Spectroscopic factors for bound *s*-wave states derived from neutron scattering lengths

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A simple and model-independent method is described to derive neutron single-particle spectroscopic factors of bound *s*-wave states in $^{A+1}Z = {}^{A}Z \otimes n$ nuclei from neutron scattering lengths. Spectroscopic factors for the nuclei ^{13}C, ^{14}C, ^{16}N, ^{17}O, ^{19}O, ^{23}Ne, ^{37}Ar, and ^{41}Ar are compared to results derived from transfer experiments using the well-known disorted wave Born analysis and to shell model calculations. The scattering length of ^{14}C is calculated from the $^{15}C_{g.s.}$ spectroscopic factor. [S0556-2813(97)04503-2]

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Spectroscopic factors (SF's) are an important ingredient for the calculation of direct transfer reaction cross sections in the distorted wave Born approximation (DWBA) and capture reaction cross sections in the direct capture (DC) model. Usually, SF's can be determined experimentally by the ratio of the measured transfer reaction cross section to the cross section calculated with the DWBA,

$$C^2S_i = \sigma_i^{\rm expt}/\sigma_i^{\rm DWBA}, \tag{1}$$

for each final state i. In the case of neutron transfer mainly (d,p) reactions were analyzed to determine the neutron single-particle SF. This determination has relatively large uncertainties because the optical potentials of both the entrance and exit channels have to be known accurately for a reliable DWBA calculation. Usually one obtains SF's with uncertainties of up to 20%. However, in many cases systematic deviations exceeding the claimed uncertainties can be found when the results of various experiments [different transfer reactions like (d,p), (^{4}He,^{3}He), (^{7}Li,^{6}Li), etc., at different energies] are compared (see, e.g., Table 8 of Ref. [1] or Table II of Ref. [2]).

Recently, our group showed that a model-independent method exists to extract SF's from the thermal neutron capture cross section [3]:

$$C^2S_i = \sigma_i^{\rm expt}(n_{\rm th},\gamma)/\sigma_i^{\rm DC}(n_{\rm th},\gamma). \tag{2}$$

This method has very limited uncertainties because at thermal energies the neutron optical potential can be adjusted properly to the scattering length. However, because the thermal (n,γ) cross section is dominated by incoming *s* waves and $E1$ transitions, this procedure works well only for bound p waves in the residual nucleus.

In this work we present a simple and model-independent procedure for the extraction of SF's of bound *s* waves from the scattering length b. In this work we use the free nuclear scattering length b, which is related to the bound scattering length by $b = (b_{\rm bound} - Zb_{ne}) \cdot A/(A+1)$ with the neutron-electron interaction length $b_{ne} = (-1.38 \pm 0.03) \times 10^{-3}$ fm [4,5]. These SF's are very important for the calculation of the (n,γ) cross section at astrophysically relevant energies in the order of several keV where transitions from incoming p waves to bound *s* and *d* waves become comparable to the transitions from the incoming *s* wave to bound p waves [6,7].

The method can be applied to light and intermediate nuclei with only one bound *s* wave or a strong *s*-wave state close to the neutron separation threshold. In these cases the scattering length can be interpreted as the very broad positive-energy wing of the *s*-wave subthreshold state. The comparison of the calculated width assuming a single-particle configuration and the experimental width of this subthreshold state leads to the SF

$$C^2S = \Gamma^{\rm expt}/\Gamma_{\rm sp}^{\rm calc}. \tag{3}$$

This calculation is performed in the following way.

First, the wave function of the subthreshold state is calculated using a neutron-nucleus optical potential. The potential strength (parameters V_0 or λ; see below) is adjusted to reproduce the binding energy of the bound state (taking into account the Pauli principle by $q = 2n + l$ where q, n, and l are the oscillator, radial node, and angular momentum quantum numbers). In this work both Woods-Saxon (WS)

$$V_{\rm WS}(r) = V_0[1 + \exp(r - R/a)]^{-1}, \tag{4}$$

with $R = R_0 A_T^{1/3}$, $R_0 = 1.25$ fm, and $a = 0.65$ fm, and folding potentials [8–10]

$$V_F(r) = \lambda \int\int \rho_P(r_P)\rho_T(r_T) v_{\rm eff}(s,\rho,E) d^3r_P d^3r_T \tag{5}$$

were used; the results practically do not depend on the chosen parametrization of the optical potential. In this sense this method is model independent.

Second, we calculate the single-particle scattering length $b_{\rm sp}^{\rm calc}$ and the width $\Gamma_{\rm sp}^{\rm calc}$ from the optical potential which was adjusted to the bound state energy E_B (note that $E_B < 0$). The scattering phase shift $\delta_{l=0}(E)$ is related to the scattering length b and the width of the resonance by the following well-known equations:

$$k \cdot b = -\sin[\,\delta_{l=0}(E = 25\ \text{meV})] \tag{6}$$

and

TABLE I. Spectroscopic factors of bound s-wave states of ^{13}C, ^{14}C, ^{16}N, ^{17}O, ^{19}O, ^{23}Ne, ^{37}Ar, and ^{41}Ar derived from the scattering length, from different transfer experiments, and from the shell model.

Nucleus	J^π	E_x (keV)	$q=2n+l$	C^2S [a]	C^2S^{expt}	Ref.	C^2S^{calc}	Ref.
^{13}C	$1/2^+$	3089	2	0.966 ± 0.015	0.65 – 1.2	[18–20]	0.85	a
^{14}C	1^-	6094	2	0.894 ± 0.020	0.43 – 0.87	[21,22,1]	0.76 – 0.85	a, [23–25]
^{14}C	0^-	6903	2	0.931 ± 0.020	1.02	[21]	0.96 – 1.00	a, [23–25]
^{16}N	0^-	120	2	1.012 ± 0.020	≈0.46	[26]	0.95	[16]
^{16}N	1^-	397	2	0.969 ± 0.020	≈0.52	[26]	0.96	[16]
^{17}O	$1/2^+$	870	2	0.989 ± 0.010	0.45 – 1.96	[27–32,2]	1.0	a, [2]
^{19}O	$1/2^+$	1472	2	0.919 ± 0.020	0.86 – ≈1	[2,33]	0.7 – 0.9	a, [2,34]
^{23}Ne	$1/2^+$	1017	2	0.698 ± 0.030	0.37 – 0.70	[35–37]	0.654	a
^{37}Ar	$1/2^+$	8789	4	0.530 ± 0.010	–		–	
^{41}Ar	$1/2^+$	6098	4	0.180 ± 0.010	–		–	

[a]This work.

$$\tan[\delta_{l=0}(E)] = \frac{\Gamma(E)}{2(E_B - E)}, \tag{7}$$

where k is the wave number of the s wave at $E=25$ meV.

Third, the experimental width Γ^{expt} is calculated from Eqs. (6) and (7) using the experimentally determined scattering length b^{expt} [11,12]. The SF which is a measure of the single-particle strength is calculated from Eq. (3) by the ratio of Γ^{expt} and $\Gamma^{\text{calc}}_{\text{sp}}$ at the thermal energy $E=25$ meV.

Our new results are listed in Table I. The main uncertainties in this procedure are given by the experimental uncertainties of the experimental scattering lengths. The uncertainties from different potential parametrizations are practically negligible. The results agree well with different transfer experiments.

The theoretical SF's were calculated from the shell model with the code OXBASH [13]. Since we need the spectroscopic factors for a $2s_{1/2}$ transition, one-particle one-hole excitations have to be taken into account for the C isotopes. We used the interaction WBN of Warburton and Brown [14] for this purpose. For the ^{16}N states we took the interaction ZBM I [15]; the results for ^{16}N were already published in Ref. [16]. The spectroscopic factors for the O and Ne isotopes were calculated with the Universal *sd*-shell interaction of Wildenthal [17]. The shell model SF's agree well with the experimental SF's derived from scattering lengths.

In the case of $^{14}\text{C}=^{13}\text{C}\otimes n$ the SF's for two bound s-wave states ($J^\pi=0^-,1^-$) can be determined, because this procedure can be applied to both channel spins $S=0$ and $S=1$. The relevant scattering lengths can be derived from the coherent and incoherent scattering lengths on ^{13}C. The same arguments hold for the case $^{16}\text{N}=^{15}\text{N}\otimes\text{n}$. However, for the nucleus ^{16}N the agreement between the experimental SF's derived from our method and from a (d,p) transfer experiment is quite poor whereas the theoretical SF's agree well with our new SF.

In the cases of $^{37}\text{Ar}=^{36}\text{Ar}\otimes n$ and $^{41}\text{Ar}=^{40}\text{Ar}\otimes n$ subthreshold resonances at $E=-10$ keV ($E_x=8778$ keV) and $E=-1$ keV ($E_x=6098$ keV) [4] determine the scattering lengths. Unfortunately, the relatively small SF's of these states were not determined experimentally [38,39]; a calculation of these SF's is very difficult because the neutron is located in the $3s_{1/2}$ shell.

Finally, for the system $^{15}\text{C}=^{14}\text{C}\otimes n$ we can invert the procedure to predict the experimentally unknown scattering length of ^{14}C from the SF's of the ^{15}C ground state ($1/2^+$). The SF is well known both from transfer experiments [40,41] and from the shell model: We adopt $C^2S=1.0\pm0.05$. The resulting scattering length is $b=7.257\pm0.369$ fm. An experimental verification of this prediction is desirable.

In conclusion, this method for the calculation of SF's works well for several light and intermediate nuclei. Because of the model independence, the SF's presented in this work can be used as a benchmark for SF's derived from transfer reactions or determined by shell model calculations.

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