_{\text{lab}}=2$ GeV/nucleon. The calculations were performed

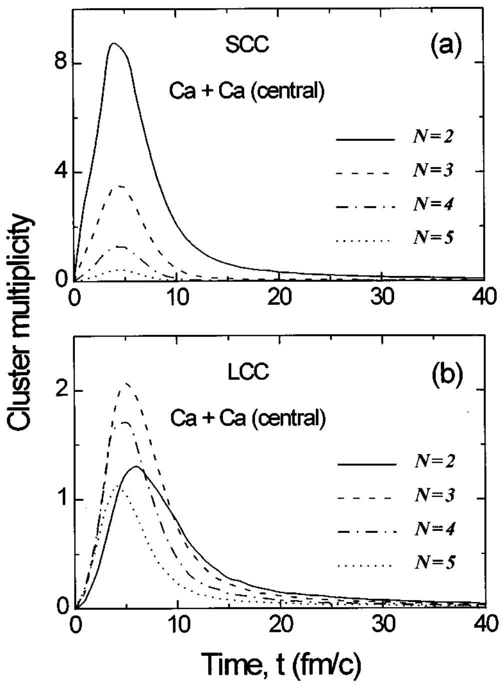

FIG. 1. Frequency of N-body collisions during the time evolution of central $^{40}Ca+^{40}Ca$ reactions at $E_{lab}=2$ GeV/nucleon. The calculations were performed under two different cluster criteria: SCC, the small-cluster formation criterion (a) and LCC, the large-cluster formation criterion (b). The frequencies of collisions involving N particles, with $N=2$, $N=3$, $N=4$ and $N=5$, are represented by the solid, dashed, dot-dashed, and dotted lines, respectively.

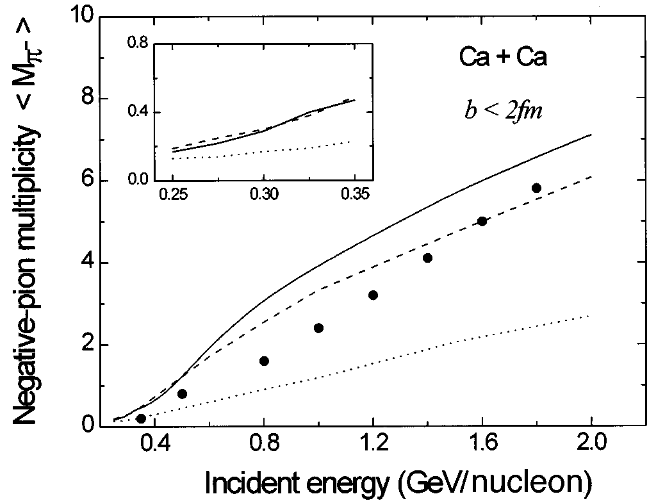

FIG. 2. Dependence of the negative-pion multiplicity on incident energy (E_{lab}) in quasicentral Ca+Ca reactions, obtained with three versions of the intranuclear cascade model: the conventional INC model [16] (solid line), the present INC model under SCC criterion (dashed line), and the present INC model under LCC criterion (dotted line). The solid circles are the data [18]. The small graph shows the pion multiplicity *versus* energy in the pion threshold energy region.

under the two different criteria (LCC and SCC) adopted in the definition of the interfering neighborhood. Under the SCC criterion, Fig. 1(a), the binary collisions dominate during the whole nuclear reaction. However, under the LCC criterion, Fig. 1(b), the relative frequency of multiparticle collisions increases significantly, with a consequent decrease in the number of binary processes; this latter is even surpassed by the frequency of three- and four-body processes. We have verified that this same qualitative behavior also holds for incident energies around 300 MeV/nucleon.

The use of either the LCC or the SCC criterion changes drastically the pion multiplicity, for energies in the range 0.25–2 GeV/nucleon. In Fig. 2 we plot the results for the negative-pion multiplicity in quasicentral (impact parameter $\leqslant 2$ fm) $^{40}Ca+^{40}Ca$ reactions, as a function of the incident energy, obtained with three different models: the conventional INC model [16] (solid line), which includes only binary collisions, and the present INC model—which includes also N-body processes—under two different assumptions for the neighborhood criterion, namely SCC (dashed line) and LCC (dotted line). The inclusion of multiparticle collisions under the SCC criterion decreases the pion production at high incident energies, having practically no effect for energies around the pion threshold (shown in the small graph in the Fig. 2). On the other hand, the LCC criterion has a much higher effect on pion production, decreasing the pion yield in the full range of energies investigated here. The experimental data [18] are displayed only to evidence the relevance of multibaryonic collisions to the pion production in relativistic heavy-ion reactions. We remark that years ago the existing discrepancy between conventional INC calculations of pion multiplicities and the data was attributed to either the presence of a compressional energy effect [18] or, alternatively, the effect of the nuclear mean field [19]. In the framework of conventional INC models, the discrepancy was only explained by claiming for a nuclear binding-energy effect [17,20].

We notice that the reduction of pion multiplicity as a consequence of the inclusion of N-body processes, as obtained in the present work, is apparently in disagreement with the effect of N-body collisions on the kaon multiplicity reported in Ref. [4]. There, the inclusion of multiparticle collisions leads to an increase in the number of the produced kaons. However, it is not intuitive that pion multiplicity should be similarly affected by the inclusion of many-body collisions in a dynamical model for heavy-ion reactions. Note that the processes of pion absorption—inverse reactions in Eqs. (6) and (7)—and of Δ and N^* recombination—inverse reactions in Eqs. (4) and (5)—are expected to affect significantly the pion yield. The inclusion of N-body processes changes the $\sqrt{s}$ distribution of baryon-baryon inelastic collisions. Hence, we display in Fig. 3 the distribution of nucleon-nucleon inelastic collisions [Fig. 3(a)], and of nucleon-delta collisions, [Fig. 3(b)]. The inclusion of many-body processes (specially in respect to the LCC criterion) decreases the fraction of inelastic nucleon-nucleon collisions, and enhances the fraction of inelastic nucleon-delta processes, in the energy region of largest Δ-recombination cross section. Then, with the inclusion of N-body processes, the combined effect reduces the final pion yield. On the other hand, kaons are not affected by similar processes, since kaons are assumed not to interact, being treated as free particles which escape the system just after they are produced. Moreover, the relevant aspect of the distribution in respect to kaon production is the change in the high energy region of the distribution tail. This tail increases

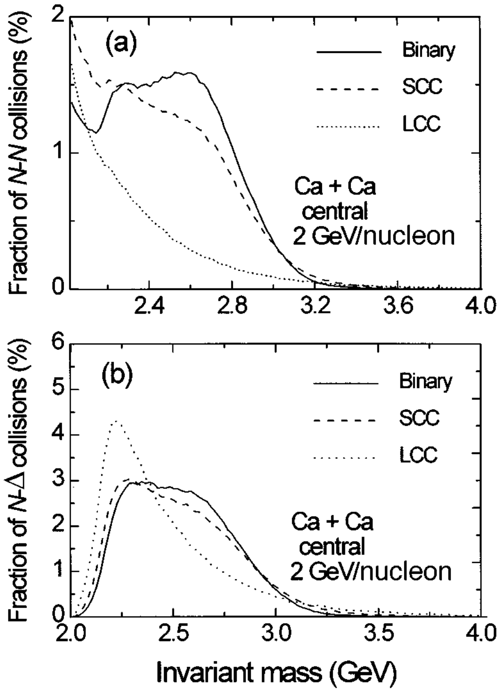

FIG. 3. $\sqrt{s}$ distribution of the inelastic nucleon-nucleon (a) and nucleon-delta (b) collisions, normalized to the total number of particle collisions. Lines are the same as in Fig. 2.

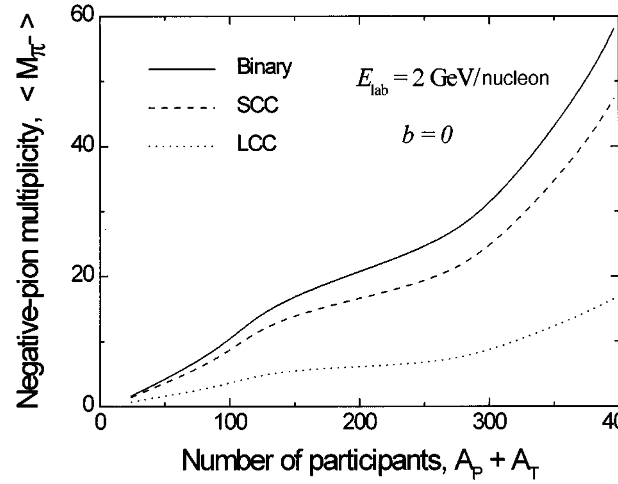

FIG. 4. Negative-pion multiplicity as a function of number of participants in central nucleus-nucleus collisions at 2 GeV/nucleon. Lines are the same as in Fig. 2.

when one considers the inclusion of many-body processes, providing an enhancement in the kaon multiplicity, as observed in Ref. [4], due to the increasing quartic power form of the kaon formation cross section [21].

We have studied the sensitivity of the present INC model to the mechanism adopted for redistributing energy and momentum among the N baryons inside a cluster. Besides the phase-space distribution described in Sec. II, we have also used the prescription adopted in Ref. [4]. The latter assumes that the total available energy is distributed among the initial particles of the cluster, following a microcanonical distribution with an effective temperature given by the average kinetic energy of the particles inside the cluster. We have found that our model is relatively insensitive to the use of either mechanism, since the pion multiplicities calculated under the two different prescriptions differ from each other by less than 2%.

We have also investigated how the effect of N-body interactions on pion multiplicity depends on the total mass of the colliding nuclear system, for two incident energies. Figure 4 shows the average negative-pion multiplicity as a function of number of participants [total mass number (A_P+A_T) of the projectile-target system], in central heavy-ion reactions at $E_{\rm lab}=2$ GeV/nucleon. From these results, and the ones calculated at $E_{\rm lab}=300$ MeV/nucleon (not shown), one can say that the effect of the inclusion of N-body processes obtained under the SCC and LCC criteria exhibits the same qualitative behavior as for the Ca-Ca reaction (Fig. 2).

At this point a few comments concerning the choice of these two geometric criteria for cluster formation, SCC and LCC, are worthwhile. The former is defined in terms of the smallest physically meaningful region of particles interfering in the baryon-baryon collision. Therefore, the magnitude of the N-body effect calculated under the SCC criterion should be regarded as the *minimum* effect of these multiparticle interactions. On the other hand, once we are not regarding long-range particle correlations, the magnitude of the N-body effect calculated under the less restrictive criterion (LCC) should be regarded as an upper bound for the same effect, since it is not physically reasonable to allow interactions between particles which do not satisfy condition (1).

IV. CONCLUSIONS AND FINAL REMARKS

In the present work, N-body processes have been included in the INC model to discuss their effect on pion production in heavy-ion reactions, for relativistic as well as for sub-

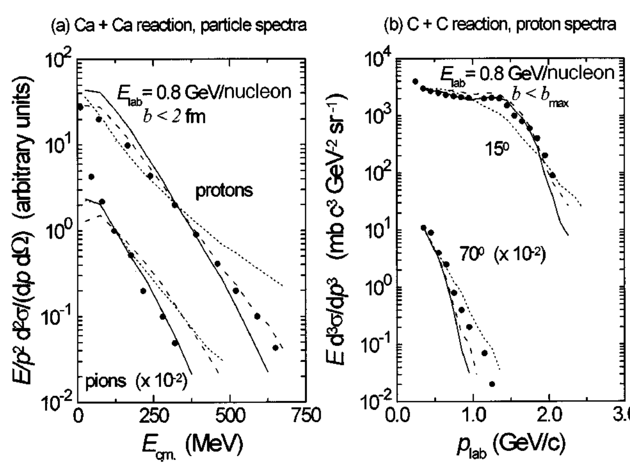

FIG. 5. Energy spectra of protons and pions as calculated with the present INC model for two different reactions at the same incident energy, $E_{\rm lab}=0.8$ GeV/nucleon. (a) refers to protons and pions emitted at $\theta_{\rm c.m.}=90°$ from quasicentral ($b\leq 2$ fm) ^{40}Ca+^{40}Ca reactions. In (b) proton spectra from inclusive ($b\leq b_{\rm max}$) ^{12}C+^{12}C reactions are shown for two values of the scattering angle: $\theta_{\rm lab}=15°$ and $\theta_{\rm lab}=70°$. In both figures, the curves are the same as in Fig. 2. Ca+Ca data, solid circles in (a), are taken from Ref. [22] and C+C data, solid circles in (b), are taken from Ref. [23].

threshold energies. We have shown that the effect of these multibaryonic collisions is noticeable, and depends strongly on the adopted criterion for cluster formation.

In order to explicit the effect of multiparticle collisions on other aspects of relativistic nucleus-nucleus reactions, we present in Fig. 5 the energy spectra for protons and pions in calcium-calcium [Fig. 5(a)] and carbon-carbon [Fig. 5(b)] reactions, calculated with the present INC model. It is clear from the figure that the inclusion of N-body processes under the SCC criterion improves the results obtained with a conventional INC model—one that includes only two-body correlations. Results obtained with our model under the LCC criterion, however, do not reproduce the data shown in Fig. 5.

As a final remark, we note that the use of *geometric* criteria for cluster formation, in the present work, raises some questions related to a possible noninvariance of our results [7]. We have carefully checked this point by running our code in two different reference frames: the laboratory frame (lab) and the center-of-mass frame (c.m.). For energies near pion threshold, we have confirmed that it is completely irrelevant to get the results in either frame. For the highest energy in our analysis, the average pion multiplicities obtained in the lab frame differ from the ones calculated in the c.m. frame by less than 3%.

We are now working on the inclusion of other elementary nucleon-nucleon inelastic channels in our model, particularly on those leading to the production of kaons and hyperons.

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