

Covariant and light-front approaches to the ρ -meson electromagnetic form factors

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The ρ -meson electromagnetic form factors are calculated, both in covariant and light-front frameworks with constituent quarks. The effect of the breakdown of rotational symmetry for the one-body current operator in the null plane is investigated by comparing calculations within light-front and covariant approaches. This allows to choose the appropriate light-front prescription, among the several ones, to best evaluate the ρ -meson form factors. [S0556-2813(97)02303-0]

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I. INTRODUCTION

Since Dirac [1], it is known that the light-front hypersurface given by $x^+ = x^0 + x^3 = 0$ (null plane) is suitable for defining the initial state of a relativistic system. Relativistic models with null-plane wave functions have becoming widely used in particle phenomenology [2]. They permit us to calculate the matrix elements of certain operators in a framework of a fixed number of constituents, while maintaining a limited covariance, under transformations that keeps the null plane invariant [3]. As the generators of rotations around the x and y axis do not belong to the stability group [4], the covariance of a composite null-plane wave function with a fixed number of constituents can be broken.

This fact has consequences, for example, in the calculation of the electromagnetic form factors of a composite spin one particle in the light front [5]. It is well known that, the J^+ ($=J^0 + J^3$) component of the current, loses its rotational invariance and consequently violates the angular condition in a light-front calculation [5,6]. The light-front matrix elements are computed with a null-plane wave function [6,7] in the Breit frame, where the vector component of the momentum transfer is along the x direction. If rotational symmetry around the x axis is valid, then $J_{zz}^+ = J_{yy}^+$, where the subscripts are the polarizations of the spin one particle in the Cartesian instant-form spin basis [8]. Such a requirement is the angular condition [5,6]. It can also be derived in the front-form spin basis, using general arguments of the parity and rotational invariance of J^+ [9].

The breakdown of rotational symmetry implies that an unique way does not exist to extract electromagnetic form-factors from the light-front matrix elements of J^+ , for composite systems with spins equal to or higher than one. Consequently, in the literature, there are different extraction schemes for spin-one form factors [6–8,10].

Recently, the issue of the breakdown of rotational covariance for the one-body component of the J^+ current in a light-front model, has been discussed in the calculation of ρ -meson form factors with constituent quarks [11,12]. In these works, the importance of relativistic effects related to the constituent mass scale and the ρ -meson size was stressed. Such relativistic effects are smaller in a test case of an S -wave deuteron system and the violation of the angular condition is quite small [13].

The judgement of the different prescriptions for obtaining electromagnetic form factors in the noncovariant light-front calculation, which corresponds to use a specific ρ -meson null-plane wave function, could be done in principle by a comparison between the results of the noncovariant and covariant calculations, within the same model. It is our aim in this work to calculate the ρ -meson form factors from the covariant Feynman one-loop triangle diagram for the “+” component of the current in two ways; one corresponds to the covariant calculation and the other to the noncovariant light-front calculation. Below the covariant and the light-front calculations of the matrix elements of J^+ are explained.

The covariant calculation is used to integrate directly the momentum loop in the standart variables in the four-dimensional phase space, integrating in k^0 analytically. In this case, the different prescriptions used to extract form-factors from J^+ give identical results, since the matrix elements satisfy the angular condition.

The noncovariant light-front calculation corresponds to the following procedure; the matrix elements of J^+ are obtained from the one-loop triangle diagram, which is integrated analytically in the “-” component of the loop momentum ($k^- = k^0 - k^3$) in the Breit frame with momentum transfer $q^+ = 0$, and numerically in k^+ and \vec{k}_\perp . The Cauchy integration in k^- takes into account only the propagator pole which corresponds to the forward propagation of the spectator particle in the photon absorption process. This in principle should be correct, as has been discussed in the context of spin-zero bound-state particles [14]. However, we found numerically that the integration in k^- is not exact in the present case of the spin-one composite particle. The resulting matrix elements of J^+ do not satisfy the angular condition and the form factors now depend on the prescription used. Our numerical calculations show the difference between the two ways of performing the integrations. This may sound surprising, but we remind the reader that the k^- integration is in general tricky and even in simple cases taking into account only the forward propagating particle pole may give wrong results. The interested reader can find examples in Ref. [15] with such problems. The integration in k^- , taking into account only the above-mentioned pole is equivalent to the use of a wave function in the null plane to obtain form factors in the Breit frame [14].

To deal with a finite value for the triangle diagram, we introduce a covariant regulator, in the manner proposed in Ref. [16]. This allows the covariant integration to be finite on one side and on the other side the covariant regularization generates a null-plane ρ -meson quark wave function [16].

We compare the covariant and light-front results for the ρ -meson form factors and then we are able to point out the appropriate prescription to evaluate the form factors of the ρ meson with the null-plane wave function. This work is relevant in practical situations, when one is faced with the problem of which light-front prescription to use in the calculation of the ρ -meson electromagnetic form factors. In the present case, the parameters of the model are chosen such that it reproduces to some extent the ρ -meson electromagnetic properties predicted by a realistic QCD inspired model [12].

The plan of the work is the following. In Sec. II the different extraction schemes for obtaining the spin one electromagnetic form factors from J^+ is discussed, and the notation is defined. In Sec. III, the matrix elements of J^+ are obtained from the Feynman triangle diagram, and the covariant and light-front calculations of the form factors are discussed. The covariant regularization is shown to be related to the null-plane wave function. In Sec. IV, the numerical results for the ρ -meson form factors calculated in both frameworks are presented and a summary of the main findings are given.

II. ELECTROMAGNETIC CURRENT AND FORM FACTORS

The general expression of the electromagnetic current of a spin-one particle has the form [5]

$$J_{\alpha\beta}^{\mu} = \left[F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_{\alpha}q_{\beta}}{2m_{\rho}^2} \right] P^{\mu} - F_3(q^2)(q_{\alpha}g_{\beta}^{\mu} - q_{\beta}g_{\alpha}^{\mu}), \quad (1)$$

where m_{ρ} is the ρ -meson mass, q^{μ} is the momentum transfer, and P^{μ} is the sum of the initial and final momentum.

We write the matrix elements of the J^+ component of the current in the instant-form spin basis, given by Eq. (1) in the Breit frame, where $q^{\mu} = (0, q_x, 0, 0)$. The instant-form Cartesian polarization four vectors are given by

$$\epsilon_x^{\mu} = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \quad \epsilon_y^{\mu} = (0, 0, 1, 0), \quad \epsilon_z^{\mu} = (0, 0, 0, 1) \quad (2)$$

for the initial ρ -meson polarization states and

$$\epsilon_x'^{\mu} = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \quad \epsilon_y'^{\mu} = \epsilon_y^{\mu}, \quad \epsilon_z'^{\mu} = \epsilon_z^{\mu}, \quad (3)$$

for the final polarization states; $\eta = -q^2/4m_{\rho}^2$. The ρ -meson four momentum in the Breit frame are, $p_i^{\mu} = (p^0, -q_x/2, 0, 0)$ for the initial state and $p_f^{\mu} = (p^0, q_x/2, 0, 0)$ for the final state; $p^0 = m_{\rho}\sqrt{1+\eta}$.

We use the spherical polarization vectors, and in the instant-form spin basis they are given by

$$\epsilon_{\pm}^{\mu} = -(\pm) \frac{\epsilon_x^{\mu} + (-)\epsilon_y^{\mu}}{\sqrt{2}}$$

and

$$\epsilon_0^{\mu} = \epsilon_z^{\mu}. \quad (4)$$

The “+” component of the electromagnetic current in the instant-form spin basis is written as

$$J^+ = \frac{1}{2} \begin{pmatrix} J_{xx}^+ + J_{yy}^+ & -\sqrt{2}J_{zx}^+ & J_{yy}^+ - J_{xx}^+ \\ \sqrt{2}J_{zx}^+ & 2J_{zz}^+ & -\sqrt{2}J_{zx}^+ \\ J_{yy}^+ - J_{xx}^+ & \sqrt{2}J_{zx}^+ & J_{xx}^+ + J_{yy}^+ \end{pmatrix}, \quad (5)$$

where the spin projections are in the order $m = (+, 0, -)$. The first and second subscripts of the current means the polarizations of the final and initial states, respectively.

The matrix elements of “+” component of the current in the instant-form spin basis, Eq. (5), are related to the matrix elements in the front-form spin basis. For notational convenience we use I^+ to express the matrix elements in the front-form spin basis. The unitary transformation between these spin-basis is the Melosh rotation [9] (Appendix). The general form of the “+” component of the current in the front-form spin basis is written as [8]

$$I^+ = \begin{pmatrix} I_{11}^+ & I_{10}^+ & I_{1-1}^+ \\ -I_{10}^+ & I_{00}^+ & I_{10}^+ \\ I_{1-1}^+ & -I_{10}^+ & I_{11}^+ \end{pmatrix}. \quad (6)$$

We express the matrix elements in the front-form spin basis in terms of matrix elements of the current in the instant-form spin basis by using the results of the Appendix,

$$\begin{aligned} I_{11}^+ &= \frac{J_{xx}^+ + (1+\eta)J_{yy}^+ - \eta J_{zz}^+ + 2\sqrt{\eta}J_{zx}^+}{2(1+\eta)}, \\ I_{10}^+ &= \frac{\sqrt{2}\eta J_{xx}^+ + \sqrt{2}\eta J_{zz}^+ + \sqrt{2}(\eta-1)J_{zx}^+}{2(1+\eta)}, \\ I_{1-1}^+ &= \frac{-J_{xx}^+ + (1+\eta)J_{yy}^+ + \eta J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{2(1+\eta)}, \\ I_{00}^+ &= \frac{-\eta J_{xx}^+ + J_{zz}^+ + 2\sqrt{\eta}J_{zx}^+}{(1+\eta)}. \end{aligned} \quad (7)$$

The angular condition $J_{zz}^+ = J_{yy}^+$ can be written in the front-form spin basis (see Appendix), giving its usual form [6,9]

$$\Delta(q^2) = (1+2\eta)I_{11}^+ + I_{1-1}^+ - \sqrt{8}\eta I_{10}^+ - I_{00}^+ = 0. \quad (8)$$

In general, the light-front impulse approximation to the electromagnetic current does not satisfy such a condition [6,11–13], this fact led to different extraction schemes of the form factors from the matrix elements of the current [6–8,10]. Let us review the prescriptions existing in the literature for calculating the form factors for the spin-one particle, from the matrix elements $I_{m'm}^+$

The charge G_0 , magnetic G_1 , and quadrupole G_2 form factors are obtained from linear combinations of the covari-

ant form factors F_1 , F_2 , and F_3 [8] (see Appendix). Below, we give the different prescriptions for obtaining the form factors, which are also written in terms of matrix elements in the Cartesian instant-form spin basis. Such a spin basis is used because it facilitates the algebraic manipulations of the covariant amplitude for the photon absorption process, and it is completely equivalent to the front-form spin basis.

In Ref. [6], the elimination of the matrix element I_{00}^+ gives the following prescription to calculate the form factors:

$$\begin{aligned} G_0^{\text{GK}} &= \frac{1}{3}[(3-2\eta)I_{11}^+ + 2\sqrt{2}\eta I_{10}^+ + I_{1-1}^+] \\ &= \frac{1}{3}[J_{xx}^+ + 2J_{yy}^+ - \eta J_{yy}^+ + \eta J_{zz}^+], \\ G_1^{\text{GK}} &= 2\left[I_{11}^+ - \frac{1}{\sqrt{2}\eta}I_{10}^+\right] = J_{yy}^+ - J_{zz}^+ + \frac{J_{zx}^+}{\sqrt{\eta}} \\ G_2^{\text{GK}} &= \frac{2\sqrt{2}}{3}[-\eta I_{11}^+ + \sqrt{2}\eta I_{10}^+ - I_{1-1}^+] \\ &= \frac{\sqrt{2}}{3}[J_{xx}^+ + J_{yy}^+(-1-\eta) + \eta J_{zz}^+]. \end{aligned} \quad (9)$$

In Ref. [7], they have obtained

$$\begin{aligned} G_0^{\text{CCKP}} &= \frac{1}{3(1+\eta)}\left[\left(\frac{3}{2}-\eta\right)(I_{11}^+ + I_{00}^+) + 5\sqrt{2}\eta I_{10}^+ \right. \\ &\quad \left. + (2\eta - \frac{1}{2})I_{1-1}^+\right] \\ &= \frac{1}{6}[2J_{xx}^+ + J_{yy}^+ + 3J_{zz}^+] \end{aligned}$$

$$G_1^{\text{CCKP}} = \frac{1}{(1+\eta)}\left[I_{11}^+ + I_{00}^+ - I_{1-1}^+ - \frac{2(1-\eta)}{\sqrt{2}\eta}I_{10}^+\right] = \frac{J_{zx}^+}{\sqrt{\eta}},$$

$$\begin{aligned} G_2^{\text{CCKP}} &= \frac{\sqrt{2}}{3(1+\eta)}[-\eta I_{11}^+ - \eta I_{00}^+ + 2\sqrt{2}\eta I_{10}^+ - (\eta+2)I_{1-1}^+] \\ &= \frac{\sqrt{2}}{3}[J_{xx}^+ - J_{yy}^+]. \end{aligned} \quad (10)$$

The prescription of Brodsky and Hiller [10] to obtain the form factors is

$$\begin{aligned} G_0^{\text{BH}} &= \frac{1}{3(1+\eta)}[(3-2\eta)I_{00}^+ + 8\sqrt{2}\eta I_{10}^+ + 2(2\eta-1)I_{1-1}^+] \\ &= \frac{1}{3(1+2\eta)}[J_{xx}^+(1+2\eta) + J_{yy}^+(2\eta-1) \\ &\quad + J_{zz}^+(3+2\eta)], \end{aligned}$$

$$\begin{aligned} G_1^{\text{BH}} &= \frac{2}{(1+2\eta)}\left[I_{00}^+ - I_{1-1}^+ + \frac{(2\eta-1)}{\sqrt{2}\eta}I_{10}^+\right] \\ &= \frac{1}{(1+2\eta)}\left[\frac{J_{zx}^+}{\sqrt{\eta}}(1+2\eta) - J_{yy}^+ + J_{zz}^+\right], \end{aligned}$$

$$\begin{aligned} G_2^{\text{BH}} &= \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2}\eta I_{10}^+ - \eta I_{00}^+ - (\eta+1)I_{1-1}^+] \\ &= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^+(1+2\eta) - J_{yy}^+(1+\eta) - \eta J_{zz}^+]. \end{aligned} \quad (11)$$

According to Ref. [8] the electromagnetic form factors are obtained from the matrix elements J_{xx}^+ , J_{zz}^+ , and J_{yy}^+ :

$$\begin{aligned} G_0^{\text{FFS}} &= \frac{1}{3(1+\eta)}[(2\eta+3)I_{11}^+ + 2\sqrt{2}\eta I_{10}^+ - \eta I_{00}^+ \\ &\quad + (2\eta+1)I_{1-1}^+] = \frac{1}{3}[J_{xx}^+ + 2J_{yy}^+], \end{aligned}$$

$$G_1^{\text{FFS}} = G_1^{\text{CCKP}},$$

$$G_2^{\text{FFS}} = G_2^{\text{CCKP}}. \quad (12)$$

The low-energy ρ -meson observables, mean square radius, magnetic moment, and quadrupole moment, are given by [7]

$$\begin{aligned} \langle r^2 \rangle &= \lim_{q^2 \rightarrow 0} \frac{6[G_0(q^2) - 1]}{q^2}, \quad \mu = \lim_{q^2 \rightarrow 0} G_1(q^2), \\ Q_2 &= \lim_{q^2 \rightarrow 0} 3\sqrt{2} \frac{G_2(q^2)}{q^2}, \end{aligned} \quad (13)$$

respectively.

III. COVARIANT AND LIGHT-FRONT CURRENTS

The ρ -meson electromagnetic form factors are obtained in the impulse approximation. It includes only a one-body current operator, and the amplitude for the photon absorption is given by the Feynman triangle diagram, with the photon leg attached to one of the quarks. We compute only the ‘‘good’’ component of the current (J^+), which is diagonal in the null-plane Fock state. The pair creation diagram is in principle suppressed for J^+ [17].

The spinor structure of the $\rho - q\bar{q}$ vertex is written in the following form:

$$\Gamma^\mu(k, k') = \gamma^\mu - \frac{m_\rho}{2} \frac{k^\mu + k'^\mu}{p^r k_r + m_\rho m - i\epsilon}, \quad (14)$$

where the ρ meson is on-mass-shell, and its four momentum is $p^\mu = k^\mu - k'^\mu$, the quark momenta are given by k^μ and k'^μ , and their mass by m . Equation (14) reduces to the vertex given in Ref. [18] for an on-mass-shell quark. This vertex corresponds to a relative S -state quark-antiquark wave function [5,18]. Above, we wrote the spinor structure of the ver-

tex. The complete null-plane wave function comes from the regularization factor and the denominator of the propagator, as will be clear in the following.

The impulse approximation to J^+ , is given by the Feynman triangle diagram, and we assume the constituent quark as a Dirac pointlike particle,

$$J_{ji}^+ = \iota \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\epsilon_j'^\alpha \Gamma_\alpha(k, k-p_f)(\not{k} - \not{p}_f + m) \gamma^+ (\not{k} - \not{p}_i + m) \epsilon_i^\beta \Gamma_\beta(k, k-p_i)(\not{k} + m)]}{[(k-p_i)^2 - m^2 + \iota\epsilon][k^2 - m^2 + \iota\epsilon][(k-p_f)^2 - m^2 + \iota\epsilon]} \Lambda(k, p_f) \Lambda(k, p_i), \quad (15)$$

where J_{ji}^+ is written in the Cartesian instant-form spin basis, and $\epsilon_j'^\alpha$ is the final polarization four vector [Eq. (3)], and ϵ_i^β is the initial four-vector polarization [Eq. (2)]; the subscripts i and j stand for x , y , and z .

The regularization function,

$$\Lambda(k, p) = \frac{N}{[(k-p)^2 - m_R^2 + \iota\epsilon]^2}, \quad (16)$$

was chosen to turn Eq. (15) finite. The special form of the regulator allows us to identify a null-plane wave function similar to the one proposed for the pion in Ref. [16]. They used a monopole form factor instead of a dipole. The normalization factor N is found by imposing $G_0(0) = 1$.

The covariant calculation of the form factors is performed with Eq. (15), which is analytically integrated in the k^0 complex plane. The integration over \vec{k} is done numerically. The angular condition is satisfied exactly by the covariant calculation, as it should be. This does not remain true in the light-front calculation. Also for $q^2 = 0$, $J_{xx}^+(0) = J_{yy}^+(0) = J_{zz}^+(0)$. The matrix elements of the current satisfy current conservation $q^\mu J_\mu(q^2) = 0$, as we verified explicitly.

The light-front calculation, allows us to integrate analytically in the complex plane of the k^- variable [14]. In principle, the pair diagrams are not present with $q^+ = 0$. The pole which contributes to the integration is

$$k^- = \frac{k_\perp^2 + m^2 - \iota\epsilon}{k^+}, \quad (17)$$

for $p^+ > k^+ > 0$, where $p^+ = p^0$ is the energy of the ρ meson in the Breit frame. This pole belongs to the lower complex semiplane, where no other pole is present. Equation (17), is the on-mass-shell condition for the spectator quark, in the process of photon absorption. However, the J^+ matrix elements do not satisfy the angular condition. Thus, different prescriptions of calculating the form factors will give different results. The analytical integration in k^- for the ρ meson including only the spectator particle pole is not exact, as our numerical calculations have shown. In the next section we show our numerical results.

The null-plane wave function of the ρ meson appears after the substitution of the on-mass-shell condition, Eq. (17), in the propagator of the quark that absorbs the photon and in the corresponding regulator,

$$\frac{1}{[(k-p)^2 - m^2 + \iota\epsilon][(k-p)^2 - m_R^2 + \iota\epsilon]^2} = \frac{1}{(1-x)} \frac{1}{(1-x)^2 (m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2}, \quad (18)$$

where $x = k^+/p^+$. The free quark-antiquark mass squared is given by

$$M_0^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m^2}{1-x} - p_\perp^2. \quad (19)$$

The function M_R^2 is given by

$$M_R^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_R^2}{1-x} - p_\perp^2. \quad (20)$$

The null-plane wave function is obtained from the combination of denominators in Eq. (18) with the spinor structure of the vertex, Eq. (14). We leave out the phase-space factor $1/(1-x)$. The resulting expression is evaluated in the center of mass system,

$$\Phi_i(x, \vec{k}_\perp) = \frac{N}{(1-x)^2 (m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \times \vec{\epsilon}_i \cdot \left[\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2} + m} \right], \quad (21)$$

the polarization state is given by $\vec{\epsilon}_i$. The wave function corresponds to an S -wave state [18].

TABLE I. Results for the low-energy electromagnetic ρ -meson observables, for the covariant (COV) and light-front calculations. The light-front extraction schemes to obtain the form factors are given by Refs. [6] (GK), [7] (CCKP), [8] (FFS), and [10] (BH). In the last column, the results of Ref. [12] are given.

Model	COV	GK	CCKP	BH	FFS	Ref. [12]
$\langle r^2 \rangle$ (fm ²)	0.37	0.37	0.38	0.40	0.39	0.35
μ	2.14	2.19	2.17	2.15	2.48	2.26
Q_2 (fm ²)	0.052	0.050	0.051	0.051	0.058	0.024

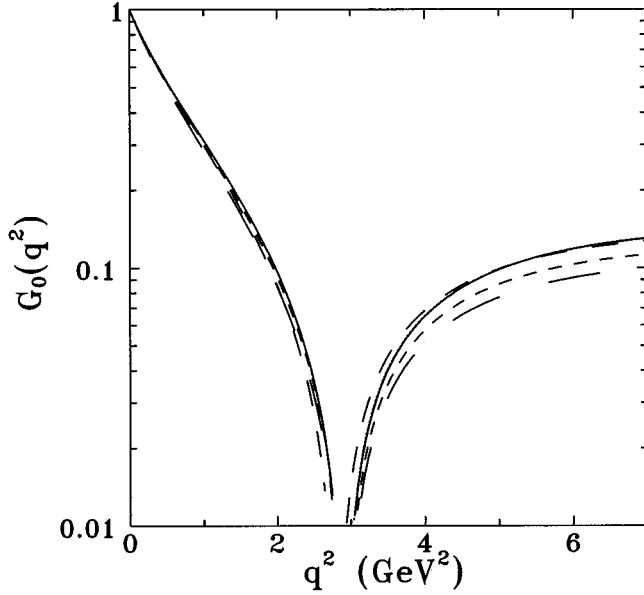


FIG. 1. Charge form factor $G_0(q^2)$ for the ρ meson as a function of q^2 , calculated with covariant and light-front schemes. The solid line is the covariant calculation. Results for the different light-front extraction schemes, Ref. [6] (GK) (dotted line) (it is not possible to distinguish from the covariant calculation), Ref. [7] (CCKP) (short-dashed), Ref. [8] (FFS) (dashed), and Ref. [10] (BH) (long-dashed).

IV. DISCUSSION

The constituent quark model for the ρ -meson null-plane wave function has two parameters, the constituent quark mass m and the regulator mass m_R . The ρ -meson mass is 0.77 GeV. In this case, the composite wave function corresponds to a bound state, which imposes a lower bound for the regulator and constituent quark masses, such that

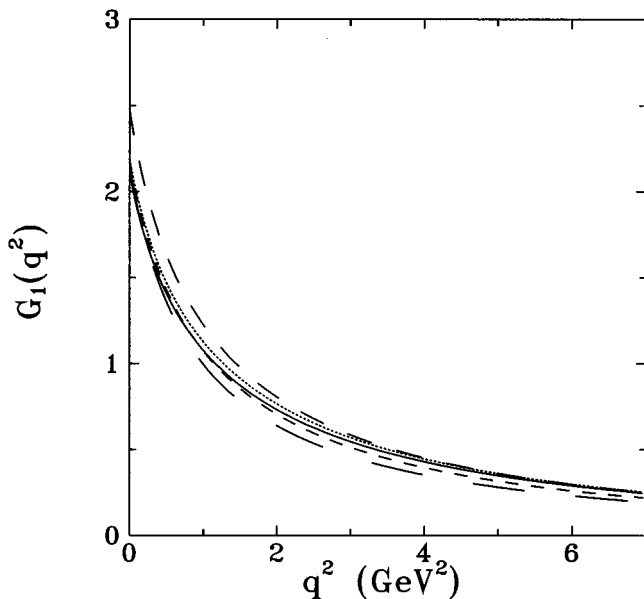


FIG. 2. Magnetic form factor $G_1(q^2)$ for the ρ meson as a function of q^2 , calculated with covariant and light-front schemes. The curves are labeled according to Fig. 1.

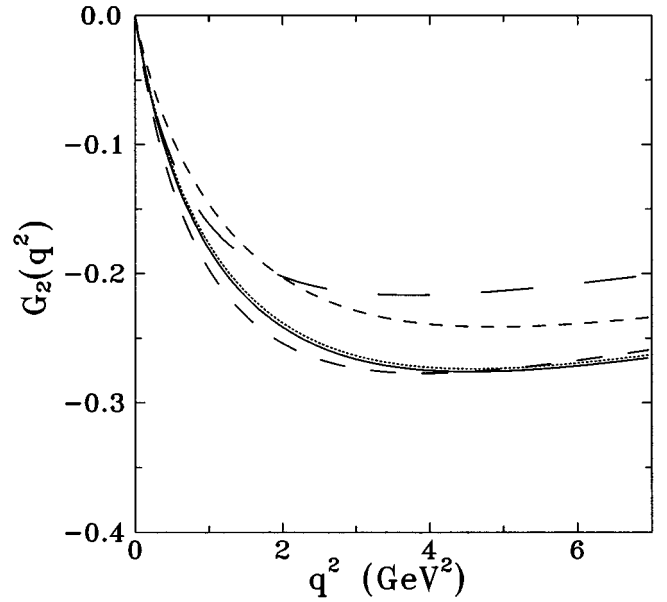


FIG. 3. Quadrupole form factor $G_2(q^2)$ for the ρ meson as a function of q^2 , calculated with covariant and light-front schemes. The curves are labeled according to Fig. 1.

$$m > \frac{m_\rho}{2}, \quad m_R + m > m_\rho.$$

The scale of the model is obtained by adjusting the parameters to get a mean square radius of about 0.35 fm^2 and $G_2(q^2 \sim 5 \text{ GeV}^2) \sim -0.25$, as calculated in Ref. [12], with pointlike constituent quarks. They used a wave function in the null plane which is dominated by one-gluon exchange at short distances and linear confinement at large distances.

In the nonrelativistic limit, the quadrupole form factor vanishes for an S -state wave function. The nonzero values of G_2 are a consequence of the relativistic nature of the model, and for this reason we consider it in the parameter fit. We used the covariant calculation for the form factors to get the parameters $m = 0.43 \text{ GeV}$ and $m_R = 1.8 \text{ GeV}$.

The low-energy electromagnetic parameters are calculated using the different light-front prescriptions with the light-front calculation of J^+ matrix elements and are compared with the covariant results. In Table I, we show the values of $\langle r^2 \rangle$, μ , and Q_2 . The mean square radius calculated in the light-front scheme has values at most 10% higher than the covariant result of 0.37 fm^2 . The magnetic moment obtained in the covariant calculation is 2.19, which can be compared with the nonrelativistic value of 2. In Ref. [12] they obtained 2.26. The light-front calculations for the magnetic moment give values with a spread of 15% above the covariant result. The quadrupole moment in the covariant calculation is 0.052 fm^2 , somewhat higher than the value quoted in Ref. [12]. The light-front calculations are within 10 to 15% of the covariant result for the low-energy parameters.

In Fig. 1, we observe that the charge form factor G_0 is sensitive to the different light-front prescriptions. The calculations show a zero placed around 3 GeV^2 consistent with Ref. [12]. We found an increasing discrepancy among the several prescriptions and the covariant results, for momentum transfers above the zero crossing. The Grach-

Kondratyuk (GK) prescription gives results in agreement with the covariant calculation, while the Brodsky-Hiller (BH) results are about 30% below at higher q^2 .

The differences between the various light-front calculations for the magnetic form factor and the covariant results are not so pronounced, as shown in Fig. 2. At small momentum transfers the Frankfurt-Frederico-Strikman (FFS) prescription has a value about 15% higher than the covariant result, in agreement with the results of Table I. In the momentum range considered the GK prescription is consistent with the covariant calculation.

The relativistic effects in the model are the origin of G_2 , and thus it is more sensitive to the difference between the light-front prescriptions. In Fig. 3, the values of G_2 calculated in the light front with prescriptions given by Chung, Coester, Keister, and Polizou (CCKP) and BH are 20% lower than the covariant result. The calculations with GK combination of the currents, present the best consistency with the covariant results among the four prescriptions tested.

We conclude that in the scale of the ρ -meson bound state, adjusted using a parametrization which reproduces the size and quadrupole form factor, of an effective constituent quark model which embodies gluon exchange and confinement, the prescription for a light-front calculation of the form factors as given by the work of Grach and Kondratyuk [6] shows consistence with the covariant results. We have used a vertex for the ρ meson, that was amenable to covariant integration and reproduced to some extent the size properties of a physically inspired null-plane wave function.

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APPENDIX

The Melosh rotation for the spin-one particle is given by

$$R_M = \begin{pmatrix} [(1 + \cos\theta)/2] & -(\sin\theta/\sqrt{2}) & [(1 - \cos\theta)/2] \\ (\sin\theta/\sqrt{2}) & \cos\theta & -(\sin\theta/\sqrt{2}) \\ [(1 - \cos\theta)/2] & (\sin\theta/\sqrt{2}) & [(1 + \cos\theta)/2] \end{pmatrix}, \quad (\text{A1})$$

where $\cos\theta = (\sqrt{1 + \eta})^{-1}$ and $\sin\theta = -\sqrt{\eta/(1 + \eta)}$.

The matrix elements of the current in the instant-form spin basis (J^+), Eq. (5), and in the front-form spin basis (I^+), Eq. (6), are related by the Melosh rotation

$$R_M^\dagger I^+ R_M = J^+. \quad (\text{A2})$$

The instant-form matrix elements are expressed in terms of the light-front matrix elements using the above equation as [8],

$$\begin{aligned} J_{xx}^+ &= \frac{1}{1 + \eta} [I_{11}^+ + 2\sqrt{2}\eta I_{10}^+ - \eta I_{00}^+ - I_{1-1}^+], \\ J_{zx}^+ &= \frac{\sqrt{2}}{1 + \eta} \left[\frac{\sqrt{2}\eta}{2} I_{11}^+ + (\eta - 1) I_{10}^+ + \sqrt{\frac{\eta}{2}} I_{00}^+ - \frac{\sqrt{2}\eta}{2} I_{1-1}^+ \right], \\ J_{yy}^+ &= I_{11}^+ + I_{1-1}^+, \\ J_{zz}^+ &= \frac{1}{1 + \eta} [-\eta I_{11}^+ + 2\sqrt{2}\eta I_{10}^+ + I_{00}^+ + \eta I_{1-1}^+]. \end{aligned} \quad (\text{A3})$$

The relations between the form factors G_0 , G_1 , and G_2 and the covariant form factors F_1 , F_2 , and F_3 , are given by

$$\begin{aligned} G_0 &= -\frac{2}{3} m_\rho \sqrt{1 + \eta} [3F_1 + 2\eta(F_1 + F_3 + (1 + \eta)F_2)], \\ G_1 &= 2m_\rho \sqrt{1 + \eta} F_3, \\ G_2 &= -4\frac{\sqrt{2}}{3} m_\rho \eta \sqrt{1 + \eta} [F_1 + (1 + \eta)F_2 + F_3]. \end{aligned} \quad (\text{A4})$$

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