Homologous states and the structure of nuclei in the lead region

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The structure of both low-lying and highly excited states and electromagnetic transitions in ^{206}Pb and ^{205}Tl has been studied within the framework of the shell model. The calculation predicts the occurrence of states in ^{206}Pb with a structure homologous to parent states in ^{205}Tl, a phenomenon experimentally reported in these systems via $(\vec{p},\alpha)$ reactions, in analogy with other regions of the mass table. This feature is not restricted to configurations where the two neutron holes are coupled to angular momentum zero, but also applies to higher configurations corresponding to angular momentum recoupling of the two neutron holes. The calculated results, obtained with both a modified surface delta interaction and Kuo-Herling interaction, are in good agreement with the experimental data, further supporting the ability of these interactions to describe nuclear properties in the lead region. [S0556-2813(97)06305-X]

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I. INTRODUCTION

Recently the nuclei in the lead region have been the subject of both experimental and theoretical studies. In particular, considerable interest has been raised by the determination of the proton occupation number of the different orbitals [1] and by the experimental evidence, also in this region, of the occurrence of homologous multipole states (HMS's), excited in $(\vec{p},\alpha)$ reactions [2–4].

As is well known, the low-lying energy level structure of the different Pb isotopes can be adequately described by the shell model. Yet the sensitivity of the calculation to the choice of the residual interaction, in particular in the region of the high excited states, deserves further investigations. For example, the simplest phenomenological residual interaction used in this region is the modified surface δ interaction (MSDI) [5]. For like-particle nuclei or like-hole nuclei, as, for example, ^{206,205,204}Pb [6], the MSDI is known to be a good one among the residual interactions. But whether the MSDI and other interactions are good enough also for high excited states of nuclei is an open question.

The study of levels in the region of high excitation energy is rather complex and so far there are no systematic theoretical investigations in this region. The high excited states discussed here are mainly homologous multipole states and stretched states. In connection with the findings of the ^{208}Pb$(\vec{p},\alpha)^{205}$Tl and ^{209}Bi$(\vec{p},\alpha)^{206}$Pb reactions [4], let us denote the one-proton-hole–two-neutron-hole $(1\pi h\text{-}2\nu h)$ states in ^{205}Tl as parent states (for example, ground state with $J^{\pi}=3/2^{+}$) and the homologous multipole states in ^{206}Pb as the multiplets of $(1\pi p\text{-}1\pi h\text{-}2\nu h)$ states obtained by the coupling of the $h_{9/2}$ proton with the parent states. Because the separation energy $S_p(h_{9/2})$ of the $h_{9/2}$ proton is relatively small and similar to the value in ^{209}Bi, we expect that the $h_{9/2}$ proton will act as a spectator and therefore to find these homologous states rather pure.

The concept of the homologous state has been experimentally investigated for some years. Recently we studied theoretically the properties of the homologous multipole states of a number of nuclei in the region with $A\approx 90$ [3]. A similar study is performed in detail in the present paper in the region $A\approx 208$, in particular for ^{206}Pb and for the parent states in ^{205}Tl. We will not restrict our search to states in ^{206}Pb homologous to parent states corresponding to configurations where the two neutron holes are coupled to angular momentum zero, but also include configurations corresponding to recoupling of the two neutron holes. We will also explore in detail the role of configurations involving the intruder $h_{11/2}$ proton orbital.

The calculations of the sequence of levels and of the electromagnetic transitions have been performed within the framework of the shell model using as residual interaction the MSDI [7] and Kuo-Herling (KH) [8] interaction, utilizing the large-basis shell model code OXBASH [9]. In Sec. II we introduce the shell model Hamiltonian and the model space. Section III gives the calculated results for ^{205}Tl; these include low-lying energy levels, the wave functions of relative parent levels (both for the lowest $1/2^{+}$ and $3/2^{+}$ states, of nearly single-proton hole nature, and for the $5/2_1^{+}$, $7/2_1^{+}$, and $5/2_2^{+}$ states, of more complicated nature), and the electromagnetic transitions between parent states. Section IV discusses, at first, the low-lying states of ^{206}Pb. The calculated low-lying levels cover a large fraction of the set of experimental levels in ^{206}Pb, and so this offers a good test for residual interactions. As a second point, we discuss the homologous multipole states. A classification of the homologous states into different categories is proposed, according to the properties of the corresponding parent states. We then discuss the effect of enlarging the model space and of adding

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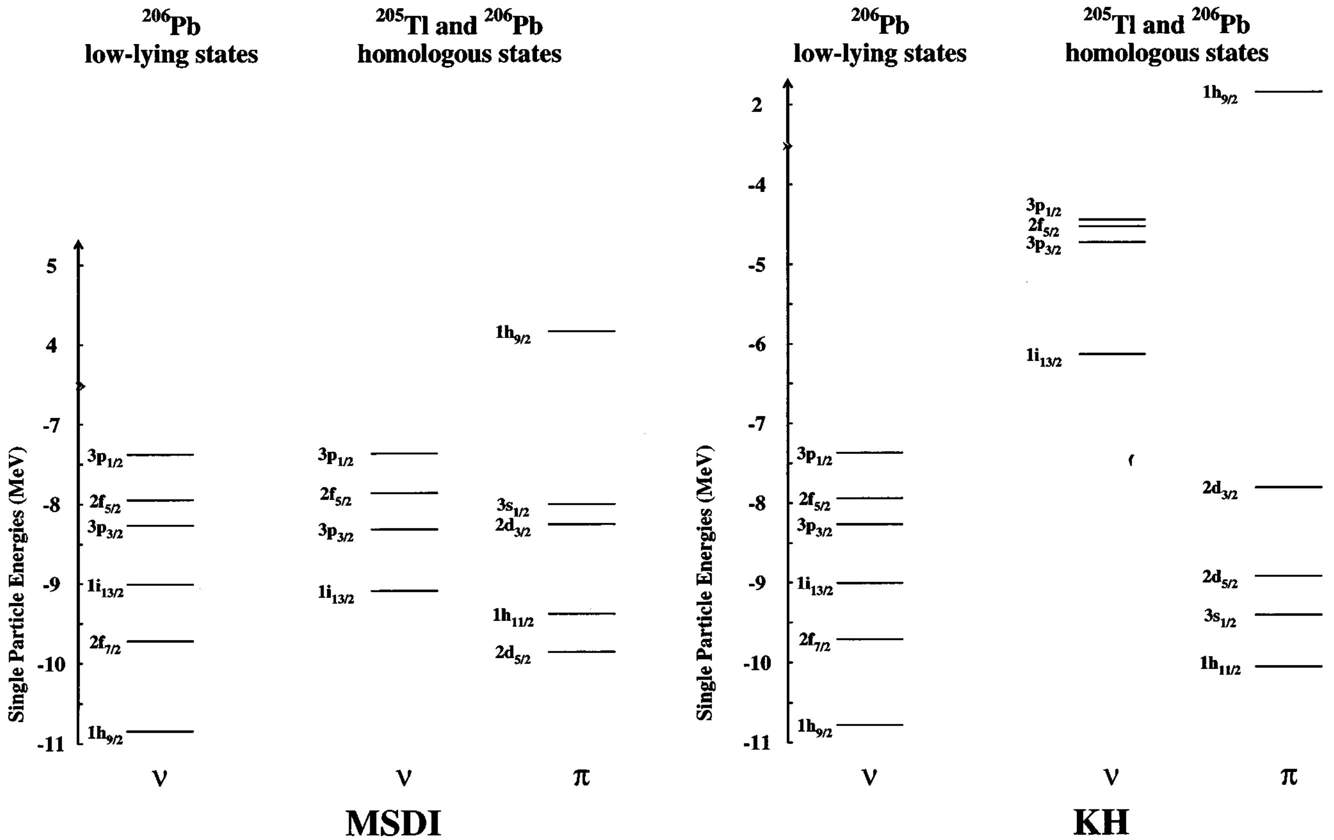

FIG. 1. Proton and neutron single-particle levels used in the shell model calculation, for the MSDI (left) and KH interaction (right).

a quadrupole-quadrupole term to the MSDI on the properties of homologous multipole states. Section V contains the conclusions.

II. SHELL MODEL

In the second-quantization representation the shell model Hamiltonian can be written as

$$H=H_{\text{core}}+\sum_{i}\epsilon_i a_i^{+}a_i+\sum_{i\geqslant j,k\geqslant l}V_{ijkl}a_i^{+}a_j^{+}a_l a_k. \tag{1}$$

We have assumed as the core the doubly closed system ^{208}Pb, and indicated as ϵ_i the energies of the single-particle states. For the calculation associated with the low-lying states in ^{206}Pb, i.e., for the states of two-neutron holes character, experimental ϵ_i values were used, taken as the single-hole energies in the neutron-hole nucleus ^{207}Pb. More precisely, the model space [sp energy (MeV)] used here is $\nu 1h_{9/2}(-10.85)$, $\nu 2f_{7/2}(-9.72)$, $\nu 1i_{13/2}(-9.01)$, $\nu 3p_{3/2}(-8.27)$, $\nu 2f_{5/2}(-7.95)$, and $\nu 3p_{1/2}(-7.38)$. For the calculation of the parent states in ^{205}Tl and of the homologous states in ^{206}Pb, instead, different sets of sp energy were used for the different interactions. The model space [sp energy (MeV)] used for the KH interaction [10] is $\pi 2d_{5/2}(-8.919)$, $\pi 2d_{3/2}(-7.805)$, $\pi 3s_{1/2}(-9.410)$, $\pi 1h_{11/2}(-10.056)$, $\pi 1h_{9/2}(2.155)$, $\nu 2f_{5/2}(-4.528)$, $\nu 3p_{3/2}(-4.727)$, $\nu 3p_{1/2}(-4.442)$, and $\nu 1i_{13/2}(-6.133)$, while for the MSDI, with values adjusted to give best fitting to data, the assumed model space is $\pi 2d_{5/2}(-9.861)$, $\pi 1h_{11/2}(-9.385)$, $\pi 2d_{3/2}(-8.258)$, $\pi 3s_{1/2}(-8.007)$, $\pi 1h_{9/2}(4.163)$ and $\nu 1i_{13/2}(-9.097)$, $\nu 3p_{3/2}(-8.318)$, $\nu 2f_{5/2}(-7.865)$, and $\nu 3p_{1/2}(-7.368)$ [Table I(a)]. For a better visualization of the different sets of single-particle levels, these are displayed in Fig. 1.

The quantities $V_{ijkl}=\langle ij|V|kl\rangle$ in the third term denote the two-body matrix elements, where $|ij\rangle$ is an antisymmetrized two-particle state in the m scheme, and $i(j,k,l)$ represents a complete set of quantum number $n_i l_i j_i(n_j l_j j_j, \ldots)$. As a first choice for the residual interaction we take the modified surface delta interaction V^{MSDI} of the form [5]

$$V^{\text{MSDI}}(1,2)=-4\pi\sum_{T}A_T\delta(\mathbf{r}(1)-\mathbf{r}(2))\delta(r(1)-R_0)+B(\tau(1)\cdot\tau(2))+C, \tag{2}$$

where $\mathbf{r}(1)$ and $\mathbf{r}(2)$ are the position vectors of the interacting neutron and proton holes (or particle), R_0 is the nuclear radius, and τ is the nucleon isospin operator. The quantities A_T, B, and C are the strength parameters and in our case are taken as in Ref. [7] for the low-lying two-neutron-hole states in ^{206}Pb and as in Ref. [11] for the states in ^{205}Tl and for the homologous states in ^{206}Pb. An appealing feature of the MSDI is provided by its simplicity and limited number of parameters. The introduction of other terms and/or the extension of the model space in the MSDI [7] can be easily implemented. Note that for ^{206,205,204}Pb [6] already the calculated

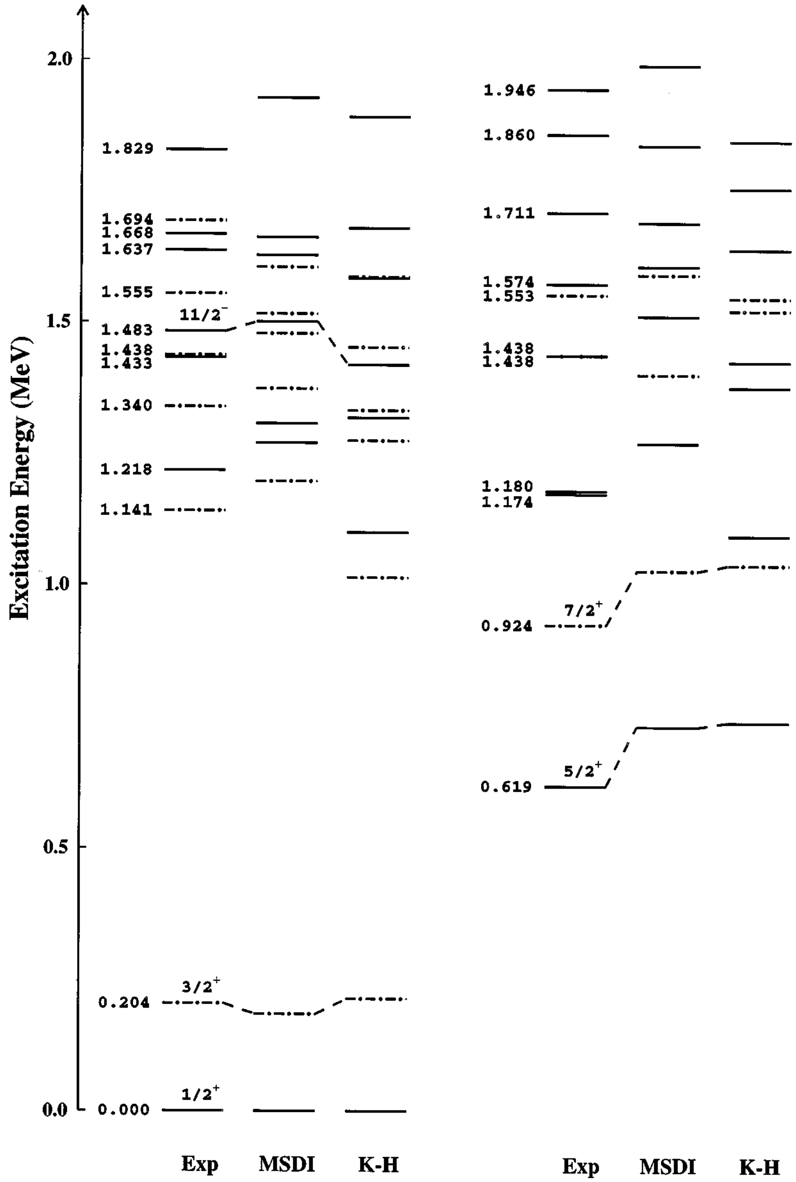

FIG. 2. Comparison between experimental and theoretical lowest eigenvalues, corresponding to the low-lying levels of ^{205}Tl. Theoretical eigenvalues are obtained using both the MSDI and KH interaction.

results with the four-parameter MSDI (taken as in Ref. [9]) are in surprisingly good agreement with the experiment. In order to discuss the homologous states we may introduce an additional $\mathbf{Q}\cdot\mathbf{Q}$ quadrupole term, which is the leading term in the multipole expansion of the long-range force and should therefore be important for the MSDI, which includes the short-range δ force only.

The Kuo-Herling interaction, also used here, is a realistic interaction. We refer to Ref. [8] for details. We only recall here that the KH interaction was derived by reaction matrix techniques from a free nucleon-nucleon potential with renomalization. The corresponding two-body matrix elements (TBME) can be written in leading order as a sum of a bare term and a term (with negative sign) due to 1p-1h core polarization. We have here neglected a further renomalization due to 2p-2h excitation.

Given the single-hole (-particle) energies and associated TBME, the eigenvalues and corresponding eigenvectors can be obtained by diagonalizing the Hamiltonian matrix. The lowest eigenvalues, i.e., those corresponding to the low-lying levels of ^{205}Tl and ^{206}Pb, are presented in Fig. 2 and Fig. 3, respectively, and the levels of homologous multipole states of ^{206}Pb in Fig. 4. In the case of the results shown in Fig. 4, two sp energies have been slightly adjusted [$\pi 1h_{11/2}(-9.485)$ and $\pi 1h_{9/2}(4.263)$]. With this choice the relative levels move down about 0.1 MeV, in better agreement with the data, but without practically changing the associated wave functions.

The main components of the wave functions of the corresponding levels of ^{205}Tl and ^{206}Pb are listed in Tables I–III.

III. LOW-LYING LEVELS IN ^{205}Tl

The calculated low-lying levels with excitation energy $E_{\mathrm{exc}}<2.0$ MeV and spin and parity $J^{\pi}=1/2^{+}$, $3/2^{+}$, $5/2^{+}$, $7/2^{+}$, $11/2^{-}$ in ^{205}Tl are shown in Fig. 2. For a better understanding, we have separated in the figure the levels of $J^{\pi}=1/2^{+}$, $3/2^{+}$, $11/2^{-}$, which have mainly the two neutron holes coupled to $J'(2\nu h)=0$, shown on the left-hand side,

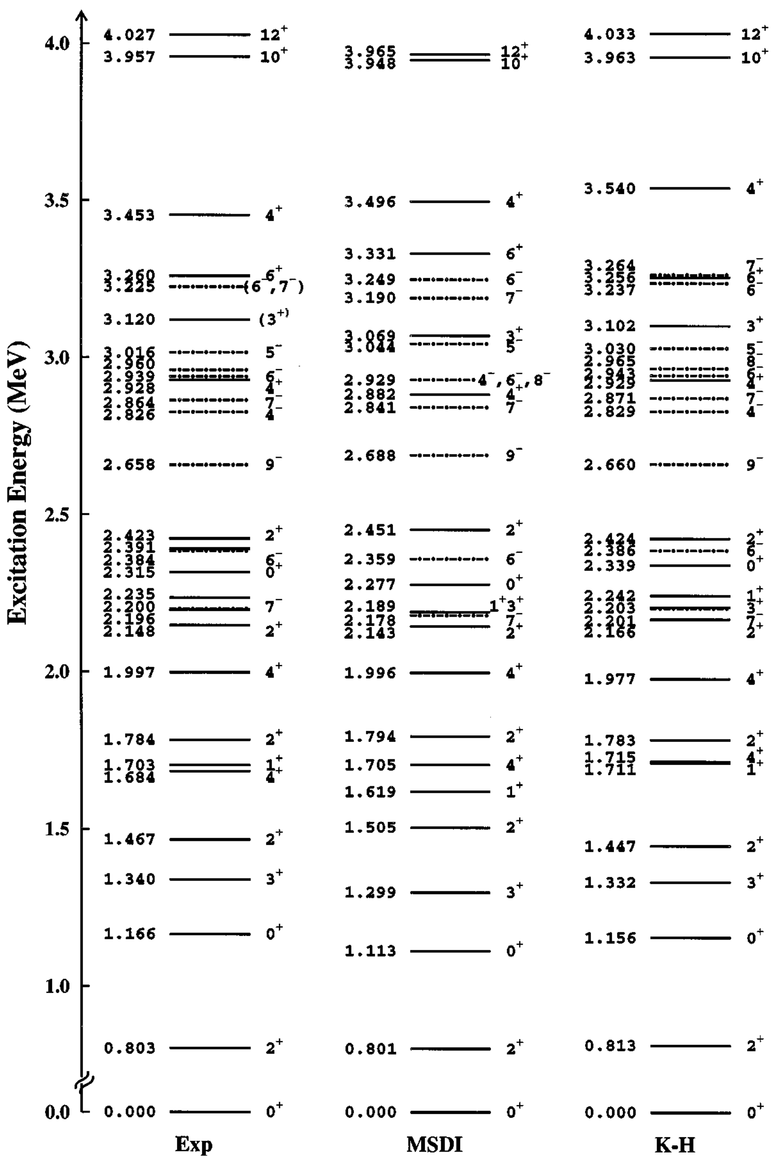

FIG. 3. The same as Fig. 2 for ^{206}Pb low-lying levels with two-neutron hole character.

from the levels of $J^{\pi}=5/2^{+}$, $7/2^{+}$, mainly of $J'(2\nu h)=2$ character, shown on the right-hand side. The comparison shows that the calculated results are in overall agreement with the adopted experimental levels [12].

The wave functions $|\pi j_1^{-1},\nu(j_2^{-1}\otimes j_3^{-1})J',J^{\pi}\rangle$, of some selected levels, e.g., those that will act as parent states, are listed in Tables I(a) and I(b) for the case of the MSDI. As is apparent from these tables, the states $1/2_1^{+}$, $3/2_1^{+}$, and $11/2_1^{-}$ are relatively pure $1\pi h$ configurations $s_{1/2}^{-1}(P\simeq 71\%)$, $d_{3/2}^{-1}(P\simeq 72\%)$, and $h_{11/2}^{-1}(P\simeq 85\%)$, respectively. In Table I(a), for example, the value 11.150 for the occupation number of the $h_{11/2}$ orbital in the first state $11/2^{-}$ arises from a wave function whose 85% corresponds to $1\pi h$ configuration $(h_{11/2}^{-1})$, while all the other configurations amount to about 15% only. One should also note that for these states the dominant $2\nu h$ configuration $(j_2^{-1}\otimes j_3^{-1})J'$ corresponds to total coupled spin $J'(2\nu h)=0$ [Tables I(b) and II(a)]. For example, for the $1/2_1^{+}$, $3/2_1^{+}$, and $11/2_1^{-}$ states the major components of the wave functions are $\nu(p_{1/2}^{-2})J'=0$ [41% (to be compared with the value 36% given in Ref. [11]) for $1/2_1^{+}$ states, 34% for $3/2_1^{+}$ state, and 41% for $11/2_1^{-}$ state].

The situation is different in the case of the $5/2_1^{+}$ and $7/2_1^{+}$ states. In this case for most components the coupled angular momentum of the two-neutron hole is $J'(2\nu h)=2$. As seen in Table I(a), the occupation numbers associated with these states are similar to those of the $1/2^{+}$ and $3/2^{+}$ states, with a single $1\pi h$ configuration for the proton orbital $s_{1/2}^{-1}(P\simeq 64\%)$ and $d_{3/2}^{-1}(P\simeq 79\%)$, and with a similar $2\nu h$ configuration, aside from the spin J'. For state $5/2_1^{+}$, the probability $P(d_{5/2}^{-1})$ of the single hole $d_{5/2}^{-1}$ is about 10% only in the experimental reaction [13], while in the (p,p') and (d,d') experiments there is evidence for a strong contribution of $|\pi s_{1/2}\otimes 2^{+}\rangle$ component [14]. These results are in agreement with our findings, $P(d_{5/2}^{-1})=14.1\%$ [Table I(a)]. The two $5/2^{+}$ and $7/2^{+}$ states, on the other hand, should be strongly populated in an ideal reaction of pickup of a proton from 2_1^{+} state of ^{206}Pb, since the $2\nu h$ configuration in these states is just similar to the 2_1^{+} state of ^{206}Pb. As stated

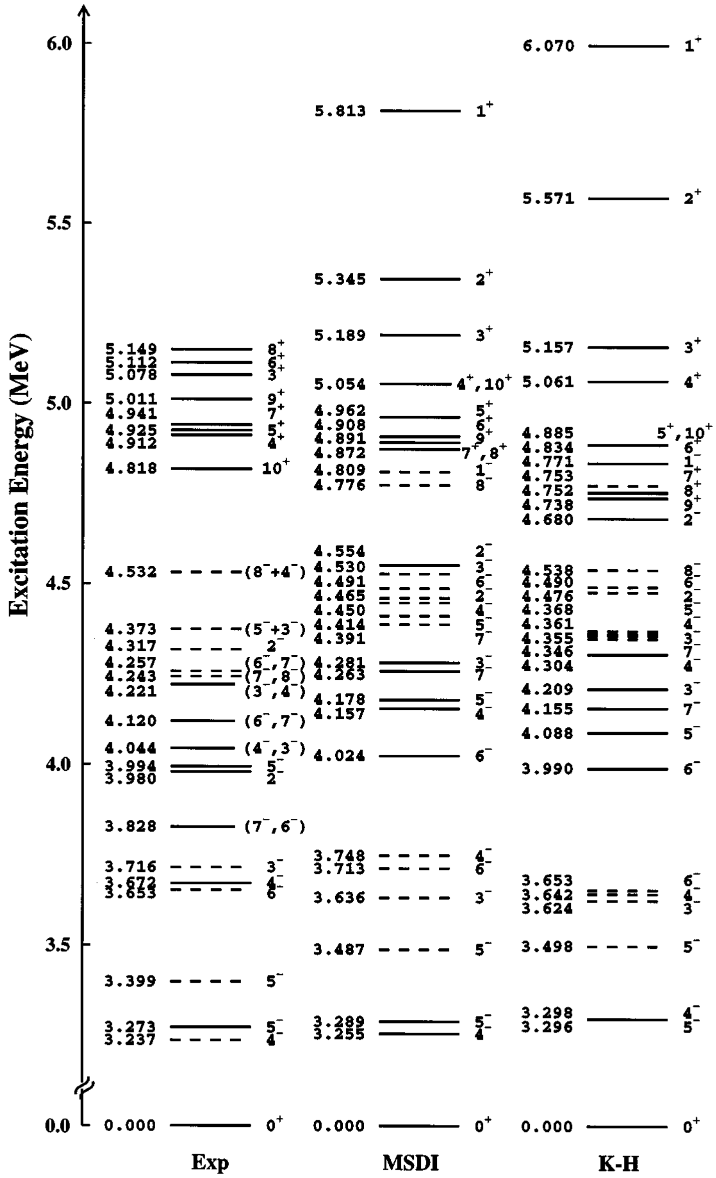

FIG. 4. Comparison between experimental and theoretical homologous multipole states of ^{206}Pb with 1pp-1ph-2nh character.

above, the $2\nu h$ configuration of the $1/2^+_1$ and $3/2^+_1$ states of ^{205}Tl has instead a structure similar to the 0^+_1 state of ^{206}Pb, while the $9/2^+_1$ state has the two-neutron holes in a configuration which is similar to the 3^+_1 state.

Essentially the same features are obtained using the KH interaction. There are only small variations in the different coefficients associated with the $1\pi h$ configuration. So, for example, we obtain for $s^{-1}_{1/2}(P\simeq 77\%)$ and $d^{-1}_{3/2}(P\simeq 79\%)$, for $5/2^+_1$ and $7/2^+_1$ states, values again close to those of $1/2^+_1$ and $3/2^+_1$ states. These states $1/2^+_1$, $3/2^+_1$, and $11/2^-_1$ have a more pure hole character, with values $P(s^{-1}_{1/2})\simeq 85\%$, $P(d^{-1}_{3/2})\simeq 81\%$, and $P(h^{-1}_{11/2})\simeq 95\%$.

The different character of $1/2^+_1$, $3/2^+_1$, and $11/2^-_1$ states, one the one hand, and of $5/2^+_1$ and $7/2^+_1$ states, on the other, is also reflected in different spectroscopic factors and electromagnetic transitions. The former states have large spectroscopic factors S, $S(s_{1/2},0^+_1)$, $S(d_{3/2},0^+_1)$, and $S(h_{11/2},0^+_1)$, values still consistent with the experimental S(expt) [10] [Table I(c)]. The latter states have large $S(s_{1/2},2^+_1)$, $S(d_{3/2},2^+_1)$ and small $S(d_{5/2},0^+_1)$, $S(g_{7/2},0^+_1)$, where $0^+_1(2^+_1)$ labels the $0^+_1(2^+_1)$ state of ^{206}Pb. Note, however, that the $g_{7/2}$ orbital has not taken into account in the model space of the 50–82 proton shell and this fact may partly explain a predicted value of $S(g_{7/2},0^+_1)$ smaller than the experimental one.

For the reduced transition probabilities connecting these two kinds of states, we expect the relation

$$B(E2,5/2^+\to 1/2^+)\simeq B(E2,7/2^+\to 3/2^+)$$

$$>B(E2,7/2^+\to 5/2^+)\simeq B(E2,3/2^+\to 1/2^+)$$

because the transitions between $5/2^+_1\to 1/2^+_1$ and $7/2^+_1\to 3/2^+_1$ correspond to a recoupling of the $2\nu h$ configuration, while the others are related to a change of the $1\pi h$ configuration only (see Fig. 5). The calculated values seem to partially satisfy the relation. We in fact obtain for the former $B(E2,5/2^+\to 1/2^+)=289\ e^2\ \mathrm{fm}^4$ $\simeq B(E2,7/2^+\to 3/2^+)=323\ e^2\ \mathrm{fm}^4$, values larger, as ex-

TABLE I. (a) Average number of particles in each j level of ^{205}Tl. The proton orbitals $d_{5/2}$, $d_{3/2}$, $s_{1/2}$, $h_{11/2}$ and the neutron orbitals $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $i_{13/2}$ have been considered. E_{exc} and J^{π} are, respectively, the excitation energy and spin and parity of the low-lying levels of ^{205}Tl. (b) Wave functions of low-lying states of ^{205}Tl. Each line corresponds to a state of the basis, which partition order is reported in the last column. [NO] is the number of unpaired nucleons; P% is the percent occupation contributing more than 0.5%. E_{exc} and J^{π} are, respectively, the excitation energy and spin and parity of the low-lying states of ^{205}Tl. (c) Experimental (S^{expt}) and theoretical (S^{th}) spectroscopic factors for low-lying states of ^{205}Tl, with excitation energy E_{exc} and spin and parity J^{π}.

(a) Average number of particles in each j level

	SPE	-9.861	-8.258	-8.007	-9.385	-7.865	-8.318	-7.368	-9.097
J^{π}	E_{exc} (MeV)	$\pi d_{5/2}$	$\pi d_{3/2}$	$\pi s_{1/2}$	$\pi h_{11/2}$	$\nu f_{5/2}$	$\nu p_{3/2}$	$\nu p_{1/2}$	$\nu i_{13/2}$
$1/2^+$	0.000	5.928	3.789	1.292	11.991	5.415	3.780	0.940	13.866
$3/2^+$	0.184	5.969	3.281	1.783	11.967	5.376	3.769	0.998	13.856
$11/2^-$	1.501	5.989	3.879	1.983	11.150	5.537	3.815	0.957	13.691
$5/2^+$	0.732	5.859	3.792	1.358	11.990	5.052	3.719	1.273	13.957
$7/2^+$	1.028	5.985	3.206	1.851	11.958	4.965	3.727	1.373	13.935
$5/2^+$	1.271	5.985	3.715	1.313	11.987	4.980	3.927	1.105	13.987

(b) The wave functions of low-lying states of ^{205}Tl

J^{π}	$1/2^+$	$3/2^+$	$5/2^+$	$7/2^+$	$5/2^+$								
E (MeV)	0.000	0.184	0.732	1.028	1.271	Partition order of orbits							
[NO]			p (%)			$d_{5/2}$	$d_{3/2}$	$s_{1/2}$	$h_{11/2}$	$f_{5/2}$	$p_{3/2}$	$p_{1/2}$	$i_{13/2}$
[1]			0.55			5	4	2	12	6	4	2	12
[1]			1.01			5	4	2	12	6	2	2	14
[3]				3.73		6	4	2	11	5	4	2	13
[1]		5.13		1.04		6	3	2	12	6	4	2	12
[3]		2.55			0.75	6	4	2	11	6	4	1	13
[3]		1.36	1.11			5	4	2	12	5	3	2	14
[3]	0.68					6	4	2	11	6	3	2	13
[3]	0.65	0.51	7.80	6.50	4.67	6	3	2	12	5	3	2	14
[1]	5.81		0.88			6	4	1	12	6	4	2	12
[1]	0.62	0.63	4.70	0.68	0.77	5	4	2	12	4	4	2	14
[3]	4.17	0.71	1.95			5	4	2	12	5	4	1	14
[1]	2.12	16.18	4.84	9.09	3.00	6	3	2	12	4	4	2	14
[3]	11.76	7.87	6.18	52.49	20.20	6	3	2	12	5	4	1	14
[3]	1.86		0.81			5	4	2	12	6	3	1	14
[3]		0.63	1.90	8.69	0.54	6	4	1	12	5	3	2	14
[3]	5.65	3.57	1.48	9.15	0.62	6	3	2	12	6	3	1	14
[1]	18.03	2.29	6.22	5.66	0.69	6	4	1	12	4	4	2	14
[1]	0.57	4.97		1.10		6	3	2	12	6	2	2	14
[3]		12.75	43.94		66.61	6	4	1	12	5	4	1	14
[3]		5.15	10.09		0.84	6	4	1	12	6	3	1	14
[1]	5.70	0.58	1.14			6	4	1	12	6	2	2	14
[1]	41.27					6	4	1	12	6	4	0	14
[1]		33.65				6	3	2	12	6	4	0	14

(c) The spectroscopic factor S of low-lying states of ^{205}Tl

J^{π}	$1/2^+$	$3/2^+$	$5/2^+$	$7/2^+$	$5/2^+$	$11/2^-$
E (MeV)	0.000	0.204	0.619	0.924	1.180	1.484
S^{expt} [16]	1.27	1.96	0.44	0.08		7.35
S^{expt} [12]	1.09	1.61	0.38			6.31
S^{expt} [14]	1.10	1.6	0.3			5.3
j	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$g_{7/2}$	$d_{5/2}$	$h_{11/2}$
$S^{\mathrm{th}}(j,0_1^+)$	1.443	2.356	0.499	0.000	0.002	8.554
j			$s_{1/2}$	$d_{3/2}$		
$S^{\mathrm{th}}(j,2_1^+)$			0.776	1.168		

TABLE II. (a) Wave functions of HMS($5/2^+$) of ^{206}Pb. First and second columns are related to the parent state and display the number of unpaired nucleons and the percent occupation. The other columns give the percent occupation, contributing more than 1%, for each member of the multiplet of homologous multipole states. First and second (th) and last and last but one (expt) give, respectively, excitation energy $E_{\rm exc}$ and spin and parity J^π for parent and homologous states. The basis states considered are the same as those appearing in Table I(b) in the column of parent state ($5/2^+$). (b) The same as (a) for HMS($7/2^+$) of ^{206}Pb. (c) The same as (a) for HMS($11/2^-$) of ^{206}Pb.

The wave functions of HMS($5/2^+$) of ^{206}Pb

J^π	$5/2^+$	7^-	6^-	5^-	4^-	3^-	2^-
E (MeV)	0.732	4.358	4.117	4.268	4.254	4.378	4.651
[NO]	p (%)			p (%)			
[1]	0.55						
[3]	1.11	1.13	1.09				
[1]	0.88			1.92	1.10		
[3]	7.80	7.10	7.25	6.53	6.65	6.44	2.94
[1]	4.70	4.42	3.48	4.02	2.82	3.04	
[3]	1.95	2.47	1.81	2.61	1.79	1.55	
[1]	4.84	5.11	6.39	4.93	6.55	7.98	9.68
[3]	1.90	5.98	3.72	3.19	4.35	3.88	1.96
[3]	6.18		11.82	8.65	19.14	6.81	30.35
[3]	0.81						
[1]	1.01						
[1]	6.22	7.93	6.72	9.44	7.32	5.56	10.83
[3]	43.94	35.91	34.26	32.69	26.96	29.51	32.67
[3]	1.48	2.60	3.12	2.22	4.58	1.33	2.83
[1]	1.14	1.10	1.00	1.99	1.33		
[1]	10.09	7.37	8.86	6.38	6.52	7.32	5.08
[1]	3.95	3.59	2.50	3.38	1.91	1.75	
E (expt)	0.619	4.12	3.828	3.994	4.044	4.221	3.980
J^π(expt)		(6^-7^-)	(6^-7^-)	5^-	(4^-3^-)	(3^-4^-)	2^-

(b) The wave functions of HMS($7/2^+$) of ^{206}Pb

J^π	$7/2^+$	8^-	7^-	6^-	5^-	4^-	3^-	2^-	1^-
E (MeV)	1.028	4.873	4.487	4.586	4.510	4.546	4.627	4.560	4.905
[NO]	p (%)				p (%)				
[3]	3.73	3.60	3.12	2.08	3.34	1.79	1.37	2.73	2.24
[1]	1.04	1.05		3.54	1.35	1.67	1.06		
[1]	0.68	1.00						1.74	
[3]	6.50	10.12	8.32	5.05	6.66	5.03	3.12	7.06	1.75
[1]	9.09	10.65	10.55	13.58	10.47	9.41	7.42	6.14	8.33
[3]	8.69	9.29	7.29	5.24	8.18	4.49	1.76	6.98	2.56
[3]	52.49	46.82	45.01	31.86	47.84	30.52	43.81	43.56	66.56
[1]	5.66	5.53	4.76	5.98	6.88	6.52	2.02	3.05	13.58
[1]	1.10	1.06	1.14	2.76	1.64	1.60	1.04		
[3]	9.15	6.62	8.86	4.29	8.23	6.14	3.44	9.27	2.05
E (expt)	0.924	4.532	4.243	4.257	4.373			4.317	
J^π (expt)		(8^-+4^-)	$(7^-,8^-)$	$(6^-,7^-)$	(5^-+3^-)			2^-	

(c) The wave functions of HMS($11/2^-$) of ^{206}Pb

J^π	$11/2^-$	10^+	9^+	8^+	7^+	6^+	5^+	4^+	3^+	2^+	1^+
E (MeV)	1.501	5.068	4.909	4.886	4.887	4.923	4.977	5.068	5.202	5.356	5.833
[NO]	p (%)						p (%)				
[1]	7.97	6.79	6.86	6.47	6.54	6.49	6.75	7.03	7.44	8.14	5.76
[3]	1.48	3.32	2.36	2.13	1.97	2.04	2.00	1.90	1.69	1.30	2.77
[3]	1.00	1.87	1.80	1.69	1.53	1.67	1.89	1.98	1.76	1.20	1.36
[3]	1.13		1.25	1.10	1.23	1.23	1.33	1.32	1.23	1.01	
[1]	17.28	17.75	16.33	16.40	15.88	16.33	16.76	17.68	18.24	19.34	16.01
[3]	0.66										
[3]	8.61	12.53	13.17	14.41	14.52	14.70	14.31	13.12	11.47	7.61	17.54

TABLE II (*Continued*).

J^{π}	$11/2^{-}$	10^{+}	9^{+}	8^{+}	7^{+}	6^{+}	5^{+}	4^{+}	3^{+}	2^{+}	1^{+}
E (MeV)	1.501	5.068	4.909	4.886	4.887	4.923	4.977	5.068	5.202	5.356	5.833
[NO]	p (%)					p (%)					
[3]	1.28	1.11	1.43	1.12	1.26	1.12	1.20	1.09	1.13		
[3]	9.52	6.54	9.61	8.02	8.82	8.12	8.37	7.89	8.18	7.88	10.84
[3]	3.68	4.80	5.31	5.71	5.76	5.86	5.74	5.19	4.38	2.68	4.23
[1]	5.60	5.27	5.17	5.12	5.02	5.10	5.25	5.47	5.61	5.88	4.42
[1]	40.90	35.66	34.60	35.37	35.28	35.07	34.41	35.46	37.23	42.43	32.41
E (expt)	1.484	4.818	5.011	5.149	4.941	5.112	4.925	4.912	5.078		

pected, than the values obtained for the latter $B(E2,7/2^{+}\to 5/2^{+})=34.1\ e^{2}\ \mathrm{fm}^{4}$, but comparable with $B(E2,3/2^{+}\to 1/2^{+})=245\ e^{2}\ \mathrm{fm}^{4}$. These calculated relations are, however, consistent with the available experimental [12] values, which are $B(E2,5/2^{+}\to 1/2^{+})=389\ e^{2}\ \mathrm{fm}^{4}$ and $B(E2,3/2^{+}\to 1/2^{+})=375\ e^{2}\ \mathrm{fm}^{4}$. In the calculation of the $B(E2)$'s natural values of parameters have been taken, i.e., effective charge $e_{\pi}=1$ and $e_{\nu}=0$. The change of parameter values, in general, does not cause any great change for relative magnitude of $B(E2)$.

At last, we comment upon the second $J^{\pi}=5/2_{2}^{+}$ state, predicted at $E=1.271$ MeV, which may be associated with one of the two levels at 1.174 MeV and 1.180 MeV, reported in Nuclear Data Sheets [12]. Our calculations predict only one state, and according to its wave function this state has a large parentage with the 3_{1}^{+} state of ^{206}Pb. Consequently this state will not be produced in a direct proton pickup $^{206}\mathrm{Pb}(d,{}^{3}\mathrm{He})^{205}\mathrm{Tl}$ or in $^{206}\mathrm{Pb}(e,e'p)^{205}\mathrm{Tl}$ reaction [15], and consistently has not been seen in the $^{208}\mathrm{Pb}(p,\alpha)^{205}\mathrm{Tl}$ reaction either.

IV. LEVELS IN ^{206}Pb

We first consider the low-lying states in ^{206}Pb of two-neutron-hole character, shown in Fig. 3. The agreement with experimental data [16] is very good, not only for the results obtained with the HK interaction, but also for those obtained with the simpler residual interaction MSDI. This good agreement also extends to a few high excited states, more precisely those of stretched character (Fig. 3). In this case there is no configuration mixing in the model space selected, and the values of the TBME can be directly determined by the energies of the stretched states. When we extend the model calculation to other lead isotopes [6] and to other neighboring nuclei, the agreement is still good.

This overall agreement shows that both interactions, KH and MSDI, are sufficiently adequate for describing the low-lying structure of nuclei in the lead region, although the results of the KH interaction are, as expected, in general slightly better than those of the MSDI. Given the good quality of the agreement, unknown values of the spin and parity J^{π} reported in Nuclear Data Sheets [16] can be assigned on the basis of the calculation, e.g., 2.196 MeV(expt), 3^{+}, 2.235(expt), 1^{+}, and 2.960(expt), 8^{-}. Instead, no calculated level can be associated with the experimental 2.391 MeV level. Aside from this, the levels of $E_{\mathrm{exc}}<3.0$ MeV have been completely observed, except the states of collective character, e.g., the 3^{-} state of 2.648 MeV.

A good agreement for the low-lying states is a necessary requirement for an equivalent agreement for the homologous multipole states. In fact these may be viewed as proton excitation over the ground state of ^{206}Pb; for example, in the case of the HMS$(11/2^{-})$ one proton is promoted from the $s_{1/2}$ orbital to the $h_{11/2}$ orbital. Similarly, the quality of the description of the low-lying states of ^{206}Pb is reflected in the description of states in ^{205}Tl. For example, as tested in the $^{206}\mathrm{Pb}(d,{}^{3}\mathrm{He})^{205}\mathrm{Tl}$ and $^{206}\mathrm{Pb}(e,e'p)^{205}\mathrm{Tl}$ reactions [15], the wave functions $|1/2^{+}\rangle$ of the ground state of ^{205}Tl should be approximately expressed in the form

$$|1/2^{+}\rangle=a|3s_{1/2}^{-1}\otimes 0^{+}\rangle+b|2d_{3/2}^{-1}\otimes 2^{+}\rangle+c|2d_{5/2}^{-1}\otimes 2^{+}\rangle.$$

The expansion coefficients depend on the calculated wave functions of 0^{+} and 2^{+}. Our findings are reasonable. For example, the value $a\simeq 0.85$ obtained with the MSDI means that the $3s_{1/2}$ proton occupancy is about 70%. But the a value obtained with the KH interaction seems large. So clearly the values of the expansion coefficients are much more sensitive to the wave functions of ^{206}Pb.

The energies of the parent levels in ^{205}Tl and the homologous multipole states of ^{206}Pb, i.e., states of $1\pi p$-$1\pi h$-$2\nu h$ character constructed on a basis of the form $|\pi h_{9/2},\pi j_{1}^{-1},\nu(j_{2}^{-1}\otimes j_{3}^{-1})J',J^{\pi}\rangle$, are shown in Fig. 2 and Fig. 4. As a first remark, the agreement with experimental energy values of homologous multipole states is reasonably good for the results obtained with both the KH interaction and MSDI but not of the same quality as for the low-lying levels of ^{206}Pb. For example, the splitting of the first doublet (5^{-}), (4^{-}), corresponding to the coupling of the $h_{9/2}$ proton with the $1/2_{1}^{+}$ state of ^{205}Tl, is smaller than the experimental value. Our predicted small splitting is instead consistent with those present in the equivalent low-lying doublets of some other nuclei, as the splitting of the doublet $(5_{\mathrm{g.s.}}^{+})$, (4^{+}) in ^{208}Bi corresponding to the configuration $h_{9/2}\otimes p_{1/2}$, which is only 63 keV. Similarly small is in ^{208}Tl the splitting of $5_{\mathrm{g.s.}}^{+}$, (4^{+}) corresponding to the configuration $g_{9/2}\otimes s_{1/2}$, which amounts to 40 keV. On the converse the experimental splitting of HMS$(1/2^{+})$ is 400 keV (Fig. 4), larger than the calculated values. This discrepancy is directly related to the value of the TBME. In this respect note that the equivalent multiplet of experimental homologous multipole states associated with $\pi h_{9/2}\otimes 5/2^{+}$, compared with the experimental low-lying states [16] in ^{206}Bi, corresponding to $\pi h_{9/2}\otimes 5/2^{-}(3\nu h)$ $((6^{+})$,0.0(keV); (4^{+}),60.0; (3^{+}),70.7;

TABLE III. The wave functions of HMS($5/2_2^+$) of ^{206}Pb.

J^π	$5/2^+$	2^-	3^-	4^-	5^-	6^-	7^-
E (MeV)	1.271	4.966	4.676	4.697	4.556	4.705	4.701
[NO]	p (%)			p (%)			
[3]	0.75						
[3]	4.67	4.46	4.69	4.69	1.46	5.18	3.41
[1]	0.77	3.02		2.01			
[1]	3.00	5.84	7.93	3.09	5.32	6.89	7.70
[3]	0.54	1.03	3.42		1.07	2.19	1.96
[3]	20.20	20.69	23.54	18.01	20.29	20.50	17.75
[1]	0.69	4.40	6.30	9.06	6.41	5.60	8.00
[3]	66.61	43.48	44.24	40.85	40.38	52.53	52.85
[3]	0.62	1.27	3.36	4.80	3.50	1.93	1.04
[3]	0.84	7.02	3.06		2.45	1.93	3.37

(5^+),80.8; (7^+),141.2; (2^+),409.2) shows different relative energies, in particular for 2^+ state. However, for these states of ^{206}Bi the calculated values, obtained using the same interaction, agree well with the experiment.

With regard to the relative energy centroid

$$W=\frac{\Sigma_i E_i(2J_i+1)}{\Sigma_i(2J_i+1)}$$

of the different multiplets the experimental values for the homologous multipole states are slightly higher, $\sim 0.1-0.2$ MeV, than the corresponding energy of parent levels $3/2^+$, $5/2^+$, and $7/2^+$. Our calculated W^{th} for the homologous multipole states are generally slightly higher than W^{expt}. Only for the results obtained using the KH interaction is the centroid W^{th} of the multiplet HMS($11/2^-$) lower than the corresponding experimental value W^{expt}.

The experimental spreading intervals of the different multiplets of homologous states seem to be roughly constant, whereas the theoretical ones increase with the number of levels of the multiplet, especially for the results obtained with the KH interaction.

The sequence of levels within each multiplet of homologous multipole states differs in some cases from that of the experiment. Particularly for 4_1^-(th), 4_2^-(th) and 2_1^-(th), 2_2^-(th) the order of the level energies is inverted; e.g., 4_1^-(th) belongs to homologous multipole state $|h_{9/2}\otimes 1/2_1^+\rangle$, while according to the experiment 4_1^-(expt) belongs to homologous multipole state $|h_{9/2}\otimes 3/2_1^+\rangle$. By adding the quadrupole term $k\,\mathbf{Q}\cdot\mathbf{Q}$ (with a value of the strength $k=0.25$ MeV fm^{-4}) in the MSDI, the level sequence of 2_1^-(th) and 2_2^-(th) can be set in agreement with the experiment, but the level sequence of 4_1^-(th) and 4_2^-(th) still remains incorrect. Note that the sequence of levels within a multiplet is obviously dominated by the values of the TBME, while depending only weakly on the values of sp energies. In fact, if we only adjust the sp energies, these results do not change qualitatively.

Let us consider now the role of the mixing of configurations. The energies, associated with pure configurations of the form $|\pi h_{9/2},\pi j_1^{-1},\nu(p_1/2)^{-2},J'=0,J^\pi\rangle$, of HMS($1/2^+$) and HMS($3/2^+$) obtained with the MSDI are much higher than the experimental values and higher than the values calculated with configuration mixing. Similarly the pure HMS($d_{5/2}$) have an excitation energy higher than 5.7 MeV, as a consequence of the fact that the corresponding energy 1.86 MeV of the $5/2^+$ state, pure single-hole $d_{5/2}^{-1}$ state of ^{205}Tl is much higher than the experimental value 0.62 MeV. As a general statement, the configuration mixing has the consequence of decreasing the energies of homologous multipole states, so that the larger the dimension of Hamiltonian matrix for definite J, the larger is in general the correlation energy with respect to the uncorrelated value. For example, without mixing the level energies within HMS($11/2^-$) increase smoothly with increasing J, at variance with the results obtained with residual interaction, as

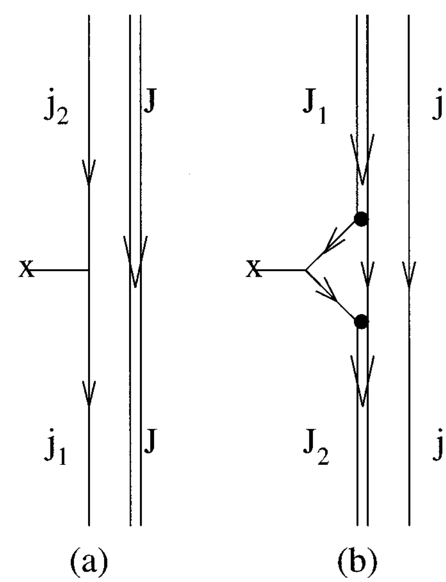

FIG. 5. Schematic graphical representation of the lowest-order contributions to the inelastic excitation of a system composed by a hole state (single arrowed line) coupled to a paired two-hole state (double arrowed line). In (a) the electromagnetic field acts on the unpaired hole; in (b) the excitation takes place after the phonon two-hole state has been decomposed into a two-hole state. Cf., e.g., [20].

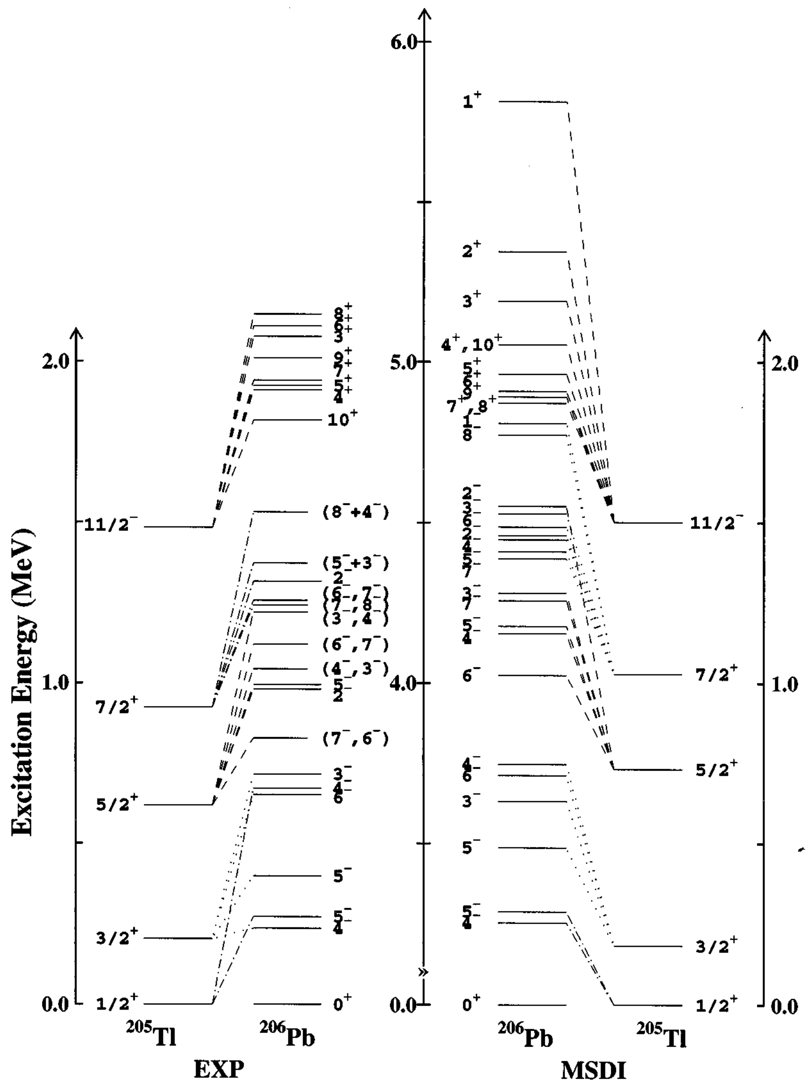

FIG. 6. Comparison between experimental and theoretical (obtained using MSDI) energy spectra of homologous multipole states of ^{206}Pb and their parent states in ^{205}Tl.

listed in the Table II(c) and shown in Fig. 4.

The wave functions of selected homologous multipole states are listed in Table III. One can see that, in spite of their complexity, they have a weak-coupling character, being similar to the wave functions of parent states. Precise tests of these functions may be difficult in view of the scarce experimental data, especially with regard to electromagnetic transitions. From the point of view of transfer reactions, where data are available, the homologous multipole states display different behaviors. Some, as the homologous states belonging to $J^{\pi}=1/2^{+}$, $3/2^{+}$, $5/2^{+}$, $7/2^{+}$, $11/2^{-}$ parent states in ^{205}Tl, are populated directly in (p,α) and/or $(d,{}^{3}\mathrm{He})$ reactions. Other homologous multipole states, on the other hand, cannot be populated in these reactions, as, for example, the state associated to the parent $5/2_{2}^{+}$ state.

More generally, according to the properties of known parent states of the form $|\pi j_1^{-1},\nu(j_2^{-1}\otimes j_3^{-1})J',J^{\pi}\rangle$, one can attempt a classification of the homologous multipole states at least into four groups.

(1) The first group includes the states HMS($1/2_1^{+}$), HMS($3/2_1^{+}$), and HMS($11/2_1^{-}$). In these cases the two neutron holes are mainly coupled to $J'(2\nu h)=0$, and the third hole state is $j_1^{-1}=s_{1/2}^{-1}$, $d_{3/2}^{-1}$, and $h_{11/2}^{-1}$ for the major component of the configuration, as in the $1/2_1^{+}$, $3/2_1^{+}$, and $11/2_1^{-}$ parent states in ^{205}Tl.

(2) The second group includes the states which correspond to the recoupling of the neutron holes to $J'(2\nu h)=2$, with $j_1^{-1}=s_{1/2}^{-1}$, $d_{3/2}^{-1}$, having as parent states in ^{205}Tl the $5/2_1^{+}$ and $7/2_1^{+}$ states.

(3) Other states correspond to other coupling angular momenta, as, for example, $J'(2\nu h)=3$. This is, for example, the case of the HMS($9/2_1^{+}$), homologous to the $9/2_1^{+}$ parent state in ^{205}Tl.

(4) And last, states with no simple structure, but still clearly homologous to equivalently complicated parent states. This is for example the case of the HMS($5/2_2^{+}$), associated with the $5/2_2^{+}$ parent state in ^{205}Tl.

As for the electromagnetic reduced transition probabilities, those among different members of the same multiplet of homologous multipole states are very similar, since all states have similar structure, aside from total angular momentum. On the other hand, for transitions connecting the states belonging to different homologous multipole states, one finds relations similar to those valid for the corresponding parent states, previously discussed, consistently with the spectator role of the extra unpaired nucleon.

As a final point, let us try to estimate the effect of a further improvement of the residual interaction and of the extension of the model space. For ^{205}Tl and homologous multipole states of ^{206}Pb the inclusion of the $\mathbf{Q}\cdot\mathbf{Q}$ term into the MSDI does not change appreciably the overall picture, leaving unchanged most of the properties. We only point out some specific relevant features.

(1) The 2^- states $2_1^-(5/2^+)$ ($E=4.25$ MeV) and $2_2^-(7/2^+)$ ($E=4.57$ Mev) are now in the correct order.

(2) Most of the energies remain nearly unchanged; only the energy of some states increases.

(3) The structure of wave functions becomes more similar to that of the KH interaction.

As for the extension of the model space, this has been tested in two ways. For the proton, by adding $f_{7/2}$ (with occupancy 0 or 1) to $d_{5/2}(5,6)$, $d_{3/2}(3,4)$, $s_{1/2}(1,2)$, $h_{11/2}(11,12)$, and $h_{9/2}(0,1)$. In this way there is one particle in either the $f_{7/2}$ or $h_{9/2}$ orbital, and one hole in one of the other four orbitals. For the neutron, we have added $f_{7/2}(6,8)$ to the original model space. In both cases, no appreciable changes were detected in the calculated values.

V. SUMMARY AND CONCLUSIONS

We have discussed in this paper the occurrence of homologous multipole states in ^{205}Tl and ^{206}Pb, due to different circumstances. The first is the spectator role which must be played by the extra particle. In our case the proton separation energy S_p [17] of ^{209}Bi is 3.798 MeV, which coincides, in a shell model treatment, with the binding energy $S_p(h_{9/2})$ of the $h_{9/2}$ orbital. This value is much smaller than the value 7.255 MeV of S_p in ^{206}Pb. We therefore expect the $h_{9/2}$ proton, as a good spectator, to couple weakly to the parent states in ^{205}Tl, giving rise to corresponding states of the multiplets with wave functions that have a character rather similar to that of the wave functions of the parent states in ^{205}Tl. As a second point, the validity of the homology depends on the configuration mixing and should be greater for pure configuration states. For example, there is only one intruder orbital, namely, $h_{11/2}$, in the 50–2 proton shell, and so the $11/2^-$ state in ^{205}Tl is found to be a pure $\pi h_{11/2}^{-1}$ configuration ($P=85\%$), purer than that of $1/2^+$ and $3/2^+$ states. As a consequence, we expect that the concept of homology will better apply to HMS$(11/2^-)$ of ^{206}Pb and the parent $11/2^-$ state, in spite of the fact that the excitation energy of the HMS$(11/2^-)$ is rather high, nearly 5 MeV. Finally, from an experimental point of view, the actual observation of these states relies on sufficiently large cross sections for the processes that are natural candidates for their study, as, e.g., the three-nucleon transfer (p,α) reaction.

For all these reasons, the possible occurrence of homology states in the region $A\approx208$ is more promising than that of nuclei in the region $A\approx90$, where these states have been clearly identified as predicted by theoretical shell model calculations [3]. The calculations discussed in this paper for the pair of system ^{205}Tl and ^{206}Pb confirm our expectations. A number of homologous multipole states are clearly singled out in ^{206}Pb in close correspondence with parent states in ^{205}Tl, not only for pure $[1\pi h, J'(2\nu h)=0]$ configuration or for the intruder $h_{11/2}$-orbital configuration, but also for configurations where the two neutron holes are coupled to nonzero angular momentum $[J'(2\nu h)\neq 0]$. We quote as an example the $5/2^+$ or $7/2^+$ state, usually assumed as pure $1\pi h$ configurations, but predicted by our calculations as $s_{1/2}$ or $d_{3/2}$ proton hole coupled to the 2_1^+ state of ^{206}Pb, i.e., $J'(2\nu h)=2$. All these homologous states have very characteristic wave functions and are therefore clearly singled out, in spite of their high excitation energy and of the density of neighbor levels.

Our predictions are in good agreement with the experimental findings, obtained with (p,α) transfer reaction. For a simpler summary of the results we compare in Fig. 6 experimental energies and theoretical predictions (obtained with the MSDI) for a selected number of states, namely, the homologous states in ^{206}Pb and their parent states in ^{205}Tl.

The validity of the concept of homology should also apply to other neighbor nuclei, as, for example, ^{206}Tl. In this case one expects excited states which arise from the weak coupling of the $g_{9/2}$ neutron with the parent states in ^{205}Tl. Our preliminary calculations [18] do indeed show the occurrence of these states, with probably clearer signature that in the ^{206}Pb case. The experimental search of these states represents an interesting and challenging line of research.

Our last comment is associated with the relevance of these features with respect to the residual interaction [19]. As shown by our calculations, energies of highly excited states and spreading widths of different multiplets depend critically on the choice of the residual interaction (see Fig. 4 for the difference between the KH interaction and MSDI). But in addition to energy systematics, electromagnetic transitions [such as $B(E\lambda)$ and/or $B(G-T)$] are also sensitive to the choice of the interaction, as well as to the model space used in the calculation. So transfer reactions, which have been so far the main source of information on the occurrence of the phenomenon of homology, must be necessary coupled to other probes in order to obtain a consistent test of the validity of the residual interaction.

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