Influence of spin-rotation measurements on partial-wave analyses of elastic pion-nucleon scattering

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Recent measurements of spin-rotation parameters in elastic $\pi^+ p$ scattering are in marked disagreement with predictions of the Carnegie-Mellon–Berkeley and Karlsruhe-Helsinki analyses. Using the method of Barrelet, we show how this discrepancy can be removed. We then show how this Barrelet transformation alters the partial-wave amplitudes. The effect of unitarity and analyticity constraints is also considered. [S0556-2813(97)02504-1]

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I. INTRODUCTION

The excited states of the nucleon have been studied mainly through partial-wave analyses of πN total, elastic, and charge-exchange scattering data. The Review of Particle Properties [1] lists about 20 such states below 2 GeV. Many of these resonances have a three-star or lower rating, which implies their existence is at most ''likely'' and their properties are uncertain. Most of these states were either found or verified in the Karlsruhe-Helsinki (KH) [2] and Carnegie-Mellon–Berkeley (CMB) [3] analyses.

In this paper we consider the effect of spin-rotation (SR) measurements on partial-wave analyses. In particular, we focus on the KH and CMB analyses, as no SR data were available in the resonance region when these analyses were performed. Measurements of R and A between 300 and 600 MeV were recently made at LAMPF [4] and the Petersburg Nuclear Physics Institute (PNPI) [5]. These quantities were at least qualitatively predicted by the existing analyses. This was not so surprising. Earlier studies [6] had suggested that the imposition of sufficient unitarity and analyticity constraints would remove those ambiguities due to the absence of SR data. However, more recent Institute for Theoretical and Experimental Physics (ITEP)-PNPI measurements [7] of A for $\pi^+ p$ at 1.3 GeV *are* surprising. These data, taken between 120 and 140 degrees, suggest an angular dependence very different from the KH and CMB predictions.

In Sec. II, we briefly review the problems that arise when analyses use incomplete sets of data (no spin-rotation measurements). The method of Barrelet [8] is used to isolate and modify one zero trajectory which may be responsible for the poor fit to the 1.3-GeV SR data. The effect of a transformed trajectory on the unitarity and analyticity of the KH and CMB solutions is also considered. In Sec. III, we show that the above procedure produces an improved agreement between the SR data and the KH analysis. Here we also compare the partial-wave amplitudes of the original and modified KH solution. Finally, in Sec. IV, we summarize our results and suggest extensions of the present work.

II. AMPLITUDE AMBIGUITIES AND THE BARRELET METHOD

The KH and CMB analyses were performed prior to the existence of spin-rotation measurements in the resonance region. These analyses had cross section (differential and total) and polarization data. However, without further theoretical input, it is clear that this dataset is insufficient. This is easier to see if we work with transversity amplitudes ($F^{\pm} = F \pm iG$) constructed from the spin-flip (G) and spin-non-flip (F) amplitudes. In this representation, the differential cross section ($d\sigma/d\Omega$) and polarization (P),

$$\frac{d\sigma}{d\Omega} = |F^+|^2 + |F^-|^2, \tag{1}$$

$$P\frac{d\sigma}{d\Omega} = |F^+|^2 - |F^-|^2, \tag{2}$$

determine $|F^{\pm}|$, leaving an undetermined relative phase. (In addition to the relative phase between transversity amplitudes, there is also an undetermined overall phase [9].) The total cross sections further constrain the forward scattering amplitudes.

The symmetry $F^+(-\theta) = F^-(\theta)$ can be used to express the two transversity amplitudes in terms of a single function. Barrelet [8] showed that it was useful to parametrize this function as a product

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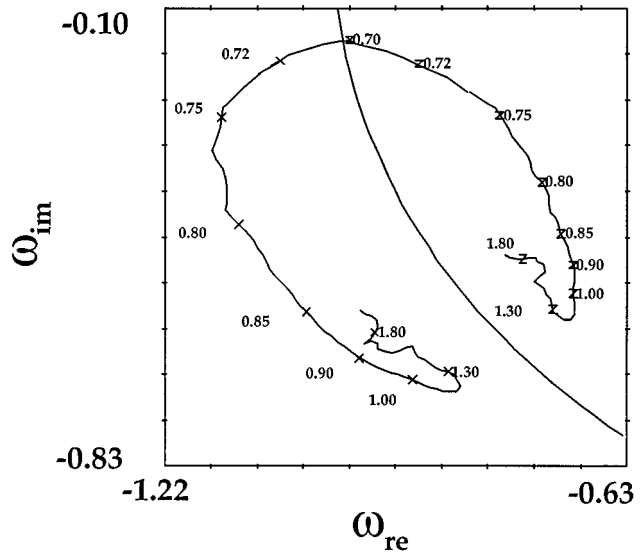

FIG. 1. Comparison of the original and transformed $F^+(w)$ zero trajectories which cross the unit circle near 0.7 GeV. Energy values (in GeV units) are marked by x and z symbols on the original and transformed trajectories, respectively.

$$F(w)=\frac{F(1)}{w^N}\prod_{i=1}^{2N}\frac{w-w_i}{1-w_i}, \tag{3}$$

in terms of the variable $w=e^{i\theta}$. The unit circle $|w|=1$ corresponds to the physical region where θ is real and equal to the center-of-mass scattering angle.

In writing Eq. (3), we have implicitly assumed that $F(w)$ can be represented by a finite polynomial. The use of this form, in analyzing scattering data directly, has been criticized by Höhler [9]. He notes that the above product should contain another factor $\mathfrak{R}(w)$ which accounts for both the distant zeros and the effects of branch cuts and poles which are known to exist but cannot be described by a polynomial. The violation of unitarity can also be a problem.

In the present work, we have applied the Barrelet method to the KA84 [2] solution. (Results for the CMB [3] and KH80 [2] solutions are similar.) Since the KH and CMB analyses have employed unitarity and analyticity constraints, we expect that they can be represented by the product given in Eq. (3), with the additional factor suggested in Ref. [9].

The operation $w_i\rightarrow 1/w_i^*$ for a single term in the above product preserves both the cross section and polarization. This represents an ambiguity that can be resolved by SR measurements. When a zero trajectory crosses the unit circle, w_i and $1/w_i^*$ are equal. At this point, an alternate zero trajectory (with $w_i\rightarrow 1/w_i^*$) can emerge. Whether this new trajectory connects reasonably smoothly to the original one also depends on the angle at which the trajectory crosses the unit circle.

We have identified one particular trajectory which can be linked to the discrepancy found in the SR measurements. This trajectory crosses the unit circle at about 700 MeV and remains influential through the remainder of the resonance region. When transformed, it produces an improved description of the SR data. This trajectory appears in both the KH and CMB solutions. The original and transformed trajectories for the KA84 solution are displayed in Fig. 1. A detailed comparison of the original and transformed solutions is given in Sec. III.

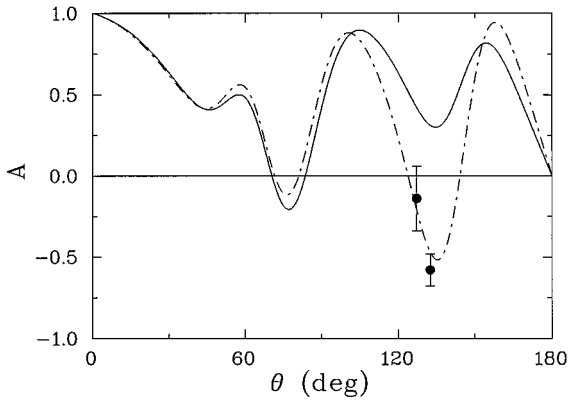

FIG. 2. The spin-rotation parameter A for π^+p elastic scattering at 1.3 GeV. The original KA84 solution (solid line) is compared to the Barrelet-transformed solution (dot-dashed line). Data are taken from Ref. [7].

If SR data had become available at the time of the KH and CMB analyses, they would have been fitted along with the constraints from dispersion relations. We have found that the Barrelet transformation, applied to these solutions, results in a good fit to SR measurements without altering the fit to the remaining database. If this operation has a minimal effect on the dispersion relation constraints, we can take the variation in partial-wave amplitudes, displayed in Sec. III, as a guide to the results expected in a full partial-wave analysis.

As the transformation of roots does not alter the forward amplitude, none of the forward dispersion-relation integrals are affected. Gauging the effect on other dispersion relations is more difficult, since we have only the A measurement for elastic π^+p scattering ($I=3/2$). A change in the $I=3/2$ partial-waves would result in a readjustment of the $I=1/2$ amplitudes in a fit to the full database. As a simple test, we recalculated the πNN coupling constant (f^2) using the π^+p amplitudes in a fixed-t dispersion relation. The value of

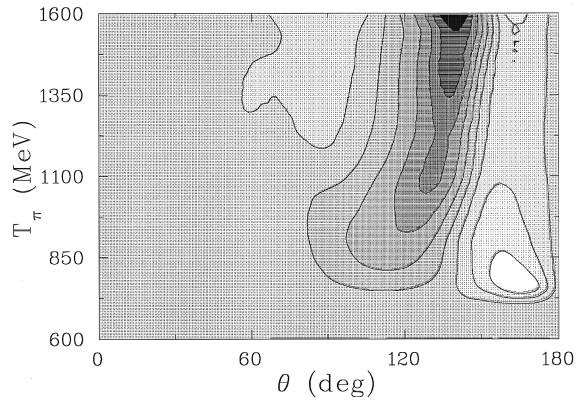

FIG. 3. Contour plot of differences between the modified and original KA84 solutions (see text). The white and black regions correspond to differences less than -0.48 and greater than $+1.15$, respectively. Neighboring contours differ by approximately 0.2.

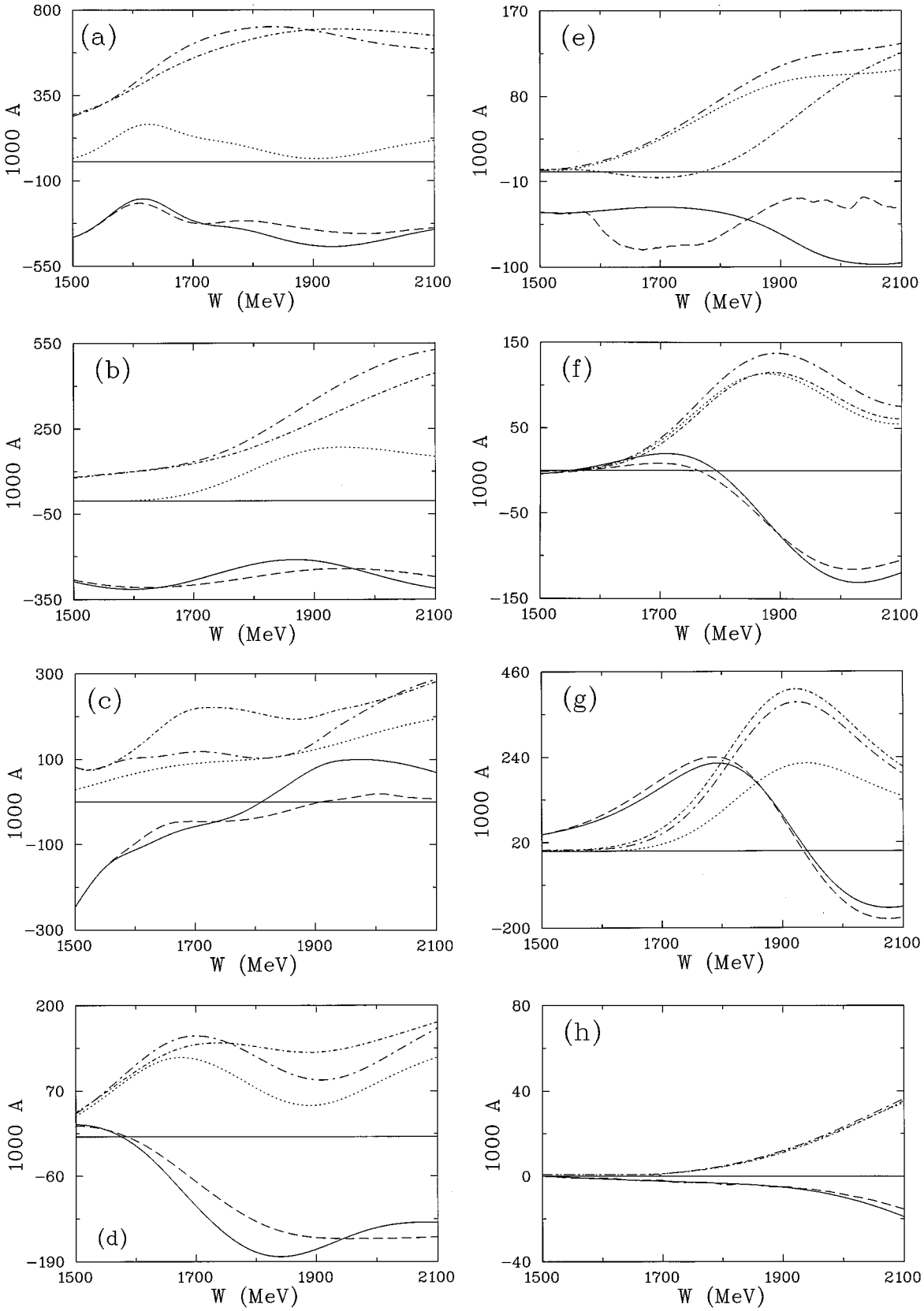

FIG. 4. Partial-wave amplitudes for the KA84 solution. The real (solid line) and imaginary (long dot-dashed line) parts of the original solution are compared to the real (dashed line) and imaginary (short dot-dashed line) parts of the Barrelet-transformed solution. The dotted line gives the unitarity constraint ($\mathrm{Im}T\text{-}|T|^2$) for the original solution. (a) S_{31}, (b) P_{31}, (c) P_{33}, (d) D_{33}, (e) D_{35}, (f) F_{35}, (g) F_{37}, (h) G_{37}, (i) G_{39}, (j) H_{39}, (k) $I_{3\ 11}$.

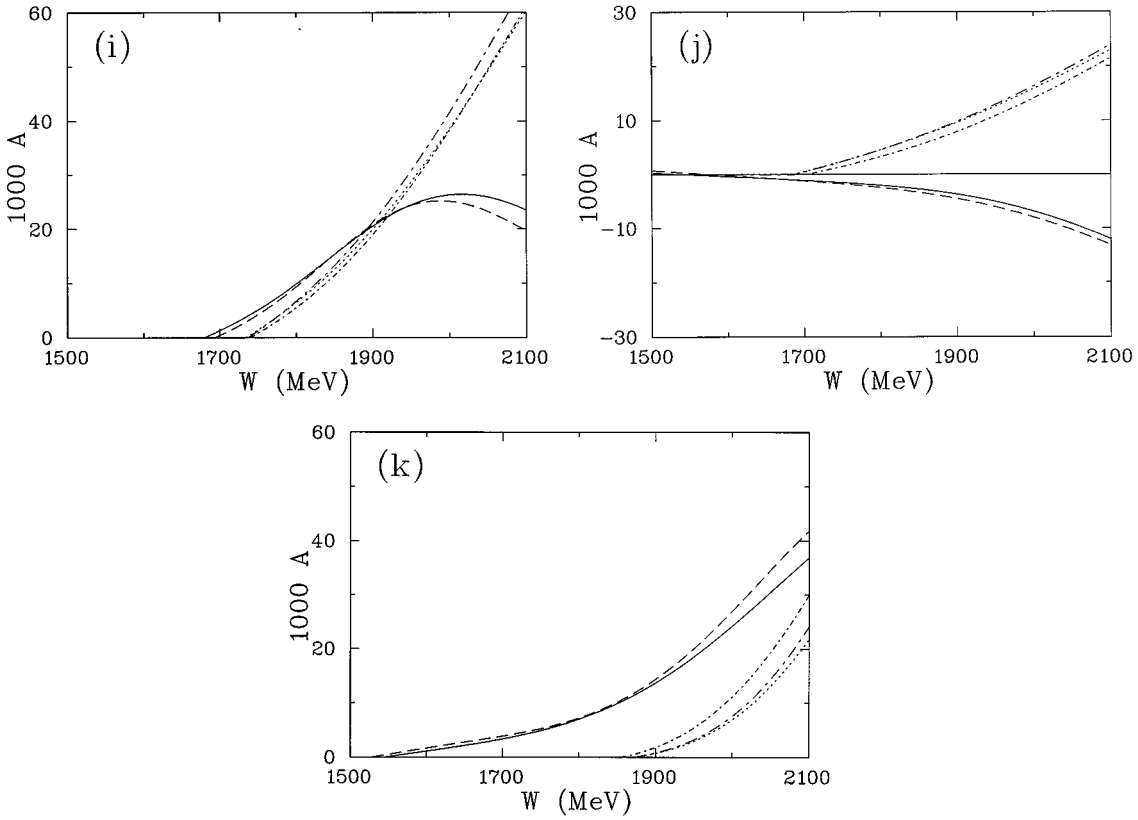

FIG. 4. (*Continued*).

f^2 was shifted systematically by less than 1% over a range of t values. While this change in f^2 is within its uncertainty, some small readjustment of the amplitudes, due to the dispersion-relation constraints, would occur in a full analysis [10].

Finally, we should note that the Barrelet transformation can result in a violation of unitarity. This can be a serious problem if data are analyzed directly. Here we are again helped by the fact that we *start* with amplitudes which are unitary. The transformed amplitudes have been checked for violations of unitarity, and one example is given in the next section.

III. PARTIAL-WAVE AMPLITUDES AND OBSERVABLES

As mentioned in Sec. II, the Barrelet transformation leaves invariant the polarization and the differential and total cross sections. In Fig. 2, we show how much better the transformed amplitudes describe the $\pi^+ p$ A measurements at 1.3 GeV. This change in the prediction for A starts at about 700 MeV and persists through the resonance region. The other sets of R and A measurements just missed this effect, as they extended up to only 600 MeV. A contour plot of differences between the predictions of the original and transformed solutions is displayed in Fig. 3. This transformation also removes disagreements with the analysis of Ref. [11] at back angles. The KH and CMB solutions have resolved ambiguities in the same way [12] and, therefore, Figs. 2 and 3 are qualitatively the same when the CMB solution is used [13].

Differences between the original and transformed KH amplitudes are shown in Fig. 4. Partial waves with clear resonance signatures show the same qualitative behavior in both solutions. Weaker resonance signals are more significantly affected in the S, P, and D waves. The higher partial waves are again qualitatively similar in the two solutions.

The D_{35} partial wave provides an example of a case where the Barrelet transformation results in a small violation of unitarity. (The imaginary part is negative.) Luckily, this problem occurs at energies where the imaginary part is very small.

IV. CONCLUSIONS

We have shown that the Barrelet transformation of one particular zero trajectory has the effect of greatly improving the KH and CMB descriptions of recent spin-rotation measurements. The effect of this transformation on the partial-wave amplitudes serves as a guide to results which would be expected in a full analysis. A further readjustment to satisfy unitarity and analyticity constraints should also be expected. This is particularly true for the D_{35} partial wave.

This transformation, applied to the KH and CMB solutions, also results in an improved agreement between the KH, CMB, and Virginia Tech [11] predictions for A in $\pi^+ p$ scattering at back angles, over a wide range of energies. It is important to have further measurements in the neighborhood of 1.3 GeV to determine whether the modified solutions make improved predictions.

There are numerous resonance candidates with masses near the center-of-mass energy of the ITEP-PNPI measurements [7]. From Fig. 4, we see that the well-established F_{35} (1905) and F_{37} (1950) resonances appear clearly in both the original and transformed solutions. Weaker structures in the S, P, and D waves have been altered significantly in the transformed solution. This added uncertainty should be factored into future comparisons with quark-model predictions.

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