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Relativistic dynamics and the deuteron axial form factor

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(Received 31 October 1996)

Matrix elements of the deuteron axial current are calculated in the covariant spectator scheme of Gross, and compared to a corresponding calculation using light-front dynamics. The results confirm the dominant role in both approaches of the deuteron D state in observable effects beyond the nonrelativistic limit. [S0556-2813(97)00105-2]

PACS number(s): 24.10.Jv, 25.30.Bf, 25.30.Pt

The deuteron has been the testing ground for increasingly refined theories of the nucleon-nucleon dynamics, from potentials to meson exchange to quark physics. It is also one of the simplest venues for implementing and studying theories which incorporate relativistic formulations of the dynamics. Almost all of such formulations have concentrated on electron scattering, including the deuteron form factors $A(Q^2)$, $B(Q^2)$, and the tensor polarization T_{20}. They include models inspired by meson-nucleon field theory, such as the Bethe-Salpeter equation [1] and the Gross equation [2,3], and those based upon direct interactions and light-front dynamics [4–6]. The results for the electric form factor $A(Q^2)$ indicate rather small effects from relativity, going beyond a few percent only at momentum transfers of several GeV^2. For the magnetic form factor $B(Q^2)$, calculations based on the Gross equation exhibit sensitivity to negative-energy P-state admixtures [3], and light-front calculations show marked dependence upon the choice of matrix elements $\langle p'\mu'|I^+(0)|p\mu\rangle$ of the electromagnetic current operator $I^\mu(x)$ used to extract the form factor [4].

It is also important to understand the role of relativistic dynamics in the deuteron axial current. This subject was explored in considerable detail by Frederico *et al.*, within the framework of light-front dynamics [7]. Their primary findings were that (1) the axial form factor is very sensitive to the choice of matrix element $\langle p'\mu'|A^+(0)|p\mu\rangle$ of the axial current operator $A^\mu(x)$ used to extract the form factor $F_A(Q^2)$—as much or more so than the magnetic form factor $B(Q^2)$—and (2) this sensitivity is connected almost entirely to the deuteron D state. By itself, sensitivity to the choice of matrix element reflects the fact that the axial current operator $A^+(0)$ must contain contributions from two-body operators as a consequence of full rotational covariance at the operator level [8]. The method of imposing covariance implicitly determines the nature of the two-body currents. One approach is to select specific matrix elements for the form factor and to let covariance determine the others. This was done explicitly by Frankfurt *et al.*, for the deuteron electromagnetic and weak currents by organizing their matrix elements according to a ''goodness'' vs ''badness'' criterion—a hierarchy which then dictates the choice of matrix elements for each electroweak form factor [9]. Another possible choice is the scheme of Karmanov [5,6], who sets up a manifestly covariant framework and extracts amplitudes using a linear combination of the ''+'' and ''⊥'' components of the current which do not depend upon the orientation of the light front in order to achieve rotational covariance. This procedure has been applied to matrix elements of the electromagnetic current.

Given the large uncertainties associated with light-front axial current matrix elements, it is useful and desirable to examine such matrix elements in a scheme which is relativistically covariant without reference to light-front dynamics. In this paper, we consider the manifestly covariant scheme of Gross [10]. At issue is whether (1) there are significant differences between this approach and the nonrelativistic limit and (2) whether the D state plays a significant role in any such relativistic effects.

The covariant calculation proceeds along the lines described in detail for electromagnetic currents by Arnold *et al.* [2]. The matrix element consists of a momentum loop integral between deuteron-neutron-proton vertices, with the spectator nucleon constrained to its mass shell:

$$G^\mu(Q^2)=\int\frac{d\mathbf{p}}{2E_{\mathbf{p}}(2\pi)^3}\mathrm{Tr}[S^T(p)C\overline{\Gamma}^\nu(p',P_d')\xi_\nu'^{*}\times S(P_d'-p)\gamma^\mu\gamma_5 S(P_d-p)\Gamma^\lambda(p,P_d)\xi_\lambda C],\tag{1}$$

where $p=(E_{\mathbf{p}},\mathbf{p})$ is the spectator momentum, P_d and P_d' the initial and final deuteron momenta, respectively, $E_{\mathbf{p}}=\sqrt{m^2+\mathbf{p}^2}$, and $S(p)=(\gamma\cdot p-m)^{-1}$. The four-vector G^μ is related to form factors via [7]

$$G^\mu(Q^2)=2P_d^0\mathbf{S}F_A(Q^2)-2\mathbf{q}(\mathbf{S}\cdot\mathbf{q})\left[F_P(Q^2)+\frac{F_A(Q^2)}{4(P_d^0+M_d)}\right],\tag{2}$$

where $\mathbf{S}$ is the deuteron spin. In this work we consider only $F_A(Q^2)$. It can be extracted by choosing $\mathbf{q}$ to lie along the z axis, and noting that $G_{23}^1=-2iP_d^0F_A$.

The nucleon axial current is taken to be pure $\gamma^\mu\gamma_5$. One could also supply a nucleon axial form factor which depends

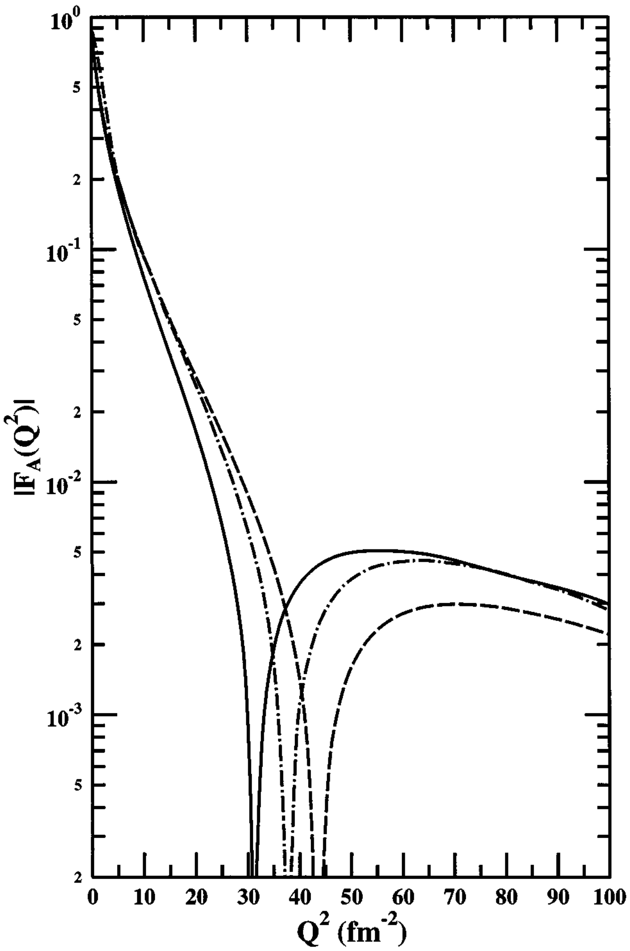

FIG. 1. Deuteron axial form factor from the Gross $\lambda=0.2$ (solid line) and light-front (dashed line) relativistic formulations, and the nonrelativistic limit (dot-dashed line), using only the full configurations of each calculation.

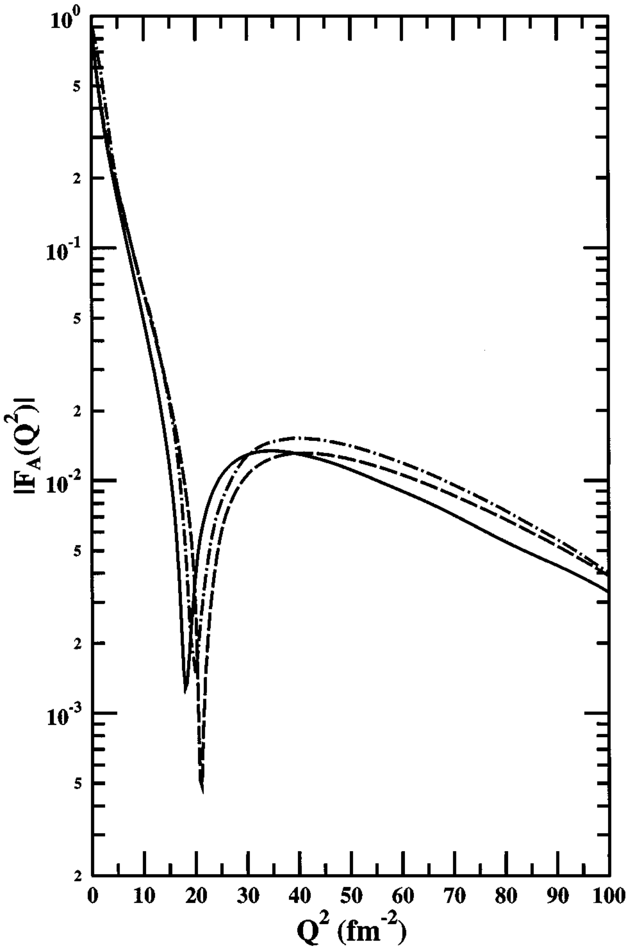

FIG. 2. Deuteron axial form factor from the Gross $\lambda=0.2$ (solid line) and light-front (dashed line) relativistic formulations, and the nonrelativistic limit (dot-dashed line), using only the S-wave contribution.

upon Q^2 (as well as other variables which describe the extent to which the struck nucleons are off their mass shells), but for purposes of comparison these are omitted in the results which are shown, and the isoscalar nucleon axial coupling constant is set to unity.

Within this scheme, we employ a family of deuteron vertex functions $\Gamma(p,P_d)$ obtained by Buck and Gross [11] for a range of values of a parameter λ, which gives the relative strength of pseudoscalar (PS) vs pseudovector (PV) coupling ($\lambda=0$ is pure PV; $\lambda=1$ is pure PS). These vertex functions do not represent the extent of fits to nucleon-nucleon observables found in more recent solutions to the Gross equation [3], but they are adequate for the sensitivity studies presented here [12].

For comparison purposes, we include results from a light-front calculation. The axial form factor is extracted from matrix elements

$$A_{\mu'\mu}:=\langle P_d'\mu'|A^+(0)|P_d\mu\rangle. \tag{3}$$

Although only the deuteron form factor $F_A(Q^2)$ contributes to matrix elements of $A^+(0)$, there are two independent light-front one-body matrix elements, which we take without loss of generality to be A_{10} and A_{11}. The extracted value of $F_A(Q^2)$ is quite sensitive to the choice of $A_{\mu'\mu}$ [7], as noted above. In the end, the resolution to this ambiguity is a dynamical issue which depends upon two-nucleon currents via meson exchange or other mechanisms involving explicit quark degrees of freedom. The results reported here represent the choice of Frankfurt *et al.* [9], who employ a linear combination:

$$F_A(Q^2)=(1+\eta)^{-1}(A_{11}+\sqrt{2\,\eta}A_{10}), \tag{4}$$

where $\eta=Q^2/4M_d^2$. Recognizing the ambiguities described above, there is no preference associated with this particular choice.

Figure 1 compares results for $F_A(Q^2)$ from the Gross and nonrelativistic approaches and those from a light-front approach. All configurations in the deuteron have been included. The results of the Gross and light-front schemes differ substantially from each other and from the nonrelativistic calculation. For Q^2 even as low as 1 GeV^2, there is a noticeable difference among the calculations. Figure 2 shows that the contribution to $F_A(Q^2)$ from the deuteron S state is essentially identical among the different relativistic formulations as well as the nonrelativistic limit. This effect was ob-

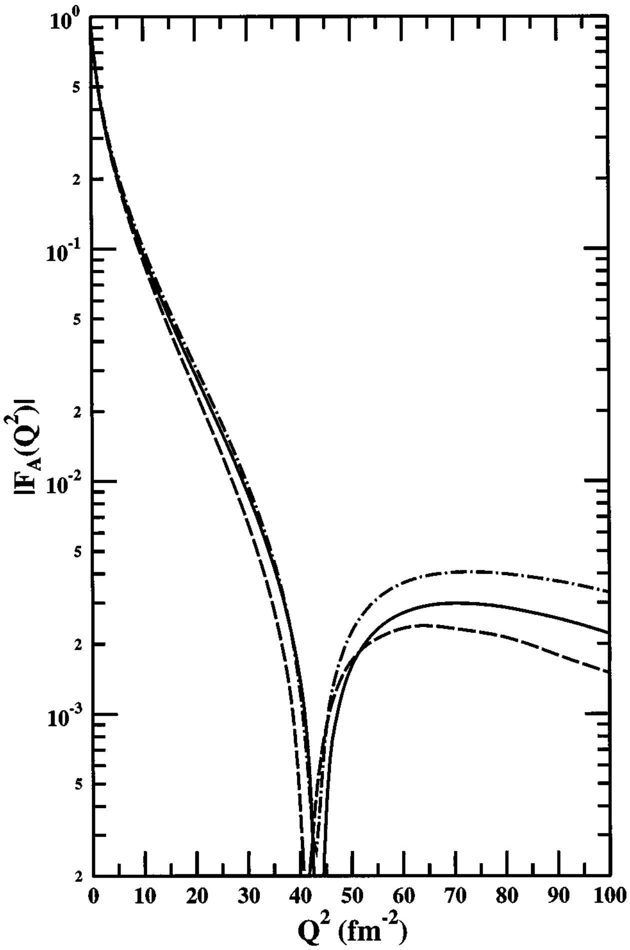

FIG. 3. Deuteron axial form factor from the light-front relativistic formulation, using the Paris (solid line), Nijmegen (dashed line), and Bonn-R (dot-dashed line) potentials.

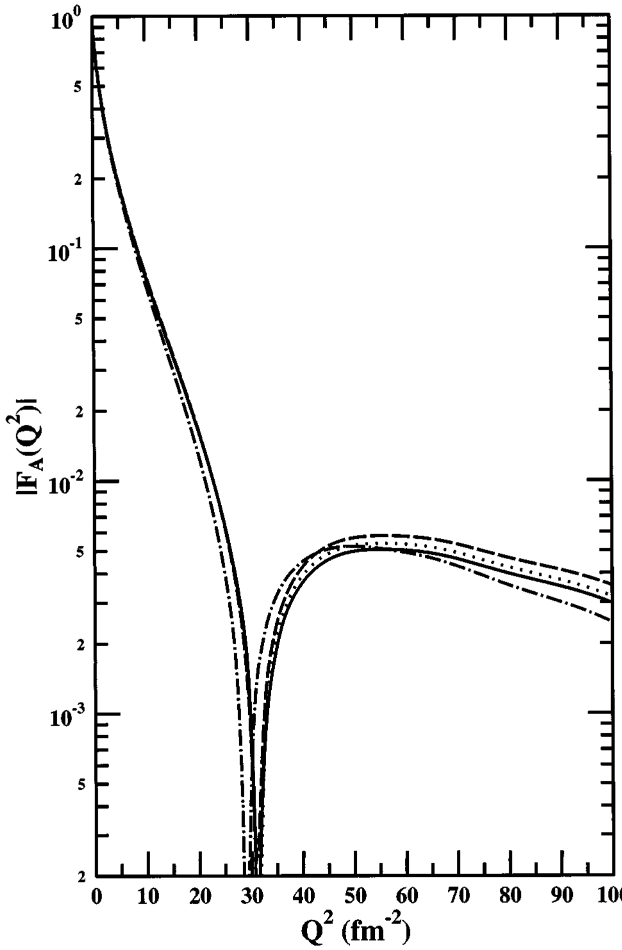

FIG. 4. Deuteron axial form factor from the Gross relativistic formulation, using Buck-Gross wave functions for $\lambda = 0.0$ (dashed line), $\lambda = 0.2$ (solid line), $\lambda = 0.4$ (dot-dashed line), and $\lambda = 0.2$ with P states omitted (dotted line).

served for light-front dynamics by Frederico *et al.* [7], but is evidently also true for the Gross approach as well.

Part of this sensitivity can be understood from the fact that the S-state contribution to $F_A(Q^2)$ has a node near $Q^2 = 18$ fm^{-2}. When the D state is included, all of the calculations shown exhibit constructive interference which pushes the node to higher Q^2. This interference then depends upon the precise manner in which the D-state contribution is implemented.

The interference effect from S and D states suggests that there might also be a sensitivity to the choice of momentum wave function, but in fact this is a relatively minor effect. The light-front results shown in Figs. 1 and 2 use wave functions from the Paris [13] potential. Figure 3 compares the same light-front calculation using the Nijmegen [14] and Bonn-R [15] potentials. There is a slight movement of the minimum in $F_A(Q^2)$ and its secondary maximum, but this effect is smaller than the differences observed between the forms of relativistic dynamics, and their difference from the nonrelativistic limit.

Of course, it may be possible to choose a particular linear combination of the light-front matrix elements A_{10} and A_{11} which yields a functional form of $F_A(Q^2)$ which matches that of the Gross calculation. In the absence of further resolution of this ambiguity, the conclusion which remains is that both the light-front and the Gross calculations show significant relativistic effects, with the D state playing the dominant role.

A calculation of current matrix elements is not complete without an accompanying analysis of possible contributions from two-body currents. Nonrelativistic calculations can require two-body currents if the interaction carries charge, as with pion exchange, but relativistic calculations can require additional two-body contributions because the current four-vector operator must satisfy dynamically dependent conditions of relativistic covariance.

One distinctive feature of the Gross equation is that it automatically includes contributions which manifest themselves as two-body currents via pair terms (Z graphs) in the nonrelativistic limit. The light-front calculations presented here are based on a Hamiltonian with fixed particle number, rather than a field theory, and therefore do not automatically contain such terms. One might then expect that this difference in content between the two relativistic approaches explains the quantitative differences shown in the figures. However, further investigation reveals that the pair contribution to $F_A(Q^2)$ in the Gross approach is quite small. Figure 4 illustrates several results which should differ significantly from each other if the physics of pair terms plays an important role. This can be seen from the fact that there is little difference among the results for differing PS/PV ratios $\lambda=0.0$, 0.2, 0.4. Pseudoscalar coupling gives rise to large pair contributions which then end up as two-body currents in a calculation which does not include pair excitation. By contrast, pseudovector coupling has a separate two-body current arising from a contact (seagull) interaction, which is not included in the calculations shown. Varying λ thus illustrates the effect of the inequivalent treatment of pseudoscalar/pseudovector coupling. Furthermore, a calculation in which the contribution from the negative energy P_s and P_t states are omitted, differs little from the full calculation. This last result provides a contrast to the case of the deuteron magnetic form factor $B(Q^2)$, where the P states provide important interference effects [3].

In summary, the deuteron axial form factor $F_A(Q^2)$ has been calculated using the manifestly covariant scheme of Gross. Like the corresponding light-front calculation, relativistic effects are significant, and the effects are manifested almost entirely via the D state. Thus, the axial form factor, together with the magnetic form factor $B(Q^2)$, provides a sensitive testing ground for dynamical models, even at moderate Q^2. The manifestly covariant scheme of Gross exhibits marked sensitivity to Z-graph contributions to $B(Q^2)$, but almost none to $F_A(Q^2)$.

The author wishes to thank Dr. F. Coester and Professor F. Gross for helpful discussions. This work was supported in part by the U.S. National Science Foundation under Grant No. PHY-9319641.

[1] M. J. Zuilhof and J. A. Tjon, Phys. Lett. **84B**, 31 (1979).
[2] R. G. Arnold, C. E. Carlson, and F. Gross, Phys. Rev. Lett. **38**, 1516 (1977); Phys. Rev. C **21**, 1426 (1980).
[3] J. W. Van Orden, N. Devine, and F. Gross, Phys. Rev. Lett. **75**, 4369 (1995).
[4] L. L. Frankfurt and M. I. Strikman, Nucl. Phys. **B148**, 107 (1979); M. Chemtob, Nucl. Phys. **A382**, 317 (1982); St. Glazek, Acta Phys. Pol. B **14**, 893 (1983); I. L. Grach and L. A. Kondratyuk, Yad. Fiz. **39**, 316 (1984) [Sov. J. Nucl. Phys. **39**, 198 (1984)]; P.-L. Chung, F. Coester, B. D. Keister, and W. N. Polyzou, Phys. Rev. C **37**, 2000 (1988); L. L. Frankfurt, I. L. Grach, L. A. Kondratyuk, and M. Strikman, Phys. Rev. Lett. **62**, 387 (1989).
[5] V. A. Karmanov and A. V. Smirnov, Nucl. Phys. **A546**, 691 (1992); **A575**, 520 (1994).
[6] V. A. Karmanov and J.-F. Mathiot, Nucl. Phys. **A602**, 388 (1996).
[7] T. Frederico, E. M. Henley, and G. A. Miller, Nucl. Phys. **A533**, 617 (1991).
[8] B. D. Keister, Phys. Rev. D **49**, 1500 (1994).
[9] L. L. Frankfurt, T. Frederico, and M. Strikman, Phys. Rev. C **48**, 2182 (1993).
[10] F. Gross, Phys. Rev. C **26**, 2203 (1982).
[11] W. W. Buck and F. Gross, Phys. Rev. D **20**, 2361 (1979).
[12] F. Gross (private communication).
[13] M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).
[14] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D **17**, 768 (1978).
[15] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).