

Identical bands and quantized alignments in superdeformed nuclei

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A method that generalizes the definition of identical bands was established. More than 100 available superdeformed bands were analyzed by means of the method and the number of identical bands as a function of criteria is presented. It is found that the quantization of incremental alignments depends very sensitively on the tolerance of the criterion for identical bands. The relation between the particle-rotor model and the present method is given. [S0556-2813(97)02404-7]

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I. INTRODUCTION

Rotational bands, or regions of bands, in different superdeformed nuclei have been found to have identical (or equivalent) transition energies to within an average of about 1 keV, much smaller than expected. This means that the rotational frequencies of the two bands are very similar because the rotational frequency (dE/dI) is approximately half the transition energy for these $E2$ transitions with $\Delta I=2$, and also implies that the dynamical moments of inertia are almost equal. These twin bands are frequently called identical bands (IB's). The fascinating phenomenon is very rare in nuclear physics and is of great interest. The first case of superdeformed IB's was found in the mass-150 region (¹⁵⁰Gd-¹⁵¹Tb, ¹⁵²Dy-¹⁵¹Tb) [1]; recently many other examples of such superdeformed rotational IB's in this mass region as well as in the mass-190 region [2] and mass-130 region, have been found. The origin and the abundance of this effect have been investigated vigorously, but no definitive interpretation has emerged.

To determine whether a pair of bands is identical or not, we have to pose a judging criterion. There has been a number of criteria used in the literature for the selection of IB's. The most common of these criteria can be defined in two ways: one is from the comparison of the dynamical moment of inertia [3,4] and another is to compare the $E2$ transition energies of two bands [5-7]. In the literature, the relationship between the γ -ray energies of two bands are frequently referred to as the "quarter point," "midpoint," and "zero point" (or "identical"), which correspond to a $\Delta E_\gamma/\delta E_\gamma$ ratio 1/4, 1/2, and 0, respectively, where ΔE_γ is the difference between the energy of the γ ray in the band considered and the nearest transition energy in the reference band, δE_γ represents the difference between the energies of the two consecutive γ -ray transitions in the reference band. In this paper, we shall pay special attention on the comparison of the $E2$ transition and introduce a more general method to identify IB's. Another current focus on IB's is the quantization of incremental alignments [2]. Its statistical regularity is another part of the present study.

II. THE METHOD

The subscripts (A, B) are used to designate different bands. For example, the energy of the $E2$ transition that

depopulates the level with spin I in band A is denoted by $E_\gamma(I_A)$. For two superdeformed bands A and B , we select regions of bands that consist of $N+1$ consecutive transition energies and compare the two quantities: $E'_\gamma(I_A)$ and $E_\gamma(I_B)$ where the $E'_\gamma(I_A)$ is defined as

$$E'_\gamma(I_A) = xE_\gamma(I_A) + (1-x)E_\gamma(I_A+2),$$

$$(I_A = I_A^0 + 2k, \quad I_B = I_B^0 + 2k, \quad k = 0, 1, 2, \dots, N). \quad (1)$$

In Eq. (1), x is a parameter restricted to $[0, 1]$, band A is considered a reference band. I_A^0 and I_B^0 are the angular momentum of the band (region) head for band A and band B , respectively. The energy of the bandhead in band B satisfies the relation $E_\gamma(I_A^0) \leq E_\gamma(I_B^0) \leq E_\gamma(I_A^0 + 2)$. $E'_\gamma(I_A) - E_\gamma(I_B)$ will take different values while x gets different values, but we could find an x which renders the quantity

$$s = \sum_{k=0}^{N-1} [E'_\gamma(I_A) - E_\gamma(I_B)]^2 \quad (2)$$

to a minimum. From $ds/dx=0$, x is worked out as

$$x = \frac{\sum_k [E_\gamma(I_A+2) - E_\gamma(I_A)][E_\gamma(I_A+2) - E_\gamma(I_B)]}{\sum_k [E_\gamma(I_A+2) - E_\gamma(I_A)]^2}. \quad (3)$$

For an appropriate N and δ (usually, $N=10$ and $\delta \sim 1-2$ keV), if there exists a pair of regions in band A and band B which satisfy the relation $\Delta E'_\gamma(i) = |E'_\gamma(I_A) - E_\gamma(I_B)| \leq \delta$ for all k and a fixed x is in the region of $[0, 1]$, we treat the two bands as identical.

For a pair of IB's, the incremental alignment [8], Δi , is defined as

$$\Delta i = 2 \frac{E_{\gamma \text{ near}} - E_\gamma(I_B)}{E_\gamma(I_A+2) - E_\gamma(I_A)}, \quad (4)$$

where $E_{\gamma \text{ near}}$ is the transition energy in band A which is the closest to $E_\gamma(I_B)$ and band A is supposed to be a reference band. From Eqs. (1), (3), and (4) and a hypothesis that

$\delta E_\gamma = E_\gamma(I_A + 2) - E_\gamma(I_A)$ is almost a constant value, we could find the relation between the parameter x and the $\overline{\Delta i}$ [the arithmetic average of Δi which is equivalent to $(1/N)\Sigma\Delta i(i)$]:

$$\overline{\Delta i} \approx 2x \quad (5)$$

if $E_{\gamma \text{ near}}$ is equal to $E_\gamma(I_A + 2)$ and

$$\overline{\Delta i} \approx 2(x - 1) \quad (6)$$

if $E_{\gamma \text{ near}}$ is equal to $E_\gamma(I_A)$. The rule for choosing Eq. (5) or (6) to calculate $\overline{\Delta i}$ is Eq. (5) may be used if $2x \in [-1, +1]$, otherwise, Eq. (6) may be used.

In the following, we give three special cases: (i) $x = 1$. In this case, the transition energies in band A are equal to those of band B to within $\delta(\text{keV})$ over a spin range of $2N\hbar$. Band A and band B are directly identical, i.e., $E_\gamma(I_A) \approx E_\gamma(I_B)$, $\overline{\Delta i} \approx 0$. (ii) $x = \frac{1}{2}$. In this case, $E_\gamma(I_B) \approx \frac{1}{2}[E_\gamma(I_A) + E_\gamma(I_A + 2)]$. It means that the energies in band B fall very close to the midpoint energies of adjacent transitions in band A over N continuous γ rays, and $\overline{\Delta i} \approx +1$ or $\overline{\Delta i} \approx -1$ (in fact, $\overline{\Delta i} = 1$ is equivalent to $\overline{\Delta i} = -1$). (iii) $x = \frac{1}{4}$ or $x = \frac{3}{4}$. In this case, $E_\gamma(I_B) \approx \frac{1}{4}E_\gamma(I_A) + \frac{3}{4}E_\gamma(I_A + 2)$ or $E_\gamma(I_B) \approx \frac{3}{4}E_\gamma(I_A) + \frac{1}{4}E_\gamma(I_A + 2)$, the energies in band B fall close to the quarter point energies of adjacent transitions in band A , $\overline{\Delta i} \approx 0.5$ or $\overline{\Delta i} \approx -0.5$. In the literature, the relationship between the γ -ray energies of the considered and reference bands described in (i), (ii), and (iii) are referred to as the ‘‘zero point,’’ ‘‘midpoint’’ and ‘‘quarter point,’’ respectively. In fact, the x calculated with Eq. (3) is approximately equal to $\Delta E_\gamma / \delta E_\gamma$ or $1 + \Delta E_\gamma / \delta E_\gamma$ due to the fact that δE_γ is almost a constant value for superdeformed bands.

However, generally, we could not confine x to only these special values $1, \frac{1}{2}, \frac{1}{4}$, or $\frac{3}{4}$. It is practically impossible that x calculated from Eq. (3) equals accurately one of those four values. On the contrary, we could investigate the quantization of incremental alignments of IB's by using x if x is not limited to $1, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}$. The advantage of the introduction of the x factor in Eq. (1) is a consideration of any relationship between the γ -ray energies of the two bands in different nuclei. Therefore, the present method to define the IB's is more general, which to a large degree combines the two most common methods, namely the requirements of near equality of the γ -ray energies E_γ and the dynamic moments of inertia $J^{(2)}$ of the two bands, into one by means of comparison between quantities $E'_\gamma(I_A)$ and $E_\gamma(I_B)$.

III. RESULTS AND DISCUSSION

We have analyzed 126 superdeformed bands in 48 nuclei which are available in the literature [9–24]. There are 20 bands in the A -130 region, 56 bands in the A -150 region, and 50 bands in the A -190 region. The rule to choosing the reference band is the band whose mass number is even is regarded as the reference band if another mass number is odd, otherwise, the band whose mass number is smaller is regarded as the reference band. Since there is no general criterion for selecting IB's, we have to use two parameters N and δ to define a criterion for IB's. Different values of N and δ will give a different number of pairs of IB's. The relation of

TABLE I. The variation of IB numbers against N and δ in the A -130, A -150, A -190 regions.

A	δ (keV)	N									
		6	7	8	9	10	11	12	13	14	
130	0.5	0	0	0	0	0	0	0	0	0	0
	1.0	3	0	0	0	0	0	0	0	0	0
	1.5	5	2	0	0	0	0	0	0	0	0
	2.0	9	5	4	1	0	0	0	0	0	0
	2.5	12	9	6	4	1	0	0	0	0	0
150	3.0	13	12	7	5	3	2	1	0	0	0
	0.5	14	6	3	2	0	0	0	0	0	0
	1.0	68	36	23	10	8	6	3	3	2	2
	1.5	141	75	51	40	22	12	7	5	2	2
	2.0	220	130	79	58	37	28	17	7	2	2
190	2.5	285	191	124	85	59	39	30	15	5	5
	3.0	371	250	164	112	82	52	39	24	12	12
	0.5	40	20	9	7	4	1	0	0	0	0
	1.0	189	104	54	30	14	8	5	3	1	1
	1.5	328	219	118	70	39	19	11	6	2	2
190	2.0	416	290	169	111	68	38	17	7	4	4
	2.5	500	349	212	139	96	56	20	8	5	5
	3.0	558	399	250	166	115	73	30	8	6	6

IB pair numbers to N and δ is shown in Table I. It is apparent that the distributions of IB's in the three mass regions are different. In the mass-130 region, only 13 pairs of IB's are found under a rather tolerant criterion ($N=6$ and $\delta=3.0$ keV) and no IB's (zero) are found under many other strict criteria, while the number of IB's under the same criterion $N=6$, $\delta=3.0$ keV for the A -150 region and the A -190 region is as large as 371 and 558, respectively. There are also some differences between the A -150 region and the A -190 region. The number of IB's in the A -190 region is more than that in the A -150 region when $N \leq 11$, but most numbers of IB's in the A -190 region are less than that in the A -150 region when $N \geq 12$. The latter case may result from the fact that the collectivity of the superdeformed bands in the A -150 region is stronger than that in the A -190 region, and thus more γ -ray transitions for a rotational band in the A -150 region can be detected experimentally. Actually, the average number of the observed γ -ray transitions is 16 for the A -150 and 12 for the A -190 superdeformed rotational bands. Of course, the number of IB's in all three mass regions increases with a more tolerant criterion, i.e., a larger δ and/or a smaller N .

Though the occurrence of IB's is related to the complicated cancellation among various factors, such as moment of inertia, deformation, pairing correlation, and rotation, some insights can be gained regarding the behavior of the moment of inertia. For example, γ -ray energies should be scaled with the moment of inertia, which is proportional to $A^{5/3}$ for a rigid body. A mass difference of one unit leads to a change in the moment of inertia and consequently a change in the $|\delta E_{\gamma 1} - \delta E_{\gamma 2}|$ by ~ 1.1 keV in the $A \sim 130$ region, ~ 0.75 keV in the $A \sim 150$ region and ~ 0.4 keV in the $A \sim 190$ region in the classical limit. Thus different values of δ for defining IB's should be used for different mass regions. In other words, if we select IB's with the same δ , more pairs of IB's are expected for a large mass region, as shown in Table

TABLE II. The proof of the relation between x and an incremental alignment. Band A is a reference band, $\Delta i'$ is calculated by Eqs. (5) and (6), $\overline{\Delta i}$ is the average incremental alignment for a pair of IB's.

Band A	Band B	x	$\Delta i'$	$\overline{\Delta i}$	$ \Delta i' - \overline{\Delta i} $
$^{146}\text{Gd}(2)$	$^{148}\text{Gd}(1)$	0.2335	0.4670	0.4641	0.0029
$^{147}\text{Gd}(2)$	$^{149}\text{Gd}(6)$	0.8686	-0.2628	-0.2614	0.0014
$^{146}\text{Gd}(2)$	$^{149}\text{Gd}(5)$	0.0184	0.0368	0.0429	0.0061
$^{195}\text{Pb}(2)$	$^{197}\text{Bi}(1)$	0.8586	-0.2828	-0.2821	0.0007
$^{194}\text{Tl}(2)$	$^{195}\text{Pb}(3)$	0.5266	-0.9468	-0.9473	0.0005
$^{194}\text{Hg}(3)$	$^{195}\text{Pb}(3)$	0.6180	-0.7640	-0.7694	0.0054

I. In the following discussion, N will be restricted to 10 and δ remains a parameter.

We have given a relation between x and $\overline{\Delta i}$ in Eqs. (5) and (6) which is illustrated as an example in Table II. It is impressive that the difference between $\overline{\Delta i}$ and $2x$ or $2(x-1)$ is very small, less than 0.01, therefore $\overline{\Delta i}$ can be replaced by $2x$ or $2(x-1)$.

It has been noticed that the incremental alignment of IB's which varies around 0, ± 0.5 , ± 1.0 shows the feature of quantization since 1990 [2], but there is no demarcation line between the quantization and nonquantization of the incremental alignment of IB's. Because the incremental alignments vary against the transition energies and $\overline{\Delta i}$ cannot be precisely 0, ± 0.5 , ± 1.0 , we divide the region $[-1, +1]$ into two parts: a quantized region

$$Q = [-1, -\frac{7}{8}] \cup (-\frac{5}{8}, -\frac{3}{8}] \cup (-\frac{1}{8}, \frac{1}{8}] \cup (\frac{3}{8}, \frac{5}{8}] \cup (\frac{7}{8}, 1]$$

which is around 0, ± 0.5 , ± 1.0 and an unquantized region U which is centered at ± 0.25 , ± 0.75 . A pair of IB's is quantized only if its $\overline{\Delta i}$ is within region Q , otherwise it is unquantized. Such a division treats the quantized and unquantized IB's equally. The distributions of $\overline{\Delta i}$ in the range $[-1, +1]$ are shown in Fig. 1. Figure 1(a) shows the distribution of the incremental alignments, $\overline{\Delta i}$, which are calculated from 26 pairs of IB's found with the criterion $\delta=1.1$ keV, and Fig. 1(b) is as the same plot as Fig. 1(a), but for 105 pairs of IB's found with $\delta=2.0$ keV. It can be seen by counting the points in Fig. 1(a) that 20 incremental alignments are located in the quantized region, Q (shaded area), while only 6 in the unquantized region, U . The ratio $N_Q/N_U=3.33$, where N_Q and N_U are the numbers of incremental alignments in the Q and U regions, respectively. In Fig. 1(b) there are 54 incremental alignments located in the Q region and 51 in the U region, the ratio N_Q/N_U is approximately equal to 1. Thus, the ratio N_Q/N_U is strongly related to the strictness of the criterion for identifying IB's. Furthermore, the relationship between the γ -ray energies of the IB's is also classified in Fig. 1. For example, Fig. 1(a) shows that the 16 IB's have "quarter point" relationship since their x factors are close to $\frac{1}{4}$ or $\frac{3}{4}$, corresponding to Eqs. (6) and (7). And only one "zero point" and three "mid-point" IB's are found from Fig. 1(a). It is noticed that the fraction of "zero point" IB's increases when the criterion goes from $\delta=1.1$ to 2.0 keV [see Fig. 1(b)].

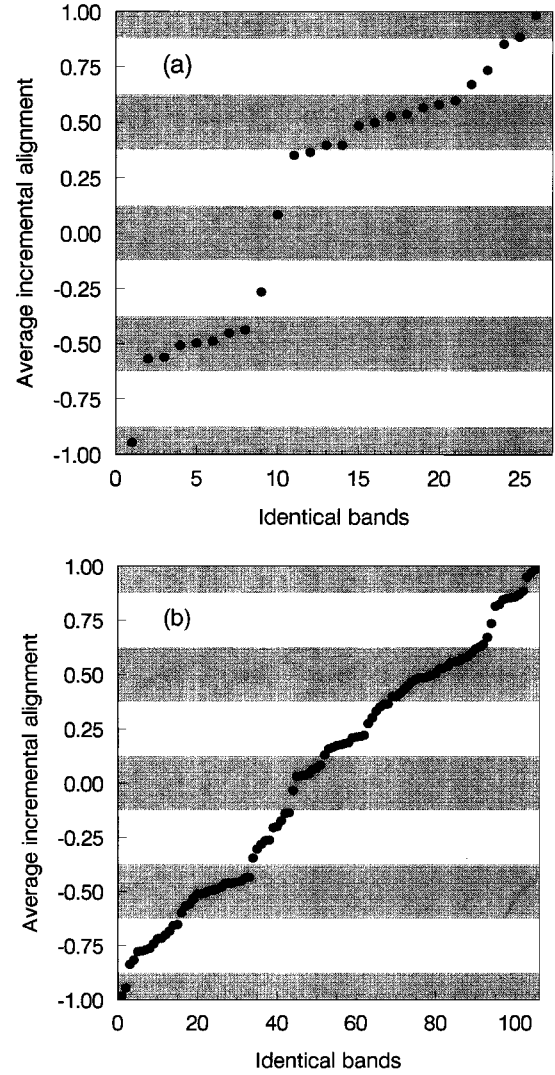


FIG. 1. The distribution of average incremental alignments in $[-1, +1]$ (a) 26 IB's when $N=10$, $\delta=1.1$ keV, (b) 105 IB's when $N=10$, $\delta=2.0$ keV.

Figure 2 shows the relation between the incremental alignment and the criterion, in which the ratio N_Q/N_U is plotted as a function of δ . The ratio $N_Q/N_U \approx 1$ when δ is larger than 1.8 keV, this indicates that the incremental alignments are almost uniformly distributed in $[-1, +1]$ and the quantization of incremental alignments is not clear. The ratio N_Q/N_U increases with decreasing δ , and it goes up to 5 when $\delta=0.9$ keV, where $N_Q=15$ and $N_U=3$. It can be seen from Fig. 2 that the ratio N_Q/N_U increases drastically and the quantization of incremental alignments of IB's becomes clear when $\delta < 1.4$ keV from the statistical point of view. It is interesting to notice that there exists a critical value δ_c defined as a value at which $N_Q/N_U=2$, when $\delta < \delta_c$, the

TABLE III. The relation between N and δ_c which is defined as a value at which $N_Q/N_U=2$.

N	6	7	8	9	10	11	12	13
δ_c (keV)	0.344	0.400	0.822	0.950	1.35	1.52	2.80	3.90

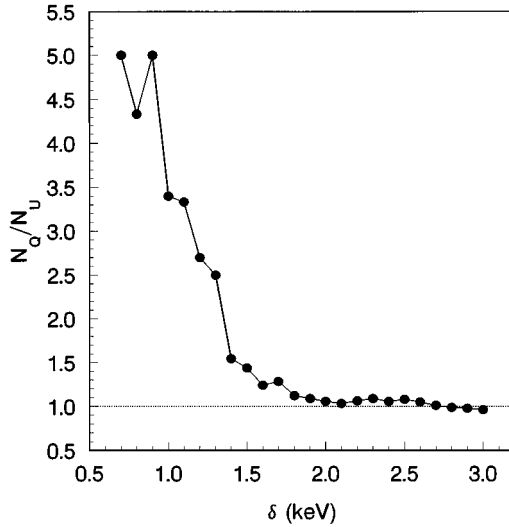


FIG. 2. The ratio of the number of quantized IB's to the number of unquantized IB's varies against δ if the IB's are selected from the standpoint of γ transition energies.

$N_Q/N_U \geq 2$, i.e., the quantized IB's dominate the selected IB's. Here, we confine N to be 10. It is obvious that the

larger N is, the greater δ_c will take. In the following, we give Table III which describes the relation between N and δ_c . Unfortunately, δ_c is just at the limit of the resolution of the current detector.

The IB selection method described above can be related to the particle-rotor model (PRM) [25]. For large deformations where the level splitting of a shell becomes large and the Coriolis interaction is relatively small, one can employ the PRM in which the valence particles with angular momentum \vec{j} coupled to a deformed core with angular momentum \vec{R} and a moment of inertia J follow the rotation of the core adiabatically. In such a strong coupling limit, the energy spectrum of an odd- A nucleus with an axial symmetry in first-order perturbation theory can be given by

$$E_{IK} = \epsilon_K + \frac{1}{2J} \left[I(I+1) - K^2 + a \left(I + \frac{1}{2} \right) (-1)^{I+1/2} \delta_{K,1/2} \right], \quad (7)$$

where $K = \Omega$ is the projection of \vec{j} on the symmetry axis, I is the total angular momentum of the nucleus, ϵ_K is the intrinsic excitation energy, and a is the decoupling parameter of the intrinsic configuration. From expression (7), one can obtain three special relations between the γ -ray energies of the nucleus and the core as in the following:

$$\begin{aligned} \text{(i)} \quad E_\gamma \left(I = R \pm \frac{1}{2} \right) &= E_\gamma^c(R) \quad \text{for } a = +1, K = \frac{1}{2}, \\ \text{(ii)} \quad E_\gamma \left(I = R \pm \frac{1}{2} \right) &= \frac{1}{2} [E_\gamma^c(R) + E_\gamma^c(R \pm 2)] \quad \text{for } a = -1, K = \frac{1}{2}, \\ \text{(iii)} \quad E_\gamma \left(I = R \pm \frac{1}{2} \right) &= \frac{3}{4} E_\gamma^c(R) + \frac{1}{4} E_\gamma^c(R \pm 2) \quad \text{for } a = 0, K \neq \frac{1}{2}, \end{aligned} \quad (8)$$

where E_γ is the transition energy in the odd- A nucleus, E_γ^c is the transition energy in the core. When the decoupling parameter a takes any other values, these special relations may be generalized as

$$E_\gamma(I) = x E_\gamma^c(R') + (1-x) E_\gamma^c(R' + 2), \quad (9)$$

where $R' = I - \frac{1}{2}$ or $I - \frac{3}{2}$ and x is restricted to $[0, 1]$. In this situation, $x = 1$ corresponds to $a = +1$, $K = \frac{1}{2}$, $x = \frac{1}{2}$ to $a = -1$, $K = \frac{1}{2}$ and $x = \frac{1}{4}$ or $\frac{3}{4}$ to $a = 0$.

For a comparison between the γ -ray energies of two bands, the transition energies of the reference bands may be constructed as Eq. (1) according to Eq. (9). Therefore, the abundance of IB's found by the present method reflects that the strong coupling picture works well for the description of superdeformed bands. Indeed, experimental studies in the $A \sim 150$ and $A \sim 190$ regions do reveal the presence of almost perfect strongly coupled structures associated with high- K excitation [26]. It is interesting to notice that a stricter criterion with a smaller δ leads to a large fraction of IB's whose incremental alignments are quantized, and this may be related to the strong coupling structure with the valence par-

ticles moving in some particular orbits. However, the challenge issue posed by the IB's remains the primary question to be investigated.

IV. SUMMARY

In summary, a more general method for selecting IB's is suggested by introducing the reference band constructed based on the strong coupling theory. The criterion in the method combines the requirements of similar transition energies and the dynamic moments of inertia into one, δ . A very good approximate relation between the x factor and the average incremental alignment is found. The category of the relations between the transition energies of IB's can be given in terms of the x factor. The quantization of the incremental alignments is found to be a very sensitive function of the tolerance of the criterion. For a given N , a small δ favors the quantization of the incremental alignments. The critical δ_c value for quantization locates, unfortunately, on the brink of the resolution of the current detectors.

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