Elimination of the vacuum instability for finite nuclei in the relativistic σ-ω model

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The σ-ω model of nuclei is studied at leading order in the $1/N$ expansion thereby introducing the self-consistent Hartree approximation, the Dirac sea corrections and the one fermion loop meson self-energies in a unified way. For simplicity, the Dirac sea is further treated within a semiclassical expansion to all orders. The well-known Landau pole vacuum instability appearing in this kind of theory is removed by means of a scheme recently proposed in this context. The effect of such a removal on the low momentum effective parameters of the model, relevant to describe nuclear matter, finite nuclei, and NN force, is analyzed. The one fermion loop meson self-energies are found to have a sizeable contribution to these parameters. However, such contribution turns out to come mostly from the Landau poles and is thus spurious. We conclude that the fermionic loop can only be introduced consistently in the σ-ω nuclear model if the Landau pole problem is dealt with properly. We comment on the possibility of a nonperturbative formulation of the model. [S0556-2813(97)02604-6]

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I. INTRODUCTION

The relativistic approach to nuclear physics has attracted much attention. From a theoretical point of view, it allows one to implement, in principle, the important requirements of relativity, unitarity, causality, and renormalizability [1]. From the phenomenological side, it has also been successful in reproducing a large body of experimental data [1–5]. In the context of finite nuclei a large amount of work has been done at the Hartree level but considering only the positive energy single particle nucleon states. The Dirac sea has also been studied since it is required to preserve the unitarity of the theory. In addition, the only limit in which the Dirac sea contribution becomes negligible corresponds to that of infinite nucleon mass, which also coincides with the nonrelativistic limit. If the sea were negligible a fully relativistic framework would be superfluous from a theoretical point of view. Actually, Dirac sea corrections have been found to be non-negligible using a semiclassical expansion which, if computed to fourth order, seems to be quickly convergent [6]. Therefore, it would appear that the overall theoretical and phenomenological picture suggested by the relativistic approach is rather reliable.

However, it has been known for ten years that such a description is internally inconsistent. The vacuum of the theory is unstable due to the existence of tachyonic poles in the meson propagators at high Euclidean momenta [7]. Alternatively, a translationally invariant mean-field vacuum does not correspond to a minimum; the Dirac sea vacuum energy can be lowered by allowing small size mean-field solutions [8]. Being a short-distance instability it does not show up for finite nuclei at the one fermion loop level and within a semiclassical expansion (which is an asymptotic large size expansion). For the same reason, it does not appear either in the study of nuclear matter if translational invariance is imposed as a constraint. However, the instability sets in either in an exact mean-field valence plus sea (i.e., one fermion loop) calculation for finite nuclei or in the determination of the correlation energy for nuclear matter (i.e., one fermion loop plus a boson loop). Unlike quantum electrodynamics, where the instability takes place far beyond its domain of applicability, in quantum hadrodynamics it occurs at the length scale of 0.2 fm that is comparable to the nucleon size and mass. Therefore, the existence of the instability contradicts the original motivation that lead to the introduction of the field theoretical model itself. In such a situation several possibilities arise. First, one may argue that the model is defined only as an effective theory, subjected to inherent limitations regarding the Dirac sea. Namely, the sea may at best be handled semiclassically, hence reducing the scope of applicability of the model. This interpretation is intellectually unsatisfactory since the semiclassical treatment would be an approximation to an inexistent mean-field description. Alternatively, and taking into account the phenomenological success of the model, one may take more seriously the spirit of the original proposal [1], namely, to use specific renormalizable Lagrangians where the basic degrees of freedom are represented by nucleon and meson fields. Such a path has been explored in a series of papers [9–11] inspired by the early work of Redmond and Bogolyubov *et al.* on nonasymptotically free theories [12,13]. The key feature of this kind of theory is that they are only defined in a perturbative sense. According to the latter authors, it is possible to supplement the theory with a prescription based on an exact fulfillment of the Källén-Lehmann representation of the two-point Green's functions. The interesting aspect of this proposal is that the Landau poles are removed in such a way that the perturbative content of the theory remains unchanged. In particular, this guarantees that the perturbative renormalizability is preserved. It is, however, not clear whether this result can be generalized to three- and higher-point Green's functions in order to end up with a completely well-behaved field theory. Although the prescription to eliminate the ghosts may seem to be *ad hoc*, it certainly agrees more with the original proposal and provides a workable calculational scheme.

The above-mentioned prescription has already been used in the context of nuclear physics. In Ref. [14], it was applied to ghost removal in the σ exchange in the NN potential. More recently, a study of the correlation energy in nuclear matter in the σ-ω model [11] and also in the evaluation of response functions within a local density approximation [9] has been explored. Although this model is rather simple, it embodies the essential field theoretical aspects of the problem while still providing a reasonable phenomenological description. We will use the σ-ω model in the present work, to estimate the binding energy of finite nuclei within a self-consistent mean-field description, including the effects due to the Dirac sea, after explicit elimination of the ghosts. An exact mean-field calculation, both for the valence and sea, does make sense in the absence of a vacuum instability but in practice it becomes a technically cumbersome problem. This is due to the presence of a considerable number of negative energy bound states in addition to the continuum states [5]. Therefore, it seems advisable to use a simpler computational scheme to obtain a numerical estimate. This will allow us to see whether the elimination of the ghosts induces dramatic changes in the already satisfactory description of nuclear properties. In this work we keep the full Hartree equations for the valence part but employ a semiclassical approximation for the Dirac sea. This is in fact the standard procedure [3–5]. As already mentioned, and discussed in previous work [6], this expansion converges rather quickly and therefore might be reliably used to estimate the sea energy up to possible corrections due to shell effects.

The paper is organized as follows. In Sec. II we present the σ-ω model of nuclei in the $1/N$ leading approximation, the semiclassical treatment of the Dirac sea, the renormalization prescriptions, and the different parameter fixing schemes that we will consider. This is done within the effective action formalism. In Sec. III we discuss the vacuum instability problem of the model and Redmond's proposal. We also study the implications of the ghost subtraction on the low-momentum effective parameters. Based on the existence of nontrivial ultraviolet fixed points, we argue that there is a perturbatively equivalent action which becomes amenable to a lattice treatment. In Sec. IV we present our numerical results for the parameters, binding energies, and mean quadratic charge radii of some closed-shell nuclei and the nucleon-nucleon potential mediated by ω-meson exchange. Our conclusions are presented in Sec. V. Explicit expressions for the zero momentum renormalized meson self-energies and related formulas are given in the Appendix.

II. σ-ω MODEL OF NUCLEI

In this section we revise the σ-ω model description of finite nuclei disregarding throughout the instability problem; this will be considered in the next section. The Dirac sea corrections are included at the semiclassical level and renormalization issues as well as the various ways of fixing the parameters of the model are also discussed here.

A. Field theoretical model

Our starting point is the Lagrangian density of the σ-ω model [1,3–5] given by

$$\mathcal{L}(x)=\overline{\Psi}(x)\{\gamma_\mu[i\partial^\mu-g_v V^\mu(x)]-[M-g_s\phi(x)]\}\Psi(x)+\tfrac{1}{2}[\partial_\mu\phi(x)\partial^\mu\phi(x)-m_s^2\phi^2(x)]-\tfrac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)+\tfrac{1}{2}m_v^2V_\mu(x)V^\mu(x)+\delta\mathcal{L}(x). \tag{1}$$

$\Psi(x)$ is the isospinor nucleon field, $\phi(x)$ the scalar field, $V_\mu(x)$ the ω-meson field, and $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$. In the former expression the necessary counterterms required by renormalization are accounted for by the extra Lagrangian term $\delta\mathcal{L}(x)$ (including meson self-couplings).

Including Dirac sea corrections requires taking care of renormalization issues. The best way of doing this in the present context is to use an effective action formalism which is manifestly renormalization group invariant. Further we have to specify the approximation scheme. The effective action will be computed at lowest order in the $1/N$ expansion, N being the number of nucleon species (with g_s and g_v of order $1/\sqrt{N}$), that is, up to one fermion loop and tree level for bosons. This corresponds to the Hartree approximation for fermions including the Dirac sea [15].

In principle, the full effective action would have to be computed by introducing bosonic and fermionic sources. However, since we will consider only stationary situations, we do not need to introduce fermionic sources. Instead, we will proceed as usual by integrating out exactly the fermionic degrees of freedom. This gives directly the bosonic effective action at leading order in the $1/N$ expansion:

$$\Gamma[\phi,V]=\Gamma_B[\phi,V]+\Gamma_F[\phi,V], \tag{2}$$

where

$$\Gamma_B[\phi,V]=\int[\tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_s^2\phi^2)-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_v^2V_\mu V^\mu]d^4x \tag{3}$$

and

$$\Gamma_F[\phi,V]=-i\ln\mathrm{Det}[\gamma_\mu(i\partial^\mu-g_vV^\mu)-(M-g_s\phi)]+\int\delta\mathcal{L}(x)d^4x. \tag{4}$$

The fermionic determinant can be computed perturbatively, by adding up the one-fermion loop amputated graphs with any number of bosonic legs, using a gradient expansion or by any other technique. The ultraviolet divergences are to be canceled with the counterterms by using any renormalization scheme; all of them give the same result after fitting to physical observables.

The effective action so obtained is uniquely defined and completely finite. However, there still remains the freedom to choose different variables to express it. Actually, the numerical value of the effective action is independent of the renormalization point [16]. We will work with fields renormalized at zero momentum. That is, the bosonic fields $\phi(x)$ and $V_\mu(x)$ are normalized so that their kinetic energy term is the canonical one. This is the choice shown above in $\Gamma_B[\phi,V]$. Another usual choice is the on-shell one, namely, to rescale the fields so that the residue of the propagator at the meson pole is unity. Note that the Lagrangian mass parameters m_s and m_v do not correspond to the physical masses (which will be denoted m_σ and m_ω in what follows) since the latter are defined as the position of the poles in the corresponding propagators. The difference comes from the fermion loop self-energy in $\Gamma_F[\phi,V]$ that contains terms quadratic in the boson fields with higher order gradients.

Let us turn now to the fermionic contribution $\Gamma_F[\phi,V]$. We will consider nuclear ground states of spherical nuclei, therefore the spacelike components of the ω-meson field vanish [5] and the remaining fields $\phi(x)$ and $V_0(x)$ are stationary. As is well known, for stationary fields the fermionic energy, i.e., minus the action $\Gamma_F[\phi,V]$ per unit time, can be formally written as the sum of single particle energies of the fermions moving in the bosonic background [15],

$$E_F[\phi,V_0]=\sum_n E_n \tag{5}$$

and

$$\{-i\alpha\nabla+g_vV_0(x)+\beta[M-g_s\phi(x)]\}\psi_n(x)=E_n\psi_n(x). \tag{6}$$

Note that what we have called the fermionic energy contains not only the fermionic kinetic energy, but also the potential energy coming from the interaction with the bosons.

The orbitals, and thus the fermionic energy, can be divided into valence and sea, i.e., positive and negative energy orbitals. In realistic cases there is a gap in the spectrum which makes such a separation a natural one. The valence energy is therefore given by

$$E_F^{\mathrm{val}}[\phi,V]=\sum_n E_n^{\mathrm{val}}. \tag{7}$$

On the other hand, the sea energy is ultraviolet divergent and requires the renormalization mentioned above [3]. The (at zero momentum) renormalized sea energy is known in a gradient or semiclassical expansion up to fourth order and is given by [6]

$$\begin{aligned}
E_0^{\mathrm{sea}}=&-\frac{N}{8\pi^2}M^4\int d^3x\left\{\left(\frac{\Phi}{M}\right)^4\ln\frac{\Phi}{M}+\frac{g_s\phi}{M}-\frac{7}{2}\left(\frac{g_s\phi}{M}\right)^2\right.\\
&\left.+\frac{13}{3}\left(\frac{g_s\phi}{M}\right)^3-\frac{25}{12}\left(\frac{g_s\phi}{M}\right)^4\right\},\\
E_2^{\mathrm{sea}}=&\frac{N}{8\pi^2}\int d^3x\left\{\frac{2}{3}\ln\frac{\Phi}{M}(\nabla V)^2-\ln\frac{\Phi}{M}(\nabla\Phi)^2\right\},\\
E_4^{\mathrm{sea}}=&\frac{N}{2880\pi^2}\int d^3x\{-11\,\Phi^{-4}(\nabla\Phi)^4\\
&-22\,\Phi^{-4}(\nabla V)^2(\nabla\Phi)^2+44\Phi^{-4}[(\nabla_i\Phi)(\nabla_iV)]^2\\
&-44\Phi^{-3}[(\nabla_i\Phi)(\nabla_iV)](\nabla^2V)-8\Phi^{-4}(\nabla V)^4\\
&+22\Phi^{-3}(\nabla^2\Phi)(\nabla\Phi)^2+14\Phi^{-3}(\nabla V)^2(\nabla^2\Phi)\\
&-18\Phi^{-2}(\nabla^2\Phi)^2+24\Phi^{-2}(\nabla^2V)^2\}.
\end{aligned} \tag{8}$$

Here, $V=g_vV_0$, $\Phi=M-g_s\phi$, and N is the isospin degeneracy of the nucleon (i.e., $N=2$ in the real world). The sea energy is obtained by adding up the terms above. The fourth and higher order terms are ultraviolet finite as follows from dimensional counting. The first two terms, being renormalized at zero momentum, do not contain operators with dimension four or less, such as ϕ^2, ϕ^4, or $(\nabla V)^2$, since they are already accounted for in the bosonic term $\Gamma_B[\phi,V]$. Note that the theory has been renormalized so that there are no three- or four-point bosonic interactions in the effective action at zero momentum [3].

By definition, the true value of the classical fields (i.e., the value in the absence of external sources) is to be found by minimization of the effective action or, in the stationary case, of the energy

$$E[\phi,V]=E_B[\phi,V]+E_F^{\mathrm{val}}[\phi,V]+E_F^{\mathrm{sea}}[\phi,V]. \tag{9}$$

Such minimization yields the equations of motion for the bosonic fields,

$$\begin{aligned}
(\nabla^2-m_s^2)\phi(x)&=-g_s[\rho_s^{\mathrm{val}}(x)+\rho_s^{\mathrm{sea}}(x)],\\
(\nabla^2-m_v^2)V_0(x)&=-g_v[\rho^{\mathrm{val}}(x)+\rho^{\mathrm{sea}}(x)].
\end{aligned} \tag{10}$$

Here, $\rho_s(x)=\langle\overline{\Psi}(x)\Psi(x)\rangle$ is the scalar density and $\rho(x)=\langle\Psi^\dagger(x)\Psi(x)\rangle$ the baryonic one:

$$\begin{aligned}
\rho_s^{\mathrm{val\,(sea)}}(x)&=-\frac{1}{g_s}\frac{\delta E_F^{\mathrm{val\,(sea)}}}{\delta\phi(x)},\\
\rho^{\mathrm{val\,(sea)}}(x)&=+\frac{1}{g_v}\frac{\delta E_F^{\mathrm{val\,(sea)}}}{\delta V_0(x)}.
\end{aligned} \tag{11}$$

The set of bosonic and fermionic equations, Eqs. (10) and (6), respectively, are to be solved self-consistently. Let us remark that treating the fermionic sea energy using a gradient or semiclassical expansion is a further approximation on top of the mean-field approximation since it neglects possible shell effects in the Dirac sea. However, a direct solution of the mean-field equations including renormalization of the sum of single-particle energies would not give a physically acceptable solution due to the presence of Landau ghosts. They will be considered in the next section.

At this point it is appropriate to make some comments on renormalization. As we have said, one can choose different normalizations for the mesonic fields and there are also several sets of mesonic masses, namely, on-shell and at zero momentum. If one were to write the mesonic equations of motion directly, by similarity with a classical treatment, there would be an ambiguity as to which set should be used. The effective action treatment makes it clear that the mesonic field and masses are those at zero momentum. In this regard, let us note that a similar set of equations derived in Ref. [9] incorrectly ignore a wave-function renormalization factor since they are used for on-shell parameters. On the other hand, since we have not included bosonic loops, the fermionic operators in the Lagrangian are not renormalized and there are no proper vertex corrections. Thus the nucleon mass M, the nuclear densities $\langle\overline{\Psi}\Psi\rangle$, and the combinations $g_s\phi(x)$ and $g_vV_\mu(x)$ are fixed unambiguously in the renormalized theory. The fermionic energy $E_F[\phi,V]$, the potentials $\Phi(x)$ and $V(x)$, and the nucleon single particle orbitals are all free from renormalization ambiguities at leading order in $1/N$.

B. Fixing of the parameters

The σ-ω and related theories are effective models of nuclear interaction, and hence their parameters are to be fixed to experimental observables within the considered approximation. Several procedures to perform the fixing can be found in the literature [2,4,5]; the more sophisticated versions try to adjust, by minimizing the appropriate χ^2 function, as many experimental values as possible through the whole nuclear table [4]. These methods are useful when the theory implements enough physical elements to provide a good description of atomic nuclei. The particular model we are dealing with can reproduce the main features of nuclear force, such as saturation and the correct magic numbers; however, it lacks many of the important ingredients of nuclear interaction, namely Coulomb interaction and ρ and π mesons. Therefore, we will use the simple fixing scheme proposed in Ref. [2] for this model.

Initially there are five free parameters: the nucleon mass (M), two boson Lagrangian masses (m_s and m_v), and the corresponding coupling constants (g_s and g_v). The five physical observables to be reproduced are taken to be the physical nucleon mass, the physical ω-meson mass m_ω, the saturation properties of nuclear matter (binding energy per nucleon B/A and Fermi momentum k_F), and the mean quadratic charge radius of ^{40}Ca. In our approximation, the equation of state of nuclear matter at zero temperature, and hence its saturation properties, depends only on the nucleon mass and on $m_{s,v}$ and $g_{s,v}$ through the combinations [3] (in fact, minus the meson exchange nucleon-nucleon potentials at zero four momentum)

$$C_s^2=g_s^2\frac{M^2}{m_s^2}, \quad C_v^2=g_v^2\frac{M^2}{m_v^2}. \tag{12}$$

At this point, there still remain two parameters to be fixed, e.g., m_v and g_s. Now we implement the physical ω-meson mass constraint. From the expression of the ω propagator at the leading $1/N$ approximation, we can obtain the value of the physical ω pole as a function of the Lagrangian parameters M, g_v, and m_v or more conveniently as a function of M, C_v, and m_v (see Appendix). Identifying the ω pole and the physical ω mass, and given that M and C_v have already been fixed, we obtain the value of m_v. Finally, the value of g_s is adjusted to fit the mean quadratic charge radius of ^{40}Ca. We will refer to this fixing procedure as the *ω-shell scheme*; the name stresses the correct association between the pole of the ω-meson propagator and the physical ω mass [17]. The above fixing procedure gives different values of m_s and g_s depending on the order at which the Dirac sea energy is included in the semiclassical expansion (see Sec. IV).

Throughout the literature the standard fixing procedure when the Dirac sea is included has been to give to the Lagrangian mass m_v the value of the physical ω mass [4,5] (see, however, Refs. [17,10,6]). Of course, this yields a wrong value for the position of the ω-meson propagator pole, which is underestimated. We will refer to this procedure as the *naive scheme*. Note that when the Dirac sea is not included at all, the right viewpoint is to consider the theory at the tree level, and the ω-shell and the naive schemes coincide.

III. LANDAU INSTABILITY SUBTRACTION

As already mentioned, the σ-ω model, and more generally any Lagrangian which couples bosons with fermions by means of a Yukawa-like coupling, exhibits a vacuum instability [7,8]. This instability prevents the actual calculation of physical quantities beyond the mean-field valence approximation in a systematic way. Recently, however, a proposal by Redmond [12] that explicitly eliminates the Landau ghost has been implemented to describe relativistic nuclear matter in a series of papers [9–11]. The main features of such a method are contained already in the original papers [12,13] and belongs to the body of some standard field theory textbooks [18,19]. Many details have also been discussed regarding its application to nuclear matter and response functions [9–11]. For the sake of clarity, we review here the method (correcting the above-mentioned error in Ref. [9]) in a way that is easily applied to the calculation of Dirac sea effects for closed-shell finite nuclei and the NN potential. We also consider some aspects of the large N expansion in the present context and argue that the ghost subtracted theory may be a perfectly well-defined quantum field theory beyond finite order perturbation theory, possibly suited to a lattice formulation.

A. Landau instability

Since the Landau instability shows up already at zero nuclear density, we will begin by considering the vacuum of the σ-ω theory. On a very general basis, namely, Poincaré invariance, unitarity, causality, and uniqueness of the vacuum state, one can show that the two point Green's function (time ordered product) for a scalar field admits the Källén-Lehmann representation [20]

$$D(x'-x)=\int d\mu^2\rho(\mu^2)D_0(x'-x;\mu^2), \tag{13}$$

where the full propagator in the vacuum is

$$D(x'-x)=-i\langle 0|T\phi(x')\phi(x)|0\rangle \tag{14}$$

and the free propagator reads

$$D_0(x'-x;\mu^2)=\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip(x'-x)}}{p^2-\mu^2+i\eta}. \tag{15}$$

The spectral density $\rho(\mu^2)$ is defined as

$$\rho(q^2)=(2\pi)^3\sum_n \delta^4(p_n-q)|\langle 0|\phi(0)|n\rangle|^2. \tag{16}$$

It is non-negative, Lorentz invariant, and vanishes for spacelike four momentum q.

The Källén-Lehmann representability is a necessary condition for any acceptable theory, yet it is violated by the

σ-ω model when the meson propagators are approximated by their leading $1/N$ term. It is not clear whether this failure is tied to the theory itself or is an artifact of the approximation —it is well known that approximations to the full propagator do not necessarily preserve the Källén-Lehmann representability. The former possibility would suppose a serious obstacle for the theory to be a reliable one.

In the above-mentioned approximation, Eq. (13) still holds both for the σ and the ω cases (in the latter case with obvious modification to account for the Lorentz structure) but the spectral density gets modified to be

$$\rho(\mu^2)=\rho^{\mathrm{KL}}(\mu^2)-R_G\delta(\mu^2+M_G^2), \tag{17}$$

where $\rho^{\mathrm{KL}}(\mu^2)$ is a physically acceptable spectral density, satisfying the general requirements of a quantum field theory. On the other hand, however, the extra term spoils these general principles. The residue $-R_G$ is negative, thus indicating the appearance of a Landau ghost state which contradicts the usual quantum mechanical probabilistic interpretation. Moreover, the δ function is located at the spacelike squared four momentum $-M_G^2$ indicating the occurrence of a tachyonic instability. As a perturbative analysis shows, the dependence of R_G and M_G on the fermion-meson coupling constant g in the weak-coupling regime is $R_G\sim\alpha/Ng^2$ and $M_G^2\sim M^2\exp(\alpha/Ng^2+\beta)$, where M is the nucleon mass, $\alpha=8\pi^2$, $\beta=8/3$ for the scalar meson, and $\alpha=12\pi^2$, $\beta=5/3$ for the vector meson. Therefore the perturbative content of $\rho(\mu^2)$ and $\rho^{\mathrm{KL}}(\mu^2)$ is the same, i.e., both quantities coincide order by order in a power series expansion of g keeping μ^2 fixed. This can also be seen in the propagator form of the previous equation

$$D(p)=D^{\mathrm{KL}}(p)-\frac{R_G}{p^2+M_G^2}. \tag{18}$$

For fixed four momentum, the ghost term vanishes as $\exp(-\alpha/Ng^2)$ when the coupling constant goes to zero. As noted by Redmond [12], it is therefore possible to modify the theory by adding a suitable counterterm to the action that exactly cancels the ghost term in the meson propagator without changing the perturbative content of the theory. In this way the full meson propagator becomes $D^{\mathrm{KL}}(p)$ which is physically acceptable and free from vacuum instability at leading order in the $1/N$ expansion.

It is not presently known whether the stability problems of the original σ-ω theory are intrinsic or due to the approximation used, thus Redmond's procedure can be interpreted either as a fundamental change of the theory or as a modification of the approximation scheme. Although both interpretations use the perturbative expansion as a constraint, it is not possible, at the present stage, to decide between them. It should be made quite clear that in spite of the seemingly arbitrariness of the no-ghost prescription, the original theory itself was ambiguous regarding its nonperturbative regime. In fact, being a nonasymptotically free theory, it is not obvious how to define it beyond finite order perturbation theory. For the same reason, it is not Borel summable and hence additional prescriptions are required to reconstruct the Green's functions from perturbation theory to all orders. As an example, if the nucleon self-energy is computed at leading order in a $1/N$ expansion, the existence of the Landau ghost in the meson propagator gives rise to a pole ambiguity. This is unlike physical timelike poles, which can be properly handled by the customary $+i\eta$ rule, and thus an additional *ad hoc* prescription is needed. This ambiguity reflects in turn in the Borel transform of the perturbative series; the Borel transform presents singularities known as renormalons in the literature [21]. In recovering the sum of the perturbative series through inverse Borel transformation a prescription is then needed, and Redmond's proposal provides a particularly suitable way of fixing such ambiguity. Nevertheless, it should be noted that even if Redmond's prescription turns out to be justified, there still remains the problem of how to extend it to the case of three and more point Green's functions, since the corresponding Källén-Lehmann representations have been less studied. As noted below, this problem cannot occur at any finite order in the $1/N$ expansion.

B. Instability subtraction

To implement Redmond's prescription in detail we start with the zero-momentum renormalized propagator in terms of the proper self-energy for the scalar field (a similar construction can be carried out for the vector field as well),

$$D_s(p^2)=[p^2-m_s^2-\Pi_s(p^2)]^{-1}, \tag{19}$$

where m_s is the zero-momentum meson mass and the corresponding renormalization conditions are $\Pi_s(0)=\Pi_s'(0)=0$. The explicit formulas for the scalar and vector meson self-energies at leading order in the large N expansion are given in the Appendix. Of course, $D_s(p^2)$ is just the inverse of the quadratic part of the effective action $K_s(p^2)$. According to the previous section, the propagator presents a tachyonic pole. Since the ghost subtraction is performed at the level of the two-point Green's function, it is clear that the corresponding Lagrangian counterterm must involve a quadratic operator in the mesonic fields. The counterterm kernel $\Delta K_s(p^2)$ must be such that cancels the ghost term in the propagator $D_s(p^2)$ in Eq. (18). The subtraction neither modifies the position of the physical meson pole nor its residue, but it will change the zero-momentum parameters and also the off-shell behavior. Both features are relevant to nuclear properties and NN potentials. This will be discussed further in the next section.

Straightforward calculation yields

$$\Delta K_s(p^2)=-\frac{1}{D_s(p^2)}\frac{R_G^s}{R_G^s+(p^2+M_G^{s\,2})D_s(p^2)}. \tag{20}$$

As stated, this expression vanishes as $\exp(-4\pi^2/g_s^2)$ for small g_s at fixed momentum. Therefore it is a genuine nonperturbative counterterm. It is also nonlocal as it depends in a nonpolynomial way on the momentum. In any case, it does not introduce new ultraviolet divergences at the one fermion loop level. However, it is not known whether the presence of this term spoils any general principle of quantum field theory.

Proceeding in a similar way with the ω-meson field $V_\mu(x)$, the following change in the total original action is induced:

$$\Delta S=\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\phi(-p)\Delta K_s(p^2)\phi(p)-\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}V_\mu(-p)\Delta K_v^{\mu\nu}(p^2)V_\nu(p), \qquad (21)$$

where $\phi(p)$ and $V_\mu(p)$ are the Fourier transform of the scalar and vector fields in coordinate space $\phi(x)$ and $V_\mu(x)$, respectively. Note that at the tree level for bosons, as we are considering throughout, this modification of the action is to be added directly to the effective action —in fact, this is the simplest way to derive Eq. (20). Therefore, in the case of static fields, the total mean-field energy after ghost elimination reads

$$E=E_F^{\rm val}+E_F^{\rm sea}+E_B+\Delta E, \qquad (22)$$

where $E_F^{\rm val}$, $E_F^{\rm sea}$, and E_B were given in Sec. II and

$$\Delta E[\phi,V]=\frac{1}{2}\int d^3x\,\phi(x)\Delta K_s(\nabla^2)\phi(x)-\frac{1}{2}\int d^3x V_0(x)\Delta K_v^{00}(\nabla^2)V_0(x). \qquad (23)$$

One can proceed by minimizing the mean-field total energy as a functional of the bosonic and fermionic fields. This yields the usual set of Dirac equations for the fermions, Eqs. (6), and modifies the left-hand side of the bosonic Eqs. (10) by adding a linear nonlocal term. This will be our starting point to study the effect of eliminating the ghosts in the description of finite nuclei. We note that the instability is removed at the Lagrangian level, i.e., the nonlocal counterterms are taken to be new terms of the starting Lagrangian which is then used to describe the vacuum, nuclear matter, and finite nuclei. Therefore no new prescriptions are needed in addition to Redmond's to specify how the vacuum and the medium parts of the effective action are modified by the removal of the ghosts.

So far, the new counterterms, although induced through the Yukawa coupling with fermions, have been treated as purely bosonic terms. Therefore, they do not contribute directly to bilinear fermionic operators such as baryonic and scalar densities. An alternative viewpoint would be to take them rather as fermionic terms, i.e., as a (nonlocal and nonperturbative) redefinition of the fermionic determinant. The energy functional, and thus the mean-field equations and their solutions, coincide in the bosonic and fermionic interpretations of the new term, but the baryonic densities and related observables would differ, since they pick up a new contribution given by the corresponding formulas similar to Eqs. (11). Ambiguities and redefinitions are ubiquitous in quantum field theories, due to the well known ultraviolet divergences. However, in well-behaved theories the only freedom allowed in the definition of the fermionic determinant comes from adding counterterms which are local and polynomial in the fields (see, e.g., [22]). Since the new counterterms induced by Redmond's method are not of this form, we will not pursue such alternative point of view in what follows. Nevertheless, a more compelling argument would be needed to make a reliable choice between both possibilities.

C. Field theoretical aspects of the large *N* expansion

Perhaps the most attractive feature of the large N expansion lies in its systematic summation of perturbative Feynman diagrams while being itself a nonperturbative method. Actually, if one introduces new diagrams using as building blocks the old nucleon line and vertices and the leading $1/N$ meson propagator as new meson line, a new expansion arises. The Feynman rules remain the same besides the fact that one should not include nucleon loops attached to two meson lines, since they are already accounted for by the new meson line. The important property is that at each order of the $1/N$ expansion there are only a finite number of new diagrams. This comes about because each vertex carries a factor $1/\sqrt{N}$ and for each nucleon loop a factor N appears. Therefore, any nucleon loop subdiagram with $k\geqslant 3$ meson legs is suppressed by a factor $(1/\sqrt{N})^{k-2}$. It is noteworthy that these features remain the same after removing the ghost. In particular, this implies that no new ghost may appear within a finite order of the $1/N$ expansion, although a summation of an infinite number of diagrams may give rise to new ghost singularities. A further aspect of the method used in the case of the ω meson is given by gauge invariance since the Lorentz structure of the propagator remains unchanged. As a consequence the perturbative renormalizability (in the $1/N$ sense) of the theory is maintained, since the longitudinal components of the ω propagator still cancel due to the coupling to the conserved baryonic current. A similar remark can be made for the case of quantum electrodynamics. The ghost subtraction together with the large N expansion render the theory Borel summable in the coupling constant at any finite order in the $1/N$ expansion. Nothing is of course implied for the Borel summability of the $1/N$ series. One should note that in spite of having a nonlocal looking Lagrangian, the physical requirement of causality is fulfilled. All these remarks indicate that the ghost subtracted theory is perfectly consistent, i.e., it satisfies the general requirements of an acceptable relativistic field theory within a $1/N$ expansion formulation.

The elimination of the ghost has a further, although related, advantage. This can be seen by computing the analogous to the Gell-Mann–Low Ψ function [23], defined as

$$\Psi(\alpha)=\frac{d\alpha}{d\ln(m^2-q^2)}, \qquad (24)$$

where the momentum dependent and renormalization group invariant effective charge $\alpha(q^2)$ is defined as

$$\alpha(q^2)=\frac{g^2}{4\pi}(q^2-m^2)D(q). \qquad (25)$$

Here m^2 is the on-shell physical meson mass and g and $D(q)$ are the renormalized coupling constant and meson propagator. The effective charge has been defined so that $\alpha(m^2)=g^2/4\pi$ with g the on-shell coupling constant. Note that the combination $g^2D(q)$ is renormalization group in-

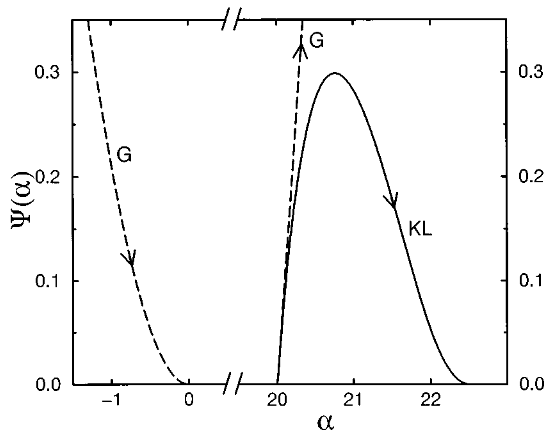

FIG. 1. The Gell-Mann–Low Ψ function for the ω meson $\Psi(\alpha)=(q^2-m_\omega^2)d\alpha/dq^2$ as a function of the effective charge $\alpha(q^2)=g_v^2(q^2-m_\omega^2)D(q^2)/4\pi$, for the ghost unsubtracted (G) and ghost free (KL) as obtained from the Källén-Lehmann representation. The arrows indicate the direction of increasing $-q^2$ starting from the on-shell point $q^2=m_\omega^2$. Note that the curve without ghost has a nontrivial ultraviolet stable fixed point, whereas the ghost unsubtracted curve has a trivial one for imaginary values of the coupling constant. We have taken $M=939$ MeV, $m_\omega=783$ MeV, and on-shell coupling constant $g_\omega=15.9$

variant, i.e., it is independent of the renormalization point used. We will see below that this combination is the NN meson exchange potential at leading order in $1/N$. The relevance of the Ψ function relies on the possibility of studying nontrivial ultraviolet fixed points [24]. We remind the reader that the zeros of this function are the renormalization group fixed points. In Fig. 1 we display $\Psi(\alpha)$ for the particular case of the ω meson before and after ghost elimination and for the same value of the renormalized parameters, namely, $M=939$ MeV, $m_\omega=783$ MeV, and on-shell coupling constant $g_\omega=15.9$ (see the Appendix for the relation between zero momentum and on-shell parameters). By construction, in both cases the first positive zero corresponds to the on-shell point and they have the same slope there. However, both curves exhibit a completely different behavior as the four momentum transfer goes into the Euclidean region. As can be clearly seen, the vacuum unstable theory does not show an ultraviolet fixed point, whereas the ghost free theory possesses one which corresponds to a double zero of Ψ. Actually, near the fixed point Ψ behaves as $(\alpha-\alpha_\infty)^2/3\pi$, indicating a rather slow approach to the large momentum coupling constant. One should say that the unstable theory also has a ultraviolet fixed point, namely at zero coupling, which, however, is approached from imaginary values of the coupling constant. Finally, the Källén-Lehmann representation guarantees that, apart from the on-shell point, $\Psi(\alpha)$ will not have other zeros before $-q^2\to+\infty$. Therefore, despite the nonlocality of the action, the ghost-free theory seems to be well defined in a nonperturbative way. This opens the possibility of a full nonperturbative calculation using, say, a lattice regularization method. We note here that some attempts have been carried out in this direction but with the naive action and in the valence approximation [25].

D. Application to finite nuclei

In this section we will take advantage of the smooth behavior of the mesonic mean fields in coordinate space which allows us to apply a derivative or low-momentum expansion. The quality of the gradient expansion can be tested *a posteriori* by a direct computation. The practical implementation of this idea consists of treating the term ΔS by expanding each of the kernels $\Delta K(p^2)$ in a power series of the momentum squared around zero

$$\Delta K(p^2)=\sum_{n\geqslant 0}(p^2)^n\Delta K_{2n}. \tag{26}$$

The first two terms are given explicitly by

$$\Delta K_0=-\frac{m^4R_G}{M_G^2-m^2R_G},$$

$$\Delta K_2=\frac{m^2R_G(m^2-m^2R_G+2M_G^2)}{(M_G^2-m^2R_G)^2}. \tag{27}$$

The explicit expressions of the tachyonic pole parameters M_G and R_G for each meson can be found below.

Numerically, we have found that the fourth and higher orders in this gradient expansion are negligible as compared to zeroth and second orders. In fact, in Ref. [6] the same behavior was found for the correction to the Dirac sea contribution to the binding energy of a nucleus. As a result, even for light nuclei, E_4^{sea} in Eq. (8) can be safely neglected. Furthermore, it has been shown [26] that the fourth order term in the gradient expansion of the valence energy, if treated semiclassically, is less important than shell effects. So, it seems to be a general rule that, for the purpose of describing static nuclear properties, only the two lowest order terms of a gradient expansion need to be considered. We warn, however, that the convergence of the gradient or semiclassical expansion does not imply convergence to the exact mean-field result, since there could be shell effects not accounted for by this expansion at any finite order. Such effects certainly exist in the valence part [26]. Even in such a seemingly safe case as infinite nuclear matter, where only the zeroth order has a nonvanishing contribution, something is left out by the gradient expansion since the exact mean-field solution does not exist due to the Landau ghost instability (of course, the situation may change if the Landau pole is removed). In other words, although a gradient expansion might appear to be exact in the nuclear matter case, it hides the very existence of the vacuum instability.

From the previous discussion it follows that the whole effect of the ghost subtraction is represented by adding a term ΔS to the effective action with the same form as the bosonic part of the original theory $\Gamma_B[\phi,V]$ in Eq. (3). This amounts to a modification of the zero-momentum parameters of the effective action. The new zero-momentum scalar field (i.e., with canonical kinetic energy), mass, and coupling constant in terms of those of the original theory are given by

$$\hat{\phi}(x)=(1+\Delta K_2^s)^{1/2}\phi(x),\quad \hat{m}_s=\left(\frac{m_s^2-\Delta K_0^s}{1+\Delta K_2^s}\right)^{1/2},$$

$$\hat{g}_s=(1+\Delta K_2^s)^{-1/2}g_s. \tag{28}$$

The new coupling constant is obtained recalling that $g_s\phi(x)$ should be invariant. Similar formulas hold for the vector meson. With these definitions [and keeping only $\Delta K_{s,v}(p^2)$ to second order in p^2] one finds[1]

$$E_B[\hat{\phi},\hat{V};\hat{m}_s,\hat{m}_v]=E_B[\phi,V;m_s,m_v]+\Delta E[\phi,V;m_s,m_v],$$

$$E_F[\hat{\phi},\hat{V};\hat{g}_s,\hat{g}_v]=E_F[\phi,V;g_s,g_v]. \quad (29)$$

The bosonic equations for the new meson fields after ghost removal are hence identical to those of the original theory using

$$\hat{m}^2=m^2M_G^2\frac{M_G^2-m^2R_G}{M_G^4+m^4R_G}, \quad \hat{g}^2=g^2\frac{(M_G^2-m^2R_G)^2}{M_G^4+m^4R_G}, \quad (30)$$

as zero-momentum masses and coupling constants, respectively. In the limit of large ghost masses or vanishing ghost residues, the reparametrization becomes trivial, as it should be. Let us note that although the zero-momentum parameters of the effective action $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$ are the relevant ones for nuclear structure properties, the parameters $m_{s,v}$ and $g_{s,v}$ are the (zero-momentum renormalized) Lagrangian parameters and they are also needed, since they are those appearing in the Feynman rules in a perturbative treatment of the model. Of course, both sets of parameters coincide when the ghosts are not removed or if there were no ghosts in the theory.

To finish this section we give explicitly the fourth order coefficient in the gradient expansion of ΔE, taking into account the rescaling of the mesonic fields, namely,

$$\frac{\Delta K_4}{1+\Delta K_2}=-\frac{R_G(M_G^2+m^2)^2}{(M_G^4+m^4R_G)(M_G^2-m^2R_G)} -\frac{Ng^2}{\alpha\pi^2M^2}\frac{m^2R_G(m^2R_G-2M_G^2)}{M_G^4+m^4R_G}, \quad (31)$$

where α is 80 for the scalar meson and 60 for the vector meson. As already stated, for typical mesonic profiles the contribution of these fourth order terms are found to be numerically negligible. Simple order of magnitude estimates show that squared gradients are suppressed by a factor $(RM_G)^{-2}$, R being the nuclear radius, and therefore higher orders can also be neglected. That the low-momentum region is the one relevant to nuclear physics can also be seen from the kernel $K_s(p^2)$, shown in Fig. 2. From Eq. (21), this kernel is to be compared with the function $\phi(p)$ that has a width of the order of R^{-1}. It is clear from the figure that at this scale all the structure of the kernel at higher momenta is irrelevant to ΔE.

E. Fixing of the parameters after ghost subtraction

As noted in Sec. II, the equation of state at zero temperature for nuclear matter depends only on the dimensionless quantities C_s^2 and C_v^2, that now become

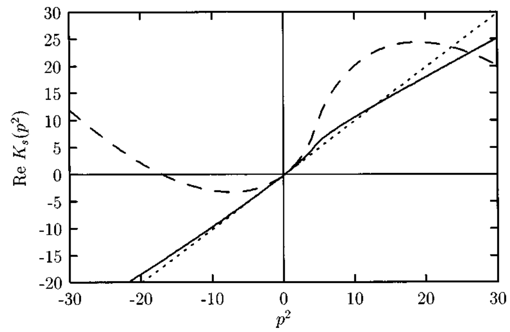

FIG. 2. Real part of the inverse scalar meson propagator $K_s(p^2)$ as a function of the squared four momentum using the no-ghost (sea 2nd) set of parameters of Table I. The dashed line represents the one-loop result without ghost subtraction. The solid line is the result after ghost elimination. The dotted line shows the free inverse propagator. In all cases the slope at zero momentum is unity. Units are in nucleon mass.

$$C_s^2=\hat{g}_s^2\frac{M^2}{\hat{m}_s^2}, \quad C_v^2=\hat{g}_v^2\frac{M^2}{\hat{m}_v^2}. \quad (32)$$

Fixing the saturation density and binding energy to their observed values yields, of course, the same numerical values for C_s^2 and C_v^2 as in the original theory. After this is done, all static properties of nuclear matter are determined and thus they are insensitive to the ghost subtraction. Therefore, at leading order in the $1/N$ expansion, to see any effect one should study either static nuclear matter properties at higher orders as done in Ref. [11] or finite nuclei. In Ref. [10] response functions have been computed within a local density approximation. In this paper we focus on finite nuclei structure.

It is remarkable that if all the parameters of the model were to be fixed exclusively by a set of nuclear structure properties, the ghost subtracted and the original theories would be indistinguishable regarding any other static nuclear prediction, because bosonic and fermionic equations of motion have the same form in both theories. They would differ, however, far from the zero-four-momentum region where the truncation of the ghost kernels $\Delta K(p^2)$ at order p^2 is no longer justified. In practice, the predictions will change after ghost removal because the ω-meson mass is quite large and is one of the observables to be used in the fixing of the parameters. If the ω-meson mass were larger, the zero-momentum region would be dominated by the ghost pole resulting in an odd sign for the slope of the on-shell renormalized propagator at zero momentum. This would imply negative values for the zero-momentum parameters g_v^2 and m_v^2. Likewise, if the on-shell ω coupling constant were sufficiently strong the ghost pole would shift towards the region of smaller momentum yielding a similar result as before. This imposes some upper bounds regarding the admissible values of the on-shell parameters for the theory with ghosts. On the contrary, the ghost-free theory does not exhibit these constraints.

To fix the parameters of the theory we choose the same observables as in Sec. II. Let us consider first the vector meson parameters $\hat{m}_v$ and $\hat{g}_v$. We proceed as follows.

[1]Note that $E_{B,F}[\,]$ refer to the functionals (the same at both sides of the equations) and not to their value as is also usual in physics literature.

(1) We choose a trial value for g_v (the zero-momentum coupling constant of the original theory). This value and the known physical values of the ω-meson and nucleon masses, m_ω and M, respectively, determines m_v (the zero-momentum mass of the original theory), namely

$$m_v^2=m_\omega^2+\frac{Ng_v^2}{4\pi^2}M^2\left\{\frac{4}{3}+\frac{5}{9}\frac{m_\omega^2}{M^2}-\frac{2}{3}\left(2+\frac{m_\omega^2}{M^2}\right)\times\sqrt{\frac{4M^2}{m_\omega^2}-1}\arcsin\left(\frac{m_\omega}{2M}\right)\right\}. \tag{33}$$

(This, as well as the formulas given below, can be deduced from those in the Appendix.)

(2) g_v and m_v provide the values of the tachyonic parameters R_G^v and M_G^v. They are given by

$$M_G^v=\frac{2M}{\sqrt{\kappa_v^2-1}},$$

$$\frac{1}{R_G^v}=-1+\frac{Ng_v^2}{12\,\pi^2}\left\{\left(\frac{\kappa_v^3}{4}+\frac{3}{4\,\kappa_v}\right)\ln\frac{\kappa_v+1}{\kappa_v-1}-\frac{\kappa_v^2}{2}-\frac{1}{6}\right\}, \tag{34}$$

where the quantity κ_v is the real solution of the following equation (there is an imaginary solution which corresponds to the ω-meson pole):

$$1+\frac{m_v^2}{4M^2}(\kappa_v^2-1)+\frac{Ng_v^2}{12\,\pi^2}\left\{\left(\frac{\kappa_v^3}{2}-\frac{3\,\kappa_v}{2}\right)\ln\frac{\kappa_v+1}{\kappa_v-1}-\kappa_v^2+\frac{8}{3}\right\}=0. \tag{35}$$

(3) Known g_v, m_v, M_G^v, and R_G^v, the values of $\hat{m}_v$ and $\hat{g}_v$ are obtained from Eqs. (30). They are then inserted in Eqs. (32) to yield C_v^2. If necessary, the initial trial value of g_v should be readjusted so that the value of C_v^2 so obtained coincides with that determined by the saturation properties of nuclear matter.

The procedure to fix the parameters m_s and g_s is similar but slightly simpler since the physical mass of the scalar meson m_σ is not used in the fit. Some trial values for m_s and g_s are proposed. This allows us to compute M_G^s and R_G^s by means of the formulas

$$M_G^s=\frac{2M}{\sqrt{\kappa_s^2-1}},$$

$$\frac{1}{R_G^s}=-1-\frac{Ng_s^2}{8\,\pi^2}\left\{\left(\frac{\kappa_s^3}{2}-\frac{3\,\kappa_s}{2}\right)\ln\frac{\kappa_s+1}{\kappa_s-1}-\kappa_s^2+\frac{8}{3}\right\}, \tag{36}$$

where κ_s is the real solution of

$$1+\frac{m_s^2}{4M^2}(\kappa_s^2-1)-\frac{Ng_s^2}{8\,\pi^2}\left\{\kappa_s^3\ln\frac{\kappa_s+1}{\kappa_s-1}-2\,\kappa_s^2-\frac{2}{3}\right\}=0\,. \tag{37}$$

One can then compute $\hat{m}_s$ and $\hat{g}_s$ and thus C_s^2 and the mean quadratic charge radius of ^{40}Ca. The initial values of m_s and g_s should be adjusted to reproduce these two quantities. We will refer to the set of masses and coupling constants so obtained as the *no-ghost scheme* parameters.

F. Application to the nucleon-nucleon potential

Traditionally, the σ-ω model is meant to be applied to finite nuclei. However, the most immediate consequence of the ghost subtraction can be found by looking at the momentum dependence of the nucleon-nucleon interaction. We remind that, for the quantum field theory to make sense, it is absolutely mandatory to get rid of the ghost. Clearly, since the number of parameters of the model is small, some predictive power can be attained by examining the NN force throughout a sensible range of energies and conclusions might be drawn whether ghost subtraction à la Redmond is experimentally favored at momentum transfers well below the occurrence of the ghost. Due to the fact that we are dealing with a model where the degrees of freedom are nucleons and mesons, the obvious framework to undertake this comparison is the Bonn one-meson exchange potential (OBEP) [27]. This allows us to compare each mesonic contribution to the potential in a separate way, instead of comparing the outcoming phase shifts. The only subtlety arises from the absence of experimental error bars in the OBEP parameters so we cannot judge quantitatively the quality of the model and our approximation, namely, the large N limit. We note that in the Hartree mean-field approximation, symmetric closed-shell nuclei data constrain only the σ and ω mesons parameters. The coupling and masses of the remaining meson (i.e., π, η, ρ, and δ) are not fixed by the abovementioned nuclei. It would be interesting to relax the isospin zero constraint in order to obtain a more realistic description of finite nuclei and simultaneously to impose some conditions on the ρ meson parameters. In this work, it is not our aim to achieve a fully realistic description of NN scattering data but rather to get some insight into the implications of removing the ghost. In addition, this comparison should be done within the same energy range used in the OBEP fits. More specifically, we compare the OBEP reduction of the full Bonn potential with the potential obtained in the leading large N expansion with and without ghost subtraction. While in the former one considers elementary mesons with phenomenological form factors, in our case the mesons are dressed through their coupling to virtual NN pairs and no additional form factors in the meson-nucleon vertices appear, as dictated by the $1/N$ expansion. In the spirit of the model such phenomenological form factors are not necessary due to its renormalizability and would arise naturally when computing next to leading $1/N$ contributions. Of course, they should be required to account for the underlying nucleon substructure at sufficiently high momentum.

IV. NUMERICAL RESULTS AND DISCUSSION

A. Finite nuclei

As explained in Sec. II, the parameters of the theory are fitted to five observables. For the latter we take the following numerical values: $M=939$ MeV, $m_\omega=783$ MeV, $B/A=15.75$ MeV, $k_F=1.3\ \mathrm{fm}^{-1}$, and 3.82 fm for the mean quadratic charge radius of ^{40}Ca.

TABLE I. Zero momentum renormalized Lagrangian parameters in several schemes. Masses are in MeV. The meaning of the labels no sea, sea 0th, sea 2nd, ω-shell, and no-ghost are as in Table III. The naive scheme corresponds to not including the meson self-energy. The numbers in brackets are the zero-momentum parameters of the effective action for the no-ghost scheme $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$. In all cases, m_σ and m_ω stand for the poles in the meson propagators after including the one fermion loop self-energy and using the corresponding Lagrangian parameters. Note that by construction the vector meson parameters coincide in the sea 0th and sea 2nd cases.

		g_s	m_s	m_σ	g_v	m_v	m_ω
no sea		9.062	449.9	439.8	13.81	783	673.6
	ω-shell	6.153	382.8	379.8	11.78	910.8	783
sea 0th	no-ghost	5.996	370.9	368.3	14.86	978.6	783
		(5.928)	(368.8)		(10.17)	(786.1)	
	naive	5.922	368.4	365.9	10.13	783	711.2
	ω-shell	6.846	425.9	420.3	11.78	910.8	783
sea 2nd	no-ghost	6.664	410.8	406.5	14.86	978.6	783
		(6.544)	(407.1)		(10.17)	(786.1)	
	naive	6.536	406.6	402.6	10.13	783	711.2

If the Dirac sea is not included at all, the numerical values that we find for the nuclear matter combinations C_s^2 and C_v^2 are

$$C_s^2 = 357.7, \quad C_v^2 = 274.1. \tag{38}$$

The corresponding Lagrangian parameters are shown in Table I. There we also show m_σ and m_ω that correspond to the position of the poles in the propagators after including the one-loop meson self-energy. They are an output of the calculation and are given for illustration purposes.

When the Dirac sea is included, nuclear matter properties fix the following values:

$$C_s^2 = 227.8, \quad C_v^2 = 147.5. \tag{39}$$

Note that in nuclear matter only the zeroth order $E_0^{\rm sea}$ is needed in the gradient expansion of the sea energy, since the meson fields are constant. The (zero-momentum renormalized) Lagrangian meson masses $m_{s,v}$ and coupling constants $g_{s,v}$ are shown in Table I in various schemes, namely, ω-shell, no-ghost, and naive schemes, previously defined. The scalar meson parameters differ if the Dirac sea energy is included at zeroth order or at all orders (in practice zeroth plus second order) in the gradient expansion. For the sake of completeness, both possibilities are shown in the table. The numbers in brackets in the no-ghost scheme are the zero-momentum parameters of the effective action, $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$ (in the other schemes they coincide with the Lagrangian parameters). Again m_σ and m_ω refer to the scalar and vector propagator-pole masses after including the one fermion loop self-energy for each set of Lagrangian parameters. Table II shows the ghost masses and residues corresponding to the zero-momentum renormalized propagators. The no-ghost scheme parameters have been used.

TABLE II. Residue (up to a sign) and mass (in MeV) of the ghosts in the zero-momentum renormalized meson propagators using the no-ghost sets of Lagrangian parameters in Table I.

	R_G^s	M_G^s	R_G^v	M_G^v
sea 0th	1.748	4605	0.6090	1457
sea 2nd	1.584	3863	0.6090	1457

TABLE III. Binding energy per nucleon of some closed-shell nuclei computed in several ways: not including the Dirac sea in the parameter fixing (no-sea), including the Dirac sea at lowest order in a gradient expansion (sea 0th), including the Dirac sea at all orders (sea 2nd), and the experimental values (exp.). The entry ω-shell corresponds to use the set of parameters that reproduce the ω-meson mass after including the meson self-energy. The entry no-ghost corresponds to the parameters obtained by applying Redmond's prescription.

	B/A (MeV)					
	no sea	sea 0th		sea 2nd		exp.
${}^A_Z$X		no-ghost	ω-shell	no-ghost	ω-shell	
${}^{40}_{20}$Ca	6.28	6.00	6.10	6.33	6.43	8.55
${}^{56}_{28}$Ni	7.24	6.51	6.60	6.80	6.90	8.64
${}^{90}_{40}$Zr	8.36	7.99	8.07	8.22	8.30	8.71
${}^{132}_{50}$Sn	8.81	8.43	8.50	8.62	8.69	8.36
${}^{208}_{82}$Pb	9.84	9.55	9.61	9.70	9.76	7.87

The binding energies per nucleon (without center-of-mass corrections) and mean quadratic charge radii (without convolution with the nucleon form factor) of several closed-shell nuclei are shown in Tables III and IV for the ω shell and for the naive and no-ghost schemes (these two latter schemes give the same numbers), as well as for the case of not including the Dirac sea. The experimental data are taken from Refs. [28–30].

From Table I it follows that the zero-momentum vector meson mass m_v in the ω-shell scheme is considerably larger than the physical mass. This is somewhat unexpected. Let us recall that the naive treatment, which neglects the meson self-energy, is the most used in practice. It has been known for a long time [31,17] that the ω-shell scheme is, as a matter of principle, the correct procedure but on the basis of rough estimates it was assumed that neglecting the meson self-

TABLE IV. Mean quadratic charge radii (MQCR) of several closed-shell nuclei. Meaning of the labels and experimental values as in Table III.

	MQCR (fm)					
	no sea	sea 0th		sea 2nd		exp.
${}^A_Z$X		no-ghost	ω-shell	no-ghost	ω-shell	
${}^{40}_{20}$Ca*	3.48	3.48	3.48	3.48	3.48	3.48
${}^{56}_{28}$Ni	3.72	3.79	3.79	3.79	3.80	
${}^{90}_{40}$Zr	4.22	4.23	4.24	4.25	4.25	4.27
${}^{132}_{50}$Sn	4.60	4.66	4.66	4.68	4.68	
${}^{208}_{82}$Pb	5.35	5.39	5.39	5.41	5.41	5.50

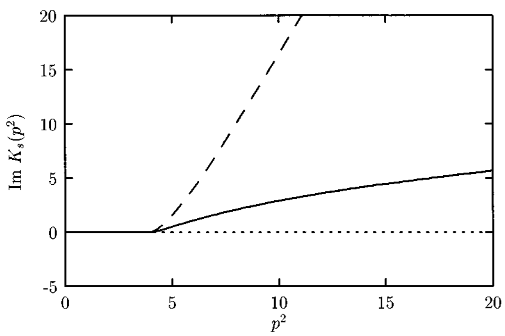

FIG. 3. Imaginary part of the inverse scalar meson propagator $K_s(p^2)$. Units and meaning of the lines as in Fig. 1.

energy would be a good approximation for the meson mass. We find here that this is not so.

Regarding the consequences of removing the ghost, we find in Table I that the effective parameters $\hat{m}_{s,v}$ and $\hat{g}_{s,v}$ in the no-ghost scheme are similar, within a few per thousand, to those of the naive scheme. This similarity reflects in turn on the predicted nuclear properties: the results shown in Tables III and IV for the no-ghost scheme coincide, within the indicated precision, with those of the naive scheme (not shown in the table). It is amazing that the outcoming parameters from such a sophisticated fitting procedure, namely, the no-ghost scheme, resemble so much the parameters corresponding to the naive treatment. We believe this result to be rather remarkable for it justifies *a posteriori* the nowadays traditional calculations made with the naive scheme.

The above observation is equivalent to the fact that the zero-momentum masses $\hat{m}_{s,v}$ and the propagator-pole masses $m_{\sigma,\omega}$ are very similar in the no-ghost scheme. This implies that the effect of removing the ghosts cancels to a large extent with that introduced by the meson self-energies. Note that separately the two effects are not small; as was noted above m_v is much larger than m_ω in the ω-shell scheme. To interpret this result, it will be convenient to recall the structure of the meson propagators. In the leading $1/N$ approximation, there are three kinds of states that can be created on the vacuum by the meson fields. Correspondingly, the spectral density functions $\rho(q^2)$ have support in three clearly separated regions, namely, at the ghost mass squared (in the Euclidean region), at the physical meson mass squared, and above the NN pair production threshold $(2M)^2$ (in the timelike region). The full meson propagator is obtained by convolution of the spectral density function with the massless propagator $(q^2+i\eta)^{-1}$ as follows from the Källén-Lehmann representation, Eq. (13). The large cancellation found after removing the ghosts leads to the conclusion that, in the zero-momentum region, most of the correction induced by the fermion loop on the meson propagators, and thereby on the quadratic kernels $K(p^2)$, is spurious since it is due to unphysical ghost states rather than to virtual NN pairs. This can also be seen from Figs. 2 and 3. There, we represent the real and imaginary parts of $K_s(p^2)$ respectively, in three cases, namely, before ghost elimination, after ghost elimination, and the free inverse propagator. In all three cases the slope of the real part at zero momentum is equal to one and the no-ghost (sea second) set of parameters from Table I has been used. We note the strong resemblance of the free propagator and the ghost-free propagator below threshold. A similar result is obtained for the vector meson.

One may wonder how these conclusions reflect on the sea energy. Given that we have found that most of the fermion loop is spurious in the meson self-energy it seems necessary to revise the sea energy as well since it has the same origin. Technically, no such problem appears in our treatment. Indeed the ghost is found in the fermion loop attached to two meson external legs, i.e., terms quadratic in the fields. However, the sea energy used, namely, $E_0^{\rm sea}+E_2^{\rm sea}$, does not contain such terms. Quadratic terms would correspond to a mass term in $E_0^{\rm sea}$ and a kinetic energy term in $E_2^{\rm sea}$, but they are absent from the sea energy due to the zero-momentum renormalization prescription used. On the other hand, terms with more than two gradients were found to be negligible [6]. Nevertheless, there still exists the possibility of ghostlike contributions in vertex functions corresponding to three or more mesons, similar to the spurious contributions existing in the two-point function. In this case the total sea energy would have to be reconsidered. The physically acceptable dispersion relations for three or more fields have been much less studied in the literature hence no answer can be given to this possibility at present.

B. Nucleon-nucleon potential

In all cases the potential at zero-four-momentum transfer yields not only the corresponding scattering length in Born approximation, but also the relevant parameters $C_{s,v}^2$ involved in the description of nuclear matter. These numbers turn out to depend dramatically on the inclusion of the Dirac sea and also, to a less extent, on the precise value of the Fermi momentum. The OBEP values $C_s^2=271$ (obtained by averaging the isospin zero and isospin one channels) and $C_v^2=192$ are closer to the ones where the Dirac sea is included [see Eqs. (38) and (39)]. In fact, in the sea included case, a good agreement between the nuclear matter results and the corresponding OBEP can be achieved by choosing a slightly different value of the Fermi momentum from that used above, namely, $k_F=1.2\ {\rm fm}^{-1}$. Similarly, the no-sea case can reproduce the OBEP values of $C_{s,v}^2$ by taking $k_F=1.4\ {\rm fm}^{-1}$. However, one should keep in mind that if heavier mesons than the low lying ones were introduced in the spirit of the OBEP, lower values of the potentials at zero momentum could be accommodated without destroying the typical values of the potentials at higher momenta. Such a framework would presumably make compatible the nucleon-nucleon and nuclear matter data including the Dirac sea.

The effect of subtracting the ghost can be best exemplified by analyzing the ω-exchange potential. Since in the region $0\leqslant q^2\leqslant m_\omega^2$ no direct experimental data are available, the extrapolation of the OBEP to that region requires further assumptions. To account for the uncertainty in the OBEP form factors in the timelike region, we adopt two extreme cases regarding the value of the ω-meson coupling constant, namely, we either trust the OBEP coupling constant at zero momentum or else at the on-shell point. In both cases we keep fixed the physical mass of the meson. We always compare the field theoretical model potentials (without phenomenological form factors) with the Bonn potential (with phenomenological form factors). In Fig. 4 we adjust the

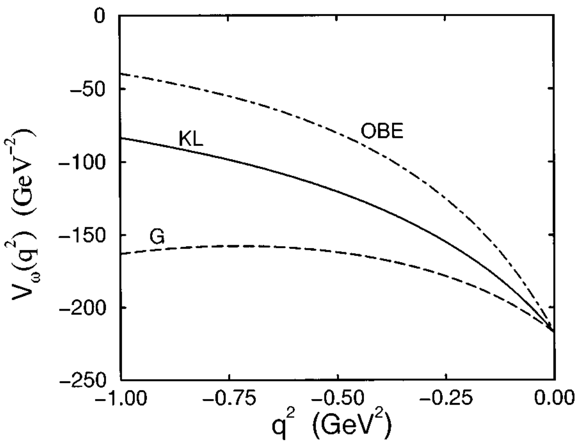

FIG. 4. ω-exchange contribution to the NN potential for the ghost subtracted (KL), ghost unsubtracted (G), and one-boson exchange (OBE) potentials. The coupling constants have been chosen in order to reproduce the value of the OBEP at zero momentum.

ω-exchange potential to the OBEP at zero-four momentum both for ghost unsubtracted (G) and ghost free (KL) potentials. This corresponds to $g_\omega^{\mathrm{G}}=9.87$, $g_\omega^{\mathrm{KL}}=11.5$, and $g_\omega^{\mathrm{OBE}}=15.9$. As we see the nonelimination of the ghost produces a sizable effect in the low-energy region used to determine the OBEP parameters ($-q^2\leqslant 0.6$ GeV^2). Note that the ghost curve will eventually display an unphysical pole at higher momentum transfers. On the other hand, the ghost-free potential is virtually identical to the free boson exchange potential in the spacelike region. The difference with the OBEP stems mainly from the phenomenological form factor in the latter.

In Fig. 5 we consider the opposite point of view, i.e., we adjust the on-shell coupling constant to the OBEP value ($g_\omega^{\mathrm{OBE}}=15.9$). Here, the effect for the ghost unsubtracted curve is even more dramatic than in the previous case since the coupling constant renormalized at zero momentum would became imaginary. This follows from the opposite slope at the origin. Also in this case the onset of the instability occurs

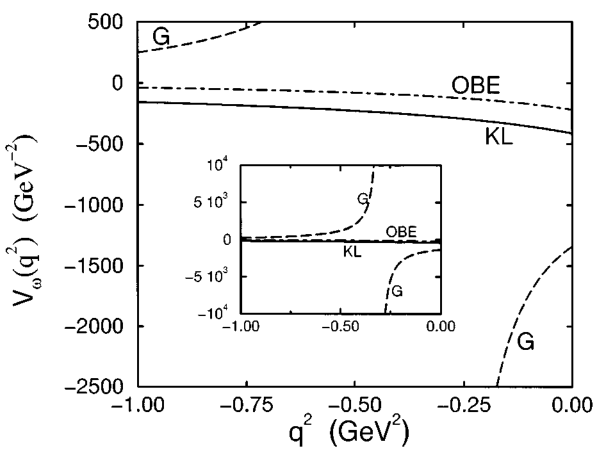

FIG. 5. ω-exchange contribution to the NN potential for the ghost subtracted (KL), ghost unsubtracted (G), and one-boson exchange (OBE) potentials. The coupling constant has been chosen in order to reproduce the pole and residue of the OBEP at the meson on-shell point (not shown in the figure).

at much lower ghost masses $M_G^v=0.305$ GeV^2, which lies well within the momentum range used in the OBEP analysis of NN scattering data.

Altogether the ghost subtracted potential resembles the OBEP much more than the ghost unsubtracted one. The similarity can be qualitatively understood by observing that the ghost-free propagator is numerically indistinguishable from the free one. In other words, most of one-loop effect goes into producing the ghost contribution. The difference between the ghost subtracted and OBE potentials stems from the inclusion of phenomenological form factors. Our results support the idea that the elimination of the ghost is not only a desirable procedure from the quantum field theoretical point of view but also produces NN potentials which compare favorably with phenomenological meson exchange fits. Similar conclusions follow for other mesons, including the σ meson, with renormalizable couplings due to the common one-loop nature of the corresponding self-energies. Particularly interesting in this regard is the pion since its coupling to the nucleon is best known, the form factor effects are negligible at the momentum scales under discussion and the extrapolation to the on-shell point is least sensitive as compared to other mesons. This should be done with the pseudoscalar coupling as required by renormalizability. Here the effect of the ghost subtraction goes in the right direction although it is less pronounced as compared to the ω case.

V. CONCLUSIONS AND FINAL REMARKS

We summarize our points. In the present paper, we have studied the consequences of eliminating the vacuum instabilities which take place in the σ-ω model. This has been done using Redmond's prescription which imposes the validity of the Källén-Lehmann representation for the two-point Green's functions. We have discussed possible interpretations to such a method and have given arguments to regard Redmond's method as a nonperturbative and nonlocal modification of the starting Lagrangian. In fact, no obstruction is met to formulate the theory within a large N expansion. Moreover, it seems that a nonperturbative definition of the σ-ω model is allowed due to the existence of nontrivial ultraviolet fixed points in the large N limit after ghost elimination. This point deserves further investigation for it opens the otherwise unexpected possibility of a lattice formulation beyond a valence approximation.

Numerically we have found that, contrary to the naive expectation, the effect of including fermionic loop corrections to the mesonic propagators (ω-shell scheme) is not small. However, it largely cancels with that of removing the unphysical Landau poles. *A priori*, this is a rather unexpected result which in fact seems to justify previous calculations carried out in the literature using a naive scheme. Actually, as compared to that scheme and after proper readjustment of the parameters to selected nuclear matter and finite nuclei properties, the numerical effect becomes rather moderate on nuclear observables. The two schemes, naive and no-ghost, are, however, completely different beyond the zero-four-momentum region and for instance predict different values for the vector meson mass. Also important is the effect upon the outcoming ω exchange contribution to the NN potentials as confronted with well-established parametrizations such as the one-boson exchange potential. Here the inclusion of the Dirac sea is favored to achieve agreement between nuclear matter data and the nucleon-nucleon potentials at low momentum. On the other hand, only the ghost-free model is able to reproduce the momentum dependence of the OBEP in a region including zero momentum and the meson pole. The theory with ghosts exhibits a very weird behavior for reasonable values of the coupling constants.

Therefore it seems that, in this model, most of the fermionic loop contribution to the meson self-energy is spurious. The inclusion of the fermionic loop in the meson propagator can only be regarded as an improvement if the Landau ghost problem is dealt with simultaneously. We have seen that the presence of Landau ghosts does not reflect on the sea energy but it is not known whether there are other spurious ghostlike contributions coming from three- or higher-point vertex functions induced by the fermionic loop.

Our calculation involves a semiclassical estimate of the Dirac sea contribution to the mean-field energy. As was shown in a previous work, the semiclassical expansion numerically converges very quickly in a way that the series can be truncated at second order. From a variational point of view, the mean-field solution can be regarded as a local minimum of the energy functional. In the theory with ghosts the occurrence of the instability in the vacuum sector—through the formation of small size inhomogeneities—suggests a lowering of the energy for finite nuclei configurations also. As a consequence, the semiclassical solution, which is obtained within a large size expansion, can hardly be interpreted as an approximation to the inexistent exact solution. On the contrary, in the ghost-free theory the local minimum obtained within a semiclassical treatment is expected to be an approximation of the true minimum. This aspect can only be made more precise after a full Hartree calculation including the negative energy discrete and continuum levels. The subtlety is that Dirac sea shell effects are missed by the semiclassical expansion and they may turn the series from convergent to asymptotic. A detailed investigation of this point is left for future research.

A model of nucleons and mesons can only be considered as an approximation to the real world since it ignores the underlying subnuclear degrees of freedom. Nevertheless it is rewarding that it is able to reproduce semiquantitatively a diversity of physical situations of direct interest to nuclear physics. At the same time this is achieved by a relativistic quantum field theory which only after elimination of the ghosts incorporates well-established principles and allows for a systematic expansion. In our view this combination of phenomenological and theoretical consistency in the σ-ω model makes it worthwhile to pursue the approach adopted in this work by inclusion of the remaining low lying mesons and baryons as well as considering higher orders in the large N expansion in a systematic way.

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APPENDIX

1. Meson self-energies in the leading 1/N expansion

As stated in the main text, the leading order in the $1/N$ expansion (N being the number of nucleon species) is achieved by considering one-fermion loop and zero-boson loop Feynman graphs in the effective action. This corresponds to compute the meson self-energies at the one-loop approximation.

For the σ meson, the bare self-energy in terms of the Lagrangian coupling constant is obtained as

$$\Pi_{B,s}(p^2;M,\xi,\varepsilon)=-i\xi^{2\varepsilon}\int\frac{d^{4-2\varepsilon}k}{(2\pi)^{4-2\varepsilon}}\mathrm{Tr}\left\{\cdot\frac{i}{\not p+\not k-M+i\eta}\times ig_s\frac{i}{\not k-M+i\eta}ig_s\cdot\right\}. \tag{A1}$$

Imposing zero-momentum renormalization we get

$$\Pi_s(p^2)=\Pi_{B,s}(p^2)-\Pi_{B,s}(0)-\Pi'_{B,s}(0)p^2 =-\frac{g_s^2N}{4\pi^2}\left\{\left(2M^2-\frac{1}{2}p^2\right)I\left(\frac{p^2}{M^2}\right)+\frac{p^2}{3}\right\}, \tag{A2}$$

where the function $I(y)$ is defined as

$$I(y)=\int_0^1 dx\ln[1-yx(1-x)-i\eta] =\begin{cases}\kappa\ln\dfrac{\kappa+1}{\kappa-1}-2, & y<0\\ 2\kappa\arcsin(\sqrt{y/4})-2, & 0<y<4\\ \kappa\ln\dfrac{1+\kappa}{1-\kappa}-2-i\pi\kappa, & 4<y,\end{cases}$$

where $\kappa=|1-(4/y)|^{1/2}$.

The ω-meson self-energy is obtained in a similar way but taking care of its Lorentz structure,

$$\Pi^{\mu\nu}_{B,v}(p^2;M,\varepsilon,\xi) =-i\xi^{2\varepsilon}\int\frac{d^{4-2\varepsilon}k}{(2\pi)^{4-2\varepsilon}}\mathrm{Tr}\left\{\frac{i}{\not p+\not k-M+i\eta}(-ig_v)\gamma^\mu\times\frac{i}{\not k-M+i\eta}(-ig_v)\gamma^\nu\right\}, \tag{A3}$$

which is highly simplified by baryonic current conservation,

$$\Pi^{\mu\nu}_{B,v}(p^2)=\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2}\right)\Pi_{B,v}(p^2). \tag{A4}$$

The explicit expression of the ω-meson self-energy renormalized at zero momentum is

$$\Pi_v(p^2)=\Pi_{B,v}(p^2)-\Pi_{B,v}(0)-\Pi'_{B,v}(0)p^2 = \frac{Ng_v^2}{12\pi^2}\left\{(2M^2+p^2)I\left(\frac{p^2}{M^2}\right)+\frac{p^2}{3}\right\}. \quad \text{(A5)}$$

2. Poles and residues

The relation between the Lagrangian mass of a boson m and its physical mass $m_{\rm sh}$ is given by

$$m_{\rm sh}^2-m^2-\Pi(m_{\rm sh}^2)=0\,, \quad \text{(A6)}$$

where the self-energy $\Pi(p^2)$ is assumed to be renormalized at zero momentum. Likewise for the coupling constants,

$$g_{\rm sh}^2=\frac{g^2}{1-\Pi'(m_{\rm sh}^2)}. \quad \text{(A7)}$$

From the expression of its self-energy given above we find that the σ-meson physical pole m_σ can be obtained in terms of the Lagrangian parameters g_s and m_s by solving the transcendental equation

$$m_s^2=m_\sigma^2-\frac{Ng_s^2}{4\pi^2}M^2\left[4-\frac{4}{3}\frac{m_\sigma^2}{M^2}+\left(-4+\frac{m_\sigma^2}{M^2}\right)\times\sqrt{\frac{4M^2}{m_\sigma^2}-1}\,\arcsin\frac{m_\sigma}{2M}\right]. \quad \text{(A8)}$$

Similarly, the equation to solve for the ω particle is

$$m_v^2=m_\omega^2+\frac{Ng_v^2}{4\pi^2}M^2\left[\frac{4}{3}+\frac{5}{9}\frac{m_\omega^2}{M^2}-\frac{2}{3}\left(2+\frac{m_\omega^2}{M^2}\right)\times\sqrt{\frac{4M^2}{m_\omega^2}-1}\,\arcsin\frac{m_\omega}{2M}\right]. \quad \text{(A9)}$$

It is interesting to note that sometimes the combination $C^2=M^2g^2/m^2$ is taken to be fixed by nuclear matter properties. This allows one to write the Lagrangian coupling constant g as a function of C and the Lagrangian mass m. Inserting the ω version of this expression into the previous equation permits us to solve the Lagrangian mass in terms of C_v and the physical ω mass m_ω. If Eqs. (A8) and (A9) are conveniently extended to the $m_{\rm sh}$-complex plane they can be used to obtain the Landau ghost masses as well [better expressions for numerical calculation are found in the main text in Eqs. (37) and (35)].

Once a Landau pole has been computed, the value of its zero-momentum residue $-R_G$ is easily obtained as

$$R_G=-[1-\Pi'(-M_G^2)]^{-1}. \quad \text{(A10)}$$

The particular expressions of this equation for the σ and ω meson are given in Eqs. (36) and (34), respectively.

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