Subthreshold resonance in the ${}^6\text{Li}(d,\alpha){}^4\text{He}$ reaction and its astrophysical implications

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(Received 4 June 1996; revised manuscript received 5 November 1996)

The total cross section of the ⁶Li(d , α)⁴He reaction has been measured for deuteron energies between 50 and 180 keV. From a detailed distorted-wave Born approximation analysis of the angular distributions and the excitation function up to 1 MeV it was possible to determine the strength of a subthreshold resonance that dominates the cross section at sub-Coulomb energies and contributes significantly to the increase of the astrophysical *S* factor at low energies. Consequently the electron screening energy we have determined for the 6 Li(d, α)⁴He reaction is considerably smaller than the value given in previous works which overestimate the theoretical predictions. In addition the stellar reaction rate has been calculated up to a temperature of $T₉=3$. $[$ S0556-2813(97)00403-2]

PACS number(s): 25.45.Hi, 24.30.Gd, 26.35.+c, 27.20.+n

I. INTRODUCTION

The recently developed inhomogeneous models of primordial nucleosynthesis $[1-4]$ and its predictions of significant abundances of nuclides heavier than ⁴He have stimulated many low-energy studies of nuclear reactions on light nuclei $[5]$. In contrast to the standard big bang model the inhomogeneous models assume a fluctuation of baryon density. Consequently, the primordial nucleosynthesis proceeds differently in high-density, neutron-poor and low-density, neutron-rich regions. The relative high abundance of ${}^{2}H$ in neutron-rich regions $[6]$ (for some parameters of the inhomogeneous models comparable to that of 4 He) causes that the deuteron induced reactions can play an important role in creation and destruction of chemical elements in the early universe, as has lately been shown for 7 Li and 8 Li isotopes $[7-9]$.

From the experimental point of view the determination of the astrophysically relevant reaction rates needs knowledge of the reaction cross section at energies far below the Coulomb barrier which therefore is strongly reduced due to the rapidly decreasing penetrability. The experimental situation gets even more difficult at energies low enough that the electron screening effect contributes significantly $[10]$. The ambiguities connected to the extraction of the cross section for bare nuclei as it is required for astrophysical applications can be limited if the value of the screening energy is known. The enhancement of the cross section due to electron screening has recently been verified experimentally for several light nuclear systems $[11–15]$. The results for the screening energies generally overestimate the theoretical values $[16]$. In the case of nuclear reactions on lithium targets a significant discrepancy has also been observed. Particularly the screening energy derived from the ⁶Li(d, α)⁴He reaction amounting to 340 ± 110 eV [12,13] overestimates the theoretical value of 186 keV calculated within the Born-Oppenheimer approximation $[17]$. This difference could be due to the fitting procedure (polynomial fit to the measured cross section) used to derive the experimental value for the screening energy.

In order to get more precise information about the electron screening contribution as well as the cross section for bare nuclei an investigation of the reaction mechanisms

at such low energies is necessary. Our recent study $[18]$ of the deuteron stripping reactions on ${}^{6}Li$ suggests that the ⁶Li(d, α)⁴He reaction may be dominated at low energies by a broad 2^+ subthreshold resonance in the compound nucleus ⁸Be. This resonance is composed of two isospin mixed states that cause a decrease of the branching ratio between the mirror stripping reactions (d, n) and (d, p) on ⁶Li for incident energies below 200 keV. One of these mixed states predominantly with an isospin 0 should have a large α width and should be observed in the ⁶Li (d, α) ⁴He reaction. However, new measurements of the cross section ratio between the ⁶Li(*d*, α)⁴He and ⁶Li(*d*,*p*)⁷Li reactions [19] do not provide any arguments for the suggested resonance reaction mechanism. The measured ratio was found to be independent of the incident energy and amounts to 5.3 in favor of the α channel. This agrees with the data obtained in older measurements at higher energies $[20]$.

Consequently the deuteron induced reactions on ${}^{6}Li$ have been described as nonresonant: either within *R*-matrix theory $(energy-independent parametrization)$ [21] or in the frame of distorted-wave Born approximation $(DWBA)$ theory [22]. Both cross-section calculations were not able to describe the experimental excitation functions at sub-Coulomb energies.

To investigate the reaction mechanism and its astrophysical consequences an analysis demands a consistent data set over a wide deuteron energy range. There exist the precise cross-section and angular distribution data by Elwyn *et al.* $[20]$ up to 1 MeV on the one hand and the data by Engstler *et al.* [12,13] down to very low energies on the other. Due to systematic uncertainties up to 40% for part of the data by Engstler *et al.*, these data had to be normalized by the authors to the data by Elwyn *et al.* For our analysis it was necessary to confirm this overall data set. Therefore we performed a thin target measurement described below of the ⁶Li(d, α)⁴He reaction for deuteron energies between 50 and 180 keV relevant for astrophysical applications. Our analysis then will show that the low-energy cross section for this reaction is a sum of a direct mechanism component and a subthreshold resonance contribution. The evaluation of the resonance contribution allows to determine the astrophysical *S* factor at zero deuteron energy as well as the stellar reaction

FIG. 1. Experimental setup.

rate, yielding an appropriate value for the electron screening energy.

II. EXPERIMENTAL PROCEDURE

The experimental setup is shown in Fig. 1. The magnetically analyzed deuteron beam from the cascade accelerator was impinged on a thin ⁶LiF target (10 μ g/cm² on a 10 μ g/cm² carbon backing, corresponding to 2–3 keV deuteron energy loss) focused to a spot of about 2 mm in diameter. The 8.2 MeV α particles were detected by a 100-mm² Canberra PIPS-detector being placed at an angle of 150° with respect to the beam in 10-cm distance from the target. Since the experiment was designed to also check the possibility of detecting the recoil nuclei from the (d,n) and (d,p) reactions for future experiments there was no protective foil in front of the detector. To avoid pileups from elastically scattered deuterons we therefore used for spectroscopic purpose a fast timing amplifier connected to a stretcher (Ortec 542) that adapted the pulse length for analog-to-digital converter processing. The latter determined our pulse-pair resolution to be better than 100 ns. A typical spectrum obtained in the present experiment is presented in Fig. 2. For a correct mea-

FIG. 2. Charged particle spectrum measured at $E_d^{\text{lab}} = 160 \text{ keV}$. The peaks labeled p_0 and p_1 are from the ⁶Li(*d*,*p*)⁷Li reaction and ⁷Li stands for the recoil nucleus. The *dd* line corresponds to elastic deuteron scattering.

FIG. 3. Astrophysical *S* factors for the ⁶Li(d, α)⁴He reaction from the present work and from previous measurements $[12,13,20]$; the data from Refs. $[12,13]$ do not include systematic errors.

surement of the beam current it was necessary to take care of deuterons (up to 50%) that have changed their charge state when traversing the target. Therefore the target was electrically connected to a surrounding cylinder box. The true beam current was the sum of the Faraday cup plus the cylinder current (see Fig. 1).

III. RESULTS

The measured cross sections for the ⁶Li(d, α)⁴He reaction assuming an isotropic angular distribution for deuteron energies below 200 keV $[20]$ were converted into astrophysical *S* factors according to the relation

$$
S(E) = E_{\text{c.m.}} \sigma(E) \exp(2\pi \eta), \tag{1}
$$

where $2\pi\eta=31.29Z_1Z_2(\mu/E_{\text{cm}})^{1/2}$ is the Sommerfeld parameter $(Z_1$ and Z_2 are the charge number of the projectile and the target nucleus, respectively, μ is the reduced mass in amu, and $E_{\text{c.m.}}$ the center-of-mass energy in keV). Our results including all systematic errors amounting up to 10% mainly due to target thickness and beam energy uncertainties are presented in Fig. 3 together with the data from the other authors $[12,13,20]$. All measurements are in good agreement which confirms the consistency of the data set over the whole energy range. A strong increase of the *S* factors with decreasing deuteron energy can be observed.

As in the case of the ${}^6\text{Li}(d,p){}^7\text{Li}$ and ${}^6\text{Li}(d,n){}^7\text{Be}$ reactions [18] we expect a significant contribution of a broad 2^+ subthreshold resonance in the low-energy region (below 60 keV the influence of the electron screening effect must additionally be taken into consideration). This *s*-wave resonance, according to Ajzenberg-Selove $[23]$, has a rather large width of 800 keV and lies 80 keV below the reaction threshold. Other subthreshold resonances are not expected to contribute significantly mainly because of their small α -particle partial widths or their unfavorable J^{π} assignments $|23|$.

In order to determine the resonance contribution we have first calculated the direct reaction component in the frame of zero range DWBA with a conventional finite range parameter of 0.65 and a nonlocality (local energy approxi-

TABLE I. Optical model and bound state parameters for the DWBA calculations.

	Initial channel	Final channel	Bound state
V (MeV)	-115	-132	fitted (-70)
r_V (fm)	0.9	1.07	1.25
a_V (fm)	0.9	0.669	0.695
W (MeV)		-3.0	
r_{W} (fm)		2.07	
w_W (fm)		0.5	
$4W_D$ (MeV)	26.8		
r_D (fm)	2.46		
a_D (fm)	0.45		

mation β =0.54) parameter [24]. Two angular momentum transfers were considered $(l=0,2)$ with the corresponding spectroscopic factors $C^2S = 1.032$ and 0.087, respectively [22]. The optical model parameters for the deuteron channel are from [25] and for the α - α channel from [26]. They are listed in Table I together with the bound state parameters [27]. To get the zero range strength factor D_0^2 the experimental angular distributions $[20]$ for six deuteron energies between 369 and 975 keV have been fitted by the expression

$$
(d\sigma/d\Omega)_{\text{exp}} = D_0^2 (d\sigma/d\Omega)_{\text{DWBA}} + \sigma_c, \qquad (2)
$$

where $(d\sigma/d\Omega)_{\text{DWBA}}$ is the theoretical angular distribution for the direct component and σ_c a constant which reflects the isotropic resonance contribution. The results are summarized in Table II and shown in Fig. 4. As is expected, the values for D_0^2 are constant within errors, the weighted average being $D_0^2 = (2.1 \pm 0.1)$ 10³ MeV² fm³. Subtracting the calculated direct reaction contribution from the measured *S*-factor values one gets the compound resonance component which can be fitted by a Lorentz curve (*s*-wave resonance). The result is presented in Fig. 5. With a fixed value of the resonance width Γ =800 keV the fitting procedure yields the resonance energy $E_R = (-50 \pm 20)$ keV. Only the experimental points for deuteron energies greater than 60 keV have been taken into account, since for smaller energies the electron screening effect cannot be neglected (see Sec. IV). In Fig. 6 the DWBA calculation result for the direct component and the sum curve of the direct component and the resonance contribution describing the data are presented.

In summary we can conclude that the 2^+ subthreshold resonance plays an important role for understanding the excitation function of the ⁶Li(d, α)⁴He reaction at low energies. In Fig. 7 the total cross section for this reaction is shown. A comparison with the *S*-factor curves (Fig. 6) allows us to state that a prominent structure in the cross section at deuteron energies around 600 keV is a result of the subthreshold resonance, the strength of which is shifted to higher deuteron energies due to the Coulomb penetration effect.

The derived resonance energy $E_R = (-50 \pm 20)$ keV agrees with the value $E_R = (-100 \pm 80)$ keV obtained from the analysis of the ${}^6\text{Li}(d,p){}^7\text{Li}$ and ${}^6\text{Li}(d,n){}^7\text{Be}$ reactions $[18]$ and confirms the importance of the resonance contribution for both stripping reactions.

TABLE II. Parameters of the fit to the angular distributions $[20]$ according to relation (2) .

$E_{d,lab}$ [keV]	D_0^2 [10 ³ MeV ² fm ³]	σ_c mb
975	2.1 ± 0.2	1.59 ± 0.05
875	2.3 ± 0.2	1.70 ± 0.05
773	1.9 ± 0.3	1.95 ± 0.07
673	2.0 ± 0.3	2.04 ± 0.09
570	1.7 ± 0.4	2.08 ± 0.11
369	1.1 ± 0.7	1.56 ± 0.15

IV. ELECTRON SCREENING EFFECT AND STELLAR REACTION RATES

For astrophysical applications it is important to extrapolate the measured *S*-factor values to zero deuteron energy *S*(0). However, the direct experimental determination of this value might be difficult because at very low energies, the measured cross sections do not represent the bare nuclei values. The cross section is increased due to the screening effect arising from the electrons surrounding the target nuclei. In the simplest picture, the gain of electronic binding energy (called screening energy U_e) can be transferred to the relative motion of the colliding nuclei which then penetrate the Coulomb barrier with a slightly higher energy $E_{\text{eff}} = E + U_e$ $[10]$. The enhancement of the cross section (or of the astrophysical S factor) is given using the relation

$$
f = \frac{\sigma(E + U_e)}{\sigma(E)} = \frac{S(E + U_e)}{S(E)} \frac{E}{E + U_e} \frac{\exp[-2\pi \eta (E + U_e)]}{\exp[-2\pi \eta (E)]}
$$

$$
\approx \exp\left[\pi \eta (E) \frac{U_e}{E}\right], \quad \text{for } U_e \ll E. \tag{3}
$$

In an adiabatic limit, i.e., the electrons take the lowest energy state of the combined projectile and target ''molecular'' system, the value for the screening energy U_e can be calculated from a static atom model. In the case of the lithium plus hydrogen ion system it amounts to 186 eV $[17]$ which is in disagreement with the experimentally determined averaged value of 420 ± 120 eV \vert 12,13. The latter has been obtained by fitting a polynomial expansion to the data together with the exponential enhancement factor according to relation (3) .

In difference to works $|12,13|$ we can use the knowledge about the mechanism of the ⁶Li(d, α)⁴He reaction to estimate the experimental value of the screening energy U_e . In Sec. III we have determined the parameters of the subthreshold resonance and the strength factor D_0^2 of the direct contribution taking into account only the experimental cross sections at deuteron energies above 60 keV for which the electron screening effect can be neglected. The experimental *S* factors for energies below 60 keV $\lceil 12,13 \rceil$ show a characteristic increase due to the electron screening effect.

As is pointed out in the paper by Langanke *et al.* [28] the experimental values of the *S* factors at such low energies depend strongly on the stopping power needed for the determination of the effective projectile energy. The recently observed deviation from the velocity proportionality of the stopping power below about 15 keV for the $H+He$ system [29] resulting from the minimum energy transfer necessary

FIG. 4. Comparison between the experimental [20] and theoretical angular distributions for the ⁶Li(d,a)⁴He reaction for deuteron energies between 369 and 975 keV.

for the electron capture by the projectile $\lceil 30 \rceil$ reduced the screening energy derived from experiment [28]. This is due to the very high excitation energy for He. However, in the case of other target materials, for which the excitation energy is significantly smaller, the velocity proportionality should be valid also at lower energies (see $[31]$ for alkaline metals). This is experimentally very well verified for many target materials down to about 10 keV (tables by Anderson and Ziegler [32]). So we assume that the *S* factors given by Engstler *et al.* $\lceil 12,13 \rceil$ down to about 20 keV need not to be corrected with respect to the stopping power values.

A fit to the low-energy data using relation (3) and our theoretical *S* factors is presented as a solid line in Fig. 8. It leads to a value for the screening energy of $U_e = 130 \pm 20$ eV. This value is significantly smaller than the averaged value U_e =340 \pm 110 eV obtained previously for the ⁶Li+*d* system in Refs. $[12,13]$. It means that our analysis of the data based on the subthreshold resonance contribution yields a

FIG. 5. Lorentz curve fit to the resonance part of the measured *S* factors for the ⁶Li(d, α)⁴He reaction.

FIG. 6. Comparison of the experimental and theoretical *S* factors for the ⁶Li(d, α)⁴He reaction. The dashed line represents the DWBA calculation for the direct reaction component. The solid line is the *S* factor predicted taking an additional resonance contribution into account.

stronger increase of the bare nuclei *S* factors than results from the polynomial fit of Refs. $[12,13]$.

On the other hand, our experimental value of the screening energy is even lower than the theoretical one of 186 eV minus about 22 eV due to the ionic binding of the LiF target. This result agrees with recent dynamical approaches to the screening effect $[33,34]$ according to which the screening energy depends strongly on the projectile energy. Its value varies between the adiabatic approximation maximum limit for low energy and a sudden approximation minimum limit for high energy. In the latter case the target electrons do not change their orbits during a collision and therefore the gain in electron binding energy corresponds only to an increase of a combined target nucleus charge which amounts to 114 eV $(calculated according to the prescription given in [33])$ for the system of a deuteron plus 6 Li atom. The time dependent Hartree-Fock calculations of Ref. [34] for the systems $d+{}^{2}H$ and $d+{}^{3}He$ predict a smooth transition between the sudden and adiabatic approximations at the intermediate energy region. The maximum value of the screening energy for

FIG. 7. Total cross sections for the ⁶Li(d, α)⁴He reaction. The description corresponds to that given for Fig. 6.

FIG. 8. Fit of the experimental cross-section enhancement factor due to the electron screening effect. The dashed line represents the bare nuclei *S* factors corresponding to the solid line in Fig. 6.

these cases is achieved only at a beam energy of 20 keV. Unfortunately, there are no comparable calculations for the Lithium atom.

Another consequence of the subthreshold resonance contribution is a significantly larger value of the *S* factor at zero deuteron energy. Our value amounts to $S(0) = 23.0 \pm 2.0$ MeV *b* compared with 17.4 MeV *b* from Refs. [12,13]. For astrophysical applications it might be useful to parametrize the energy dependence of the *S* factor for a wide energy range. Due to the resonance contribution we had to choose a special form of the parametrization for deuteron energies up to 1 MeV:

$$
S(E_{\text{c.m.}}) = S(0) \exp(-4.838 \times 10^{-3} E_{\text{c.m.}} + 1.3586 \times 10^{-6} E_{\text{c.m.}}^2)
$$
 (4)

where $E_{\text{c.m.}}$ is the center of mass energy in keV.

According to the standard prescription $[35]$ we have calculated numerically the stellar reaction rates which are presented in Fig. 9 (solid curve). The resulting curve can be

FIG. 9. Stellar reaction rates for the ${}^6\text{Li}(d, \alpha)^4$ He reaction (solid curve). The dashed curve represents the contribution from the direct reaction component only.

fitted with a usual polynomial expansion up to a temperature of $T_9 = 3$:

$$
N_A \langle \sigma v \rangle = 1.51 \times 10^{11} T_9^{-2/3} \exp(-10.1116 T_9^{-1/3})
$$

×(1.0+3.14 T_9^{1/3} - 10.44 T_9^{2/3} + 10.64 T_9
-4.78 T_9^{4/3} + 0.871 T_9^{5/3}), (5)

where T_9 is the temperature in 10⁹ K and $N_A \langle \sigma v \rangle$ is in cm^3 s $^{-1}$ mol $^{-1}$.

V. CONCLUSIONS

The analysis of the angular distributions $[20]$ and the total cross sections $([20,12,13]$ and present work) of the 6 Li(*d*, α)⁴He reaction indicates a strong contribution of the 2^+ $[E_x(^8\text{Be})=22.2$ MeV] subthreshold resonance at low deuteron energies. This confirms similar results obtained in the study of the low-energy mirror reactions ${}^{\circ}$ Li (d,p) ^TLi and ${}^{6}Li(d,n)^{7}$ Be published previously [18]. A prominent structure observed in the cross section at deuteron energies around 600 keV (Fig. 7) is a result of the resonance strength shift to higher deuteron energies due to the Coulomb penetration effect. The resonance contribution also leads to a significantly larger value for the bare nuclei astrophysical *S* factor at zero deuteron energy and consequently to a lower value for the electron screening energy which now is somewhat below its maximum limit corresponding to the adiabatic approximation of the atomic collision.

Using these results we have determined the stellar reaction rates for the ⁶Li(d, α)⁴He reaction for temperatures up to $T_9 = 3$.

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