Influence of short and long range correlations on the charge densities and radii of Ca nuclei

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The empirical data for the charge (proton) density differences and the isotope shifts of the even Ca nuclei are analyzed by means of a simple phenomenological model. The role of short and long range correlations in the description of the data is examined. Certain types of short and long range correlations are accounted for. Short range correlations are approximated through a Jastrow-type correlation function while for long range correlations the fluctuations of the nuclear surface are considered. The analysis shows that the combined effects of these correlations improves the description of the experimental data. [S0556-2813(97)01804-9]

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The study of Ca nuclei attracted interest for theoretical study long ago. This is due to the rich experimental information available in this region. The even Ca isotopes have been investigated by many methods such as Coulomb excitation, muon spectroscopy, hadron scattering, and electron scattering [1–4]. Because of these methods, valuable information has been obtained about their charge and mass distributions. Furthermore, laser spectroscopy [5,6] provides very precise mean square (MS) radii. The data indicate that for a satisfactory theoretical description nuclear correlations beyond the mean field approach are necessary. These nuclear correlations, which represent modifications of the mean field picture, can be attributed to a coupling of the Hartree-Fock ground state to low-lying collective modes and to short range correlations (SRC's) due to hard collisions between nucleons at small distances.

In a very recent publication [7] the role of short range correlations in reproducing the empirical data for the charge (proton) density differences of even Ca nuclei was examined. In that approach the cluster expansion [8,9] truncated at the two-body term was employed and SRC's of the Jastrow type [10] were considered. We recall that expressions for the correlated charge form factors, $F_{\rm ch}(q)$, densities and moments of the s-p and s-d shell nuclei have been derived [11–15] in the framework of the factor cluster expansion of Ristig, Ter Low, and Clark [8,9] using the Jastrow ansatz for the correlated wave functions. This type of correlation is characterized by the correlation parameter λ_{nlS} which enters the normalized correlated wave functions of the relative motion:

$$\psi_{nlS}(r)=N_{nlS}[1-\exp(-\lambda_{nlS}r^2/b^2)]\phi_{nl}(r), \qquad (1)$$

where N_{nlS} are the normalization factors, $\phi_{nl}(r)$ are the harmonic oscillator (HO) wave functions for the relative motion, and $b=\sqrt{2}b_1$ ($b_1=\sqrt{\hbar/m\omega}$) is the corresponding harmonic oscillator parameter. In such an approach the expression for the point proton form factor, $F(q)$, takes the form

$$F(q)=F_1(q)+F_2(q), \qquad (2)$$

where $F_1(q)$ is the contribution of the one-body term to $F(q)$, which can be written easily in closed form [7,16]. The contribution of the two-body term $F_2(q)$ to the form factor $F(q)$ can be expressed in a rather simple way in a closed form by means of matrix elements:

$$A_{nlS}^{n'l'S'}(j_k)=\langle\psi_{nlS}|j_k(qr/2)|\psi_{n'l'S'}\rangle. \qquad (3)$$

These are simple polynomials and exponential functions of q^2 [11–15]. The correlation parameter λ_{nlS} is taken as state independent ($\lambda_{nlS}=\lambda$). Then the charge form factor $F_{\rm ch}(q)$ is written $F_{\rm ch}(q)=f_p(q)\times f_{\rm c.m.}(q)\times F(q)$ with $f_p(q)$ and $f_{\rm c.m.}(q)$ being the corrections due to the finite proton size [11] and the center-of-mass motion [17], respectively. The interesting feature of the method is the possibility of finding an analytic form for the correction to the uncorrelated charge (proton) density distribution by means of a Fourier transform of $F_2(q)$. Thus the correlated proton density distribution is written as

$$\rho_{\rm SRC}(r)=\rho_1(r)+\rho_2(r). \qquad (4)$$

We note here that the use of HO orbitals for the uncorrelated proton density distribution, although it is a simplification, has certain advantages. The correction of the center of mass motion can be done exactly. Most of the calculations are analytic and closed form expressions can be derived for various quantities such as the correlated form factor and density distribution as well as the moments of the density. In addition the computation time is reduced considerably.

The parameters of the model were adjusted to reproduce the experimental isotope shifts [5,6] of Ca nuclei. For each nucleus there are two parameters: the HO parameter b_1 and the ''actual'' correlation parameter μ, ($\mu=\sqrt{b_1^2/\lambda}$). In Ref. [7] it was assumed that these parameters for the Ca isotopes can be written as

$$\mu(A_c+n)=\mu(A_c)+\delta\mu(A_c+n), \qquad (5)$$

$$b_1(A_c+n)=b_1(A_c)+\delta b_1(A_c+n), \qquad (6)$$

where $\mu(A_c)$ and $b_1(A_c)$ are the parameters of the corresponding closed shell nucleus ($A_c=40$). The differences $\delta\mu$ and δb_1 express the change in the parameters due to the addition of extra neutrons (n).

The parameters $\mu(A_c)$ and $b_1(A_c)$ were determined by a fit to the data for the charge form factor of ^{40}Ca. Thus the problem was reduced to the determination of the differences

δb_1 and $\delta\mu$. The changes δb_1 were determined using isospin-dependent theoretical expressions for the oscillator parameters [18], while $\delta\mu$ were adjusted to reproduce the experimental data for the isotope shifts of Ca nuclei. It turned out that the calculated values for the differences of the density distributions exhibited the correct trend. It should be noted, however, that the comparison with the data was not very good in all cases. The maximum for the proton density difference of ^{48}Ca - ^{40}Ca (see Fig. 4 of [7]) was not reproduced well. This was an indication that additional correlations were necessary to improve the agreement with the experiment.

Recently, the effect of fluctuations of the nuclear surface was also included in the model [19]. It was shown that the combined effects of SRC's and of the surface fluctuation correlations (SFC's) improved the description of the experimental charge form factors of ^{16}O and ^{40}Ca nuclei. The SFC's belong to the category of the so-called long range correlation (LRC's) which are defined as modifications of an independent particle model wave function due to low-lying collective exited states of nuclei, such as rotations, vibrations, and giant resonances. It is noted that the role of ground state (long range) correlations was a matter of detailed investigation long ago [20–25]. Esbensen and Bertcsh [24] have shown that fluctuations of the nuclear surface due to zero-point motions coming from low-lying collective states affect the ground state charge density. Barranco and Broglia [25], in perhaps the most fundamental approach, have found that the ground state correlations associated with the surface modes of the Ca isotopes are important and qualitatively explain the observed dependence of the MS radii on the mass number.

In this paper we extend the study of Ref. [7] by including in the formalism the effect of SFC's. Our aim is to study in the framework of a simple phenomenological model the combined effects of SRC's and LRC's by analyzing the available empirical data of even Ca nuclei, in other words to investigate the effects of SFC's on the charge (proton) density differences and the radii of Ca nuclei, while simultaneously approximating SRC's through the Jastrow correlation factor. It is noted that SRC's and/or LRC's have been considered phenomenologically in theoretical studies [26–29] for the description of $(e,e'p)$ knockout reactions [30,31].

In the present work we follow Ref. [24]; i.e., we consider the ground state correlations which are introduced due to zero-point motion of collective surface vibrations. According to [24] the proton (or charge) density of a nucleus, deformed through the zero-point fluctuations, has the form (see also [20–22] for a rather similar expression)

$$\rho_{\rm cor}(r)=\frac{1}{\sqrt{2\pi}\sigma}\int \rho_1(r-\xi)\exp\left[-\frac{(\xi-s_0)^2}{2\sigma^2}\right]d\xi, \quad (7)$$

where $\rho_1(r)$ is the uncorrelated density, s_0 is a correction needed to conserve the number of particles in the correlated ground state, and σ is a measure of the effect of the zero-point fluctuations. The value of σ is related to β_λ, the deformation parameters for the states of multipolarity λ, with the relation

TABLE I. The values of the HO parameter b_1 (in fm) and the SRC parameter μ (in fm) for the even Ca nuclei. The values in parentheses correspond to the results of Ref. [7], where the effect of SFC's was not considered. In the last column the quantity σ (in fm) for the SFC's of the even Ca nuclei is listed. The values are taken from Ref. [25].

A	b_1	μ	σ
40	1.785 (1.860)	0.638 (0.499)	0.638
42	1.780 (1.855)	0.669 (0.548)	0.636
44	1.776 (1.851)	0.605 (0.566)	0.605
46	1.772 (1.848)	0.691 (0.545)	0.555
48	1.770 (1.845)	0.704 (0.528)	0.457

$$\sigma^2\simeq\frac{R_0^2}{4\pi}\sum_\lambda \beta_\lambda^2(\tau=0),$$

while the β_λ parameters can be determined from the values of $B(E_\lambda)$ [24,32].

The study of the combined effect of SFC's and SRC's is done by substituting in Eq. (7) the uncorrelated proton density distribution $\rho_1(r)$ with $\rho_{\rm SRC}(r)$ [formula (4)] where the effect of SRC's is accounted through Jastrow-type correlations. The three parameters of the model are determined in a way similar to that of Ref. [7]. The additional parameter σ due to SFC's is written as

$$\sigma(A_c+n)=\sigma(A_c)+\delta\sigma(A_c+n). \quad (8)$$

The parameters $\mu(A_c)$ and $b_1(A_c)$ are again determined by a fit to the data of the charge form factor of ^{40}Ca, while $\sigma(A_c)$ is taken from Ref. [25]. We recall that the quantity σ for the even-even Ca nuclei (see last column of Table I) was defined in [25] by means of the collective ''non-energy-weighted sum rule'' (NEWSR) for different multipolarities up to $\lambda=5$.

The differences $\delta\sigma$ and δb_1 are calculated from the σ values listed in Table I and the isospin-dependent expressions of the HO parameters [18], respectively. Then, the changes $\delta\mu$, are determined as in [7], i.e., adjusted to reproduce the experimental isotope shifts of Ca nuclei. The expression for the charge radii of Ca isotopes [formula (8) of Ref. [7]], after the inclusion of the SFC effect takes the form

$$\langle r^2\rangle_{\rm ch}=C_{\rm HO}\left(1-\frac{1}{A}\right)b_1^2+C_{\rm SRC}\frac{\mu^3}{b_1}+C_{\rm SFC}\sigma^2+r_p^2+\frac{N}{Z}r_n^2, \quad (9)$$

where r_p^2 and r_n^2 are the proton and neutron MS charge radii, respectively. For the latter the value $r_n^2=-0.116$ fm^2 is used [33]. The constants $C_{\rm HO}$, $C_{\rm SRC}$, and $C_{\rm SFC}$ for the case of Ca nuclei take the values $C_{\rm HO}=3$, $C_{\rm SRC}=12.4673$, and $C_{\rm SFC}=2.4308$. Finally, it should be noted that in all calculations of the MS charge radii of Ca isotopes the proton and neutron MS charge radii were taken into account [33].

In Figs. 1(a) and 1(b) the quantity $\Delta\rho_{\rm ch}(40+2n)r^2$ for the charge distribution differences of ^{42}Ca-^{40}Ca and ^{44}Ca-^{40}Ca, respectively, calculated in the present approach (SRC+SFC, short-dashed line) is compared with the empirical data (solid lines). For the sake of comparison the

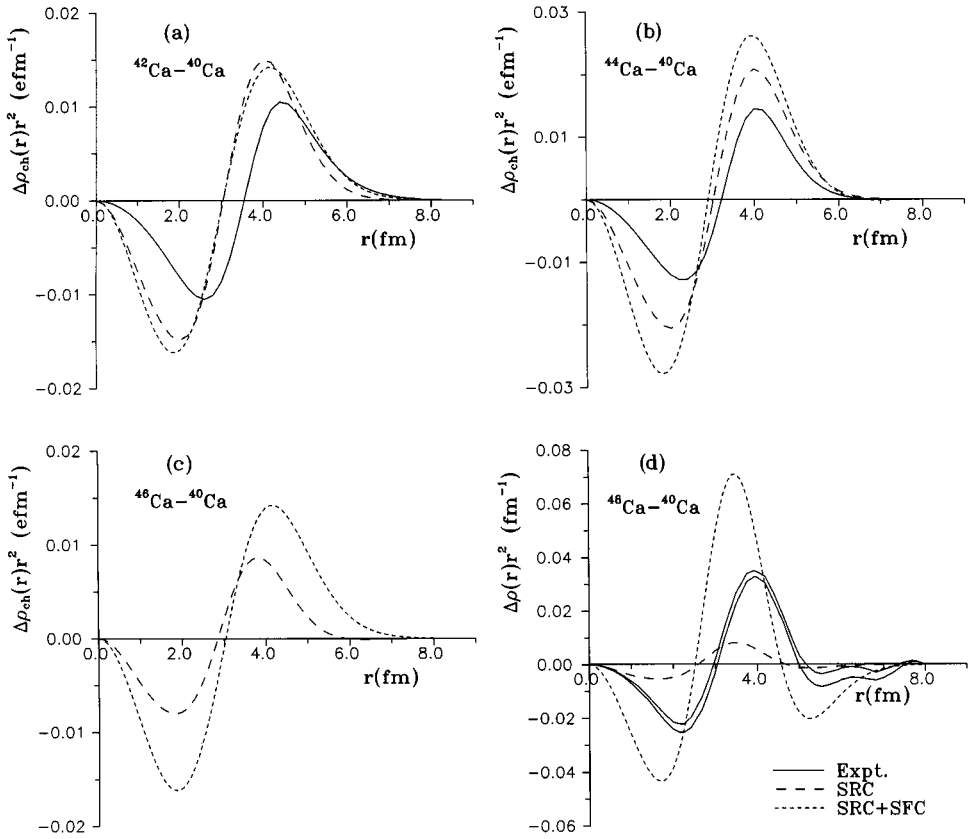

FIG. 1. The differences of the charge (proton) distributions of ^{42}Ca-^{40}Ca (a), ^{44}Ca-^{40}Ca (b), ^{46}Ca-^{40}Ca (c), and ^{48}Ca-^{40}Ca (d) multiplied by r^2 (short-dashed line), calculated in the present approach including the effect of SFC's and using the method of Ref. [7] are compared with the results (long-dashed line) of Ref. [7] where the effect of SFC's was not included. The experimental data (solid line) taken from Refs. [1,2] are also shown.

results of Ref. [7] (SRC, long-dashed lines), where only SRC's were considered, are also shown. The same is also in Fig. 1(d) for the difference ^{48}Ca-^{40}Ca. In this case the available experimental values correspond to the proton density distributions. The two solid lines correspond to the upper and lower values of the proton density difference. Finally, in Fig. 1(c) the prediction of the model for the charge density difference of ^{46}Ca-^{40}Ca is shown. It is seen that the trend is again reproduced rather well; however, the introduction of SFC's does not improve in general the comparison with the experiment. The charge density difference ^{42}Ca-^{40}Ca does not differ much from the one of Ref. [7]. It is slightly improved near the surface, but becomes less satisfactory in the interior. The agreement of the difference ^{44}Ca-^{40}Ca with the data becomes worse compared with the previous calculations [7]. Finally, for the ^{48}Ca-^{40}Ca proton density difference the maximum is overestimated and the behavior near the surface is not good. One could say that the use of the values of Table I for the σ parameters is rather restrictive for the present model. This is because by fixing the σ parameters and ''forcing'' the changes $\delta\mu$ to reproduce the empirical data for the isotope shifts, the so-determined parameters μ do not probably reflect correctly the influence of SRC's.

In Table I the values of the parameters b_1 (first column) and μ (second column) of even-even Ca nuclei are shown. The values in parentheses correspond to the ones calculated in [7], where the effect of SFC's was not taken into account. It is seen that using for the parameter σ the values of Table I (last column) the effect of SRC's increases significantly (more than 20%), while the oscillator parameters are slightly decreased. It is also noted that the quality of the fit for the charge form factor of ^{40}Ca does not improve. Moreover, the MS charge radius of ^{40}Ca is 3.62 fm, while from the fit of Ref. [7] it was found to be 3.43 fm, which compares fairly well to the empirical value (3.48 fm). It is interesting to note that repeating the fit and treating this time σ as a free parameter the charge form factor of ^{40}Ca is better reproduced (especially the third minimum) (see also [19]) and the charge radius is 3.50 fm. It seems that some more freedom is necessary for the determination of the parameters μ and σ in order to adjust themselves to the optimum values.

Next, we have tried another possibility. The determination of the parameters of the model was done using the available data for the charge (proton) density differences of the even Ca nuclei and then the calculated differences of the charge radii were compared with the experimental isotope shifts. It is clear that such an experimental input provides much more detailed information on nuclear structure than the isotope shifts measurements which give information only about the changes of the MS radii.

The parameters are determined by an overall (global) fit of the correlated charge (proton) density differences

$$\Delta\rho_{\rm cor}(40+2n)=\rho_{\rm cor}(40+2n)-\rho_{\rm cor}(40) \qquad (10)$$

to the experimental data.

More specifically the expression

$$\Delta\rho_{\rm cor}(40+2n)r^2, \quad n=1,2,4, \qquad (11)$$

is fitted to the experimental charge distribution differences of ^{42}Ca-^{40}Ca and ^{44}Ca-^{40}Ca ($n=1,2$) and the proton density difference of ^{48}Ca-^{40}Ca ($n=4$). The experimental values are taken from Refs. [1] and [2], respectively.

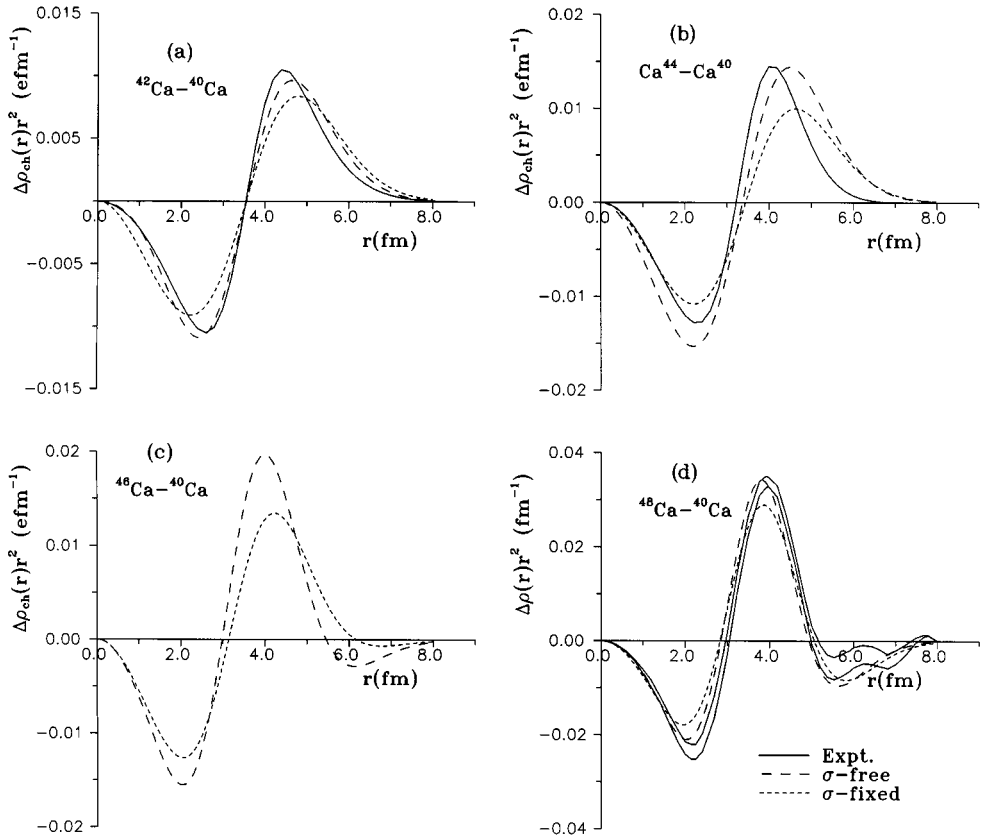

FIG. 2. The same as in Fig. 1, with the parameters of the model, however, determined from a global fit to the experimental charge (proton) density differences of the even Ca nuclei. The long-dashed line corresponds to a fit where the parameter σ is considered free, while the short-dashed line to a fit where this parameter takes values from Table I. The solid line corresponds to the experiment.

In Table II the parameters b_1, μ, and σ obtained from the fitting procedure are listed. An additional fit was also performed fixing the parameters σ and using the values of Table I. The corresponding b_1 and μ values are shown in parentheses. In the same table our predictions for the parameters of the ^{46}Ca nucleus are also given. These values are determined by an interpolation of the calculated values for ^{44}Ca and ^{48}Ca nuclei.

In Fig. 2 the charge (proton) differences are presented in the same order as in Fig. 1. The solid lines correspond to the experiment. The long-dashed line corresponds to the fit with the three parameters free. It is seen that the theoretical curves compare well with the experiment. The short-dashed lines describe the fit with two free parameters (b_1 and μ) and the σ parameters taken from Table I. It is seen that the description of the empirical data is less good but more satisfactory than in the first case (short-dashed lines of Figs. 1). This is because, in the present case, there is more freedom in the determination of the parameters b_1 and μ in contrast with the method of Ref. [7]. This may also indicate that there is a delicate balance between the effects of SRC's and SFC's and that probably there is some overlap between them.

It is interesting also to see how well the calculated isotope shifts of the even Ca nuclei compare with the experiment. It is seen in Fig. 3 that the theoretical values (squares), calculated with the correlation parameters μ and σ free, give a

TABLE II. The values of the HO parameter b_1 (in fm), the SRC parameter μ (in fm) and the SFC parameter σ (in fm) for the even Ca nuclei obtained from a global fit to the charge (proton) density differences. The values in parentheses correspond to a similar fit, where the values of the parameter σ are taken from the last column of Table I.

A	b_1	μ	σ
40	1.966 (1.998)	0.5805 (0.5683)	0.5284
42	2.011 (2.002)	0.5788 (0.5968)	0.4333
44	1.985 (2.010)	0.6158 (0.5975)	0.4748
46	1.993 (2.011)	0.6143 (0.6069)	0.3401
48	2.002 (2.011)	0.6129 (0.6168)	0.2054

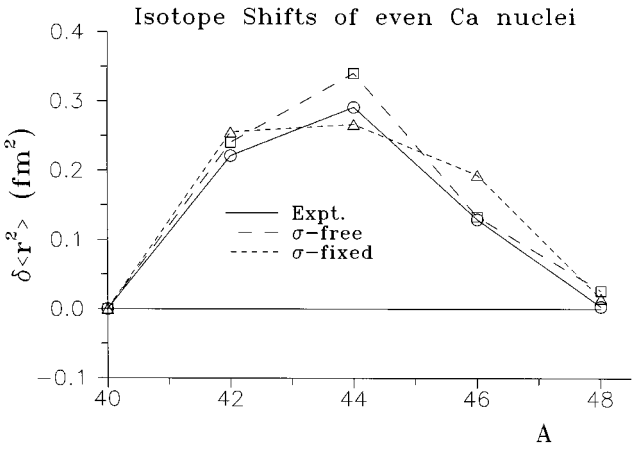

FIG. 3. The isotope shifts of even Ca nuclei, calculated in the present work considering σ as free parameter (squares) or using for σ the values of Table I (triangles). The circles correspond to the empirical data obtained from the laser spectroscopy measurements [5].

good description of the data (circles). In addition the isotope shift of ^{46}Ca is predicted correctly. The well-known parabolic behavior of the charge radii of Ca nuclei is reproduced. The triangles correspond to the isotope shifts calculated with the parameters σ being fixed. The description is less satisfactory.

A few more comments are appropriate.

(i) The analysis shows that by considering only SRC's (b_1 and μ free parameters) the calculated charge (proton) density differences, though they have the correct trend, are not very good. The charge form factors reproduce well all the diffraction minima. The mean square charge radii exhibit a parabolic behavior. However, the maximum is in the wrong place (^{42}Ca).

(ii) Accounting only SFC's (b_1 and σ free parameters) the charge (proton) density differences are not reproduced well. Especially for the difference ^{44}Ca-^{40}Ca the comparison with the experiment is very bad. The calculated isotope shifts have the correct trend, but the value for ^{44}Ca is unnaturally large. Finally the third diffraction minimum in the charge form factors is not reproduced at all.

(iii) The values of parameters σ in Table I give a measure of the effect of zero-point fluctuations and they are directly connected to the deformation parameters β_λ. Treating, however, σ as a free parameter, one cannot directly compare the obtained best fit values for σ to the ones of Table I. In such a case, σ describes effectively SFC's and perhaps its role is mixed with the other two parameters or possibly other effects are taken into account effectively. This is also seen from Table II. The values of the σ parameters, though they present the same decreasing trend, differ from those of Table I calculated using the collective NEWSR [25].

(iv) One cannot make a clear distinction between SRC's and SFC's. Their roles probably have some overlap and this makes difficult the determination of the parameters μ and σ which give a measure of their effects. Furthermore, perhaps other effects ''hidden'' in the empirical data might be accounted. Finally, it should be noted that due to the normalization condition defining N_{nlS}, in relation (1), LRC's are also present in the wave functions Ψ_{nlS} although their effects are not significant [34].

In summary, we have analyzed the empirical data of the even Ca nuclei using a simple phenomenological model where certain types of SRC's and LRC's have been accounted for. The combination of SRC's and SFC's improves the description of the density differences and reproduces the parabolic behavior of the charge radii. The analysis also shows that there is a balance between the SRC's and SFC's. It seems that the putative roles of the mean field, short range correlations and surface fluctuation effects get mixed up to some degree. The interplay between SRC's and SFC's improves the correction to the independent particle model, leading thus to a better description of the experimental data of Ca nuclei. One should keep in mind, however, the simplicity of the present phenomenological approach.

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