$E2/M1$ ratio of the $\Delta\leftrightarrow\gamma N$ transition within the chiral constituent quark model

Peng-Nian Shen,[1,2,3] Yu-Bin Dong,[1,2] Zong-Ye Zhang,[1] You-Wen Yu,[1] and T.-S. H. Lee[4]
[1]*Institute of High Energy Physics, Academia Sinica, Beijing 100039, China*
[2]*China Center of Advanced Science and Technology (World Laboratory), Beijing 100080, China*
[3]*Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China*
[4]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843*
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The chiral constituent quark model is applied to predict the magnetic $M1$ and electric $E2$ amplitudes of the $\Delta\leftrightarrow\gamma N$ transition. It is found that the one-meson-exchange quark-quark potential due to the chiral fields enhances the $E2/M1$ ratio by a factor of about 2. The predicted M_{1^+} amplitude and the $E2/M1$ ratio $\simeq -1.0\%$ are within the range determined from a recent analysis of the data of pion photoproduction. [S0556-2813(97)03204-4]

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I. INTRODUCTION

An approximate way to realize the nonperturbative physics governed by the breaking of chiral symmetry in QCD is to assume that in the low and intermediate energy regions a baryon consists of a constituent-quark sector and a meson sector. This notion, first discussed by Manohar and Georgi [1], has been the basis for developing various chiral constituent quark models for baryon-baryon interactions [2–4]. The same idea has been used recently by Glozman and Riska [5] to study the baryon spectroscopy. In this work, we investigate the consequence of such an approach in determining the $\Delta\leftrightarrow\gamma N$ transition which has been the focus of recent investigations [7–14] of pion photoproduction on the nucleon. The central issue is the ratio between the electric $E2$ and magnetic $M1$ amplitudes of the $\gamma N\leftrightarrow\Delta$ transition, which measures the deformation of the Δ state.

The $\gamma N\leftrightarrow\Delta$ transition has been calculated within the constituent quark model [15–19]. A common feature of these investigations is that the predicted $M1$ amplitude is much smaller than the value listed by the Particle Data Group [21]. This was understood in Ref. [14] that the discrepancy is due to the nonresonant meson-exchange mechanism which cannot be separated from the direct photoexcitation of the nucleon to the Δ state in a purely phenomenological amplitude analysis of the pion photoproduction data. In this work we will further verify this result of Ref. [14] from the point of view of the chiral constituent quark model. Our focus will be on how the $E2/M1$ ratio is influenced by the quark-quark interaction due to the chiral fields.

In Sec. II, we first present a chiral constituent quark model for describing the masses and wave functions of the nucleon and the Δ states. The formula for calculating the $\Delta\rightarrow\gamma N$ transition are then given explicitly. To compare our predictions with the data, we establish in Sec. III the relationship between the constructed chiral constituent quark model and the effective Hamiltonian formulation [14] of pion photoproduction. Results and discussions are given in Sec. IV.

II. THE $\Delta\leftrightarrow\gamma N$ TRANSITION WITHIN THE CHIRAL CONSTITUENT QUARK MODEL

Following the approach of Ref. [4], we assume that the chiral constituent quark model can be defined by the following Hamiltonian:

$$H=H_0'+B_0+\sum_i\left(m+\frac{p_i^2}{2m}\right)+\sum_{i>j}(V_{ij}^{\rm OGE}+V_{ij}^{\rm conf})+\sum_i\{[h_{\pi q,q}(i)+h_{\sigma q,q}(i)+h_{\gamma q,q}(i)]+(\mathrm{H.c.})\},\qquad(1)$$

where H_0' includes the free Hamiltonians for mesons and photons and the associated photon-meson couplings, m is the constituent quark mass, B_0 is the zero point energy, $V_{ij}^{\rm OGE}$ and $V_{ij}^{\rm conf}$ are, respectively, the one-gluon-exchange potential and the confinement potential. The meson-quark vertex interactions $h_{\pi q,q}$ and $h_{\sigma q,q}$ are due to the linear realization [4] of the spontaneously breaking of chiral symmetry. The photon-quark coupling vertex is denoted as $h_{\gamma q,q}$. In Eq. (1), (H.c.) denotes taking the Hermitian conjugate of the term on its left. To investigate the πN and γN reactions, it is necessary to cast the above Hamiltonian in terms of hadronic degrees of freedom. This is obviously a difficult task. In a phenomenological approach of Refs. [3,4], one simply assumes that Eq. (1) can lead to the following ''effective'' Hamiltonian:

$$H=H_B+H_0'+\sum_{B,B'}[(h_{\pi B,B'}+h_{\gamma B,B'})+(\mathrm{H.c.})],\qquad(2)$$

where $B,B'=N,\Delta$ are eigenstates of a one-baryon Hamiltonian H_B. The vertex interactions in Eq. (2) are calculated from the quark operators $h_{\pi q,q}$ and $h_{\gamma q,q}$ of Eq. (1):

$$f_{\pi B,B'}=\langle B|\sum_i f_{\pi q,q}(i)|B'\rangle,\qquad(3)$$

$$f_{\gamma B,B'}=\langle B|\sum_i f_{\gamma q,q}(i)|B'\rangle.\qquad(4)$$

To evaluate the above matrix elements, we need to define H_B to generate the wave functions for N and Δ. We follow Ref. [4] to further assume that H_B includes a one-meson-exchange interaction V_{ij}^{chiral} due to the chiral coupling terms $h_{\pi q,q}$ and $h_{\sigma q,q}$ in Eq. (1). It has the following form:

$$H_B=B_0+\sum_i\left(m+\frac{p_i^2}{2m}\right)+\sum_{i>j}\left(V_{ij}^{\text{OGE}}+V_{ij}^{\text{conf}}+V_{ij}^{\text{chiral}}\right), \quad (5)$$

where

$$V_{ij}^{\text{chiral}}=V_{ij}^{\pi}+V_{ij}^{\sigma}, \quad (6)$$

with

$$V_{ij}^{\pi}=\frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m^2}\frac{\Lambda^2}{\Lambda^2-m_\pi^2}m_\pi(\vec{\tau}_i\cdot\vec{\tau}_j)\times\left(\left[Y(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)+\left[Y_2(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}Y_2(\Lambda r_{ij})\right]S_{ij}\right), \quad (7)$$

and

$$V_{ij}^{\sigma}=-\frac{g_{ch}^2}{4\pi}\frac{\Lambda^2}{\Lambda^2-m_\sigma^2}m_\sigma\left[Y(m_\sigma r_{ij})-\frac{\Lambda}{m_\sigma}Y(\Lambda r_{ij})\right]. \quad (8)$$

In the above equations, Λ is the cutoff parameter for regularizing the potential at short distances and we have introduced the following notations

$$S_{ij}=3(\vec{\sigma}_i\cdot\hat{r}_{ij})(\vec{\sigma}_j\cdot\hat{r}_{ij})-\vec{\sigma}_i\cdot\vec{\sigma}_j, \quad (9)$$

$$Y(x)=\frac{e^{-x}}{x}, \quad (10)$$

$$Y_2(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x). \quad (11)$$

The one-gluon-exchange interaction in Eq. (5) takes the following familiar form

$$V_{ij}^{\text{OGE}}=(\lambda_i^c\cdot\lambda_j^c)\frac{\alpha_s}{4}\times\left[\frac{1}{r_{ij}}-\frac{\pi}{m^2}\left(1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\delta(\vec{r}_{ij})-\frac{1}{4m^2r_{ij}^3}S_{ij}\right], \quad (12)$$

where λ_i^c is the color SU(3) matrix. We note here that within quantum chromodynamics a rigorous expression of the one-gluon-exchange interaction should also include a spin-orbital component. It was demonstrated in Ref. [6] that this force is very disruptive in the constituent quark model. It can lower the energies of the P-wave baryon states by about 500 MeV and is completely unacceptable to the data. While some possible reasons were speculated in Ref. [6] to explain how the spin-orbital component of the one-gluon exchange is cancelled by other dynamics within quantum chromodynamics, it remains to be an unsettled theoretical issue. For our present purpose, we follow the common practice and also neglect the spin-orbital component in the chiral constituent quark model. In other words, Eq. (12) should be considered as a phenomenological residual quark-quark interaction and is not related to the gluon-exchange dynamics in a simple way. Therefore, the hyperfine coupling constant α_s of Eq. (12) can be treated as an adjustable parameter in our calculation.

To examine the model dependence, we consider two possible confinement potentials

$$V_{ij}^{\text{conf}}(1)=-a_c(\lambda_i^c\cdot\lambda_j^c)r_{ij}, \quad (13)$$

$$V_{ij}^{\text{conf}}(2)=-a_c(\lambda_i^c\cdot\lambda_j^c)\text{erf}(\mu r_{ij}), \quad (14)$$

where erf is the error function. Both confinement potentials were used [4] in the investigation of NN potential.

Our first task is to calculate the energies and wave functions for the N and Δ states. This is done by exactly diagonalizing the one-baryon Hamiltonian Eq. (5) in a chosen model-space spanned by the harmonic-oscillator basis functions. The resulting baryon wave functions in the rest frame are then written as

$$\Phi_B(\vec{\rho},\vec{\lambda})=\sum_\alpha C_{\alpha,B}\phi^{(\alpha)}(\vec{\rho},\vec{\lambda},b), \quad (15)$$

where $B=N,\Delta$, $\phi^{(\alpha)}$ is the harmonic oscillator wave function with a length parameter b and the quantum numbers

$$\alpha=[\Sigma(SI),NL^P]_J. \quad (16)$$

Here Σ denotes the spin-flavor symmetry characterized by the total spin S and total isospin I, N is the energy quantum number, L the total orbital angular momentum, P the parity, and J the total angular momentum. The internal coordinates in Eq. (15) of the three-quark system are defined by

$$\vec{\rho}=\sqrt{\frac{1}{2}}(\vec{r}_1-\vec{r}_2), \quad (17)$$

$$\vec{\lambda}=\sqrt{\frac{1}{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3). \quad (18)$$

The corresponding center of mass coordinate is defined by

$$\vec{R}=\sqrt{\frac{1}{3}}(\vec{r}_1+\vec{r}_2+\vec{r}_3). \quad (19)$$

By using Eqs. (17)–(19), we get the following useful relation for the later calculations of the $\Delta\leftrightarrow\gamma N$ transition

$$\vec{r}_3=-\sqrt{\frac{2}{3}}\vec{\lambda}+\sqrt{\frac{1}{3}}\vec{R}. \quad (20)$$

With the model Hamiltonian Eq. (2), the $\gamma N\rightarrow\Delta$ transition is defined by $f_{\gamma N,\Delta}$. To evaluate $f_{\gamma N,\Delta}$, Eq. (4), we follow Refs. [15,16] to deduce a one-body current operator $f_{\gamma q,q}$ from taking the nonrelativistic limit of the usual $\bar{\psi}\gamma_\mu\psi A^\mu$ coupling. As discussed in Ref. [17], this is valid to a large extent since an explicit calculation of relativistic corrections did not lead to a drastic change of the nonrelativistic results of Ref. [15]. Explicitly, we have (with the normalizations of Goldberger and Watson [20])

$$\langle \vec{p}_i'|f_{\gamma q,q}|\vec{p}_i\rangle = \frac{1}{\sqrt{2k}}\frac{1}{\sqrt{(2\pi)^3}}\frac{e_i}{2m} \times(\vec{p}_i+\vec{p}_i'-i\vec{\sigma}_i\times\vec{k})\cdot\vec{\epsilon}^*_\lambda(\vec{k})\,\delta(\vec{p}_i'-\vec{p}_i-\vec{k}), \quad (21)$$

where $\epsilon_\lambda(\vec{k})$ is the polarization vector of the photon with momentum $\vec{k}$. It is more convenient to carry out calculations in coordinate space. The corresponding operator can be written as

$$\langle \vec{r}_i'|f_{\gamma q,q}|\vec{r}_i\rangle = \frac{1}{\sqrt{2k}}\frac{1}{\sqrt{(2\pi)^3}}\frac{e_i}{2m}[-i\overleftarrow{\nabla}e^{i\vec{k}\cdot\vec{r}_i}\delta(\vec{r}_i-\vec{r}_i') + i\,\delta(\vec{r}_i-\vec{r}_i')e^{i\vec{k}\cdot\vec{r}_i}\vec{\nabla}-i\vec{\sigma}_i\times\vec{k}\,\delta(\vec{r}_i-\vec{r}_i') \times e^{i\vec{k}\cdot\vec{r}_i}]\cdot\vec{\epsilon}^*_\lambda(\vec{k}). \quad (22)$$

Taking into account the symmetry properties of the N and Δ wave functions and using Eq. (22), Eq. (4) for the $\gamma N\rightarrow\Delta$ transition takes the following expression:

$$\langle\Psi_{\vec{p}_N,N}|f_{\gamma N,\Delta}|\Psi_{\vec{p}_\Delta,\Delta}\rangle = 3\int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3' d\vec{r}_3[\Psi^*_{\vec{p}_N,N}(\vec{r}_1,\vec{r}_2,\vec{r}_3') \times\langle\vec{r}_3'|f_{\gamma q,q}|\vec{r}_3\rangle\Psi_{\vec{p}_\Delta,\Delta}(\vec{r}_1,\vec{r}_2,\vec{r}_3)], \quad (23)$$

where $\vec{p}_N$ and $\vec{p}_\Delta$ are, respectively, the momenta of the nucleon and the Δ. Each total wave function in Eq. (23) is a product of a plane-wave for the center of mass motion and the internal wave function defined by Eq. (15)

$$\Psi^*_{\vec{p}_N,N}(\vec{r}_1,\vec{r}_2,\vec{r}_3) = \frac{1}{(2\pi)^{3/2}}e^{i\vec{p}_N\cdot\vec{R}}\Phi^*_N(\vec{\rho},\vec{\lambda}),$$

$$\Psi_{\vec{p}_\Delta,\Delta}(\vec{r}_1,\vec{r}_2,\vec{r}_3) = \frac{1}{(2\pi)^{3/2}}e^{-i\vec{p}_\Delta\cdot\vec{R}}\Phi_\Delta(\vec{\rho},\vec{\lambda}). \quad (24)$$

Substituting Eqs. (22) and (24) into Eq. (23) and using the properties Eq. (19), we obtain in the Δ rest frame ($\vec{p}_\Delta=0$ and $\vec{p}_N=-\vec{k}$)

$$\langle\Psi_{\vec{p}_N,N}|f_{\gamma N,\Delta}|\Psi_{\vec{p}_\Delta,\Delta}\rangle = \delta(\vec{p}_N+\vec{k}-\vec{p}_\Delta) \times\sum_{i=1}^{4}[\vec{\epsilon}^*_{\lambda_\gamma}(\vec{k})\cdot\vec{O}^i_{N,\Delta}(\vec{k})], \quad (25)$$

where

$$\vec{O}^i_{N,\Delta}(\vec{k}) = \int d\vec{\lambda}\,d\vec{\rho}\,\Phi^*_N(\vec{\rho},\vec{\lambda})\hat{O}_i(\vec{\lambda},\vec{k})\Phi_\Delta(\vec{\rho},\vec{\lambda}), \quad (26)$$

with

$$\hat{O}_1(\vec{\lambda},\vec{k}) = C\sqrt{\frac{2}{3}}\overleftarrow{\nabla}_\lambda e^{-i\sqrt{2/3}\vec{k}\cdot\vec{\lambda}}, \quad (27)$$

$$\hat{O}_2(\vec{\lambda},\vec{k}) = -C\sqrt{\frac{2}{3}}e^{-i\sqrt{2/3}\vec{k}\cdot\vec{\lambda}}\vec{\nabla}_\lambda, \quad (28)$$

$$\hat{O}_3(\vec{\lambda},\vec{k}) = C(\vec{k}\times\vec{\sigma})e^{-i\sqrt{2/3}\vec{k}\cdot\vec{\lambda}}, \quad (29)$$

$$\hat{O}_4(\vec{\lambda},\vec{k}) = C\frac{i}{3}\vec{k}e^{-i\sqrt{2/3}\vec{k}\cdot\vec{\lambda}}. \quad (30)$$

Here we have defined an overall constant

$$C = i\left(\frac{3}{2\pi}\right)^{3/2}\frac{3}{\sqrt{2k}}\frac{e_3}{2m}. \quad (31)$$

Recalling the spin-isospin quantum numbers of N and Δ and using the standard angular-momentum algebra, we can write Eq. (26) as

$$\vec{O}^i_{N,\Delta}(\vec{k}) = \vec{O}^i_{J_N M_N, J_\Delta M_\Delta}(\vec{k}) = \sum_{jm_j}\sum_L \vec{y}^{jm}_{L1}(\hat{k})\langle J_N M_N|J_\Delta j M_\Delta m_j\rangle \times\langle\Phi_{J_N}||(\hat{O}_i)_{j,L}||\Phi_{J_\Delta}\rangle, \quad (32)$$

where

$$\vec{y}^{jm_j}_{L1}(\hat{k}) = \sum_{M_L m}\langle jm_j|L1M_Lm\rangle\vec{\epsilon}_m Y_{LM_L}(\hat{k}), \quad (33)$$

with

$$\vec{\epsilon}_{+1} = \frac{-1}{\sqrt{2}}(\hat{x}+i\hat{y}),$$

$$\vec{\epsilon}_{-1} = \frac{+1}{\sqrt{2}}(\hat{x}-i\hat{y}),$$

$$\vec{\epsilon}_0 = \hat{z}.$$

From the above definitions Eqs. (26)–(33), it is straightforward to derive formulas for calculating the reduced matrix elements $\langle\Phi_{J_N}||(\hat{O}_i)_{j,L}||\Phi_{J_\Delta}\rangle$ of Eq. (32). Choosing $\vec{k}$ in the z axis of the rest frame of the Δ, the magnetic $M1$ and electric $E2$ of the $\gamma N\rightarrow\Delta$ transition amplitudes can then be calculated from these reduced matrix elements by the following well-known expressions

$$M_{1+} = \frac{-1}{2\sqrt{3}}(3A_{3/2}+\sqrt{3}A_{1/2}), \quad (34)$$

$$E_{1+} = \frac{1}{2\sqrt{3}}(A_{3/2}-\sqrt{3}A_{1/2}), \quad (35)$$

where the helicity amplitudes are defined by

$$A_{1/2}=\left(\frac{3}{2\pi}\right)^{-3/2}\sum_{i=1}^{4}\left[\vec{\epsilon}_{+1}(\vec{k})\cdot\vec{O}^{i}_{J_N M_N=-1/2,J_\Delta M_\Delta=1/2}(\vec{k})\right], \tag{36}$$

$$A_{3/2}=\left(\frac{3}{2\pi}\right)^{-3/2}\sum_{i=1}^{4}\left[\vec{\epsilon}_{+1}(\vec{k})\cdot\vec{O}^{i}_{J_N M_N=1/2,J_\Delta M_\Delta=3/2}(\vec{k})\right]. \tag{37}$$

III. THE $\Delta\leftrightarrow\gamma N$ TRANSITION IN THE DYNAMICAL MODEL OF PION PHOTOPRODUCTION

We now return to Hamiltonian Eq. (2) to consider πN and γN reactions. Here we need to address two problems. First, extensive earlier studies [14,8–11,22] have indicated that a realistic description of πN scattering and $\gamma N\rightarrow\pi N$ reaction can be obtained only when the couplings with ρ and ω vector mesons are included. In the approach of Ref. [14], this is equivalent to extending Eq. (2) to the following form:

$$H=H_B+H_0'+\sum_{B,B'}\left[(h_{\pi B,B'}+h_{\gamma B,B'}+h_{\rho B,B'}+h_{\omega B,B'})+(\mathrm{H.c.})\right]. \tag{38}$$

Here H_0' now also contains the kinetic energy terms of vector mesons and the associated photon-meson couplings.

The second problem is that the vertex interactions in Eq. (38) can generate infinite number of mesons during πN scattering. It is necessary to introduce appropriate approximations to solve the scattering problem. In Ref. [14], this problem is solved by using an effective Hamiltonian approach. The essential step is to use a unitary transformation to eliminate the ‘‘virtual’’ process $\alpha N\leftrightarrow N$ with $\alpha=\pi,\rho,\omega$ in the equations of motion such that there is no mass renormalization problem for the nucleon in the coupled $\Delta\oplus\pi N\oplus\gamma N$ space. Details are discussed in Ref. [14].

Starting with the Hamiltonian Eq. (38) and performing the unitary transformation up to second order in coupling constants, one can show that the πN and γN reactions within the coupled $\Delta\oplus\pi N\oplus\gamma N$ space can be described by the following effective Hamiltonian:

$$H_{\mathrm{eff}}=H_0+H_{\mathrm{st}}+H_{\mathrm{e.m.}}, \tag{39}$$

where H_0 is the free Hamiltonian for π, N, Δ, and photon. The pion acquires its physical mass. The physical mass of the nucleon and the bare mass m_Δ^0 of the Δ are identified with the masses of the ground state and the first excited state of the chiral constituent quark model Hamiltonian H_B, Eq. (5). The interaction terms in Eq. (39) are

$$H_{\mathrm{st}}=V_{\pi N,\pi N}+h_{\pi N,\Delta}+h_{\Delta,\pi N},$$

$$H_{\mathrm{e.m.}}=V_{\gamma N,\pi N}+V_{\pi N,\gamma N}+f_{\gamma N,\Delta}+f_{\Delta,\gamma N}. \tag{40}$$

It is important to note that the vertex $h_{\pi N,N}$ of Eq. (38) is not present in the effective Hamiltonian Eq. (40). Its effects have been absorbed in the mass term of H_0 and the πN potential $V_{\pi N,\pi N}$. This is a great simplification in deriving πN scattering formulation from Eqs. (39) and (40). The two body interaction terms $V_{\pi N,\pi N}$ and $V_{\gamma N,\pi N}$ consist of direct and exchange nucleon terms, Δ exchange terms, meson-exchange terms, and contact terms. The details of the effective Hamiltonian Eqs. (39) and (40) are given in Ref. [14].

It is straightforward [14] to derive the formulation for πN scattering from the effective Hamiltonian Eqs. (39) and (40). The resulting πN scattering amplitude can be written as

$$T_{\pi N,\pi N}(E)=t_{\pi N,\pi N}(E)+\overline{h}_{\pi N,\Delta}(E)G_\Delta(E)\overline{h}_{\Delta,\pi N}(E). \tag{41}$$

The nonresonant (background) term $t_{\pi N,\pi N}(E)$ of the above equation is determined only by the πN potential

$$t_{\pi N,\pi N}(E)=V_{\pi N,\pi N}+V_{\pi N,\pi N}G_{\pi N}(E)t_{\pi N,\pi N}(E), \tag{42}$$

with

$$G_{\pi N}(E)=\frac{P_{\pi N}}{E-H_0+i\delta}, \tag{43}$$

where $P_{\pi N}$ is the projection operator for the πN subspace. The resonant term in Eq. (41) is determined by the dressed $\Delta\leftrightarrow\pi N$ vertices and the dressed Δ propagator. They are defined by

$$\overline{h}_{\Delta,\pi N}(E)=h_{\Delta,\pi N}[1+G_{\pi N}(E)t_{\pi N,\pi N}(E)], \tag{44}$$

$$\overline{h}_{\pi N,\Delta}(E)=[t_{\pi N,\pi N}(E)G_{\pi N}(E)+1]h_{\pi N,\Delta}, \tag{45}$$

and

$$G_\Delta(E)=\frac{P_\Delta}{E-m_\Delta^0-\Sigma_\Delta(E)}, \tag{46}$$

where P_Δ is the projection operator for the Δ state. The Δ self-energy is

$$\Sigma_\Delta(E)=h_{\Delta,\pi N}(E)G_{\pi N}(E)\overline{h}_{\pi N,\Delta}(E). \tag{47}$$

It is also straightforward to derive from the effective Hamiltonian Eq. (39) the $\gamma N\rightarrow\pi N$ amplitude up to the first order in electromagnetic coupling constant. The resulting form can be cast [14] into

$$T_{\pi N,\gamma N}(E)=t_{bg}(E)+\overline{h}_{\pi N,\Delta}G_\Delta(E)\overline{f}_{\Delta,\gamma N}(E), \tag{48}$$

with

$$t_{\mathrm{bg}}(E)=[1+t_{\pi N,\pi N}(E)G_{\pi N}(E)]V_{\pi N,\gamma N}, \tag{49}$$

$$\overline{f}_{\Delta,\gamma N}(E)=f_{\Delta,\gamma N}+\overline{h}_{\Delta,\pi N}G_{\pi N}(E)V_{\pi N,\gamma N}. \tag{50}$$

Clearly, the ‘‘background term’’ t_{bg} does not involve the Δ excitation. The second term of the dressed $\Delta\leftrightarrow\gamma N$ of Eq. (50) is due to the presence of the direct nonresonant reaction mechanism $V_{\pi N,\gamma N}$.

It is common to parametrize [10] the bare vertex by the following form:

$$f_{\Delta,\gamma N}=\overline{w_\Delta^\mu}\Gamma_{\mu\nu}u_N\epsilon^\nu, \tag{51}$$

where w_Δ^μ is the Rarita-Schwinger spinor and u_N is the nucleon spinor. The transition tensor is written as

TABLE I. The parameters for the Hamiltonian defined by Eqs. (5)–(14). The linear and Erf confinement potentials are, respectively, defined by Eqs. (13) and (14). The $E2/M1$ ratio R_{EM} is defined by Eq. (55). N is the energy quantum numbers for the oscillator wave functions included in the diagonalization of the Hamiltonian. m is quark mass, b is the oscillator parameter. The amplitudes M_{1^+} and E_{1^+} are in units of $(\mathrm{GeV})^{1/2}\times 10^{-3}$.

		$m = 0.35$ GeV and $b = 0.465$ fm			
V_{ij}^{chiral} ($\Lambda=1$ GeV)		set to zero		Included	
V_{ij}^{conf}		Linear	Erf	Linear	Erf
α_c		0.0352 $(\mathrm{GeV})^2$	0.435 GeV	0.0203 $(\mathrm{GeV})^2$	0.253 GeV
μ^{-1}			0.625 fm		0.625 fm
α_s of V_{ij}^{OGE}		1.151	1.151	0.668	0.668
$N=0,2,4$	M_{1^+}	184	175	176	168
	E_{1^+}	-1.12	-1.16	-1.92	-1.68
	R_{EM}	-0.61%	-0.66%	-1.09%	-1.0%
Analysis of Ref. [14]:		$M_{1^+} = 179.5 \pm 4.5$,	$E_{1^+} = 0.0 \pm 2.28$	$R_{EM} = (0.0 \pm 1.3)\%$	

$$\Gamma_{\mu\nu}=G_M K_{\mu\nu}^M + G_E K_{\mu\nu}^E, \tag{52}$$

where $K_{\mu\nu}^M$ and $K_{\mu\nu}^E$ are the kinematic factors which were given explicitly in Ref. [10]. It can be shown that the $M1$ and $E2$ amplitudes of the $\gamma N \to \Delta$ transition are, respectively, determined by the parameters G_M and G_E

$$M_{1^+}=\frac{e}{2m_N}\left(\frac{|\vec{k}|m_\Delta}{m_N}\right)^{1/2} G_M, \tag{53}$$

$$E_{1^+}=-\frac{e}{2m_N}\left(\frac{|\vec{k}|m_\Delta}{m_N}\right)^{1/2} \frac{2|\vec{k}|m_\Delta}{m_\Delta^2-m_N^2} G_E. \tag{54}$$

At the resonance energy defined by $m_\Delta = E_N(k)+k=1236$ MeV, Eqs. (53) and (54) lead to a ratio

$$R_{EM}=\left.\frac{E_{1^+}}{M_{1^+}}\right|_{m_\Delta=E_N(k)+k} = -\frac{G_E}{G_M}. \tag{55}$$

This is the quantity which reflects the deformation of the bare Δ state.

The main advance made in Ref. [14] is to calculate all terms in Eqs. (48)–(50) except the bare vertex $f_{\Delta,\gamma N}$ within a meson-exchange model with its parameters constrained by the πN data. In this way, the parameters G_M and G_E of $f_{\Delta,\gamma N}$ are determined from the fit to the very extensive $\gamma N \to \pi N$ data. The resulting ratio is $R_{EM}=0.0\pm1.3$ %. The uncertainty is mainly due to the not-well-determined coupling constant of ω meson. The detail of the analysis is given in Ref. [14]. It is important to note that the bare vertex $f_{\Delta,\gamma N}$ is precisely defined by Eq. (23) within the chiral constituent quark model. Our interest in this work is therefore to see whether the amplitudes M_{1^+} and E_{1^+} calculated from Eqs. (34)–(37) can explain the empirical values defined by Eqs. (53) and (54) as well as the ratio R_{EM} of Eq. (55).

IV. RESULTS AND DISCUSSIONS

Our first task is to choose appropriate parameters for the model Hamiltonian, defined by Eqs. (5)–(14), to calculate the masses and wave functions of the nucleon and the bare Δ state. To be consistent with the dynamical model of pion photoproduction described in Sec. III, the physical nucleon mass must be identified with the ground-state energy of the chiral constituent quark model Hamiltonian Eq. (5). To simplify the calculation, we follow Refs. [2–4] to fix the chiral coupling constant by the empirical πNN coupling constant $g_{ch}=(3/5)(m/m_N)g_{\pi NN}$. This is required to obtain the well-established one-pion-exchange component in the resonant-group calculation [2–4] of the NN potential using the model Hamiltonian Eq. (5). We consider the harmonic oscillator basis states in Eq. (15) only up to $N\leqslant 4$ excitations. This is certainly not high enough to ensure the convergence. So we employ the variational condition $\partial\langle\Phi_N|H_0|\Phi_N\rangle/\partial b=0$ (b is the length parameter of the oscillator wave function) to determine the parameters of the confinement potentials V_{ij}^{conf} defined by Eqs. (13) and (14). Furthermore, the constituent quark mass must be around 1/3 of the nucleon mass to reproduce the nucleon magnetic moment, and the oscillator parameter b must be chosen to give a reasonable nucleon radius. Therefore, the free parameters in our calculations are α_s of the one-gluon-exchange potential and the zero-point energy B_0. We adjust these two parameters to reproduce the masses of the nucleon and the Δ. Since this structure calculation is performed in the absence of the coupling with the πN decay channel, the mass of the Δ we need to reproduce is not the value 1236 MeV listed in particle data table. Rather, we fit the bare mass $m_\Delta^0 \sim 1300$ MeV of the dynamical model [14] of pion photoproduction described in Sec. III.

We have performed extensive calculations for various choices of quark mass m and oscillator parameter b. Both the models with and without couplings with the chiral fields are constructed. It is found that for $m\simeq$ 0.35 GeV and $b\simeq$ 0.5 fm, the calculation converges well as the model space for diagonalizing the one-baryon Hamiltonian Eq. (5) increases from $N=(0,2)$ to $N=(0,2,4)$. Our results for the model space with $N=(0,2,4)$ excitations are given in Table I. The predicted M_{1^+} and E_{1^+} amplitudes as well as the corresponding ratios R_{EM} defined by Eq. (55) are also listed there and compared with the empirical values of Ref. [14].

In Table I, we first observe that all of the calculated M_{1^+} values are close to the empirical values determined in Ref. [14]. Clearly, the chiral field does not influence significantly the magnetic $M1$ transition. All of the predicted M_{1^+} values are also close to the values predicted from the constituent quark models [15–19]; but they are much smaller than the value $\sim 250\times 10^{-3}$ (GeV)$^{-1/2}$ from a purely phenomenological amplitude analysis [21] of the pion photoproduction data. As discussed in Ref. [14], the discrepancy is due to the nonresonant meson-exchange mechanism as described in the second term of Eq. (50).

The main effect of the chiral field is on the electric $E2$ transition. In the absence of the chiral field, the only source of the Δ deformation is the tensor component of the one-gluon-exchange potential. The resulting deformation is clearly very small with $R_{EM}\simeq -0.6\%$ (first two columns of Table I). By including the chiral field, the M_{1^+} is only slightly reduced while the E_{1^+} is increased significantly. Consequently, the $E2/M1$ ratio is increased to $R_{EM}\simeq -1.0\%$ which is close to the value -1.3% determined in Ref. [14]. We also note that the predicted ratio is not very sensitive to the choice of the confinement potential. The values of R_{EM} predicted from using linear and error-function confinement potentials are practically identical.

In conclusion, we have applied the chiral constituent quark model to calculate the $M1$ and $E2$ amplitudes of the $\gamma N\rightarrow\Delta$ transition. It is found that the predicted $E2/M1$ ratio is enhanced by a factor of about 2 by the coupling with the chiral field. The predicted M_{1^+} amplitude and the $E2/M1$ ratio are consistent with the values determined within a dynamical model [14] of pion photoproduction.

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