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Compressibility of nuclear matter and breathing mode of finite nuclei in relativistic random phase approximation

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Isoscalar monopole modes in finite nuclei are studied in the framework of relativistic models currently used in ground state calculations. Response functions in the random phase approximation are calculated with nonlinear models for the first time. It is found that some effective Lagrangians having a bulk compression modulus in the range 280–350 MeV can predict correctly breathing mode energies in medium and heavy nuclei. It is pointed out that the parametrization NL1 ($K_\infty = 211$ MeV) leads to an anomalous behavior of the monopole response. [S0556-2813(97)05405-8]

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The issue of determining the value of the compression modulus K_∞ of nuclear matter is of great importance for obtaining the nuclear equation of state. From the experimental side, the main information at our disposal comes from energy systematics of the breathing mode measured in many nuclei across the Periodic Table. Yet, it is only possible at the present stage to ascertain the value of K_∞ to belong to the 200–350 MeV interval [1] if one tries to avoid model-dependent analyses and deduce K_∞ from an $A^{-1/3}$ expansion of the finite nucleus compressibility K_A. Thus, one has to introduce some degree of model dependence in order to establish a link between the energy of the isoscalar giant monopole resonance (GMR) in finite nuclei and the nuclear matter incompressibility. This has been done already for quite some time in the nonrelativistic framework, and the commonly accepted value of $K_\infty = 210 \pm 30$ MeV was deduced by Blaizot [2] by calculating nuclear matter properties with effective interactions which could describe satisfactorily the GMR in Hartree-Fock random phase approximation (RPA) models.

In recent years the relativistic many-body theory has met great success in predicting ground state properties of finite nuclei including unstable ones up to the nucleon drip lines. Since the early work of Walecka [3] several parametrizations of effective Lagrangians containing self-interaction terms of the meson fields or density-dependent coupling constants have been proposed [4–9], all of them aiming at a good description of nuclear ground states in a relativistic mean field, i.e., Dirac-Hartree framework. It turns out that the values of K_∞ that they predict can span a wide range, from about 200 MeV up to above 500 MeV. However, one does not know what monopole energies in finite nuclei these nonlinear or density-dependent models would give. Only a few attempts have been made to predict with relativistic models the GMR energies in nuclei, either by using the relativistic RPA (RRPA) method [10,11] or the constrained Dirac-Hartree method [12]. In both cases, however, the investigations were limited to the linear model with the parameter set of Horowitz and Serot [13] (HS) which corresponds to $K_\infty = 545$ MeV and consequently it was found that GMR energies in medium and heavy nuclei are overestimated.

The purpose of this work is to examine the more recently proposed effective Lagrangians including nonlinear terms from the point of view of their GMR predictions in nuclei. The RRPA is the appropriate framework to extend the relativistic mean field description to the nuclear excitations. Indeed, in a way similar to the nonrelativistic case, the RRPA can be seen as the small amplitude limit of the time-dependent Dirac-Hartree theory and therefore the same effective Lagrangian should be able to describe ground states and giant resonances as well. Effective Lagrangians often used successfully in Dirac-Hartree calculations all belong to the class of non-linear models with self-interaction terms in the σ field and also sometimes in the ω field. We are thus led to calculate the linear response function of nuclei in RRPA with nonlinear models. We shall discuss the GMR energies obtained with three nonlinear models often found in the literature, namely the NL-SH model of Sharma *et al.* [7], TM1 of Sugahara and Toki [8], and NL1 of Reinhard *et al.* [4].

We start from an effective Lagrangian of the form

$$\begin{aligned}\mathcal{L} = {} & \bar{\Psi}(i\gamma^\mu \partial_\mu - M_N - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \tau^a \gamma^\mu \rho^a_\mu)\Psi \\ & + \frac{1}{2}\,\partial^\mu \sigma \partial_\mu \sigma - U_\sigma(\sigma) - \frac{1}{4}\,W^{\mu\nu}W_{\mu\nu} + U_\omega(\omega) \\ & + \frac{1}{2}\,m^2_\rho \rho^{a\mu}\rho^a_\mu - \frac{1}{4}\,R^{a\mu\nu}R^a_{\mu\nu} - \bar{\Psi} e\gamma^\mu A_\mu\,\frac{1}{2}\,(1-\tau_3)\Psi \\ & - \frac{1}{4}\,F^{\mu\nu}F_{\mu\nu},\end{aligned} \tag{1}$$

where

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$$W^{\mu\nu}\equiv\partial^\mu\omega^\nu-\partial^\nu\omega^\mu,$$

$$R^{a\mu\nu}\equiv\partial^\mu\rho^{a\nu}-\partial^\nu\rho^{a\mu}+g_\rho\epsilon^{abc}\rho^{b\mu}\rho^{c\nu},$$

$$F^{\mu\nu}\equiv\partial^\mu A^\nu-\partial^\nu A^\mu, \qquad (2)$$

$$U_\sigma(\sigma)=\frac{1}{2}\,m_\sigma^2\sigma^2+\frac{1}{3}\,g_2\sigma^3+\frac{1}{4}\,g_3\sigma^4,$$

$$U_\omega(\omega)=\frac{1}{2}\,m_\omega^2\omega^\mu\omega_\mu+\frac{1}{4}\,c_3(\omega^\mu\omega_\mu)^2. \qquad (3)$$

In this Lagrangian there appear the nucleon field Ψ, the meson fields σ, ω_μ, ρ_μ^a, and the photon field A_μ. This general form contains the three nonlinear parametrizations (NL-SH, TM1, NL1) and also the linear parametrization (HS) as a special case. The coupling constants g_σ, g_ω, g_ρ, the σ meson mass m_σ and the parameters g_2, g_3, and c_3 entering the self-interaction terms U_σ and U_ω are adjusted to reproduce bulk properties of nuclear matter and ground state properties of finite nuclei. We note that the four models HS, NL-SH, TM1, and NL1 correspond to decreasing incompressibilities, the values of K_∞ being 545, 355, 281, and 211 MeV, respectively.

The present calculation of the nuclear response function in finite systems follows the method described in detail in Ref. [10]. Here, we shall only recall the main points in order to explain how the RRPA can be carried out in the case of nonlinear models. In Ref. [10] one introduced the Hartree, or unperturbed polarization operator $\Pi_0(P,Q;\mathbf{k},\mathbf{k}';E)$ and the RRPA, or perturbed polarization operator $\Pi(P,Q;\mathbf{k},\mathbf{k}';E)$. These polarization operators depend on some general one-body operators P and Q, transferred momenta $\mathbf{k}$ and $\mathbf{k}'$, and excitation energy E. For a given operator Q and momentum transfer $\mathbf{k}$ the Hartree and RRPA response functions are given by

$$R_0(Q;\mathbf{k},E)=\frac{1}{\pi}\,\mathrm{Im}\,\Pi_0(Q,Q;\mathbf{k},\mathbf{k};E),$$

$$R(Q;\mathbf{k},E)=\frac{1}{\pi}\,\mathrm{Im}\,\Pi(Q,Q;\mathbf{k},\mathbf{k};E). \qquad (4)$$

These response functions describe the distributions of transition strengths of the operator Q. Since the unperturbed ground state is treated in Hartree approximation, i.e., exchange interactions are omitted, the RRPA corresponds to the ring approximation and therefore, Π and Π_0 are related by the usual integral equation [14,15]:

$$\Pi(P,Q;\mathbf{k},\mathbf{k}',E)=\Pi_0(P,Q;\mathbf{k},\mathbf{k}',E)-\sum_i g_i^2\int d^3k_1 d^3k_2\Pi_0(P,\Gamma^i;\mathbf{k},\mathbf{k}_1,E)\times D_i(\mathbf{k}_1-\mathbf{k}_2,E)\Pi(\Gamma_i,Q;\mathbf{k}_2,\mathbf{k}',E). \qquad (5)$$

In this equation the index i runs over σ, ω, and ρ mesons, g_i and D_i are the corresponding coupling constants and meson propagators, $\Gamma^i=1$ for σ and $\Gamma^i=\gamma^\mu,\gamma^\mu\vec{\tau}$ for ω and ρ, respectively.

The meson propagators can be constructed in the following way. The meson fields in the Lagrangian can be expanded around their mean (classical) values. In the same manner that the first-order variations of the action with respect to each meson field give the field equation (Klein-Gordon equation) satisfied by that meson, the second-order variations give the inverse of the meson propagator [16]. This amounts, in practice, to take the second derivative of the Lagrangian with respect to the meson field. For instance, for the σ propagator in coordinate space we have

$$\left(\partial^\mu\partial_\mu+\frac{\partial^2U_\sigma(\sigma)}{\partial\sigma^2}\right)D_\sigma(x,y)=-\delta(x-y). \qquad (6)$$

Taking the Fourier transform of Eq. (6), we come to the expression of the propagator in momentum space:

$$(E^2-\mathbf{k}^2)D_\sigma(\mathbf{k}-\mathbf{k}',E)-\frac{1}{(2\pi)^3}\times\int S_\sigma(\mathbf{k}-\mathbf{k}_1)D_\sigma(\mathbf{k}_1-\mathbf{k}',E)d^3k_1=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}'), \qquad (7)$$

where $S_\sigma(\mathbf{k}-\mathbf{k}')$ is the Fourier transform of $\partial^2U_\sigma(\sigma)/\partial\sigma^2$:

$$S_\sigma(\mathbf{k}-\mathbf{k}')=\int e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}[m_\sigma^2+2g_2\sigma(r)+3g_3\sigma^2(r)]d^3r, \qquad (8)$$

$\sigma(r)$ being the classical value of the σ field at point $\mathbf{r}$. In the limit of the linear model one recovers the usual meson propagator which is a local function in momentum space.

In this work we are concerned with the isoscalar GMR where the isovector ρ meson plays practically no role in the residual particle-hole interaction. Therefore, in the integral equation (5) we keep only the σ and ω propagators. Of course, the ρ meson plays an essential role at the mean field level in $N\neq Z$ nuclei and its contributions must be kept in the Dirac-Hartree field when one builds the unperturbed polarization operator Π_0 [10].

The above scheme is used to calculate the isoscalar monopole strength distributions in the closed-shell nuclei ^{16}O, ^{40}Ca, ^{90}Zr, and ^{208}Pb. Assuming spherical symmetry, the Dirac-Hartree mean fields are calculated in coordinate space. Then, the unperturbed polarization operator Π_0 is constructed in momentum space. Equations (5) and (7) are multipole expanded and solved on a grid in momentum space. The continuous single-particle spectrum is discretized by the method of Ref. [10] and an averaging parameter $\Delta=2$ MeV is used to smooth out the response functions. This is easily done by replacing the excitation energy E by $E+i\Delta/2$ in all expressions. All other parameters concerning the harmonic oscillator basis used for continuum discretization, the grid in momentum space, the energy cutoffs can be found in Ref [10]. The response functions are calculated for the operator $Q=r^2Y_{00}\gamma^0$.

We first concentrate on medium and heavy nuclei where the GMR is clearly located experimentally [1]. In Fig. 1 are shown response functions calculated for ^{90}Zr and ^{208}Pb us-

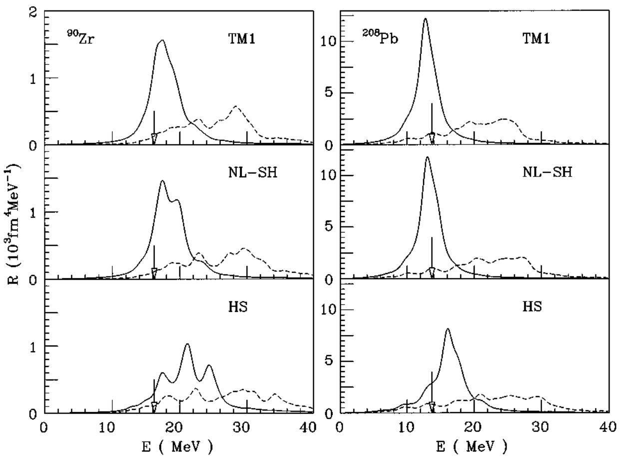

FIG. 1. Isoscalar monopole response functions $R_0(E)$ (Dirac-Hartree, dashed curves) and $R(E)$ (RRPA, solid curves) calculated in ^{90}Zr and ^{208}Pb using HS, NL-SH, and TM1 models. Arrows indicate experimental energies of the GMR.

ing the HS, NL-SH, and TM1 models. The theoretical and experimental GMR energies are indicated in Table I. It can be seen that the GMR appears as a clear collective excitation in these nuclei. Indeed, the unperturbed strengths are strongly shifted downwards by the particle-hole interaction and the distributions become rather narrow, especially in ^{208}Pb. This collective effect is, however, less marked with the HS model which has a high incompressibility than with the two nonlinear models and one can see that the RRPA response with HS exhibits structures reminiscent of the unperturbed spectrum. More quantitatively, we can characterize the degree of collectivity by the Landau damping width Γ_{Landau} of the GMR. The present calculations do not take into account any escape nor multi-particle-hole damping effects and therefore, the width of the GMR is entirely due to the Landau damping and to the averaging parameter Δ we have used. Correcting for the effect of Δ, the values of Γ_{Landau} can be deduced from the full width at half maximum of the distributions. These values are indicated in Table I and they show that NL-SH and TM1 predict similar Γ_{Landau} whereas HS is markedly different. It is worth noting that the value $\Gamma_{\mathrm{Landau}}=0.9$ MeV in ^{208}Pb is consistent with what we know experimentally about the total width and escape width of the GMR [17] and the fact that its calculated spreading width is small [18].

TABLE I. Peak energies E_{peak}, centroid energies $\overline{E}$, inverse energy-weighted moments m_{-1}, and Landau widths for finite nuclei. The values for ^{16}O and ^{40}Ca are calculated with $\Delta=5$ MeV. The units of E_{peak}, $\overline{E}$ and Γ are MeV, and m_{-1} is in fm^4 MeV^{-1}.

		E_{peak}	$\overline{E}$	m_{-1}	Γ_{Landau}
	HS	16.0	16.4	2862	2.1
	NL-SH	13.0	13.7	3998	0.9
^{208}Pb	TM1	12.8	13.3	4160	0.9
	EXP	13.7 ± 0.40			
	HS	21.2	21.4	388	5.1
	NL-SH	17.4	18.9	494	2.3
^{90}Zr	TM1	17.4	18.2	518	2.0
	EXP	16.2 ± 0.50			
	HS	24.5	24.6	86.3	8.7
^{40}Ca	NL-SH	22.0	23.4	90.7	4.6
	TM1	21.5	22.8	94.7	4.5
	HS	23.0	24.9	20.3	8.5
^{16}O	NL-SH	24.0	25.9	17.5	6.2
	TM1	23.0	25.3	18.3	6.9

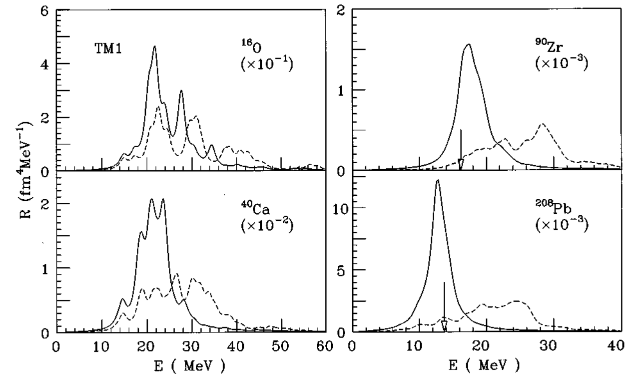

FIG. 2. Same as Fig. 1, for ^{16}O, ^{40}Ca, ^{90}Zr, and ^{208}Pb calculated with TM1.

A comparison of calculated peak energies with experiment shows that NL-SH and TM1 are doing fairly well with a slight underestimate in ^{208}Pb and overestimate in ^{90}Zr while HS predicts too high values as already found in Refs. [10,12]. Clearly, a lowering of the value of K_∞ is necessary to obtain agreement with experiment but the link between K_∞ and GMR energies in finite nuclei is not obvious since NL-SH ($K_\infty=355$ MeV) and TM1 ($K_\infty=281$ MeV) are not very different in these two nuclei. The finite nucleus incompressibility K_A differs from K_∞ by many correction terms (surface, curvature, symmetry, Coulomb) which are neither small nor model independent [1,2]. Thus, the GMR energy in ^{208}Pb predicted by the HS model is only 1.25 times that of TM1 while the ratio of $K_\infty^{1/2}$ is 1.39.

Calculations have also been performed with HS, NL-SH, and TM1 in lighter nuclei. In Fig. 2 are shown the isoscalar monopole strength distributions in ^{16}O, ^{40}Ca, ^{90}Zr, and ^{208}Pb calculated with TM1. When going from heavy to lighter systems, the trend is that collectivity becomes weaker, correlated and uncorrelated responses are closer and Landau damping becomes predominent. Hence, it becomes difficult, in a nucleus like ^{40}Ca to define theoretically a GMR energy or width. This finding completely agrees with experiment [1]. Peak energies, centroid energies of the distributions, inverse energy-weighted moments and Landau widths are reported in Table I. In order to give a quantitative description of those quantities for light nuclei, the values for ^{16}O, ^{40}Ca given in Table I are calculated with $\Delta=5$ MeV.

For the case of the NL1 model, our calculations for ^{40}Ca, ^{90}Zr, and ^{208}Pb give extremely low centroid energies and a dramatic loss of energy-weighted strength as compared with the unperturbed distribution. This might be a sign that part of the monopole strength has been shifted to the energy region below the ground state. In other words, if one diagonalizes the RRPA matrix in configuration space there would appear imaginary eigenvalues. This can be confirmed by calculating the inverse energy-weighted moment:

$$m_{-1} \equiv \sum_n \frac{|\langle 0|Q|n\rangle|^2}{E_n - E_0} = \frac{1}{2} \, \mathrm{Re}\Pi(Q,Q;\mathbf{k},\mathbf{k};E=0), \quad (9)$$

where $\{|n\rangle, E_n\}$ are the RRPA excited states and energies. In the case of the other models (HS, NL-SH, TM1) we have actually calculated m_{-1} either by relating it to the real part of the polarization operator at zero energy (they are the values shown in Table I) or by integrating the strength distributions. We generally find for the three models that the two methods agree within 2–3%. However, for the NL1 model we find that Eq. (9) gives negative values of m_{-1} for ^{40}Ca, ^{90}Zr, and ^{208}Pb. In a recent work [19] Vretenar *et al.* have also examined the question of isoscalar monopole energies in nuclei using a time-dependent relativistic mean field approach. They also noticed that the NL1 excitation energies are systematically lower than those of other parametrizations although the deviations are less dramatic in their case. The origin of the differences between the two methods deserves more investigation.

In conclusion, we have investigated the isoscalar monopole properties of finite nuclei predicted by various nonlinear relativistic models frequently used in mean field calculations. The RRPA equations for Lagrangians containing self-interaction terms have been solved for the first time. All studied models can describe correctly static ground state properties (binding energies, radii, etc.) of nuclei but not all of them are able to predict the breathing mode in those nuclei with success. We find that for some models such as TM1 and NL-SH the GMR energies in medium and heavy nuclei are correctly reproduced although their bulk incompressibilities may differ by more than 20%. For other models like NL1 the fact that the compression modulus is close to the commonly accepted value is no guarantee that the compressibility in finite nuclei will be satisfactory. This comes as a surprise and a warning that the relation between compressibilities in bulk and finite systems must be complex in the relativistic approach.

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