

Observation of a unitary cusp in the threshold $\gamma p \rightarrow \pi^0 p$ reaction

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A rigorous multipole analysis of the recent $\gamma p \rightarrow \pi^0 p$ cross-section measurement is presented. The data were taken using the photon spectrometer TAPS at the tagged photon beam of the Mainz microtron. The s and p wave multipoles were extracted using minimal model assumptions. The predicted unitary cusp for the s -wave multipole E_{0+} due to the two step $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ reaction was observed. The results are consistent with one-loop chiral perturbation theory calculations for which three low-energy constants have been determined by a fit to the data. The uncertainties in the analysis and the need for polarization observables are discussed. [S0556-2813(97)03002-1]

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I. INTRODUCTION

Experiments on photopion production from the nucleon are important because the pion is approximately a Goldstone Boson of QCD [1]. The consequences of this are a relatively small mass (due to the small up and down quark mass) and a weak πN interaction at low energies [1]. These characteristics allow a QCD-based approximation scheme known as chiral perturbation theory (ChPT) [1–3]. Using this technique extensive (one-loop) calculations for threshold photopion and electropion production have been performed [4].

For a long time there has been a predicted unitary cusp in the s wave $\gamma p \rightarrow \pi^0 p$ electric dipole amplitude, $E_{0+}(\gamma p \rightarrow \pi^0 p)$, due to the two-step $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ reaction [5]. There are two reasons. First, the electric dipole amplitude for the $\gamma p \rightarrow \pi^+ n$ reaction $E_{0+}(\gamma p \rightarrow \pi^+ n)$ is more than an order of magnitude larger than $E_{0+}(\gamma p \rightarrow \pi^0 p)$. Second, the threshold energies for the $\gamma p \rightarrow \pi^0 p$ and $\pi^+ n$ channels are different (see Table I). Therefore, the unitary cusp is isospin violating.

The magnitude of the cusp is related to

$$\beta = E_{0+}(\gamma p \rightarrow \pi^+ n) \cdot a_{\text{cex}}(\pi^+ n \leftrightarrow \pi^0 p) \quad (1)$$

where a_{cex} is the s -wave charge exchange scattering length. Recently it was pointed out that an accurate observation of the energy dependence of the unitary cusp would allow one to make a measurement of this important and previously unmeasured scattering length [6]. Furthermore it was also shown that the two-step reaction is expected to exhibit an additional isospin violation [6] as a consequence of the predicted isospin violation in a_{cex} due to the mass difference of the up and down quarks [7]. Consequently, experimental tests of the predictions of ChPT [4] and the unitary cusp with its light quark dynamics are of great importance.

The first measurement of the $\gamma p \rightarrow \pi^0 p$ threshold cross sections with a 100% duty cycle electron accelerator was performed at Mainz [8]. The data confirmed a previously measured total cross section at Saclay [9] which was obtained with a 1% duty cycle linac and consequently had larger errors. The Mainz differential cross section qualitatively showed the predicted unitary cusp for E_{0+} [10]. The

original interpretation of the differential cross section data [8,9] showed a disagreement with the low-energy theorems (LET) [11,12]. However, it was later shown [13] that when the total cross-section data were included that the results were consistent with the LET prediction [11]. Subsequently it was shown that the LET were slowly converging [4,12] and that the prediction for E_{0+} should be significantly smaller in magnitude. The exact value is not predicted by ChPT since it depends on low-energy constants which have to be evaluated from experiment [4]. It was clear that such an important measurement should be repeated with improved equipment. A subsequent experiment was performed at Mainz with the TAPS photon detector [15] where the data from threshold to 152 MeV were presented after a preliminary analysis.

In this paper a more thorough analysis from threshold to 160 MeV will be presented. The main purpose of this paper is to obtain the most accurate values of the multipoles with the minimum number of model-dependent assumptions, and to compare these results with the ChPT fit [4]. A secondary purpose is to show the model dependence of the extracted multipoles and the limits due to the fact that the existing database contains only unpolarized cross sections. The results presented here are in good agreement, as expected, with our previous publication [15]. Recently an experiment from Saskatoon has been reported [16] and will also be discussed.

II. FORMULAS AND DATA ANALYSIS

Near threshold one can safely assume that the pions are produced in s - and p -wave states. The differential and total cross sections are

$$\begin{aligned} \sigma(\theta) &= (q/k)[A + B\cos(\theta) + C\cos^2(\theta)] \\ \sigma_T &= 4\pi(q/k)[A + C/3] \end{aligned} \quad (2)$$

where q and k are the pion and photon center-of-mass momenta.

It is conventional to compare theory and experiment in terms of multipole amplitudes. These are: s -wave electric dipole E_{0+} ; p -wave magnetic dipole with $j = 1/2$ M_{1-} ; and

TABLE I. Threshold energies.

Reaction	Threshold Energy (MeV)
$\gamma p \rightarrow \pi^0 p$	144.68
$\gamma p \rightarrow \pi^+ n$	151.44

p -wave magnetic dipole and electric quadrupole amplitudes with $j = 3/2$ M_{1+} and E_{1+} . The A , B , and C coefficients are quadratic combinations of these four amplitudes. Following a previous notation [4] we define

$$P_1 = 3E_{1+} + M_{1+} - M_{1-}, \quad P_2 = 3E_{1+} - M_{1+} + M_{1-}, \quad (3)$$

$$P_3 = 2M_{1+} + M_{1-}, \quad |P_{23}|^2 = (|P_2|^2 + |P_3|^2)/2.$$

The A , B , and C coefficients are

$$A = |E_{0+}|^2 + |P_{23}|^2, \quad B = 2\text{Re}(E_{0+}P_1^*), \quad (4)$$

$$C = |P_1|^2 - |P_{23}|^2.$$

One can see that an accurate measurement of $\sigma(\theta)$ for unpolarized photons determines three linear combinations of the multipoles (A , B , and C). On the other hand, there are seven unknown parameters, namely the real and imaginary parts of the four multipoles minus one arbitrary overall phase. In the threshold region one can take advantage of the fact that the p -wave πN phase shifts are small [17] which means that the imaginary parts of the p -wave multipoles are negligible [4,6].

In order to fit the data the predicted unitary cusp in E_{0+} [4–6] must be taken into account. This is caused by the relatively strong two-step $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ reaction channel and a static isospin violating effect which occurs because of the threshold difference in the $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ reaction channels as shown in Table I. The first derivations used a single scattering K matrix approach to calculate the effect of the final state charge exchange (CEX) [5]. The ChPT calculations are basically isospin conserving but the biggest isospin nonconserving effect due to the pion mass difference has been included [4]. These approximations can be overcome by using a three-channel S -matrix approach in which unitarity and time reversal invariances are satisfied [6]. The resulting equation is

$$E_{0+}(\gamma p \rightarrow \pi^0 p) = e^{i\delta_0} [A_0 + iA_+ a_{\text{cex}} q_+], \quad (5)$$

where δ_0 is the s -wave $\pi^0 p$ phase shift, A_0 and A_+ are $E_{0+}(\gamma p \rightarrow \pi^0 p)$ and $E_{0+}(\gamma p \rightarrow \pi^+ n)$ in the absence of the final state charge exchange (CEX) reaction, and q_+ is the π^+ center-of-mass momentum in units of m_{π^+} . For photon energies $k_\gamma < k_T(\pi^+ n)$, the π^+ threshold energy, one must analytically continue $q_+ \rightarrow i|q_+|$. This switching of the amplitude from real to imaginary as the secondary threshold opens is the sign of a unitary cusp.

We can safely neglect δ_0 in the threshold region because the s -wave scattering length $a(\pi^0 p)$ is expected [7,17,18] to be very small ($\lesssim 0.01/m_\pi$). Since the effect of the $\pi^0 p$ channel is expected to be small in the $\pi^+ n$ channel we can take $A_+ \equiv E_{0+}(\gamma p \rightarrow \pi^+ n)$. With these two mild approxima-

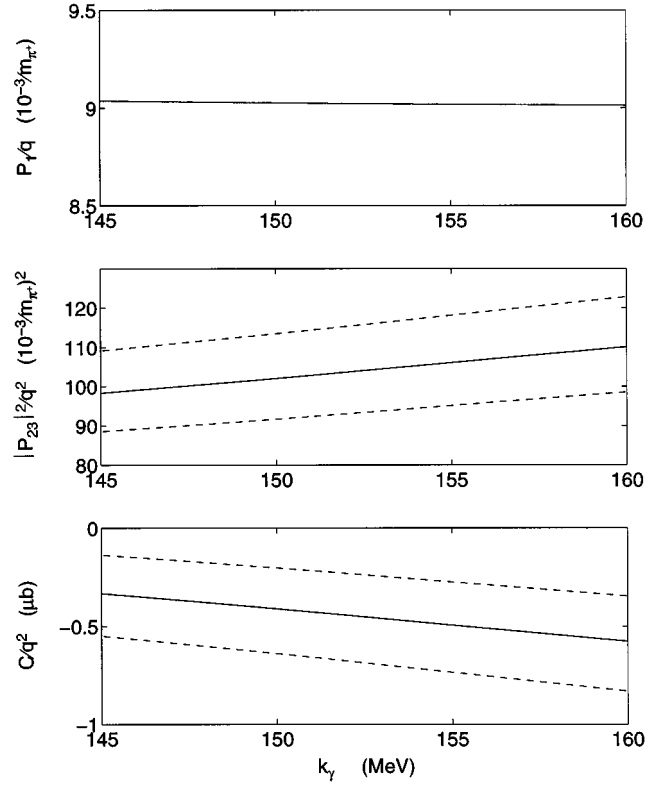


FIG. 1. ChPT predicted behavior of the p waves. Solid line represents the predicted values and the dashed lines represent a $\pm 10\%$ variation of b_p .

tions the three-channel S matrix formulation reduces to the previously obtained formulas [4,5]

$$E_{0+}(\gamma p \rightarrow \pi^0 p) = A_0(k_\gamma) + i\beta q_+, \quad (6)$$

where the only first-principles constraint that we have for A_0 is that it is a smooth function of k_γ . Note that for $k_\gamma < k_T(\pi^+ n)$, $E_{0+}(\gamma p \rightarrow \pi^0 p) = A_0(k_\gamma) - i\beta|q_+|$ is purely real. For $k_\gamma > k_T(\pi^+ n)$, $E_{0+}(\gamma p \rightarrow \pi^0 p)$ is complex with $\text{Re}E_{0+} = A_0(k_\gamma)$, a smooth function of k_γ , and $\text{Im}E_{0+} = \beta q_+$, the cusp function. The same function $A_0(k_\gamma)$ and parameter β occur both below and above $k_T(\pi^+ n)$.

To determine the p -wave multipoles we need to consider their energy dependence. It was previously assumed that they go to zero as qk for $k_\gamma \rightarrow k_T(\pi^0 p)$ [20]. Recently it has been shown that the p -wave multipoles are proportional to q (i.e., the factor of k should not be there) [4]. Numerically the difference is not large but the proper form will be used here. These threshold arguments alone cannot determine over what range of k_γ this simple energy dependence is expected to be valid. In order to see this we plot in Fig. 1 the energy dependence of the three p -wave observables as predicted by ChPT [4] up to $k_\gamma = 160$ MeV. We observe that P_1 is predicted to be very close to linear with q for the entire energy region. If all of the p -wave multipoles were linear in q then P_{23}^2 and C would be proportional to q^2 . As can be seen in Fig. 1 there is a deviation from this quadratic dependence which is approximately linear in $\Delta k_\gamma = k_\gamma - k_T(\pi^0 p)$.

The predictions of ChPT for the $\gamma N \rightarrow \pi N$ reaction depend on three low-energy constants labeled b_p , a_1 , and a_2

[4]. Of these only b_p effects the p -wave multipoles. In Fig. 1 the variation of the three observables are also shown for a $\pm 10\%$ variation of b_p . There is no dependence of P_1 on b_p . Furthermore, the slope of $|P_{23}|$ and C are approximately independent of b_p . The approximately linear deviation from the q^2 dependence of C and $|P_{23}|^2$, shown in Fig. 1, will be assumed in the analysis but with empirically determined constants.

We have performed three fits to the data. One uses the A , B , and C coefficients with the energy dependence of C specified as

$$C = q^2[\bar{C} + \tilde{C}\Delta k_\gamma], \quad (7)$$

where Δk_γ is taken for convenience to be in units of m_{π^0} .

A multipole fit was also performed with the functional form of the energy dependence of the p -wave multipoles and $\text{Im}E_{0+}$ fixed. From the discussion of the expected energy dependence of the s - and p -wave multipoles the following energy dependence was chosen:

$$\text{Im}E_{0+} = 0, \quad k < k_T(\pi^+ n),$$

$$\text{Im}E_{0+} = \beta q_+, \quad k > k_T(\pi^+ n), \quad (8)$$

$$P_1 = q\bar{P}_1, \quad |P_{23}|^2 = q^2[\bar{P}_{23} + \tilde{P}_{23}\Delta k_\gamma],$$

where the fit parameters are the values of $\text{Re}E_{0+}(k_i)$ at each photon energy, β , \bar{P}_1 , \bar{P}_{23} , and \tilde{P}_{23} .

We have also performed a unitary fit for which $E_{0+}(k_\gamma)$ is parametrized as

$$E_{0+}(k_\gamma) = A_0(k_\gamma) + i\beta q_+, \quad A_0(k_\gamma) = \bar{A} + \tilde{A}\Delta k_\gamma, \quad (9)$$

and where the values of \bar{A} , \tilde{A} , and β were taken as free parameters.

To calculate the expected value of β the best experimental value of $a(\pi^- p \rightarrow \pi^0 n) = -(0.1301 \pm 0.0059)/m_\pi$ from the observed width in the $1s$ state of pionic hydrogen atom [21] was used. This is in excellent agreement with ChPT predictions of $-(0.130 \pm 0.006)/m_\pi$ [18]. Assuming isospin is conserved $a(\pi^+ n \leftrightarrow \pi^0 p) = -a(\pi^- p \leftrightarrow \pi^0 n)$. There are no modern measurements for $E_{0+}(\gamma p \rightarrow \pi^+ n)$ so we can use the ChPT prediction of 28.2 ± 0.6 [4,14,19]. From these, one obtains $\beta = 3.67 \pm 0.18$ [14].

The analysis that will be presented here depends on the range of validity of Eq. (8). In order to insure that this analysis is accurate and as model independent as possible it was terminated at a photon energy of 160 MeV. This is sufficient to show the main features of the threshold region.

III. RESULTS

In this section the results of the analysis of the data [15] will be presented and compared with ChPT calculations [4]. The data were taken from threshold to 270 MeV. In the first publication only a preliminary analysis of the data to a photon energy of 152 MeV was presented [15]. In this publication a more thorough analysis to 160 MeV will be presented.

The empirical fits and ChPT [4] results for $\sigma(\theta)$ are compared to experimental data in Fig. 2. It should be pointed out that the errors shown in Fig. 2 are statistical only. The sys-

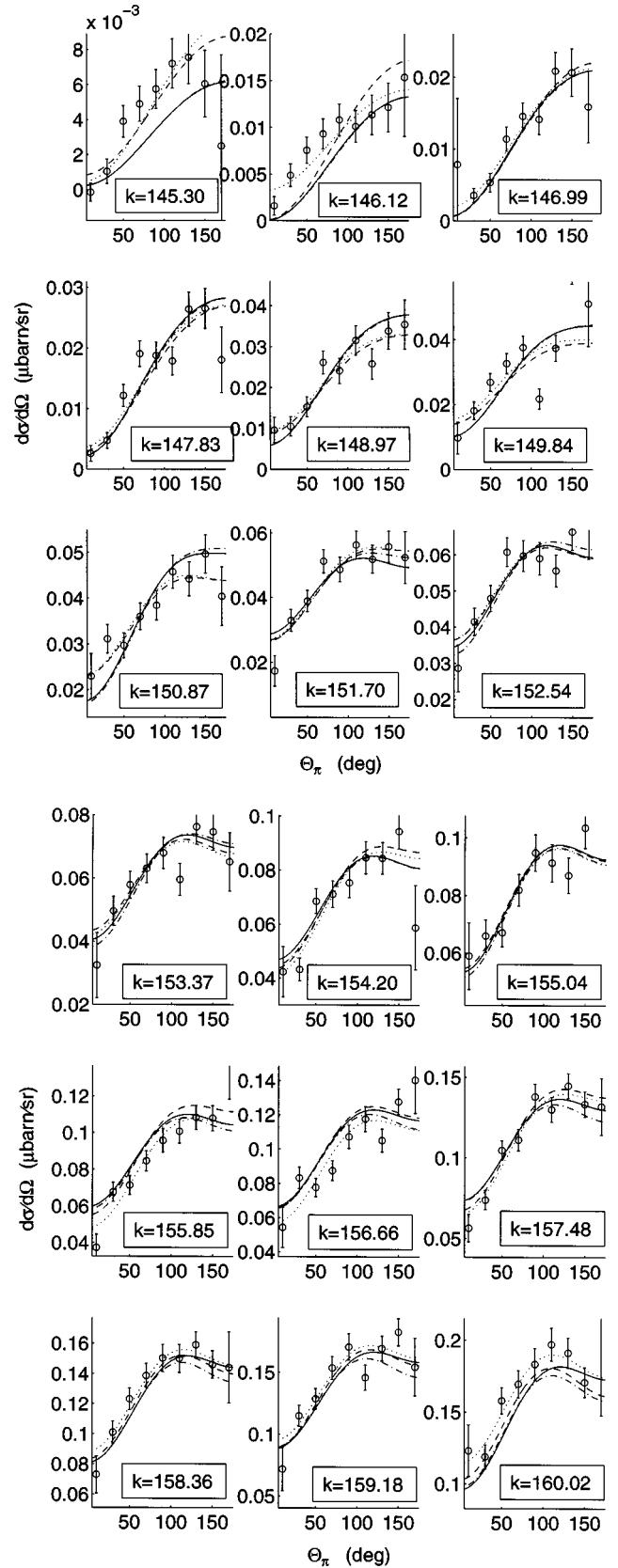


FIG. 2. Differential cross sections (COM system) with statistical errors in $\mu\text{b}/\text{sr}$ vs pion angle in degrees. The photon energies in MeV are given. The solid line represents the unitary fit, the dot-dot line the A, B, \bar{C}, \tilde{C} fit, the dash-dash line the multipole fit, and the dash-dot line the ChPT fit.

TABLE II. Parameters in Eqs. (8) and (9) for the unitary and multipole fits and ChPT [4]. Except for χ^2 all of the quoted parameters are given in units of $10^{-3}/m_{\pi^+}$. In Eqs. (8) and (9) q and Δk_γ are in units of m_{π^0} and q_+ in units of m_{π^+}

Parameter	Unitary Fit	Multipole Fit	ChPT
χ^2/DOF	2.13	1.96	2.21
β	3.76 ± 0.11	2.82 ± 0.32	2.78
\bar{P}_1	9.006 ± 0.079	9.151 ± 0.071	8.998
\bar{P}_{23}	101.9 ± 1.1	95.40 ± 0.62	97.71
\tilde{P}_{23}	41.8 ± 13.0	159.1 ± 7.3	108.5
\bar{A}	-0.12 ± 0.020		-0.41
\tilde{A}	-4.27 ± 0.28		-0.76

tematic errors are estimated to be approximately 5% [15]. For the A, B, C fit [Eq. (7)] the best fit values are $\bar{C} = -0.338 \pm 0.025 \mu b$, $\bar{C} = -1.83 \pm 0.30 \mu b$, and $\chi^2/DOF = 1.65$. For ChPT [4] there are three low-energy constants which were adjusted to obtain a best fit to the data. Qualitatively all of the fits and the ChPT calculation are very close with $\chi^2/DOF = 1.96$ for the multipole fit and 2.21 for ChPT.

The fit parameters and χ^2 results are shown in Table II and compared to ChPT. Only the fitting errors are presented.

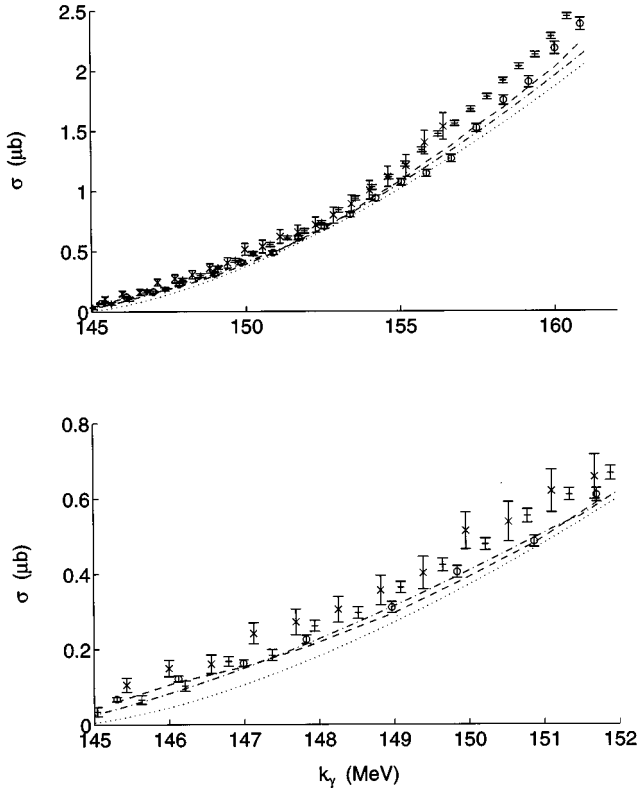


FIG. 3. Measured total cross section with statistical errors in μb vs photon energy with the ChPT calculations [4] and the empirical fits. The new Mainz (TAPS) data (circle) [15], Saskatoon (plus) [16], and older Mainz points (cross) [8] are shown. The bottom figure shows the energy region through the $\pi^+ n$ threshold in more detail. In both figures dash-dash line represents the multipole fit, dash-dot line the ChPT result, and dot-dot line only the p -wave contribution to the ChPT.

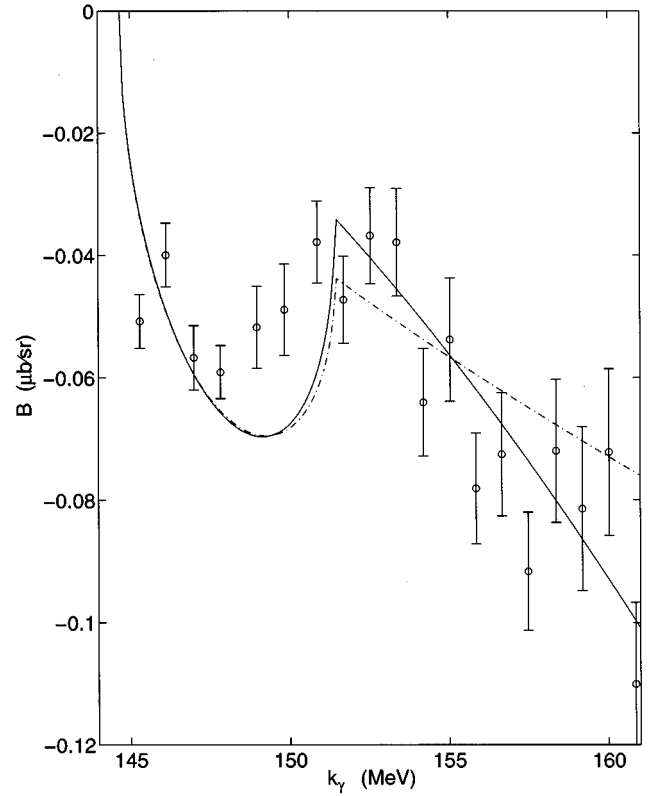


FIG. 4. B coefficient (with statistical errors) vs photon energy from fits and ChPT. Circles represent the A, B, C fit, the solid line represents the unitary fit, and the dash-dot line the ChPT.

The values presented for ChPT [4] were obtained by fitting the numerical calculations with Eqs. (8) and (9) and finding the best fit parameters. The agreement between the fitted form and numerical results is excellent.

For the p -wave multipoles one can see from Table II that the extracted values of \bar{P}_1 and \bar{P}_{23} are in good agreement with ChPT [4]. In this part of the calculation there is only one low-energy constant (b_p) which was fit to the data. On the other hand, there is a large systematic error for \tilde{P}_{23} . This can be seen by comparing the much different values obtained from the unitary and multipole fits. These results straddle the ChPT value. The fact that the next-to-leading-order slope of the p wave multipoles is not strongly constrained by the data will have consequences in the determination of $\text{Im}E_{0+}$ (see Sec. IV).

The parameters for $\text{Re}E_{0+}$ (β , \bar{A} , and \tilde{A}) are significantly different between the unitary fit and the ChPT calculation. As will be shown below (see Fig. 8 and discussion) the two resulting curves both are in reasonable agreement with the extracted values of $\text{Re}E_{0+}$ from the multipole fit. It should be pointed out that for this multipole there are two low-energy parameters (a_1 and a_2) that have been fit to the data.

The results for the total cross section are shown in Fig. 3. It can be seen that there is a discrepancy between the Saskatoon [16] and TAPS [15] results particularly for $k_\gamma > 152$ MeV. The older Mainz results [8] tend to be in better agreement with the Saskatoon data. Unfortunately the cause of this disagreement is not known.

The Saskatoon group did not publish any differential cross-section data. The only information that is presented is

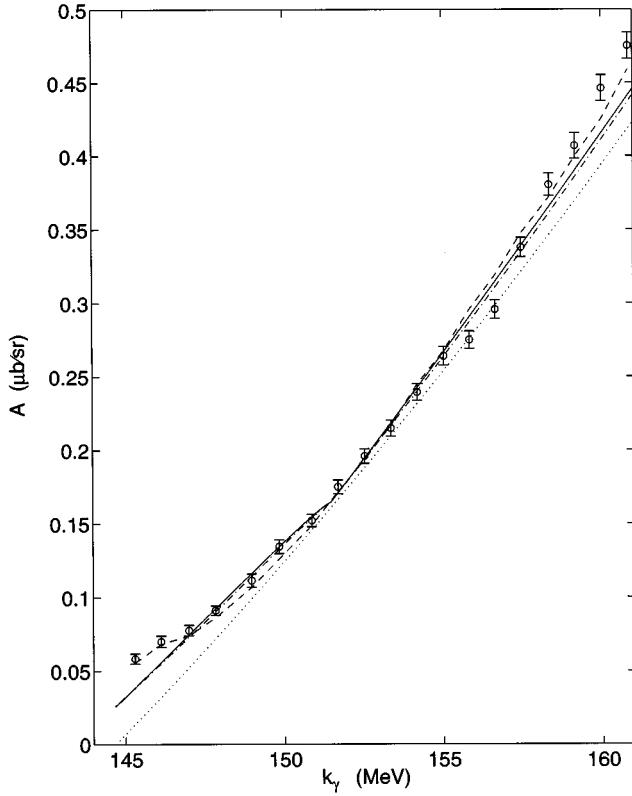


FIG. 5. A coefficient (with statistical errors) vs photon energy from fits and ChPT. Circles represent the A, B, C fit, the solid line represents the unitary fit, the dash-dash line the multipole fit, the dash-dot line the ChPT, and the dot-dot line the p -wave contribution to ChPT.

the “belt pattern” for the angular distributions. In order to use this information one has to perform a Monte Carlo calculation using predicted differential cross sections which are then compared with the “belt patterns.” We therefore cannot compare the results presented here for the differential cross sections with the Saskatoon data.

From Fig. 3 one can see that the ChPT fit is in good agreement with the TAPS data and not in agreement with the Saskatoon data. This is not surprising since the three free parameters of ChPT were fit to the TAPS data. One also notes that the p -wave contribution to the total cross section is dominant except for the first few points above threshold. This makes it difficult to see the effect of the (s wave) unitary cusp in the cross section.

The results for the B coefficient are shown in Fig. 4. This shows the effect of the predicted unitary cusp because it is an sp interference amplitude. The extracted values of the B coefficient are in reasonable agreement with the unitary fit and with ChPT [4] because only the statistical errors are shown. An estimate of the systematic errors can be inferred from the scatter of the points.

The results for the A coefficient are shown in Fig. 5. This is the most accurately measured coefficient since $\sigma(\theta=90^\circ) = (q/k)A$, and since the total cross section is proportional to $A + C/3$ and $|C| \ll A$. As a consequence the errors for A are small. In contrast to the B coefficient, the unitary cusp is hardly visible. This is due to the dominance of p waves as shown in Fig. 5. It can also be seen that the

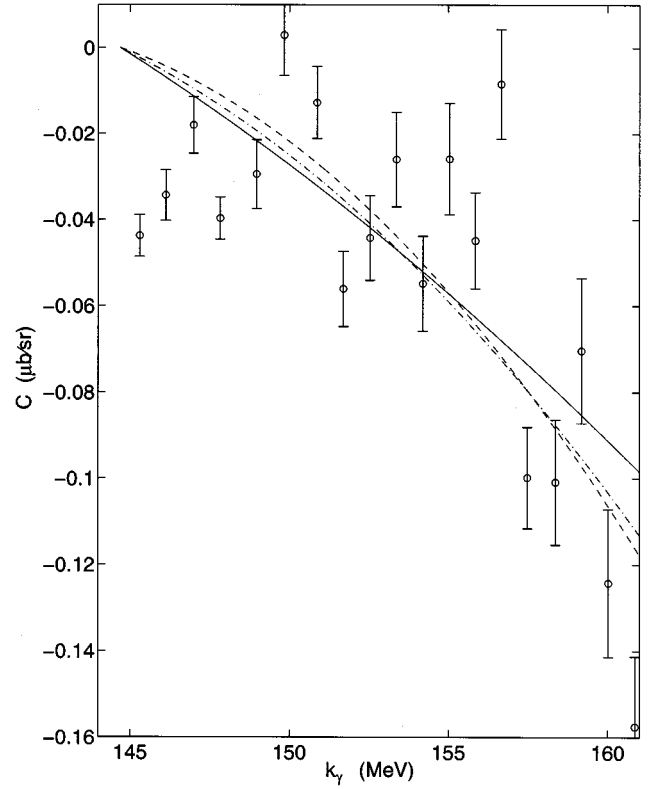


FIG. 6. C coefficient (with statistical errors) vs photon energy from fits and ChPT. Circles represent the A, B, C fit, the solid line represents the unitary fit, the dash-dash line the multipole fit, and the dash-dot line the ChPT.

ChPT calculation is in reasonable agreement with A .

The results for the C coefficient are shown in Fig. 6. It can be seen that all of the fits and the results of ChPT are in good agreement. In addition a fit in which the A , B , and C coefficients at each energy are found from a least-square fit to the data are also presented. In this case the energy dependence of the C coefficient is not constrained by theory. The scatter of these C coefficients indicate that the data do not strongly constrain the p -wave multipoles despite the fact that this is the dominant multipole in the unpolarized cross section. We conclude that although the present results are consistent with the ChPT theory calculations that the experiment with linearly polarized photons recently completed at Mainz [22] is needed for a more precise measurement of the p -wave multipoles. The results from the analysis of this experiment will be available in the next year.

The extracted values of the magnitude of

$$|E_{0+}| = \sqrt{(\text{Re}E_{0+})^2 + (\text{Im}E_{0+})^2}$$

are shown in Fig. 7. These are obtained from the unitary fit to the Mainz/TAPS data [15] and from the published Saskatoon results [16]. Despite the discrepancy in the measured total cross sections, the two data sets result in values of $|E_{0+}|$ which are in reasonable agreement. The fitting errors for the unitary fit are significantly smaller than the errors shown for the individual points of the Saskatoon data since they represent an overall fit to the data.

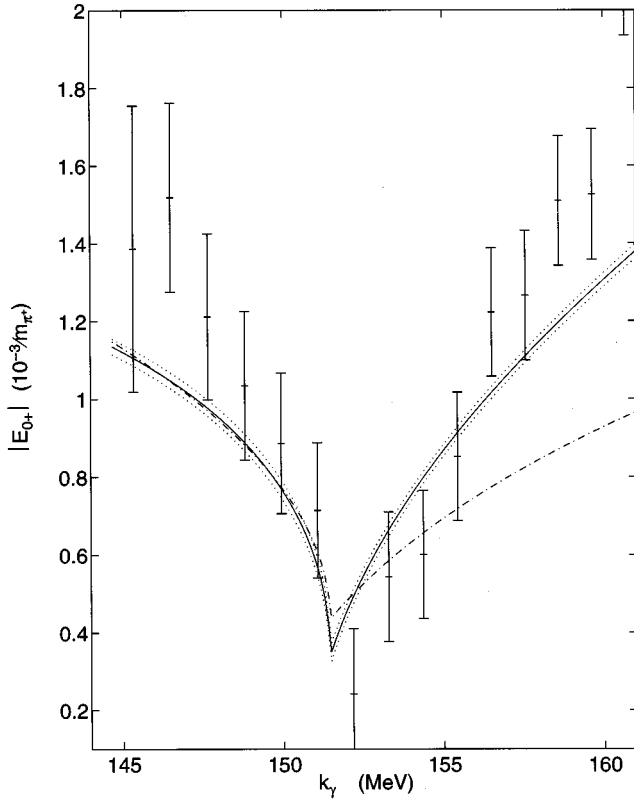


FIG. 7. $|E_{0+}|$ vs photon energy. Plus represents the Saskatoon data, the solid line represents the unitary fit, the dash-dot line the ChPT, and the dot-dot lines represent the fitting errors of the unitary fit.

The extrapolated threshold values for E_{0+} are -1.13 ± 0.04 for the unitary fit and -1.5 ± 0.1 for the multipole fit in the usual units [14]. These two values do not agree because the two analyses give a somewhat different energy dependence for E_{0+} . Taking the average of the two values and adjusting the error to reflect this disagreement, the threshold value of $E_{0+} = -1.3 \pm 0.2$ [14] was obtained. This value is in good agreement with the Saskatoon result of -1.32 ± 0.1 [14,16] but disagrees with the older prediction of the low-energy theorems [11,12] of -2.28 [14] which have been shown to be incomplete [4,12].

The $\text{Re}E_{0+}$ as extracted from the multipole fit is presented in Fig. 8. This agrees with the Saskatoon results [16] and the older Mainz results [8] within the experimental errors. The effect of the rapid energy variation of $\text{Re}E_{0+}$ below the π^+n threshold is again visible in qualitative agreement with the ChPT calculation [4] and the unitary fit. Note that the errors shown in Fig. 8 are statistical only. The magnitude of the systematic errors can be inferred from the scatter of the points.

IV. MODEL DEPENDENCE

The differential cross section for unpolarized photons in Eq. (2) shows that three independent combinations of the multipoles (A , B , and C) are measured while there are seven independent multipole parameters for the emission of s - and p -wave pions. In order to extract useful information about the multipoles from the data assumptions were made which

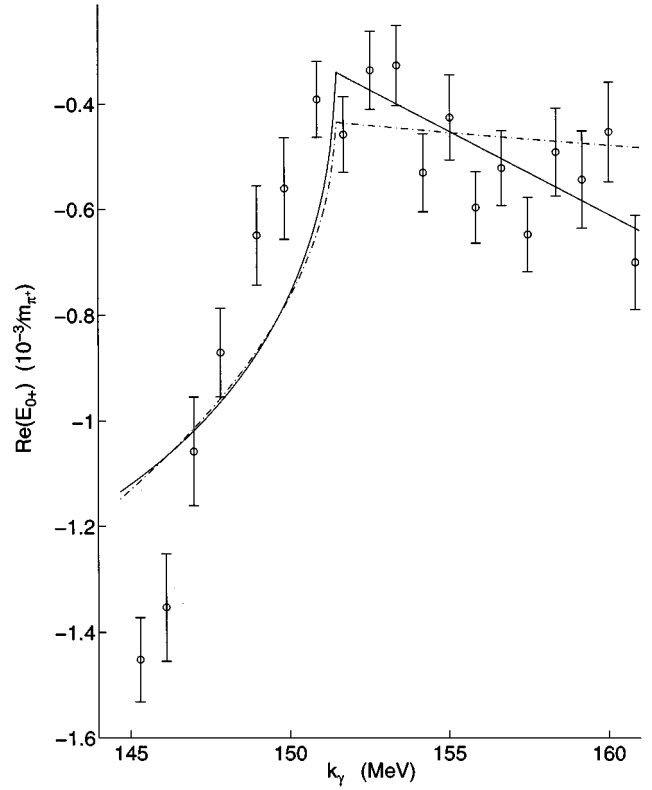


FIG. 8. $\text{Re}E_{0+}$ vs photon energy. The circles represent the multipole fit, the solid line represents the unitary fit, and the dash-dot line the ChPT fit.

follow from first principles which should cause relatively small analysis errors. In this section the sensitivity to these assumptions will be explored to obtain a measure of the uncertainties, and also to explore how future data, particularly with polarization degrees of freedom, can improve the situation.

The major model assumption that was made in this analysis is the assumption that the p -wave multipoles have the same analytic energy dependence as predicted by ChPT [4]. In part this assumption has been checked by the fact that the least-square parameters are close to those of ChPT (see Table II). In order to further check this assumption an additional fit was made which was very similar to the multipole fit [Eq. (8)] except that the p -wave multipoles were assumed to vary linearly with q [i.e., $\tilde{P}_{23}=0$ in Eq. (8)]. This fit is quantitatively similar to those in which the p -wave multipoles were assumed to vary as qk [13]. The results of doing this can be surmised by the observation that A is the best measured of the three coefficients in Eq. (2) and by noting that this determines the absolute value of E_{0+} in addition to the dominant p -wave contribution. Since $\text{Re}E_{0+}$ is determined from the B coefficient this determines $\text{Im}E_{0+}$ after a suitable subtraction of the p -wave contribution. If one assumes a smaller energy dependence in the p -wave multipole then a stronger energy dependence will emerge for $\text{Im}E_{0+}$. The results for the determination of $\text{Im}E_{0+}$ are presented in Fig. 9 and it can be seen that this is precisely what has happened. For the fit in which the p waves are assumed to be proportional to q the extracted value of $\beta = 4.51 \pm 0.20$, which is far larger than the value of $\beta = 2.82 \pm 0.32$ obtained

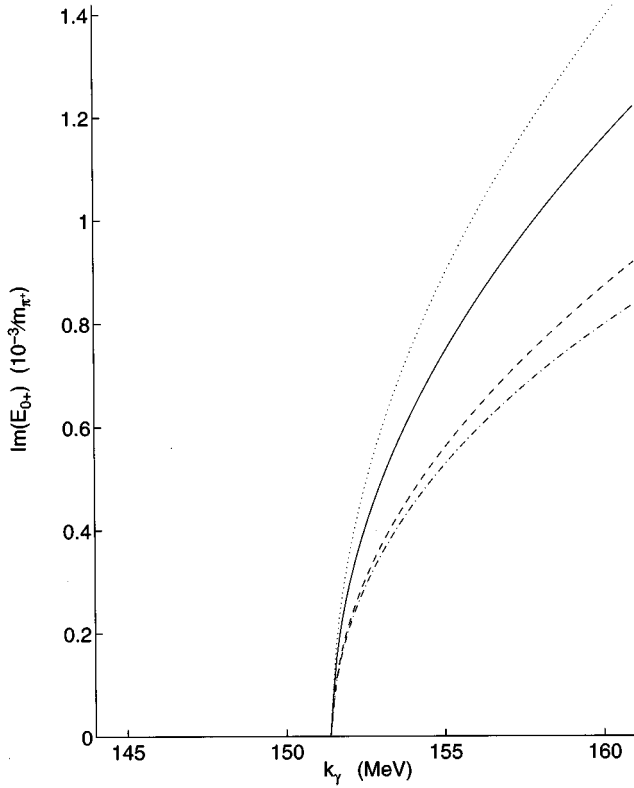


FIG. 9. $\text{Im}E_{0+}$ vs photon energy. Solid line represents the unitary fit, dash-dash line the multipole fit, dash-dot line the ChPT, and dash-dash line the fit which assumes linear dependence of p waves of q .

with the multipole fit or $\beta = 3.76 \pm 0.11$ for the unitary fit. The two latter fits use the p -wave energy dependence of Eq. (8). These results for β indicate a strong correlation between β and \tilde{P}_{23} . This is to be expected since the only information about these parameters is obtained from A [see Eqs. (4) and (8)].

From the spread in the values of β shown in Fig. 9 and Table II one concludes that the systematic error is significantly larger than the fit error. This is due to the fact that there is not sufficient information in the unpolarized cross section to determine this quantity. Taking the average of the unitary and multipole fits $\beta = 3.3 \pm 0.5$ (15%). The error reflects the difference between the results of the two analyses but does not take the entire possible range of values shown in Fig. 9. Experiments with polarized targets are required to precisely determine $\text{Im}E_{0+}$ [6].

The cusp effect is isospin breaking due in part to the threshold difference between the $\pi^0 p$ and $\pi^+ n$ channels. In the ChPT calculation [4] isospin is broken by inserting the mass difference between the charged and neutron pions by hand. This leads to a value of $\beta = 2.78$ which is significantly below the predicted value of 3.67 ± 0.19 expected from the predicted values of $E_{0+}(\gamma p \rightarrow \pi^+ n)$ and $a(\pi^+ n \leftrightarrow \pi^0 p)$ quoted in Sec. II. An improved ChPT calculation which takes isospin breaking into account in a more dynamic way seems to be required. It is also of interest to discuss how well the p -wave multipoles are measured. By comparing the values obtained from the unitary and multipole fits (Table II) it can be shown that the leading terms in the energy depen-

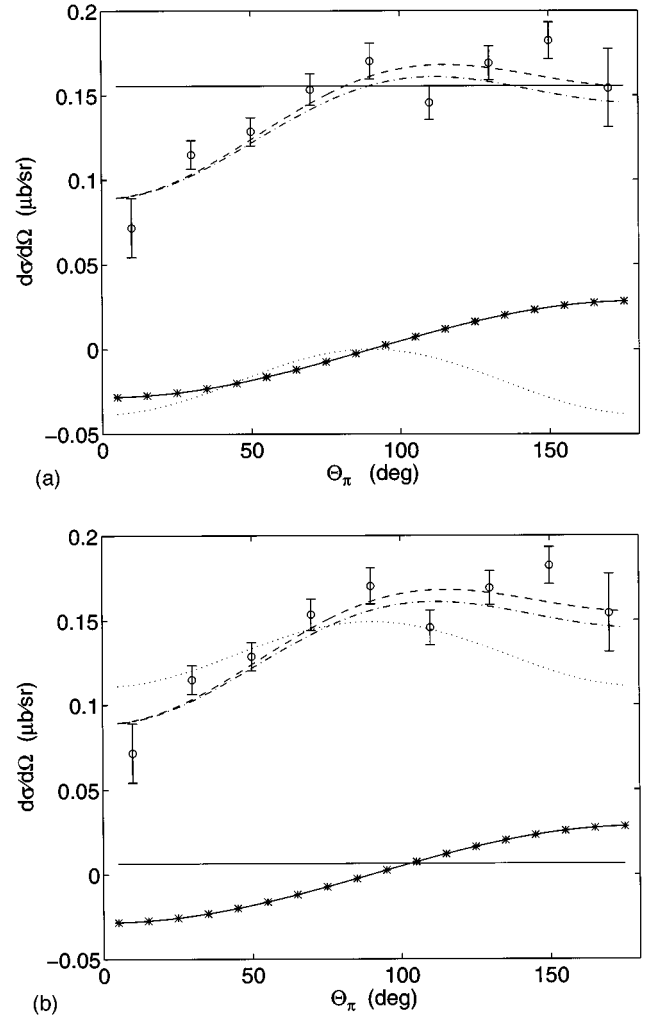


FIG. 10. $\sigma(\theta)$ at 159.1 MeV showing the contributions of the A , B , and C terms (bottom) and s and p terms (a). In both graphs the circles represent Mainz/TAPS data, the dash-dash line represents the multipole fit, and the dash-dot line the ChPT fit. The solid line represents the s wave (a) and A term (b) contributions, the dot-dot line p wave (a) and C term (b) contributions, and the stars represent sp (a) and B term (b) contributions to $\sigma(\theta)$.

dence of the p wave multipoles [\bar{P}_1 and \bar{P}_{23} in Eq. (8)] are well determined, but not the next order term [\bar{P}_{23} in Eq. (8)]. This is a relatively small effect in the magnitude of the p waves as can be seen from the three curves for C in Fig. 6. The question of how accurately the energy dependence of the p -wave multipoles can be obtained from the data can be seen by examining the relative contributions of the different terms [Eqs. (2)–(4)] to the differential cross section. As shown in Fig. 10 the p -wave multipoles are dominant. However, to extract their precise magnitude from the data is not trivial. First, as was discussed above, the best measured A coefficient has a tradeoff between $\text{Im}E_{0+}$ and the energy dependence of the p -wave multipoles. The B coefficient is an interference term between P_1 and $\text{Re}E_{0+}$ so that only the product is determined. Only the relatively small C coefficient has purely p -wave contributions. As was shown in the discussion of Fig. 6 this coefficient is less determined than B and C . The reason for this is illustrated in Fig. 10 which shows the contributions of the A , B , and C terms to $\sigma(\theta)$ at

159.1 MeV. This is a relatively high energy for this analysis so that the contribution of the C term is as large as possible. It can be seen that the C -term contribution is harder to determine with precision. It will be much better determined by the $\vec{\gamma} + p \rightarrow \pi^0 + p$ reaction with linearly polarized photons [22]. In this case if one assumes that the E_{1+} multipole is negligible the polarized photon asymmetry at 90° is $-C/A$. This will be the most precise measurement of the energy dependence of the p -wave multipoles.

V. CONCLUSIONS

We have presented a rigorous analysis of the recent TAPS/Mainz data [15]. Since there are three independent observables for the unpolarized cross section, while for s - and p -wave photopion emission there are seven independent amplitudes, some simplifying assumptions must be made. In this work these assumptions follow closely from first principles. The systematic errors of the analysis were assessed by using several assumptions; the main one is that the energy dependence of the p -wave multipoles follow approximately the same analytic form predicted by ChPT [4].

The main conclusion is that the calculation of ChPT [4] is in reasonable agreement with the data. This one-loop calculation has three low-energy constants which are fitted to the data. The threshold value of E_{0+} was determined to be -1.3 ± 0.2 in the usual units [14]. This is in agreement with

the Saskatoon result of -1.32 ± 0.1 [16]. Note that this disagrees with the predictions of the older “low energy theorems” of -2.28 [11,12] which have been theoretically shown to be incomplete [4].

The predicted unitary cusp in the s -wave electric dipole amplitude E_{0+} [4–6], due to the two-step $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$, has been observed in experiments at Mainz [8,10,15] and Saskatoon [16,23]. Only a range of β values [Eq. (1)] between approximately 2.8 and 4.5 [4] can be obtained from the unpolarized cross-section data (Sec. IV).

The new experiments on the threshold $\gamma p \rightarrow \pi^0 p$ reaction mark a significant advance in our understanding of this important reaction. The small discrepancy between the TAPS [15] and Saskatoon [16] results is not understood. A new experiment with linearly polarized photons has been performed [22] and the completion of the data analysis will allow a more complete test of the ChPT predictions [4]. The average value of $\beta = 3.3 \pm 0.5$ (15%) has been obtained. However, as discussed these depend on the assumed energy dependence of the p -wave multipoles. Further work with polarized targets is required to precisely measure β , which should provide a measure of the s -wave charge exchange scattering length $a_{\text{cex}}(\pi^+ n \leftrightarrow \pi^0 p)$ [6]. On the theoretical side we have seen tremendous progress in our understanding of threshold pion photoproduction and electroproduction. Further advances are needed to treat isospin breaking in a more rigorous way and to test the convergence of E_{0+} .

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