

Resonances in ${}^9\text{He}$ and ${}^{10}\text{He}$

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The ${}^{10}\text{He}$ nucleus, being a “maximal neutron-rich nucleus” in the ratio of neutrons to protons ($p:n=1:4$) among nuclei observed so far, is investigated within the ${}^8\text{He}+n+n$ model. The energy and decay width of the three-body system are solved by applying the complex scaling method to the ${}^8\text{He}+n+n$ Hamiltonian, where the folding-type ${}^8\text{He}-n$ potential is carefully chosen through the analysis of the experimental resonance energy 1.16 MeV of ${}^9\text{He}$ and in comparison with the ${}^9\text{Li}-n$ folding potential employed successfully in the description of the loosely bound ${}^{11}\text{Li}$ system. The calculated result ($E_r=1.8$ MeV, $\Gamma=1.4$ MeV) of the ${}^{10}\text{He}$ ground state corresponds well to the observed values. [S0556-2813(97)05105-4]

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I. INTRODUCTION

Recently, many interesting properties of neutron-rich nuclei have been revealed by the development of radioactive nuclear beam experiments [1]. One of the most interesting current topics is the observation of the ${}^{10}\text{He}$ nucleus by Korshennikov *et al.* [2]. This nucleus is a “maximal neutron-rich nucleus” in the ratio of neutrons to protons ($p:n=1:4$) among nuclei observed so far. Since ${}^{10}\text{He}$ has only one proton fewer than ${}^{11}\text{Li}$ which is a typical example of neutron-rich nuclei, the simultaneous study of both nuclei ${}^{10}\text{He}$ and ${}^{11}\text{Li}$ will give us more insight into the binding mechanism of the neutron-rich nuclei.

The observed ground state of ${}^{10}\text{He}$ is an unbound state ($S_{2n}=-1.2\pm 0.3$ MeV) [2], and decays to ${}^8\text{He}+n+n$ three-body continuum states. As a similar but a loosely bound system, ${}^{11}\text{Li}$ has successfully been described by a ${}^9\text{Li}+n+n$ three-body model. Therefore, ${}^{10}\text{He}$ is also considered to be described by a ${}^8\text{He}+n+n$ three-body model. It is very interesting and meaningful to understand the binding mechanisms of ${}^{10}\text{He}$ and ${}^{11}\text{Li}$ through a comparative study of those nuclei.

The ${}^9\text{Li}+n+n$ and ${}^8\text{He}+n+n$ systems have a difference only in their core clusters; ${}^9\text{Li}$ has one more proton compared with ${}^8\text{He}$. Hence the difference should be described by the core- n interaction. Because of the lack of one proton for ${}^9\text{Li}$, the ${}^8\text{He}-n$ interaction must have weaker binding ability in comparison with the ${}^9\text{Li}-n$ interaction. Because of this weaker ability of the ${}^8\text{He}-n$ interaction, the ${}^8\text{He}+n+n$ system is expected to have a more loosely bound or an unbound resonance three-body structure, though the ${}^9\text{Li}+n+n$ system has a loosely bound structure. Therefore, to understand those properties in the binding mechanisms of ${}^8\text{He}+n+n$ and ${}^9\text{Li}+n+n$ systems consistently, we need a framework to treat not only three-body bound states, but also resonances. However, the theoretical treatment of many-body resonances becomes more complicated because of the boundary condition in the asymptotic region.

Recently, a very powerful method to calculate resonance

energies and widths of many-body resonances has been developed [3]. This method is called the complex scaling method (CSM) [4]. The most advantageous point of this method is that we can solve many-body bound states and resonances on the same footing. This advantageous point is especially demonstrated in loosely bound nuclei such as the neutron- (proton-) rich nuclei or the hyper nuclei. Previous applications of the CSM to several neutron-rich nuclei have succeeded in the explanations of the loosely bound and resonance mechanisms [5–8]. Moreover, since many-body resonances which have large decay widths can also be solved by using this method, we can analyze the excited resonances with large decay widths in addition to the ground state [7,8]. Therefore, by using the CSM, we study more extensively the unbound ground and excited states of ${}^{10}\text{He}$ within the same framework of the ${}^8\text{He}+n+n$ model as that of the ${}^9\text{Li}+n+n$ model for ${}^{11}\text{Li}$.

In the ${}^9\text{Li}+n+n$ model, the folding potential has been used as the ${}^9\text{Li}-n$ interaction [8–10]. Recently, Katō and Ikeda investigated the ${}^9\text{Li}-n$ folding potential based on the new experimental data [11] of ${}^{10}\text{Li}$ [8]. To understand the binding mechanisms of ${}^9\text{He}$ and ${}^{10}\text{He}$ quantitatively, and to compare the results with the detailed analyses of ${}^{10}\text{Li}$ and ${}^{11}\text{Li}$ [8–10], we use the same-type ${}^8\text{He}-n$ folding potential.

The purpose of this paper is to gain an understanding of the ground and excited resonances in ${}^9\text{He}$ and ${}^{10}\text{He}$. We use the CSM to solve the many-body resonances, and use the accumulated knowledge for ${}^{10}\text{Li}$ to investigate the ${}^8\text{He}-n$ interaction. Characteristic points of our study are as follows: (i) We use a folding potential as a more realistic ${}^8\text{He}-n$ interaction which can reproduce the resonance energy of the ground state of ${}^9\text{He}$, (ii) we solve the ground state of ${}^{10}\text{He}$ as a three-body resonance by using the CSM, and (iii) we are trying for the consistent understanding of the binding mechanisms of ${}^9\text{He}$, ${}^{10}\text{He}$, ${}^{10}\text{Li}$, and ${}^{11}\text{Li}$ explicitly.

In reference to the theoretical study of ${}^{10}\text{He}$ so far, the most quantitative one has been performed with the hyperspherical coordinate method based on the ${}^8\text{He}+n+n$ model by Korshennikov, Danilin, and Zhukov [12]. Their obtained ground state energies of ${}^{10}\text{He}$ are 0.7–0.9 MeV and the decay widths are 150–300 keV. But their determination

of the parameters of the ${}^8\text{He}-n$ potential is based on the real S -matrix theory, and this usually makes three-body systems overbound in the three-body calculation based on the complex S -matrix theory.

In Sec. II, we only briefly explain the present core-nucleus + valence-neutrons model based on the cluster orbital shell model (COSM) [13] and complex scaling method (CSM) [4], whose details have been given in our previous papers [5,7,8]. In Sec. III, we analyze the difference between the ${}^8\text{He}-n$ interaction and the ${}^9\text{Li}-n$ interaction, and investigate the ${}^8\text{He}-n$ folding potential based on the analysis of the observed ground state of ${}^9\text{He}$. In Sec. IV, we calculate energies and widths of the excited states of ${}^9\text{He}$ based on the ${}^8\text{He}+n$ model used for the ground state. In Sec. V, we analyze the ground and excited resonant states ${}^{10}\text{He}$ by using the obtained ${}^8\text{He}-n$ folding potential. Section VI is devoted to summary and conclusion.

II. MODEL AND METHOD

First, we explain the core-nucleus (${}^8\text{He}$) plus valence-neutrons model. The Hamiltonian is given as

$$H = T_{\text{core}} - T_{\text{c.m.}} + \sum_{i=1}^n [T_i + U_J(r_i) + U_F(\mathbf{r}_i)] + \sum_{i>j} V_{nn}(r_{ij}), \quad (1)$$

where T_{core} , $T_{\text{c.m.}}$, and T_i are kinetic energy operators of the core nucleus, the center-of-mass of the total system, and the i neutron, respectively. For the potential U_J between the core nucleus and the i neutron, we employ a ${}^8\text{He}-n$ folding potential. The U_F term is a pseudopotential to project out the Pauli-forbidden states of the intercluster motion. For the neutron-neutron interaction V_{nn} , we employ the central part of the Minnesota force [14] with an exchange parameter $u=0.95$.

We can get solutions of the Schrödinger equation with the above Hamiltonian by solving an eigenvalue problem. We expand the wave functions by the COSM bases [13]. They are given as

$$\begin{aligned} \Phi_{JM} = & \sum_{i_1, l_1, j_1} \sum_{i_2, l_2, j_2}, \dots, \sum_{i_n, l_n, j_n} c^{i_1, l_1, j_1, \dots} \\ & \times \mathcal{A}[\phi_{l_1 j_1}^{b_{i_1}}(\mathbf{r}_1) \otimes \phi_{l_2 j_2}^{b_{i_2}}(\mathbf{r}_2) \otimes \dots \otimes \phi_{l_n j_n}^{b_{i_n}}(\mathbf{r}_n)]_{JM}, \end{aligned} \quad (2)$$

where \mathcal{A} is an antisymmetrization operator for valence neutrons and $c^{i_1, l_1, j_1, \dots}$ are coefficients for a linear combination of products of Gaussian-type wave functions with size parameters b_i . The size parameters b_i are given by a common geometric progression $b_0 \gamma^{(i-1)}$; $i=1, \dots, n_{\text{max}}$, where b_0 , γ , and n_{max} are the first term, the geometric ratio, and the number of basis functions, respectively.

Next we explain the practical prescription how to solve the resonance solutions by using the CSM. The complex scaling is defined by the following transformation of a radial coordinate (and its conjugate momentum):

$$r \rightarrow r \exp(i\theta) \quad [p \rightarrow p \exp(-i\theta)], \quad (3)$$

where θ is a scaling parameter and real values should be taken. By doing this transformation for relative coordinates between clusters, we obtain the complex scaled Hamiltonian $H(\theta)$.

According to the so-called ABC theorem [4], we can obtain resonance energies (E_r) and widths (Γ) as eigenvalues of the complex scaled Hamiltonian (non-Hermitian) $H(\theta)$; resonance eigenvalues should be complex numbers ($E_r - i\Gamma/2$) independent of the scaling parameter θ ($> \frac{1}{2} \tan[\Gamma/(2E_r)]$). Also, one of the very promising properties of the CSM is that, independently of θ , the Hamiltonian $H(\theta)$ gives the same bound-state (real and negative) eigenvalues as those of the original (nonscaled) Hamiltonian. All other eigenvalues of $H(\theta)$ except the ones of bound and resonance solutions depend on θ , and its dependence is regularly proportional to $\exp(-2i\theta)$.

To obtain the resonance solutions by diagonalizing the matrix of $H(\theta)$, some techniques might be needed because the ABC theorem is not necessary satisfied for a limited number of basis functions. Since the three-body system has many degrees of freedom itself, it is impossible to employ a large number of basis states for every degree of freedom.

Even if we use a limited number of basis functions, we can determine accurate resonance eigenvalues by searching stationary points of the eigenvalues for the scaling angle θ (“ θ -trajectory”)[8]. In addition to the “ θ -trajectory” method to search for a stationary point, we can also determine the resonance position from the b_0 dependence (“ b trajectory”) of complex eigenvalues for the resonance solutions [8]. See Refs. [7,8] for details of these methods to obtain broad width resonance solutions.

III. ${}^8\text{He}-n$ POTENTIAL

In the following, we explain the explicit form of the ${}^8\text{He}-n$ folding potential and how to determine potential parameters. The folding potential $U_J(r)$ is expressed by a sum of central term $U_J^{\text{cnt}}(r)$ and spin-orbit term $U_J^{\text{ls}}(r)$:

$$U_J(r) = U_J^{\text{cnt}}(r) + U_J^{\text{ls}}(r). \quad (4)$$

The central part of the folding potential $U_J(r)$ is constructed from a nucleon-nucleon interaction with a Gaussian form. In the present calculation, we use modified Hasegawa-Nagata (MHN) potential and Hasegawa-Nagata No. 1 (HN1) potential [15]. These nucleon-nucleon potentials have also been used in the folding potential between ${}^9\text{Li}$ and a neutron [8], and the second-range strength v_2^0 has been slightly changed to $v_2^0(1+\delta)$ [10]. The reason for tuning in v_2^0 is that this potential has been made for the normal nuclei based on the reaction-matrix calculation [15]. Therefore this potential may be slightly modified for the unstable nuclei, especially the unbound nuclei in this case. As will be discussed later, we will first use the exact same parameter δ as in the case of the ${}^9\text{Li}-n$ folding potential in the first place; we will next determine the best parameter for the ${}^8\text{He}-n$ folding potential based on the experimental data of ${}^9\text{He}$ [16,17].

By using this Gaussian-type nucleon-nucleon interaction $[\sum_n v_n^0 (W_n + B_n P^\sigma + H_n P^\tau + M_n P^{\sigma\tau}) e^{-\rho_n r^2}]$ and assuming a neutron sub-closed-shell configuration ($p_{3/2}$)⁴ for the ${}^8\text{He}$ core, the ${}^8\text{He}-n$ central potential is calculated as

$$U_J^{\text{cnt}}(r) = \sum_n \left(\frac{16}{7\lambda_n + 16} \right)^{3/2} f_n^{\text{cnt}} \left(\sqrt{\frac{8}{9}} \frac{r}{b} \right) \times \exp \left[-\frac{9\lambda_n}{7\lambda_n + 16} \left(\sqrt{\frac{8}{9}} \frac{r}{b} \right)^2 \right], \quad (5)$$

where

$$f_n^{\text{cnt}} \left(\sqrt{\frac{8}{9}} \frac{r}{b} \right) = v_n^0 \left[(8W_n + 4B_n - 6H_n - 3M_n) - \frac{16\lambda_n}{7\lambda_n + 16} (2W_n + B_n - 2H_n - M_n) + (2W_n + B_n - 2H_n - M_n) \times \frac{96(\lambda_n)^2}{(7\lambda_n + 16)^2} \left(\sqrt{\frac{8}{9}} \frac{r}{b} \right)^2 \right]. \quad (6)$$

A size parameter of $b = 1.85$ fm of the harmonic oscillator (HO) wave function for ${}^8\text{He}$ is here taken to fit the rms matter radius of the experimental data (2.49 ± 0.04 fm) [18] and λ_n is given as $2\rho_n b^2$ with the nucleon-nucleon potential range ρ_n . The parameters W_n , B_n , H_n , and M_n express the exchange character of the n th range term in the nucleon-nucleon potential.

The spin-orbit part of the folding potential $U_J(r)$ can be constructed from a nucleon-nucleon spin-orbit potential in a similar way. But its calculation is rather cumbersome. Therefore, as in the ${}^{10}\text{Li}$ case, we assume that its potential form is proportional to the gradient of the ${}^8\text{He}$ density $\rho(r)$. This leads to the form

$$U_J^{ls}(r) = \frac{1}{2} V_0^{ls} [j(j+1) - l(l+1) - \frac{3}{4}] f^{ls}(r), \quad (7)$$

where

$$f^{ls}(r) = -\frac{128}{91\sqrt{14}\pi b^3} \exp \left[-\frac{8}{7} \left(\frac{r}{b} \right)^2 \right] \left[1 + \frac{64}{7} \left(\frac{r}{b} \right)^2 \right]. \quad (8)$$

We have parameters in the folding potential $U_J(r)$: δ in $U_J^{\text{cnt}}(r)$ and V_0^{ls} in $U_J^{ls}(r)$. First, we examine the ${}^8\text{He}$ - n solutions by using the Katō-Ikeda parametrizations [8] based on the recent experimental data of ${}^{10}\text{Li}$ [binding energy (BE) = -0.42 MeV] [11]. The parameter set (I) is $\delta = 0.0442$ and $V_0^{ls} = 44.20$ (MeV fm 3) with the MHN potential. The parameter set (II) is $\delta = 0.0866$ and $V_0^{ls} = 51.52$ (MeV fm 3) with the HN1 potential. In the ${}^{10}\text{Li}$ case, the most favored one is set (I) with the MHN potential because the potential (I) is able to reproduce not only the binding energy of the ground 1^+ state of ${}^{10}\text{Li}$ but the observed first excited 2^+ state which is considered to be the spin-doublet partner of the ground state. On the other hand, these spin-doublet states almost degenerate in the result of the potential (II) [8]. This difference between the results of two potential comes from the fact that the HN1 potential does not have an odd-state potential. More detailed discussion is given in Ref. [8].

Moreover, we introduce a pseudopotential $U_F(r)$ [19] in order to project out the Pauli-forbidden states between ${}^8\text{He}$ and neutron. This pseudopotential is expressed as

TABLE I. Energies and widths of the ground state of ${}^9\text{He}$ in comparison with the previous calculations of ${}^{10}\text{Li}$. See text for the parameter sets (I) and (II).

	E_r^{calc} of ${}^9\text{He}$ (MeV)	E_r^{calc} of ${}^{10}\text{Li}$ (MeV)
I	$E_r = 1.55$, $\Gamma = 3.08$	$E_r = 0.420$, $\Gamma = 0.286^a$
II	$E_r = 1.30$, $\Gamma = 2.85$	$E_r = 0.422$, $\Gamma = 0.322^a$

^aReference [8].

$$U_F(r) = \lambda [|(0s_{1/2})\rangle\langle(0s_{1/2})| + |(p_{3/2})\rangle\langle(p_{3/2})|], \quad (9)$$

where $|(0s_{1/2})\rangle$ and $|(p_{3/2})\rangle$ are the HO wave functions occupied by neutrons in the core nucleus, and the strength λ is taken large (10^4 – 10^8) enough to project out the Pauli-forbidden states into a high energy region.

By using these parameters, δ and V_0^{ls} , we calculated the energy and width of the ground state of ${}^9\text{He}$ with the CSM. The obtained results are shown in Table I in comparison with the previous calculations of ${}^{10}\text{Li}$. The obtained resonance energies of ${}^9\text{He}$ are higher than those of ${}^{10}\text{Li}$ and the decay widths Γ are much larger. This result can easily be understood because the ${}^8\text{He}$ - n folding potential is shallower than the ${}^9\text{Li}$ - n one and the height of the centrifugal barrier of these nuclei is around 1 MeV. The resonance energies of ${}^9\text{He}$ are higher than the barrier height.

It can be seen at a glance that the results for ${}^9\text{He}$ given in Table I are in good agreement with the experimental data $E_r = 1.16$ MeV. The differences of the resonance energies are 0.2–0.4 MeV. When we take into account the experimental error 0.1 MeV, we do not have to change the parameter values. However, the obtained resonance widths are very large (~ 3 MeV).

Experimentally, we usually observed resonances with $\Gamma/2E_r \ll 1$ and it is very difficult to observe the states near to $\Gamma/2E_r = 1$ (de Broglie condition). Although the decay width has not yet been obtained experimentally, it is reasonable to think the width Γ may be smaller than 2 times the resonance energy E_r . Therefore, we examined the parameters to make the width sharper by keeping the experimental resonance energy at $E_r = 1.16$ MeV.

In Fig. 1, we show the δ dependence of energy and width in the case of set (I). The δ dependence of the resonance energy is not so strong; on the other hand, the resonance width shows the large δ dependence. Therefore, we can make the width sharper without changing the resonance energy so much. The reason for the small δ dependence of the resonance energy can be understood from the fact that the height of the centrifugal potential barrier is around 1 MeV, which is the approximately same position as its resonance energy and depends little on δ . In Fig. 2, we show the explicit form of the ${}^8\text{He}$ - n folding potential with $\hbar^2 l(l+1)/2\mu r^2$ for the case of $\delta = 0.0442$ and $\delta = 0.102$ in the MHN potential. The parameter δ changes the potential depth at 2–3 fm and does not have a large influence on the height but does influence the thickness of the potential barrier.

The δ parameters which reproduces the experimental resonance energy $E_r = 1.16$ (MeV) [12] are 0.102 for the MHN potential and 0.139 for the HN1 potential. The obtained decay width Γ is 1.62 MeV for the MHN potential

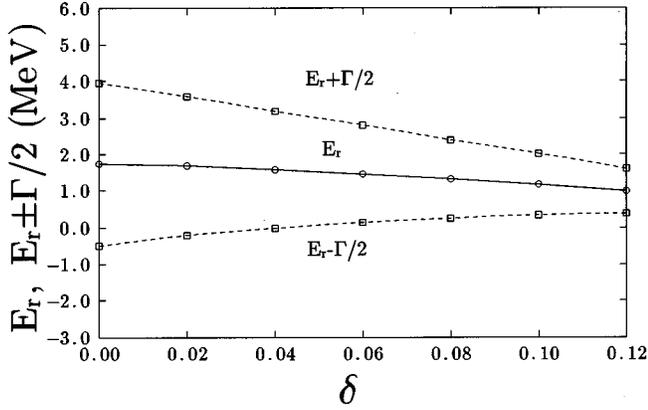


FIG. 1. The δ dependence of the energy and width for the ground state of ${}^9\text{He}$. The solid line and distance between dashed lines indicate the resonance energy and width, respectively. The using nucleon-nucleon interaction is the MHN potential.

and 2.03 MeV for the HN1 potential, respectively. Hereafter, we call these potentials with newly chosen parameter δ sets (I') and (II').

We checked the b dependence (b is the HO size parameter of the ${}^8\text{He}$ wave function) of the folding potential on the resonance energy and width of ${}^9\text{He}$, but the results are little changed the present calculation. Therefore, we fix as $b=1.85$ fm which reproduces the experimental rms matter radius (2.49 ± 0.04 fm) [18] of ${}^8\text{He}$ as mentioned before.

IV. ${}^8\text{He}+n$ EXCITED RESONANCES IN ${}^9\text{He}$

Experimentally, two excited resonances ($E_x=1.17$ MeV, $E_x=3.8$ MeV) are observed as candidates for s - and d -state excitation of a valence neutron [16]. It is very interesting to study whether these states are explained by ${}^8\text{He}+n$ two-body resonances of ${}^9\text{He}$. By using the CSM, we also calculated the excited resonances of ${}^9\text{He}$. The ${}^8\text{He}-n$ potential used in the calculation of the excited states is given by set (I').

The obtained results are presented in Table II. These results are very similar to those of ${}^{10}\text{Li}$ [8]. In the ${}^{10}\text{Li}$ case,

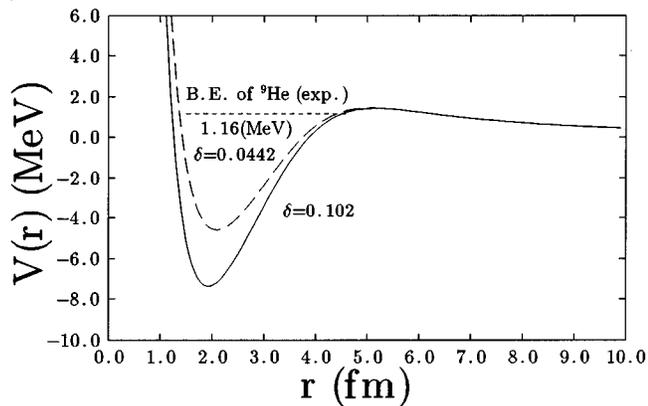


FIG. 2. The ${}^8\text{He}-n$ folding potential with the MHN potential. The solid line shows the $p_{1/2}$ potential which reproduces the experimental binding energy of ${}^9\text{He}$. The dashed line shows the $p_{1/2}$ potential which has the same parameter as ${}^{10}\text{Li}$ [8].

TABLE II. Energies and widths of d states for ${}^9\text{He}$ in the case of set (I').

l_j	E_r (MeV)	Γ (MeV)
$d_{5/2}$	7.80	11.5
$d_{3/2}$	7.46	20.2

we obtained eight resonances at $E_r=6-7$ MeV with the widths of $\Gamma=7-15$ MeV. Both results of ${}^9\text{He}$ and ${}^{10}\text{Li}$ suggest no sharp two-body resonances in the low energy region ($E_x < 4$ MeV). The obtained $d_{5/2}$ and $d_{3/2}$ resonances of ${}^9\text{He}$ indicate the small $l \cdot s$ splitting and their energies are reversed in comparison with those of usual bound states. However, the results do not mean that the spin-orbit potential is different from the usual one. The present $l \cdot s$ potential for the $d_{3/2}$ state is also more repulsive than that for the $d_{5/2}$ state, but the spin-orbit potential does not give a large influence on the $d_{3/2}$ state because the wave function of the $d_{3/2}$ state is more extended than that of the $d_{5/2}$ state. Furthermore, we cannot obtain s -wave resonances in ${}^9\text{He}$ as well as in ${}^{10}\text{Li}$. Because s states have no centrifugal barrier, there are no resonances of sharp widths in the case of the smooth Gaussian-type potential. To examine the possibility of virtual s states, we searched the parameter value of δ which barely makes a bound state or not. Such a potential is found to be about 10% deeper in comparison with the present potential depth.

V. RESONANCES IN ${}^{10}\text{He}$

We investigated the complex eigenvalues of the three-body resonances of ${}^{10}\text{He}$ ($J^\pi=0^+$) by using two parameter sets (I) and (I'). Set (I') was obtained in the previous section, Sec. III, to reproduce the observed resonance energy of ${}^9\text{He}$ [16], and set (I) given in Ref. [8].

First, in Fig. 3, we show the distribution of complex eigenvalues of $H(\theta=0.50$ rad.) with the ${}^8\text{He}-n$ potential given by set (I'). In this calculation, we employ the $[(p_{1/2})(p_{1/2})]_{0^+}$ channel wave functions in the COSM. A three-body resonance of ${}^{10}\text{He}$ is obtained at the region around $E_r [= \text{Re}(E)] = 2.1$ MeV and $\Gamma [= 2\text{Im}(E)] = 1.9$ MeV. After calculating so-called θ and b_0 trajectories [8], we obtained more accurate values $E_r=2.07$ MeV and $\Gamma=1.85$ MeV.

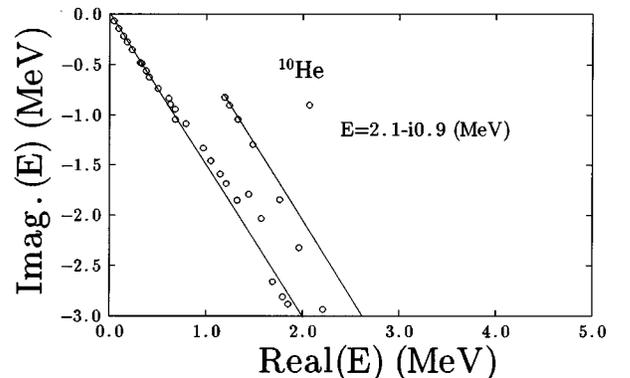


FIG. 3. The 0^+ eigenvalue distribution of the complex scaled Hamiltonian with $\theta=0.50$ rad.

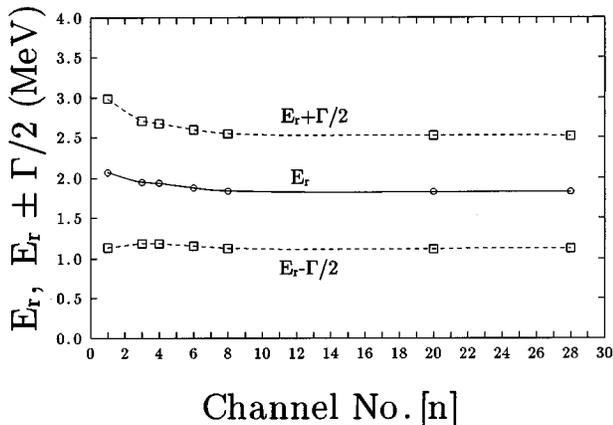
TABLE III. Energies and widths of the ${}^{10}\text{He}$ ground state.

${}^8\text{He}-n$ potential	Channel	E_r (MeV)	Γ (MeV)
(I)	$(p_{1/2})^2$	2.96	4.25
(I')	$(p_{1/2})^2$	2.07	1.85

As seen in Table III, the resonance energies obtained by using the parameter sets (I) and (I') are different by about 890 keV. This result reflects the stronger binding ability of the ${}^8\text{He}-n$ interaction of set (I') in comparison with that of set (I). In this framework, we can obtain not only three-body resonances (${}^{10}\text{He}$) but two-body resonances (${}^9\text{He}$) at the same time. We can also see the solution corresponding to the resonance of ${}^9\text{He}$ at $E_r=1.16$ MeV [$\Gamma=2\text{Im}(E)=1.62$ MeV] in Fig. 3, though another valence neutron is a continuum state. Thus, it is confirmed that this three-body calculation reproduces the two-body resonance of the subsystem ${}^9\text{He}$ at $E_r=1.16$ MeV obtained in the two-body calculation.

Next, we examined the ground state energy of ${}^{10}\text{He}$ within a larger model space in the COSM, to include the higher partial wave effects and the $n-n$ pairing energy. As we have discussed in our previous papers for ${}^6\text{He}$ and ${}^{11}\text{Li}$ [5,9], these effects make the binding energy more ~ 1 MeV bound. The above obtained results for the binding energy of ${}^{10}\text{He}$ suggest a shortage by ~ 0.8 MeV from that of experimental data ($\text{BE}=-1.2\pm 0.3$ MeV). But in the ${}^{10}\text{He}$ case, the resonance energy is near the energy region of the centrifugal barrier. Therefore, we can easily see that the resonance energy does not change so much (a few hundred keV order) as the case of bound states in ${}^6\text{He}$ and ${}^{11}\text{Li}$. However, because of the three-body effect from the $n-n$ pairing interaction, the width Γ will change.

We show the results of the resonance energies (solid line) with the obtained decay widths (dotted line) in Fig. 4. The obtained resonance energy is $E_r=1.8$ MeV and the decay width $\Gamma=1.4$ MeV for a 28-channel coupled equation. The channels used here are $(p_{1/2})_0^2 + (d_{5/2})_0^2 + (d_{3/2})_0^2 + (s_{1/2})_0^2 + (f_{7/2})_0^2 + \dots + (l=14, j=27/2)_0^2$. We can evidently see that three-body effects also appear in the obtained decay width Γ . Because of the $n-n$ interaction, the decay width Γ of ${}^{10}\text{He}$ becomes sharper than ${}^9\text{He}$. In other words,

FIG. 4. The ground state energy convergence for ${}^{10}\text{He}$.

the ground state of ${}^{10}\text{He}$ is stabler than ${}^9\text{He}$, though the calculation indicates that the resonance energy of ${}^{10}\text{He}$ is above the ${}^9\text{He}+n$ threshold.

The present calculation has been done in a large model space. Even if a larger model space including T -type basis wave function [9] is employed, the binding energy of the present calculation is estimated to increase by several 10 keV from the corresponding results of ${}^6\text{He}$ [5] and ${}^{11}\text{Li}$ [9]. Furthermore, we may also calculate in a larger model space including the core polarization of ${}^8\text{He}$. Preliminary calculations of the core excitation effects on the binding energy of ${}^{11}\text{Li}$ within a ${}^9\text{Li}^*+n+n$ model indicate that ${}^{11}\text{Li}$ is more bound by several 100 keV than that of the usual three-body model. Those improvements for the model space will make the calculated binding energy of ${}^{10}\text{He}$ near to the experimental one. However, taking into account the experimental error of the binding energy of ${}^{10}\text{He}$ (± 0.3 MeV), we think that the present result is acceptable for this stage.

Moreover, we have calculated possible excited resonances in ${}^{10}\text{He}$ assuming one-neutron-excited configurations from the $p_{1/2}$ orbit to s and d orbits. We obtain resonance solutions which have large decay widths. Here, we discuss a low-lying resonance solution of the $[p_{1/2}d_{5/2}]_3^-$ configuration which is solved at $E_r=8.5$ MeV and $\Gamma=11$ MeV. These energy and width are reasonable and consistent with the results of ${}^9\text{He}$. The calculated resonance energy of the ${}^8\text{He}+n$ system is $E_r=1.16$ MeV for the $p_{1/2}$ state and $E_r=7.80$ MeV for the $d_{5/2}$ state. So with one valence-neutron in the $p_{1/2}$ orbit and another in the $d_{5/2}$ orbit, the resonance energy of the ${}^8\text{He}+n+n$ system is expected at ~ 8.96 MeV. Furthermore, this energy may be smaller due to the interaction between valence neutrons and a three-body effect. The sharper decay width of the 3^- state of ${}^{10}\text{He}$ in comparison with that of the $d_{5/2}$ state of ${}^9\text{He}$ is also understood in the same way. Other resonance solutions are also obtained in the energy region of 7–8 MeV. A resonance solution of the 1^- state associated with a soft $E1$ resonance in this model space is predicted with the $[(p_{1/2})(d_{3/2})]_{1^-}$ channel basis function at $E_r=8.7$ MeV and $\Gamma=19$ MeV.

We checked the solution assuming that the $[(s_{1/2})(s_{1/2})]_{0^+}$ state is the ground state of ${}^{10}\text{He}$. We did not obtain the resonance pole of the ${}^{10}\text{He}$ by using the very strong potential parameter which makes the s state of ${}^{10}\text{Li}$ barely bound. On the other hand, for the $[(p_{1/2})(p_{1/2})]_{0^+}$ state, by using the same potential, we obtained the bound solution which is inconsistent with the experimental one.

VI. SUMMARY AND CONCLUSION

In this paper, we studied the ${}^8\text{He}-n$ folding potential based on the analyses of the observed ground state of ${}^9\text{He}$. We used the folding potential between ${}^8\text{He}$ and a valence neutron in the similar way as the analysis of ${}^{10}\text{Li}$ [8]. The ${}^8\text{He}-n$ folding potential has two parameters as the ${}^9\text{Li}-n$ folding potential does; one is δ which tunes the strength of the second range of the nucleon-nucleon potential, $v_2(1+\delta)$, and the other is the strength V_0^s of the spin-orbit potential. For the exactly same parameter δ and V_0^s as the case of the ${}^9\text{Li}-n$ folding potential, we obtained the consistent result with the experimental binding energy of ${}^9\text{He}$. However, this result shows a little underbinding and having a

large decay width in comparison with the experimental one. Therefore, we researched the best parameter δ of the $^8\text{He}-n$ folding potential for the experimental data of ^9He [16] in order to investigate the excited states of ^9He and the ground and excited states of ^{10}He .

By using the obtained $^8\text{He}-n$ folding potential, we calculated resonance energies of d states in ^9He at ~ 7 MeV with rather large decay widths. But s -wave resonances were not obtained in this calculation. Therefore, the observed excited states at $E_x=1.17$ MeV and $E_x=3.8$ MeV cannot be explained by the simple two-body model. If we want to explain these experimentally observed excited states, we must include ^8He core excitations such as neutron $(p_{3/2})^{-1}(p_{1/2})$ configurations.

By using 28-channel basis functions in the COSM space, we also calculated the ground state energy and width of ^{10}He . The obtained result is $E_r=1.8$ MeV and $\Gamma=1.4$ MeV. This result corresponds well to the experimental resonance

energy $E_r=1.2$ MeV and width $\Gamma\leq 1.2$ MeV. Taking into account the experimental error of the binding energy (± 0.3 MeV), the present result is acceptable, although we may be able to do a larger model space calculation including T -type basis wave function. For the excited states of ^{10}He , we calculated several resonances by using valence-neutron s , and d configurations. However, the results indicate that while the resonance energies are a little high, the decay widths are very large.

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