Pion absorption cross section for ^{2}H and ^{3}He in the Δ-isobar region: A phenomenological connection

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The absorption of π^+ on ^{3}He in the Δ region is evaluated with exact inclusion of the final state interaction among the three emerging protons. The absorption is described by a $\pi N \to \Delta$ vertex and an $N\Delta$-NN transition t matrix which are calculated from a phenomenological model for NN and πd reactions. In a calculation where the initial pion scattering effects are neglected, the predicted peaks of the pion absorption cross sections for ^{2}H and ^{3}He lie too high in energy in relation to the data. The effect of the final-state three-nucleon interaction turns out to be too small for changing the magnitude and shifting the peak position of the total absorption cross section for ^{3}He. We demonstrate that the adjustment of the peak position for the deuteron cross section by small modifications of the Δ parameters automatically leads to the correct peak position in ^{3}He. [S0556-2813(97)01405-2]

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I. INTRODUCTION

In recent years it became possible to solve the quantum mechanical three-body problem with realistic two- and three-nucleon forces [1–5]. Powerful computer facilities allowed this important step forward. Except for a few observables the theoretical predictions based on realistic NN forces agree very well with the experimental data. The exact treatment of the strong rescattering among the three particles is thereby crucial.

The Faddeev equations have been applied not only to the pure $3N$ system but also to inelastic electron scattering on ^{3}He [6–9]. The Faddeev formalism allowed one to calculate any breakup process, exclusive [8] and inclusive [9] ones. In the same manner we apply now the Faddeev equations to investigate pion absorption phenomena. The simplest reaction is pion absorption on the deuteron requiring the study of the πNN system for which a vast literature exists [10–21]. We shall not try to improve our understanding of this system, rather we shall present an exploratory calculation of pion absorption on ^{3}He which is motivated by recent experimental studies of the reaction $\pi^+ {}^3\mathrm{He} \to 3p$ by the LADS Collaboration at PSI [23–25]. In this first study we shall treat the dynamics of the incoming pion approximately. In particular we shall not allow for initial state interactions where the pion is rescattered before it is absorbed. We shall assume that pion absorption takes place in the first step by a Δ-resonance mechanism and after that the nucleons interact strongly in a $3N$ state. We shall determine the effect of this final state interaction for the total absorption cross section. To the best of our knowledge this will be the first time that an exact treatment of FSI has been performed. The choice of ^{3}He instead of ^{2}H also allows for pion absorption on two nucleons not only in isospin $t=0$ but also in $t=1$ states. Finally choosing π^+ absorption on ^{3}He generates a system of three interacting final protons, which cannot be realized in a pure $3N$ scattering process.

In this study we are interested in the Δ-resonance energy range and therefore we introduce explicitly the Δ degree of freedom. We use the phenomenological NN-$N\Delta$ model of Betz and Lee [17] which treats the $\pi N\Delta$ vertex that is responsible for pion absorption in a self-consistent way. In the present exploratory calculation we exclude for simplicity the contributions corresponding to propagating πNN intermediate states. Ohta, Thies, and Lee [26] applied a similar simplification of the model of Betz and Lee to heavier nuclei but did not include final state interactions and had to rely on simple model target wave functions, whereas here we shall use a realistic ^{3}He description. A glance at the pion absorption cross section for ^{2}H and ^{3}He reveals immediately that it peaks around $T_\pi \approx 130$ MeV, whereas the elastic and inelastic cross sections peak around 170 MeV, closer to the position of the Δ resonance in free πN scattering. We shall establish that FSI is not related to that shift in the peak position. However, it is possible to describe the energy shifts in both nuclei by a common parametrization of the underlying mechanism.

In Sec. II we briefly outline the way we use the Faddeev equations to describe the final state interaction for pion absorbtion on ^{3}He. Since it is similar in structure to inelastic electron scattering on ^{3}He we can refer to various articles [9] for more details and show only those steps which are specific to the pion absorption process. This is presented in Sec. III. In order to show how we treat the Δ particle we introduce the $N\Delta$ propagator in Sec. IV. A formalism very similar in structure to ours has been presented before in [27], though no numerical application thereof is known to us. Our numerical results are shown in Sec. V. We summarize and give an outlook in Sec. VI.

II. FORMALISM

Let us first describe a situation where the pion is absorbed on a nucleon converting it into a Δ particle which then to-

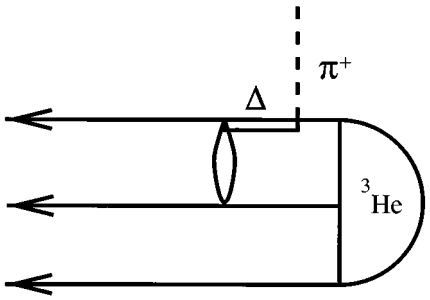

FIG. 1. The amplitude $|\Gamma\rangle$ of Eq. (2) describing π^+ absorption on a nucleon in ^{3}He leading to a Δ particle and followed by a deexcitation into two nucleons.

gether with a second nucleon undergoes an infinite number of rescatterings described by a two-body t matrix $t_{NN,N\Delta}$. That two-body t matrix obeys a coupled set of Lippmann-Schwinger equations

$$\begin{pmatrix} t_{NN,NN} & t_{NN,N\Delta} \\ t_{N\Delta,NN} & t_{N\Delta,N\Delta} \end{pmatrix} = \begin{pmatrix} V_{NN,NN} & V_{NN,N\Delta} \\ V_{N\Delta,NN} & V_{N\Delta,N\Delta} \end{pmatrix} + \begin{pmatrix} V_{NN,NN} & V_{NN,N\Delta} \\ V_{N\Delta,NN} & V_{N\Delta,N\Delta} \end{pmatrix} \begin{pmatrix} G^0_{NN} & 0 \\ 0 & G^0_{N\Delta} \end{pmatrix} \times \begin{pmatrix} t_{NN,NN} & t_{NN,N\Delta} \\ t_{N\Delta,NN} & t_{N\Delta,N\Delta} \end{pmatrix}. \quad (1)$$

In our model calculation we choose the transition potentials V from the analysis of Betz and Lee [17]. The iteration of Eq. (1) describes the consecutive transitions between the ΔN system generated by the pion absorption and the resulting NN system. The resulting amplitude has the form

$$|\Gamma\rangle = t_{NN,N\Delta} G^0_{NN\Delta} F(\pi) |\pi, {}^3\mathrm{He}\rangle, \quad (2)$$

where $G^0_{NN\Delta}$ is the free NNΔ propagator, $F(\pi)$ is the π-absorption vertex function and $|\pi, {}^3\mathrm{He}\rangle$ is the initial state. This term is depicted in Fig. 1. The amplitude Γ is the starting point for the rescattering processes among the three nucleons. Taken by itself it provides the properly symmetrized impulse approximation

$$U^{\mathrm{DWIA}} = \frac{1}{\sqrt{3}}(1+P)|\Gamma\rangle. \quad (3)$$

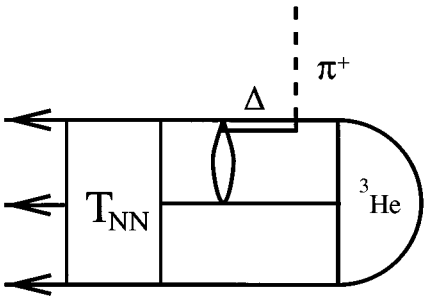

FIG. 2. The π absorption on ^{3}He as described in Fig. 1 followed by the complete $3N$ final state interaction T_{NN}.

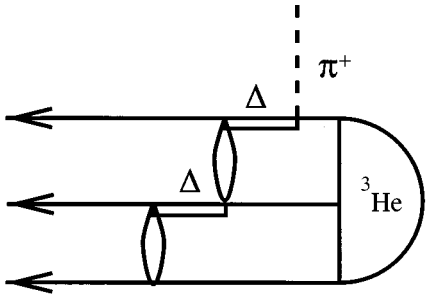

FIG. 3. Lowest order initial state interactions, ISI.

Here DWIA means distorted waves with respect to the two-body subsystem and plane wave with respect to the third particle. We use the usual permutation operator P, a sum of a cyclic and an anticyclic permutation of three objects, which is a very convenient structural element in the Faddeev treatment of three identical particles [28]. The three-nucleon rescattering amplitude

$$U^{\mathrm{rescatt}} = \frac{1}{\sqrt{3}}(1+P)T_{NN}|\Gamma\rangle \quad (4)$$

is generated by the operator T_{NN}, which obeys

$$T_{NN}|\Gamma\rangle = t_{NN,NN} G^0_{NNN} P|\Gamma\rangle + t_{NN,NN} G^0_{NNN} P T_{NN}|\Gamma\rangle. \quad (5)$$

Here T_{NN} is a three-body operator and $t_{NN,NN}$ is a two-body operator as depicted in Fig. 2. Because of our simplifying assumption a reoccurrence of a Δ-particle is not allowed, thus only the free $3N$ propagator G^0_{NNN} occurs.

Comparing Eq. (5) to the corresponding equation for inelastic electron scattering [9], we see that the driving term is modified due to the absence of the term $t_{NN,NN} G^0_{NNN}|\Gamma\rangle$. That term would double count the NN interaction, since $|\Gamma\rangle$ contains the NN interaction to inifinite order in the same particle channel. In electron scattering $|\Gamma\rangle$ is driven by the electromagnetic current operator and no double counting occurs.

For the processes discussed up to now the breakup amplitude is

$$U = U^{\mathrm{DWIA}} + U^{\mathrm{rescatt}} \quad (6)$$

and this will be investigated numerically.

So far the rescattering parts of the diagrams were three-nucleon reducible. Nonreducible diagrams shown in Fig. 3 are likely to play an important role and will be investigated numerically in a forthcoming article. Here we just present the necessary formal extensions. The Δ resulting from the the absorption of the initial pion can be absorbed and reexcited on another nucleon line, a process that can be iterated before the three-nucleon final state is reached. This is incorporated in the amplitude[1]

[1]In the $3N$ problem this quantity would be called U_0 with the index denoting the three-nucleon continuum channel. This is unnecessary here since π^+ absorption on ^{3}He has no other channels.

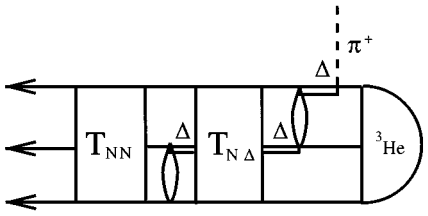

FIG. 4. The leading term of Eq. (8) representing initial state interactions acting in the Hilbert space of two nucleons and one Δ particle.

$$U^{\mathrm{ISI}}=\frac{1}{\sqrt{3}}(1+P)T^{\mathrm{ISI}}|\Gamma\rangle, \tag{7}$$

where the superscript ISI stands for initial state interaction and T^{ISI} obeys the integral equation

$$T^{\mathrm{ISI}}|\Gamma\rangle = T_{NN}t_{NN,N\Delta}G^0_{NN\Delta}PT_{N\Delta}F(\pi)|\pi,{}^3\mathrm{He}\rangle + T_{NN}t_{NN,N\Delta}G^0_{NN\Delta}PT_{N\Delta}t_{N\Delta,NN}G^0_{NNN}PT^{\mathrm{ISI}}|\Gamma\rangle \tag{8}$$

and $T_{N\Delta}$ generates all possible $N\Delta$ pairs via

$$T_{N\Delta}=t_{N\Delta,N\Delta}G^0_{NN\Delta}+t_{N\Delta,N\Delta}G^0_{NN\Delta}PT_{N\Delta}\,. \tag{9}$$

Iterating Eqs. (5), (8), and (9) one can visualize the processes contained in T^{ISI} as is shown in Figs. 4 and 5. Figure 3 is the simplest new diagram contained in Fig. 4 representing the initial state interaction. An example of an additional final state interaction [the leading term of Eq. (8)] is shown in Fig. 4.

III. CHOICE OF COORDINATES

The amplitude $|\Gamma\rangle$ contains three steps: the pion absorption by the single-particle operator $F(\pi)$, the free propagator of the (zero width) Δ particle and two nucleons, and the action of the transition operator $t_{NN,N\Delta}$ converting the $N\Delta$ system into a two-nucleon system. In our three-body context this requires the use of various sets of Jacobi momenta. The ^{3}He wave function depends on the following momenta:

$$\vec{p}\,'=\frac{1}{2}(\vec{k}\,'_2-\vec{k}\,'_3),$$

$$\vec{q}\,'=\frac{2}{3}\left(\vec{k}\,'_1-\frac{1}{2}(\vec{k}\,'_2+\vec{k}\,'_3)\right) \tag{10}$$

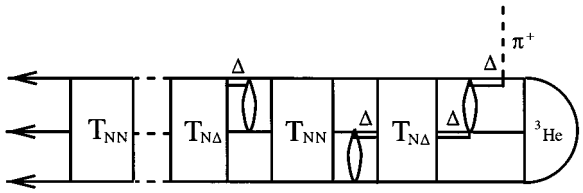

FIG. 5. The general representation of the second term of Eq. (8).

defined in terms of the individual momenta of three nucleons. After the pion absorption on nucleon 1 we describe the system consisting of two nucleons and a Δ particle by

$$\vec{p}=\frac{1}{2}(\vec{k}_2-\vec{k}_3),$$

$$\vec{q}=\frac{2M_N\vec{k}_1-(M_N+\omega)(\vec{k}_2+\vec{k}_3)}{3M_N+\omega}, \tag{11}$$

where M_N is the nucleon mass and $\omega=\sqrt{\mu^2+k_\pi^2}$ the energy of the pion[2] in the overall center-of-mass (c.m.) system. We choose the single-particle operator $F(\pi)$ to depend on the relative momentum $\vec{q}_0$ of nucleon 1 and the pion:

$$\vec{q}_0=\frac{M_N\vec{k}_\pi-\omega\vec{k}\,'_1}{M_N+\omega}=\vec{k}_\pi-\frac{\omega}{M_N+\omega}\vec{q}. \tag{12}$$

The second equality holds true in the overall c.m. system. The functional dependence of $F(\pi)$ related to the p-wave property of the Δ particle is given in the Appendix. We define

$$\langle\vec{k}_1|F|\vec{k}\,'_1\vec{k}_\pi\rangle=F(\vec{q}_0)\,\delta(\vec{k}_1-\vec{k}\,'_1-\vec{k}_\pi), \tag{13}$$

where $\vec{k}_1$ is the momentum of the Δ particle. Therefore we have

$$\langle\vec{p}\vec{q}|F|\pi,{}^3\mathrm{He}\rangle=\int d\vec{p}\,'d\vec{q}\,'\langle\vec{p}\vec{q}|F|\vec{p}\,'\vec{q}\,'\rangle\langle\vec{p}\,'\vec{q}\,'|\pi,{}^3\mathrm{He}\rangle = \int d\vec{q}\,'F(\vec{q}_0)\,\delta\left(\vec{q}-\vec{q}\,'-\frac{2}{3}\vec{k}_\pi\right)\langle\vec{p}\vec{q}\,'|\pi,{}^3\mathrm{He}\rangle. \tag{14}$$

The transition operator $t_{NN,N\Delta}$ acting between particles 1 and 2 requires another set of Jacobi momenta:

$$\vec{p}\,''=\frac{M_N\vec{k}_1-(M_N+\omega)\vec{k}_2}{2M_N+\omega}, \tag{15}$$

$$\vec{q}\,''=\frac{(2M_N+\omega)\vec{k}_3-M_N(\vec{k}_1+\vec{k}_2)}{3M_N+\omega}. \tag{16}$$

They are related to $\vec{p}$ and $\vec{q}$ by

$$\vec{p}=-\frac{3M_N+\omega}{2(2M_N+\omega)}\vec{q}\,''-\frac{1}{2}\vec{p}\,'', \tag{17}$$

$$\vec{q}=-\frac{M_N+\omega}{2M_N+\omega}\vec{q}\,''+\vec{p}\,''. \tag{18}$$

Finally we use Jacobi momenta denoted by $\vec{p}\,'''$ and $\vec{q}\,'''$ for three nucleons analogous to Eqs. (15) and (16) describing the

[2]Following [17] the quantity $M_N+\omega$ is used interchangeably with M_Δ in the resonance energy range.

three-nucleon system to the left of the $NN,N\Delta$ transition matrix, see Fig. 2. Since the transition t matrix is diagonal in $\vec{q}'''$ and $\vec{q}''$ we finally get

$$\langle \vec{p}'''\vec{q}'''|t_{NN,N\Delta}G^0_{NN\Delta}F|\pi,{}^3\mathrm{He}\rangle = \int d\vec{p}''\langle \vec{p}'''|t_{NN,N\Delta}|\vec{p}''\rangle G^0_{NN\Delta}(\vec{p}'',\vec{q}''') \times \int d\vec{p}d\vec{q}\langle \vec{p}''\vec{q}'''|\vec{p}\vec{q}\rangle\langle \vec{p}\vec{q}|F|\pi,{}^3\mathrm{He}\rangle. \quad (19)$$

IV. DRESSING THE Δ PARTICLE

So far we have introduced the momentum space representation of the $NN-N\Delta$ transition operator and the free $N\Delta$ propagator. In the Betz-Lee model [17] the $N\Delta$ propagator is dressed

$$G^0_{N\Delta} = \frac{1}{E-(M^0_\Delta-M_N)-[k^2(M_N+M^0_\Delta)/2M_NM^0_\Delta]-\Sigma_{N\Delta}(k,E)}, \quad (20)$$

where E is the c.m. system energy of the two-nucleon system, $\vec{k}$ is the $N\Delta$ relative momentum. The physical mass of the Δ particle is

$$M_\Delta = M^0_\Delta + \delta M, \quad (21)$$

where M^0_Δ is the bare mass. The energy dependent self-interaction Σ is

$$\Sigma_{N\Delta}(k,E) = \int_0^\infty \frac{F^2(k')k'^2dk'}{E+i\epsilon-H^0_{NN\pi}(k,k')-H^0_{N\pi}(k')} = \delta M - i\Gamma/2 \quad (22)$$

with

$$H^0_{NN\pi}(k,k') = \frac{k^2}{2M_N}+\frac{k^2}{2(M_N+\sqrt{\mu^2+k'^2})}, \quad (23)$$

$$H^0_{N\pi}(k') = \frac{k'^2}{2M_N}+\sqrt{\mu^2+k'^2}. \quad (24)$$

Here Γ is the energy dependent width of the Δ particle. The vertex function F contains the bare coupling constant F^0_Δ and the range parameter Λ_Δ, which are defined in the Appendix. The steps required for the partial wave representation are also described there. For the calculation below we shall allow small variations of the bare Δ parameters M^0_Δ and F^0_Δ.

V. RESULTS

In order to test the input for the pion absorption reaction on ^{3}He we recalculated the total pion πd absorption cross section on the deuteron as a function of energy in the Betz-Lee model. Table I shows the partial waves used, and the parameters for the potential are taken from [17].

In the present study we exclude for simplicity the contribution corresponding to propagating πNN states and the initial pion scattering effects. Thus, our result for the deuteron is different from the full unitary calculation of [17]. The dashed line in Fig. 6 shows the total cross section together with the data interpolated by the solid line. The dotted line shows a calculation without the $^1D_2$ partial wave, which demonstrates the importance of that wave.

TABLE I. Partial wave decomposition of NN and $N\Delta$ systems.

NN	$N\Delta$
$^1S_0$	$^5D_0$
$^3P_0$	$^3P_0$
$^3P_1$	$^3P_1$, $^5P_1$
$^3P_2$, $^3F_2$	$^3P_2$, $^5P_2$
$^1D_2$	$^5S_2$
$^3F_3$	$^5P_3$
$^1G_4$	$^5D_4$

As discussed in [17], the Betz-Lee model in our simplified approximation does not quantitatively reproduce the data. The peak position is about 30 MeV too high and the cross section is too low on the rising part below the resonance. The last feature is certainly partly related to neglecting nonresonant πN partial waves. On the other hand, the shift of the resonance has been obtained correctly in models containing explicit pion propagaton in intermediate states [17,18]. In the present paper we stick to the pure Δ model excluding explicit $NN\pi$ propagation for the reaction on the deuteron or $NNN\pi$ in the case of ^{3}He. We have therefore adjusted the bare parameters M^0_Δ and F^0_Δ of the Betz-Lee model in order to reproduce the observed energy dependence of the total cross section on the deuteron. Only small changes are needed to reproduce size and position at the resonance, see Fig. 7:

$$M^0_\Delta = 1280 \rightarrow 1260 \ \mathrm{MeV} \quad (25)$$

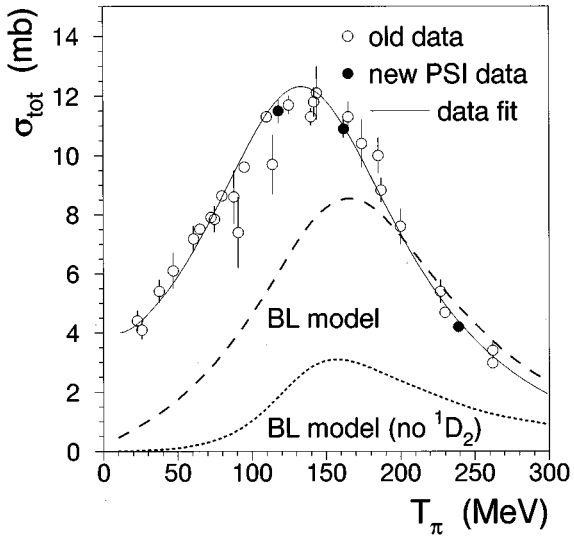

FIG. 6. Total cross section for $\pi+d\rightarrow pp$ as a function of the laboratory pion kinetic energy. The data (solid line) are taken from [30–42]. The dashed and dotted lines are calculated from the Betz-Lee potential with all partial wave set of Table I and without the $^1D_2$ wave, respectively.

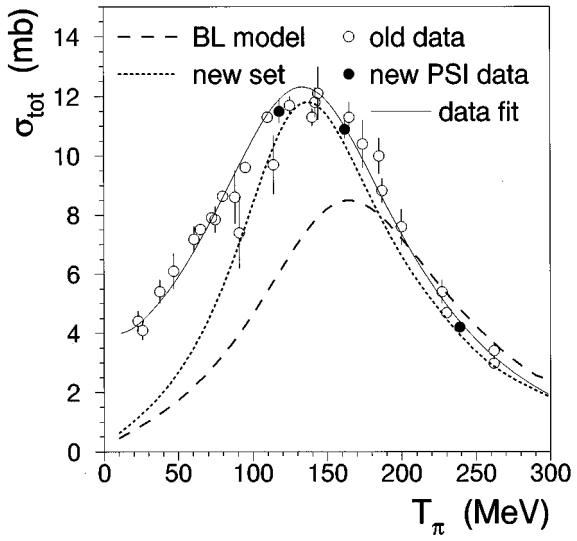

FIG. 7. Total cross section for $\pi+d\rightarrow pp$ as a function of the incident pion energy in laboratory system. The description of the lines is the same as in Fig. 6 with the exception of the dotted line which is now the theoretical prediction based on the new parameter set.

and

$$F^0_\Delta = 0.98 \rightarrow 1.00. \tag{26}$$

Since the new parameters reflect pion propagation in the absorption reaction in an effective way, it is clear that the elastic πN and the elastic πd cross sections will not be correctly described. The same is true for the NN phase shifts. As an illustration we show the effect of the new parametrization on the $^1D_2$ partial wave in Fig. 8. The Betz-Lee approach which we use also neglects the diagonal potential

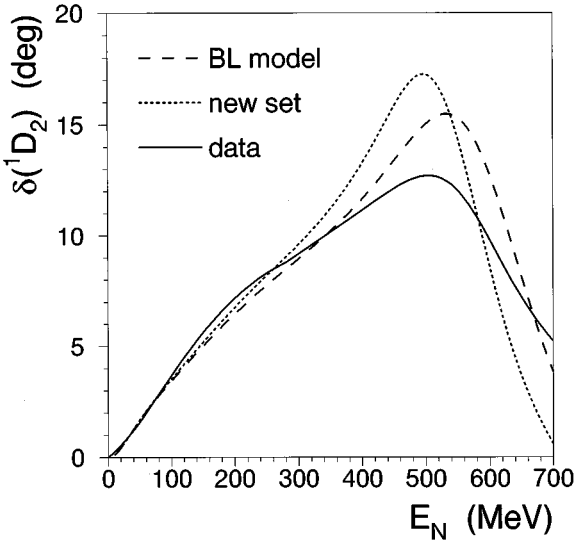

FIG. 8. The $^1D_2$ NN phase shift as a function of the laboratory kinetic nucleon energy. The data (solid line) are represented by the partial wave from the SAID analysis (see [43] and references therein). The dashed and dotted lines correspond to the Betz-Lee potential and its new parametrization, respectively.

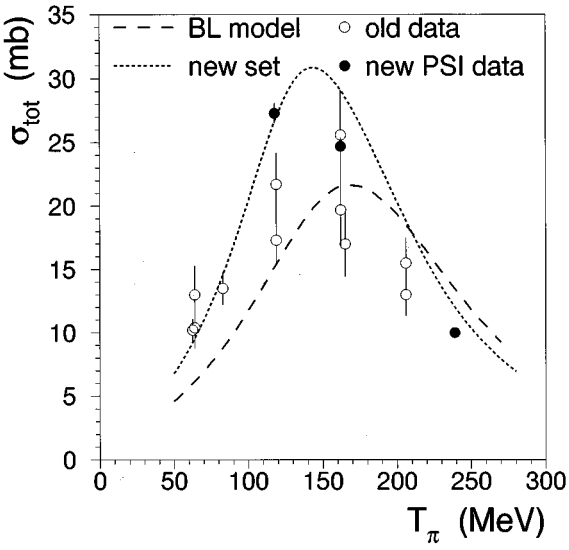

FIG. 9. Total cross section for $\pi^+{}^3\mathrm{He}\rightarrow ppp$ as a function of the laboratory pion kinetic energy. The data are from PSI [23,44] and [45–51].

$V_{N\Delta,N\Delta}$. We have verified that the inclusion of such a potential allows to shift the peak position downwards. For the time being, however, we restrict ourselves to treating M^0_Δ and F^0_Δ as the only effective parameters and retain $V_{N\Delta,N\Delta}=0$.

It is gratifying that the effective parameters of Fig. 7 also improve the description of the total cross section on ^{3}He as is shown in Fig . 9. We therefore see that the gross feature on ^{3}He falls into place once the reaction on the deuteron is properly described. In Fig. 9 the effects of final state interactions in the $3N$ continuum state are fully included. On the scale of the figure the effect of FSI is too small to be drawn (2%). For the total cross section FSI is thus negligible. For observables and kinematics which are not dominated by the two-nucleon DWIA mechanism (the quasideuteron process)

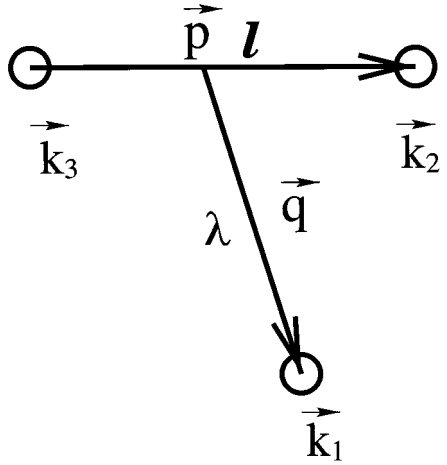

FIG. 10. Jacobi momenta and related orbital angular momenta.

significant modifications due to FSI are however to be expected.

VI. SUMMARY AND OUTLOOK

We formulated a model of pion absorption on ^{3}He in a Faddeev scheme, which includes the final state interaction among the three outgoing nucleons and which also allows for initial state interaction where more than one Δ resonance is excited (see Figs. 3–5). The numerical evaluation in this paper is restricted to the leading quasideuteron absorption term with inclusion of the final state interactions, Eq. (6), between the three protons. The phenomenological Betz-Lee model for the NN-NN and NN-$N\Delta$ systems is used. In the present exploratory calculation where the initial pion scattering and the contributions corresponding to propagating πNN states are neglected, the resulting total pion absorption cross sections for ^{2}H and ^{3}He do not agree with the data. The most striking feature is that the theoretical peak positions occur at too high pion energy in comparison to experiment. The full inclusion of the final state interaction among the three nucleons in our model has no visible effect for the total cross section, its contribution is only about 2%.

We introduced a very simple method to shift the peak position for the deuteron reaction by lowering the bare Δ mass M_Δ^0 by 20 MeV and increasing the coupling strength F_Δ^0 from 0.98 to 1.00. We demonstrated that this modification at the same time also reproduces the ^{3}He cross section within the uncertainties of the data. This points to a common dominant mechanism for pion absorption in both nuclei.

In our model with Δ-resonance excitation and propagation, the π^+ absorption in ^{3}He occurs both on $t=0$ and $t=1$ nucleon pairs since the ^{3}He wave function is fully antisymmetrized. Our approach is thus not a deuteron like model for the absorption on ^{3}He. However, we cannot expect the ratio of $t=0$ to $t=1$ contributions to be realistic, since the absorption of π^+ by a $t=1$ $^1S_0$ pair leads to a P-wave $N\Delta$ state which is highly suppressed dynamically as demonstrated in the calculation [26]. It is necessary to include non-Δ mechanisms to give a realistic description of the absorption on a $t=1$ pair. Empirically, it has been fairly established [22] that non-Δ contributions are suppressed by a factor of 10 in the resonance region. Such contributions will affect our results by interference with the dominant Δ excitation amplitudes, but we do not expect that the shift of the peaking position from this source will be as large as 30 MeV. However, this should be further examined in a a future calculation which also includes non-Δ absorption mechanisms.

The Betz-Lee model sets the transition potential $V_{N\Delta,N\Delta}$ to zero. In a forthcoming study we shall abolish that assumption and include the transition potential $V_{N\Delta,N\Delta}$, as it occurs for instance in the phenomenological V28 potential [29]. At the same time we shall investigate the importance of initial state interactions introduced in Eq. (7). The diagonal $N\Delta$ potential is expected to be important in this context.

More efforts are also needed to clarify the experimental situation for the $\pi^+{}^3$He absorption. While the new PSI data have much smaller errors than the old data, the overall agreement between different measurements of the total cross section is marginal. In particular, a better determination of the position and the width of the peak would be important for testing theoretical models.

ACKNOWLEDGMENTS

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APPENDIX: PARTIAL WAVE REPRESENTATIONS

Here we present the partial wave representation used. It is related to the choice of Jacobi coordinates of Fig. 10, see also Sec. III. For three particles (three nucleons or two nucleons and a Δ-particle) the partial wave basis in momentum space is

$$|p,q,\alpha JMTM_T\rangle = |p,q,(s_2s_3)s(ls)j(\lambda s_1)I(jI)JM(\tau_2\tau_3)t(t\tau_1)TM_T\rangle, \tag{A1}$$

where the orbital angular momenta l and λ are related to $\vec{p}$ and $\vec{q}$, and s_i, τ_i $(i=1,2,3)$ are spins and isospins, respectively.

The initial state nucleus, the ^{3}He ground state, has $J=\frac{1}{2}$ and $T=\frac{1}{2}$. The final ppp state has $T=\frac{3}{2}$. The $\pi N\Delta$ vertex function is written as

$$F(\vec{q}_0) = F(q_0)\sum_{m,\mu} |\tfrac{3}{2}\,\mu\,\tfrac{3}{2}\,m\rangle \sum_{m_1,m_N} \langle \tfrac{3}{2}\,m|1m_1\,\tfrac{1}{2}\,m_N\rangle Y^*_{1,m_1}(\hat{q}_0)\langle \tfrac{3}{2}\,\mu 1|\mu_\pi\,\tfrac{1}{2}\,\mu_N\rangle\langle \tfrac{1}{2}\,m_N\,\tfrac{1}{2}\,\mu_N|, \tag{A2}$$

where m and m_N are the z component of the spin of the Δ and the nucleon, μ and μ_N are the corresponding isospin quantum numbers. The pion enters through its orbital angular momentum z component m_1 and its isospin quantum number μ_π.

The function $F(q_0)$ is taken from the Betz-Lee model [17] [Eq. (3.5)]:

$$F(q_0) = \frac{F_\Delta^0}{\sqrt{2(M_N+\mu)}}\,\frac{q_0}{\mu}\left(\frac{\Lambda_\Delta^2}{\Lambda_\Delta^2+q_0^2}\right)^2, \tag{A3}$$

where $F_\Delta^0=0.98$ and $\Lambda_\Delta=358$ MeV/c. Using this operator F of Eq. (14) can be written as

$$\langle p,q,\alpha' J' M_I' T' M_T' | F \Psi J M T M_T = \sum_\alpha \delta_{l,l'}\delta_{s,s'}\delta_{j,j'}\delta_{t,t'}(X_1+X_2)\mathcal{I}, \tag{A4}$$

where

$$X_1 = k_\pi \sqrt{\frac{3}{4\pi}}(-)^{\lambda+J'+I+j-J-M}\sqrt{\hat\lambda\hat\lambda'\hat I\hat I'\hat J\hat J'}\sum_{\lambda_1+\lambda_2=\lambda} q^{\lambda_1}\left(\frac{2}{3}k_\pi\right)^{\lambda_2}\sqrt{\frac{\hat\lambda!}{\hat\lambda_1!\hat\lambda_2!}}\sqrt{\hat\lambda_1\hat\lambda_2}\sum_{b'}\hat b'\sum_{\mathcal{L}}\hat{\mathcal{L}}\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ b' & \lambda' & \mathcal{L}\end{Bmatrix}$$
$$\times\begin{pmatrix}\mathcal{L} & \lambda_1 & \lambda'\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\mathcal{L} & \lambda_2 & b'\\ 0 & 0 & 0\end{pmatrix}S^\alpha_{\mathcal{L}}(p,q,k_\pi)\sum_x (-)^x\hat x\begin{Bmatrix}I' & I & x\\ \lambda' & \lambda & b'\\ \frac{3}{2} & \frac{1}{2} & 1\end{Bmatrix}\begin{pmatrix}x & b' & 1\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}J & I & j\\ I' & J' & x\end{Bmatrix}\begin{pmatrix}J & x & J'\\ M & 0 & -M\end{pmatrix}, \tag{A5}$$

$$X_2 = -\epsilon q\sqrt{\frac{3}{4\pi}}(-)^{J'-J+j+I'-M}\sqrt{\hat J\hat J'\hat I\hat I'\hat\lambda\hat\lambda'}\sum_{\lambda_1+\lambda_1=\lambda} q^{\lambda_1}\left(\frac{2}{3}k_\pi\right)^{\lambda_2}\sqrt{\frac{\hat\lambda!}{\hat\lambda_1!\hat\lambda_2!}}\sqrt{\hat\lambda_1\hat\lambda_2}\sum_{bb'}\hat b\hat b'\sum_{\mathcal{L}}\hat{\mathcal{L}}\begin{Bmatrix}\lambda_1 & \lambda_2 & \lambda\\ b' & b & \mathcal{L}\end{Bmatrix}$$
$$\times\begin{pmatrix}\mathcal{L} & \lambda_1 & b\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\mathcal{L} & \lambda_2 & b'\\ 0 & 0 & 0\end{pmatrix}S^\alpha_{\mathcal{L}}(p,q,k_\pi)\begin{pmatrix}\lambda' & 1 & b\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}I' & I & b'\\ \lambda & b & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix}\frac{3}{2} & \frac{1}{2} & 1\\ b & \lambda' & I'\end{Bmatrix}\begin{Bmatrix}J & I & j\\ I' & J' & b'\end{Bmatrix}\begin{pmatrix}J & b' & J'\\ M & 0 & -M\end{pmatrix}, \tag{A6}$$

$$\mathcal{I} = \sqrt{4\hat T\hat T'}(-)^{1/2+t+T'-T-M_{T'}}\begin{Bmatrix}T & \frac{1}{2} & t\\ \frac{3}{2} & T' & 1\end{Bmatrix}\begin{pmatrix}T & 1 & T'\\ M_T & \mu_\pi & -M_T'\end{pmatrix}, \tag{A7}$$

$$\epsilon = \frac{\omega}{M_N+\omega}, \tag{A8}$$

and the notation $\hat x \equiv (2x+1)$ is used. The function $S^\alpha_{\mathcal{L}}(p,q,k_\pi)$ is defined as

$$S^\alpha_{\mathcal{L}}(p,q,k_\pi) = \int_{-1}^{1} dx P_{\mathcal{L}}(x)\frac{F(|\vec k_\pi - \epsilon\vec q|)}{|\vec k_\pi - \epsilon\vec q|}\frac{\Psi_\alpha\left(p,\left|\vec q - \frac{2}{3}\vec k_\pi\right|\right)}{|\vec q - \frac{2}{3}\vec k_\pi|^\lambda}, \tag{A9}$$

where x is the cosine between $\vec q$ and $\vec k_\pi$, and Ψ_α is the ^{3}He wave function in the basis (A1).

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