Chiral symmetry breaking in the presence of a confining interaction

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We study chiral symmetry breaking, making use of a generalized Nambu–Jona-Lasinio (NJL) model that includes a description of confinement. The Schwinger-Dyson and Bethe-Salpeter equations are solved for our model of the self-energy of a quark. We show that our analysis is consistent with the Goldstone theorem. That is, the pion has zero mass, if the current quark masses are zero. We use a confining interaction with a (Dirac) matrix structure that leads to simple equations for the self-energy and for a vertex function that serves to sum a ladder of confining interactions. We consider spacelike values of q^2, and carry out our analysis in a Euclidean momentum space. For timelike q^2, we use calculational procedures that we have developed in our earlier work in order to exhibit properties of the confining vertex. For the spacelike values of q^2 considered here, we see that the effects due to the introduction of our model of confinement are small. (However, such effects are very important for timelike q^2. Their consideration is essential, if we wish to study mesons, such as the rho and omega, in our model.) [S0556-2813(97)05003-6]

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I. INTRODUCTION

In recent years, we have seen numerous applications of the Nambu–Jona-Lasinio (NJL) model in the study of chiral symmetry breaking and in the description of hadron properties [1]. Usually, confinement is neglected, so that the theory may be applied to study the pion and the ''sigma meson.'' (The sigma meson appears at the threshold of the quark-antiquark continuum, while the rho and omega are in the continuum of the model.) In previous work, we have shown how a confining interaction (a linear potential) introduced in our momentum-space calculations removes the unitarity cut associated with the quark and antiquark going on mass shell. The resulting formalism then only has cuts when hadrons go on mass shell [2]. In our previous work we assumed that the constituent quark mass had a constant value that was obtained from the gap equation, without confinement. In the present work, we have included the confining interaction in the Schwinger-Dyson equation and we, therefore, find a self-energy for the quark that varies with k^2, the square of the quark momentum.

It is usually thought that confinement is not particularly important for the study of the low-energy hadron spectrum. In this work we wish to put such suggestions on a more quantitative basis and to obtain guidance as to the implementation of a more comprehensive treatment of low-lying mesonic states, including the pseudoscalar octet and the η'. The organization of our work is as follows. In Sec. II we describe the confining interaction that we use in this work. In Sec. III, we present equations for the quark self-energy, $\Sigma(k)=B(k^2)\not{k}+A(k^2)$. In Sec. IV, we introduce a vertex function that serves to sum a ''ladder'' of confining interactions and present results of our calculations of that quantity. In Sec. V, we discuss the vertex function associated with the entire interaction of the generalized NJL model. We show that the Goldstone theorem is satisfied, with the zero-mass pion as the Goldstone boson. Finally, Sec. VI contains a summary and some conclusions.

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II. THE NJL MODEL WITH A CONFINING INTERACTION

For our study we will use the SU(2)-flavor version of our model, where the Lagrangian is

$$\mathcal{L}=\overline{q}(x)(i\not{\partial}-m_q^0)q(x)+\frac{G_S}{2}\{[\overline{q}(x)q(x)]^2+[\overline{q}(x)i\gamma_5\bm{\tau}q(x)]^2\}+\mathcal{L}_{\rm conf}. \tag{2.1}$$

We will use

$$\mathcal{L}_{\rm conf}=[\overline{q}(x)\gamma^\mu q(x)V^C(x-y)\overline{q}(y)\gamma_\mu q(y)-\overline{q}(x)\gamma^\mu\gamma_5 q(x)V^C(x-y)\overline{q}(y)\gamma_\mu\gamma_5 q(y)]. \tag{2.2}$$

(The advantages of using this form will be made clear as we proceed.) Thus, the confining interaction is

$$\overline{V}(x-y)=V^C(x-y)[\gamma^\mu(1)\gamma_\mu(2)-\gamma^\mu(1)\gamma_5(1)\gamma_\mu(2)\gamma_5(2)]. \tag{2.3}$$

We used $V^C(r)=\kappa r\exp(-\mu r)$ in our earlier work, with μ=0.050 GeV. The parameter μ is introduced to make the Fourier transform of $V^C(r)$ less singular and, thus, facilitate our numerical calculations. We see that the potential is not absolutely confining. For example, if κ=0.20 GeV^2 and μ=0.050 GeV, the maximum value of the potential is 1.47 GeV. For μ=0.030 GeV, the maximum value is 2.45 GeV. Therefore, if we are only interested in excitations of energy less than 1 GeV, the finite height of the potential will not be important. Of course, as smaller values of μ are used, the difference between our form and a linear potential becomes less important. The Fourier transform of $V^C(r)$ is

$$V^C(\mathbf{k}-\mathbf{k}')=-4\pi\kappa\left\{\frac{2}{[(\mathbf{k}-\mathbf{k}')^2+\mu^2]^2}-\frac{8\mu^2}{[(\mathbf{k}-\mathbf{k}')^2+\mu^2]^3}\right\}. \tag{2.4}$$

When we studied Lorentz-scalar confinement, we took κ to be positive in Eq. (2.4). For the present work, we take κ to be negative, since a negative sign, that has its origin in the Dirac matrix structure of the interaction, arises when switching from scalar to vector confinement. For calculations in a Euclidean momentum space, that we report upon in this work, it is necessary to treat all the components of the momentum transfer on the same footing. Therefore, we will use

$$V_E^C(k-k')=-4\pi\kappa\left\{\frac{2}{(k_E^2+\mu^2)^2}-\frac{8\mu^2}{(k_E^2+\mu^2)^3}\right\}, \tag{2.5}$$

where k_E^μ is the momentum transfer in the Euclidean momentum space.

It is useful to write

$$\overline{V(x-y)}=V^C(x-y)\sum_{i=1}^{2}O_i(1)O_i(2) \tag{2.6}$$

with $O_1=\gamma^\mu$ and $O_2=i\gamma^\mu\gamma_5$. [See Eq. (2.3).] (We will use a bar over a letter to denote a quantity that has Dirac matrix indices.)

For the interaction of Eq. (2.3), the Lagrangian has chiral symmetry, if $m_q^0=0$. Indeed, each of the terms in Eq. (2.3) respects chiral symmetry. Our motivation in combining the terms, as in Eq. (2.3), is to achieve a particularly simple form for the self-energy and vertex functions. For example, if we write the self-energy as $\Sigma(k^2)=B(k^2)\not{k}+A(k^2)$, we find that $B(k^2)=0$. Also, if we consider a scalar or pseudoscalar vertex for the confining interaction, $F_S(q^2,q\cdot k,k^2)$ or $F_P(q^2,q\cdot k,k^2)$, we find that, when Eq. (2.3) is used, $F_P(q^2,q\cdot k,k^2)=F_P(q^2,q\cdot k,k^2)\gamma_5\tau^i$ and $F_S(q^2,q\cdot k,k^2)=F_S(q^2,q\cdot k,k^2)\cdot\mathbb{I}$. Here $\mathbb{I}$ denotes the unit matrix in the space of Dirac matrices. That is, the simple structure of the vertex operators of the NJL model is maintained in the presence of confinement, if $V(x-y)$ of Eq. (2.3) is used. If we need to refer to that interaction, we may call it a $V-A$ form. That designation is in keeping with the current practice, where one speaks of ''scalar confinement'' or ''vector confinement.''

We now write the quark propagator as

$$iS(k)=\frac{i}{\not{k}-\Sigma(k)+i\epsilon}, \tag{2.7}$$

with $\Sigma(k)=B(k^2)\not{k}+A(k^2)$. This propagator is represented by a double line in Fig. 1. In Fig. 1(a) we see the equation for the quark self-energy, $\Sigma(k)$. There, the wavy line is the confining interaction. The solid circle in Fig. 1(a) denotes the element iG_S, where G_S is defined in Eq. (2.1). In the absence of the first term of Fig. 1(a), we would reproduce the gap equation of the standard NJL model. In that case, one has a constant value for the constituent mass, $\Sigma(k)=m_q$, with $m_q\sim$250–350 MeV.

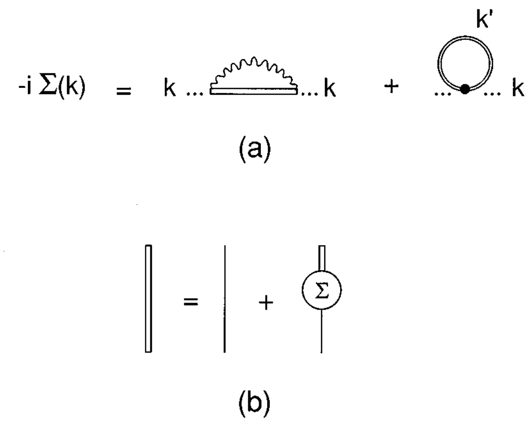

FIG. 1. (a) The figure shows the contributions to the quark self-energy, $\Sigma(k)$. Here the double line represents the quark propagator, $iS(k')=i[\not{k}-\Sigma(k')+i\epsilon]^{-1}$ and the wavy line is the confining interaction. [As a diagrammatic element, the wavy line introduces $-iV^C(k-k')$ when evaluating the diagram. The solid circle denotes a factor of iG_S.] (b) The equation for the quark propagator is given in terms of the self-energy, $\Sigma(k)$, seen in (a). The single line is the propagator of a massless quark in this figure.

In Fig. 1(b), we show the integral equation for the quark propagator, $iS(k)$, given in terms of a massless propagator (single line) and the self-energy, $\Sigma(k)$.

III. THE QUARK SELF-ENERGY

We write the quark self-energy as $\Sigma(k)=B(k^2)\not{k}+A(k^2)$. From the equation depicted in Fig. 1(b), we find that $B(k^2)=0$ and that $A(k^2)$ satisfies the equation

$$A(k^2)=i\int\frac{d^4k'}{(2\pi)^4}\frac{8V^C(k-k')+4n_cn_fG_S}{k'^2-A^2(k'^2)}A(k'^2). \tag{3.1}$$

Note that, if $V^C=0$, $A(k^2)$ is a constant. In that case, we have

$$m_q=4n_cn_fG_Si\int\frac{d^4k'}{(2\pi)^4}\frac{m_q}{k'^2-m_q^2}. \tag{3.2}$$

Thus, since $m_q^0=0$ at this point, m_q may be factored out to yield the equation

$$1=4n_cn_fG_Si\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2-m_q^2}. \tag{3.3}$$

Solutions of Eq. (3.1) have been obtained by using a Euclidean momentum space. One may use either a covariant cutoff, where $k_E^2<\Lambda_E^2$, or the Pauli-Villars regularization procedure, among other possibilities. We have done both kinds of calculations; however, the Pauli-Villars method is to be preferred, since it preserves the symmetries of the theory.

First, we note that, if $G_S=7.91$ GeV^{-2} and $\kappa=0$, and if we use a Euclidean momentum-space cutoff of $\Lambda_E=1.0$ GeV, we find $m_q=241$ MeV. Once we include a finite value for κ, we no longer obtain a constant value of the self-energy. That is, $A(k^2)$ depends upon k^2 as shown in Fig. 2. For the result shown in Fig. 2, we put $\kappa=-0.140/8$ GeV$^2=-0.0175$ GeV^2. The factor of (1/8) in our choice for

κ serves to convert a value of κ appropriate to scalar confinement to a value appropriate to the interaction given in Eqs. (2.3) and (2.4). A factor of 2 is obtained since we have two terms in Eq. (2.3) and a factor of 4 arises from the Lorentz character of the interaction.

IV. THE VERTEX FUNCTION OF THE CONFINING INTERACTION

We now consider a vertex function that sums a ''ladder'' of confining interactions. The equation for the pseudoscalar vertex is then [2]

$$\overline{F_P^i}(q,k)=\gamma_5\tau_i+\sum_{j=1}^{2} i\int\frac{d^4k'}{(2\pi)^4}V^C(k-k')O_j(1)\times S(k'+q/2)\overline{F_P^i}(q,k')S(k'-q/2)O_j(2). \quad (4.1)$$

We define $A_+(q,k')=A[(k'+q/2)^2]$ and $A_-(q,k')=A[(k'-q/2)^2)]$ and make use of the relation

$$\gamma^\mu\{[\not k'+\not q/2+A_+(q,k')]\gamma_5[\not k'-\not q/2+A_-(q,k')]\}\gamma_\mu+\gamma_5\gamma^\mu\{[\not k'+\not q/2+A_+(q,k')]\gamma_5[\not k'-\not q/2+A_-(q,k')]\}\gamma_\mu\gamma_5$$
$$=8[(k'+q/2)\cdot(k'-q/2)-A_+(q,k')A_-(q,k')]\gamma_5. \quad (4.2)$$

We also define

$$N_P(q,k')=k'^2-q^2/4-A_+(q,k')A_-(q,k'), \quad (4.3)$$

so that, if we put

$$\overline{F_P^i}(q,k)=\gamma_5\tau^iF_P(q,k), \quad (4.4)$$

we see that the function $F_P(q,k)$ satisfies the integral equation

$$F_P(q,k)=1+i\int\frac{d^4k'}{(2\pi)^4}\frac{8V^C(k-k')N_P(q,k')F_P(q,k')}{D[(k'+q/2)^2]D[(k'-q/2)^2]}. \quad (4.5)$$

In Eq. (4.5), we have used the definition $D(p^2)=p^2-A^2(p^2)$.

An entirely similar equation may be written for the scalar confining vertex, where we put $F_S(q,k)=F_S(q,k)\mathbb{I}$. Thus,

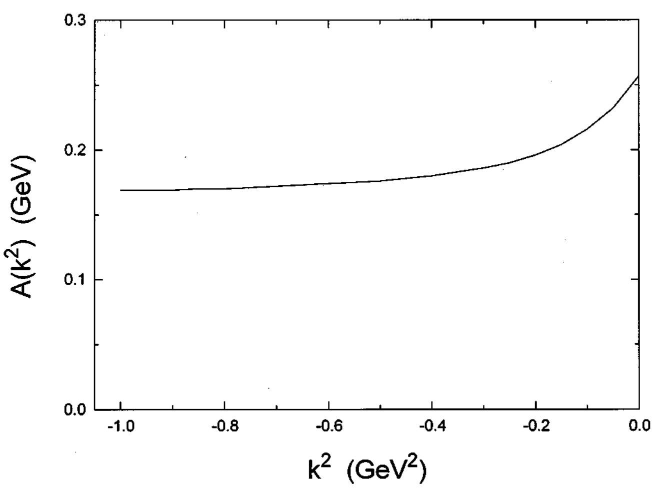

FIG. 2. Values of $A(k^2)$ are shown for $k^2\leq 0$. The calculations are made in Euclidean momentum space using a regulator function, $C(k^2)=2\Lambda^4[k^2+A^2(k^2)+\Lambda^2]^{-1}[k^2+A^2(k^2)+2\Lambda^2]^{-1}$, with $\Lambda=0.78$ GeV. Here $\kappa=-0.14/8$ GeV^2 and $G_s=8.30$ GeV^{-2}.

$$F_S(q,k)=\mathbb{I}+i\int\frac{d^4k'}{(2\pi)^4}\frac{8V^C(k-k')N_S(q,k')F_S(q,k')}{D((k'+q/2)^2)D((k'-q/2)^2)}, \quad (4.6)$$

with

$$N_S(q,k')=k'^2-q^2/4+A_+(q,k')A_-(q,k'). \quad (4.7)$$

Note the difference sign for the last term of Eqs. (4.3) and (4.7).

In the past, for $q^2>0$, we have solved the equation for $F_S(q,k)$ by completing the k_0' integral in the complex k_0' plane [2]. There are two poles in the lower-half plane arising from the quark propagators. For one pole, the quark is on its positive mass shell. For the other pole, the antiquark is on its negative mass shell. [We have also neglected any poles that would arise if we admit energy transfer in $V^C(k-k')$ and we continue to make that approximation for our calculations for timelike $q^2(q^2>0)$.] See Fig. 3.

For the function $F_P(q^2,q\cdot k,k^2)$, let us consider the case where the quark is on its mass positive shell, so that $k^0+q^0/2=E_q(\mathbf{k}+\mathbf{q})$. The resulting function is a function of only two variables. In the frame where $\mathbf{q}=0$, we define

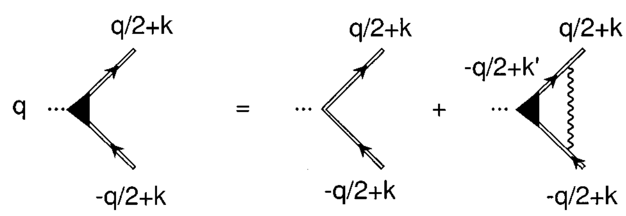

FIG. 3. The integral equation for a vertex function of the confining interaction. The solid triangular area denotes the vertex functions. [We may write integral equations for either $F_S(q,k)$ or $F_P(q,k)$.] Here the wavy line is the confining interaction $V^C(k-k')$. The resulting equation is solved with either the quark on its positive mass shell or the antiquark on its negative mass shell. (See the text.) The double line denotes the propagator $iS(k+q/2)=i[\not k+\not q/2-A_+(q,k)]^{-1}$ or $iS(k-q/2)=i[\not k-\not q/2-A_-(q,k)]^{-1}$.

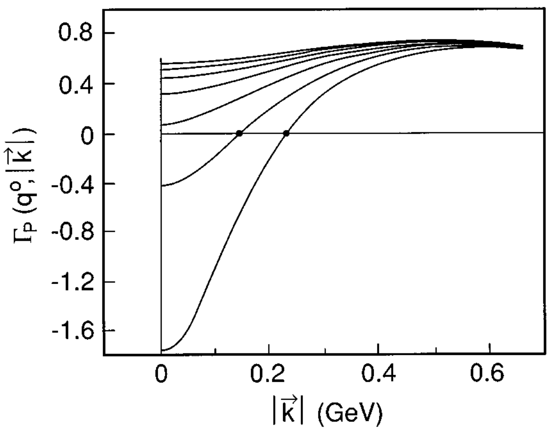

FIG. 4. The function $\Gamma_S(q^0,|\vec{k}|)$ is shown for $\kappa=-0.0175$ GeV^2 and various values of q^0. Note that $\Gamma_S(q^0,|\mathbf{k}_{0n}|)=0$, when $\mathbf{k}_{0n}^2=(q^0/2)^2-m_q^2$. Starting at the lowest curve and moving higher, we have $q^0=0.7$ GeV, $q^0=0.6$ GeV, $q^0=0.5$ GeV, $q^0=0.4$ GeV, $q^0=0.3$ GeV, $q^0=0.2$ GeV, and $q^0=0.0$ GeV. (The values for $q^0=0.1$ GeV are quite close to those for $q^0=0.0$ GeV.) The results are for a constant mass, $m_q=260$ MeV. If m_q is replaced by $A(k^2)$ only some small changes are observed near $|\mathbf{k}|=0$. A cutoff on the three-momenta has been used ($|\mathbf{k}'|<\Lambda_3$), with $\Lambda_3=0.702$ GeV. (See the text.)

$$\Gamma_P^{+-}(q^0,|\mathbf{k}|)=F_P(q^2,q\cdot k,k^2)|_{k^0+q^0/2=E_q(\mathbf{k})}. \tag{4.8}$$

Similarly, we consider the case where the antiquark is on its *negative* mass shell. In that case, we define

$$\Gamma_P^{-+}(q^0,|\mathbf{k}|)=F_P(q^2,q\cdot k,k^2)|_{k^0-q^0/2=E_q(\mathbf{k})}. \tag{4.9}$$

These functions satisfy coupled equations of the form

$$\begin{bmatrix}\Gamma_P^{+-}(q^0,|\mathbf{k}|)\\ \Gamma_P^{-+}(q^0,|\mathbf{k}|)\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}-4\int\frac{d^3k'}{(2\pi)^3}V^C(k-k')\times\begin{bmatrix}\dfrac{1}{q^0-2E_q(\mathbf{k}')}-\dfrac{1}{q^0+2E_q(\mathbf{k}')}\\ \dfrac{1}{q^0-2E_q(\mathbf{k}')}-\dfrac{1}{q^0+2E_q(\mathbf{k}')}\end{bmatrix}\times\begin{bmatrix}\Gamma_P^{+-}(q^0,|\mathbf{k}'|)\\ \Gamma_P^{-+}(q^0,|\mathbf{k}'|)\end{bmatrix}. \tag{4.10}$$

We see, however, from the last equation that

$$\Gamma_P^{+-}(q^0,|\mathbf{k}|)=\Gamma_P^{-+}(q^0,|\mathbf{k}|). \tag{4.11}$$

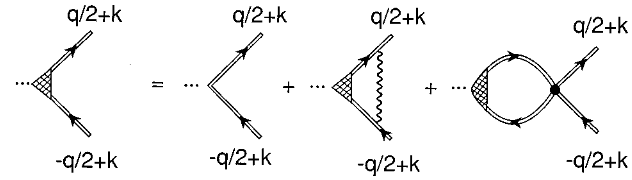

FIG. 5. The equation satisfied by the pseudoscalar-isovector vertex of the total interaction, $F_T(q,k)$, is shown as a crosshatched triangular area. The wavy line is the confining interaction, $V^C(k-k')$, and the solid circle represents a factor of iG_S. The driving term for this equation $\gamma_5\tau^j$. [See Eqs. (5.1) and (5.2).]

Discarding the superscripts, we have a single integral equation

$$\Gamma_P(q^0,|\mathbf{k}|)=1-4\int\frac{d^3k'}{(2\pi)^3}\frac{4E_q(\mathbf{k}')}{(q^0)^2-[2E_q(\mathbf{k}')]^2}\times V^C(k-k')\Gamma_P(q^0,|\mathbf{k}'|). \tag{4.12}$$

In Fig. 4 we present values of $\Gamma_P(q^0,|\mathbf{k}|)$ for several values of $q^0\geqslant0$. It may be seen from the figure that $\Gamma_P(q^0,|\mathbf{k}|)$ for large $|\mathbf{k}|$ is about 0.6–0.7. Those values are related to a cutoff placed on the three-momenta in the integral equation, $\Lambda_3=0.702$ GeV ($|\mathbf{k}'|\leqslant\Lambda_3$). However, the integral equation does not need to be regulated, since the integrals converge as $|\mathbf{k}'|\to\infty$. If one puts Λ_3 equal to several GeV, the various $\Gamma_P(q^0,|\mathbf{k}|)$ go rapidly to 1 with increasing $|\mathbf{k}|$ for all q^0 values considered here. We recall that it is the zeroes of the confining vertex functions for $k_{0n}^2=(q^0/2)^2-m_q^2$ that remove the $q\bar{q}$ cut in various vacuum polarization diagrams. Such cuts would appear for $q^2>4m_q^2$ in the absence of a model for confinement [2,3]. The zeroes of $\Gamma_P(q^0,|\mathbf{k}|)$ are indicated in Fig. 4 by small dots. For those curves without such dots, q^0 is too small ($q^0<2m_q$) to lead to on-mass-shell quarks in the absence of confinement.

V. THE VERTEX FUNCTION FOR THE TOTAL INTERACTION AND THE GOLDSTONE BOSON OF THE GENERALIZED NJL MODEL

In this section we discuss a vertex function, $\overline{F_T}(q,k)$, that includes the effects of *both* the confining interaction and the (zero-range) NJL interaction. The integral equation for that quantity is depicted in Fig. 5. That integral equation reads, in the pion channel,

$$\overline{F_T}(q,k)=\gamma_5+\sum_{j=1}^{2}i\int\frac{d^4k'}{(2\pi)^4}V^C(k-k')O_j(1)S(q/2+k')\overline{F_T}(q,k')S(-q/2+k')O_j(2)-n_cn_fiG_S\gamma_5\times\int\frac{d^4k'}{(2\pi)^4}\mathrm{Tr}[S(q/2+k')\overline{F_T}(q,k')S(-q/2+k')\gamma_5]. \tag{5.1}$$

In the last equation, we have already factored out the isospin matrix that appears in the driving term, $\gamma_5\tau^j$. We then define $F_T(q,k)=\gamma_5F_T(q,k)$ where $F_T(q,k)$ is a scalar function. We have

$$F_T(q,k)=1+i\int \frac{d^4k'}{(2\pi)^4}\frac{[8N_P(q,k')V^C(k-k')-n_cn_fG_SM(q,k')]F_T(q,k')}{D_+(q,k')D_-(q,k')}, \tag{5.2}$$

where we have put $D_+(q,k')\equiv D((q/2+k')^2)$ and $D_-(q,k')\equiv D((q/2-k')^2)$. Note that $N_P(q,k)$ was given previously in Eq. (4.3) Here, we have also defined

$$M(q,k)=4\left(\frac{q^2}{4}-k^2+A_+(q,k)A_-(q,k)\right). \tag{5.3}$$

We now consider the limit $q\rightarrow 0$. We have $D_+(0,k)=D_-(0,k)=D(k^2)$, $N_P(0,k)=D(k^2)$, and $M(0,k)=-4D(k^2)$. Therefore,

$$F_T(0,k)=1+i\int \frac{d^4k'}{(2\pi)^4}\frac{8V^C(k-k')+4n_cn_fG_S}{D(k'^2)}F_T(0,k'). \tag{5.4}$$

If there is a bound state at zero energy, the homogeneous version of Eq. (5.4) will have a solution. Thus, we consider

$$F_T(0,k)=i\int \frac{d^4k'}{(2\pi)^4}\frac{8V^C(k-k')+4n_cn_fG_S}{D(k'^2)}F_T(0,k'). \tag{5.5}$$

However, the gap equation, was

$$A(k^2)=i\int \frac{d^4k'}{(2\pi)^4}\frac{8V^C(k-k')+4n_cn_fG_S}{D(k'^2)}A(k'^2). \tag{5.6}$$

Thus, we see that $F_T(0,k)$ is proportional to $A(k^2)$ and we have a zero-mass state. The pion is the Goldstone boson, as expected.

It is sometimes useful to define the wave function

$$\psi_T(k^2)=n\,\frac{1}{k^2-A^2(k^2)}A(k^2), \tag{5.7}$$

where n is a normalization factor.

We have

$$[k^2-A^2(k^2)]\psi_T(k^2)=i\int \frac{d^4k'}{(2\pi)^4}[8V^C(k-k')+4n_cn_fG_S]\psi_T(k'^2). \tag{5.8}$$

VI. DISCUSSION

Work that has some relation to ours was carried out by Gross and Milana [4]. These authors obtained coupled equations for meson wave functions, in the presence of a confining interaction, by performing integrals in the complex k_0' plane and picking up the poles of the quark propagators, as was done in our work when q^2 was timelike. However, the formalism used in Ref. [4] does not yield a zero-mass pion unless the confining potential satisfies a constraint. [The constraint used is a relativistic generalization of the relation $V^C(r)=0$, for $r=0$.]

The appropriate Lorentz transformation properties of the confinement interaction is a matter of some uncertainty. Scalar confinement has been popular, since it provides a good representation of spin-orbit effects in heavy-quark systems. However, a number of authors have suggested that better results may be obtained with vector confinement. In particular, Münz [5] has noted that the Salpeter equation used for the study of meson structure is unstable for scalar confinement [6], while vector confinement works well. Studies of Swanson and Isgur [7] and of Szczepaniak [8] also suggest the importance of vector confinement. Further, Resag and Münz have used vector confinement and have only kept the term involving γ_0 [9]. In addition, Münz has used equal amounts of scalar and vector confinement, again keeping only the γ_0 term of the vector-confinement interaction [5].

In the work reported here, we have shown that the *V-A* form of the confining interaction yields simple equations for the quark self-energy and for the pseudoscalar vertex function. Thus, we could carry out our analysis without solving coupled, nonlinear equations for $A(k^2)$ and $B(k^2)$. We could also deal with a single scalar function $F_P(q,k)$, rather than with four such functions that characterize the pseudoscalar vertex in the general case. On the other hand, we do not recommend the *V-A* form for the study of meson spectra. We believe that vector confinement provides a more satisfactory model and also allows us to write a Lagrangian with chiral symmetry. We hope to complete a study of vector confinement in the future. The equations that will be studied are more complicated than those considered in the present study. Results of some preliminary calculations for the case of vector confinement are reported in the Appendix.

In this work we have shown that one may write a Lagrangian that has chiral symmetry when we include a model of confinement. However, our work has a number of limitations and further work is required. For example, our analysis has been made in a Euclidean momentum space. We would like to have results for timelike values of the momentum as well. One might attempt to continue the results for spacelike momenta into the timelike region, but we do not believe that the accuracy of such a procedure is known and we do not adopt that approach. On the other hand, we have made a large number of calculations in Minkowski space for timelike values of the momenta [2,3]. We have used an approach similar to that of Gross and Milana [4] in that we have only considered the singularities of the quark propagators, when completing integrals in the complex k_0' plane. That is a correct procedure, if the confining potential has no singularities in the k_0' plane. Therefore, we have neglected energy transfer when using the confining interaction in Minkowski-space calculations. Thus, we see that our Euclidean-space calculations reported here are not entirely consistent with our

Minkowski-space calculations reported upon elsewhere [2,3]. We hope to study this problem in the future. While the Lorentz structure of the confining interaction may be considered an open question, we have shown that confining interactions that preserve chiral symmetry may be used for various studies. Such potentials are to be preferred, since they are consistent with the properties that a good effective interaction for low-energy QCD should have. Their use is essential, if we are to study the effects of confinement in the case of the pseudoscalar octet (Goldstone bosons).

APPENDIX

In this appendix, we wish to study the self-energy, $\Sigma(k)=B(k^2)\not{k}+A(k^2)$, and the vertex function, $F_T(q,k)$, for pure vector confinement. In this case we find the coupled nonlinear equations

$$A(k^2)=i\int \frac{d^4k'}{(2\pi)^4}\frac{4V^C(k-k')+4n_cn_fG_S}{k'^2[1-B(k')]^2-A^2(k'^2)}A(k'^2) \tag{A1}$$

and

$$k^2B(k^2)=-i\int \frac{d^4k'}{(2\pi)^4}\frac{2(k\cdot k')[1-B(k'^2)]V^C(k-k')}{k'^2[1-B(k')]^2-A^2(k'^2)}. \tag{A2}$$

The pseudoscalar vertex function is more complicated than in the V-A model, since there are now four scalar functions of three variables to be calculated. We may write, in the general case,

$$\overline{F_T}(q,k)=\gamma_5[a_1(k,q)+\not{k}a_2(k,q)+\not{q}a_3(k,q)+\not{q}\not{k}a_4(k,q)], \tag{A3}$$

with $a_1(k,q)=a_1(k^2,k\cdot q,q^2)$, etc. However, $\overline{F_T}(q,k)$ is simpler, if we consider the case $q=0$:

$$\overline{F_T}(0,k)=\gamma_5[a_1(k^2)+\not{k}a_2(k^2)], \tag{A4}$$

where $a_1(k^2)=a_1(0,k)$, etc.

The vertex function for the total interaction satisfies the equation

$$\overline{F_T}(0,k)=\gamma_5+i\int \frac{d^4k'}{(2\pi)^4}V^C(k-k')\gamma^\mu S(k')\overline{F_T}(0,k')\times S(k')\gamma_\mu-n_cn_fiG_S\gamma_5\int\frac{d^4k'}{(2\pi)^4}\times\mathrm{Tr}[S(k')\overline{F_T}(0,k')S(k')\gamma_5]. \tag{A5}$$

We now make use of Eq. (A4) and find that

$$a_1(k^2)=1+i\int \frac{d^4k'}{(2\pi)^4}\frac{[4V^C(k-k')+4\dot{n}_cn_fG_S]a_1(k'^2)}{k'^2[1-B(k'^2)]^2-A^2(k'^2)} \tag{A6}$$

and

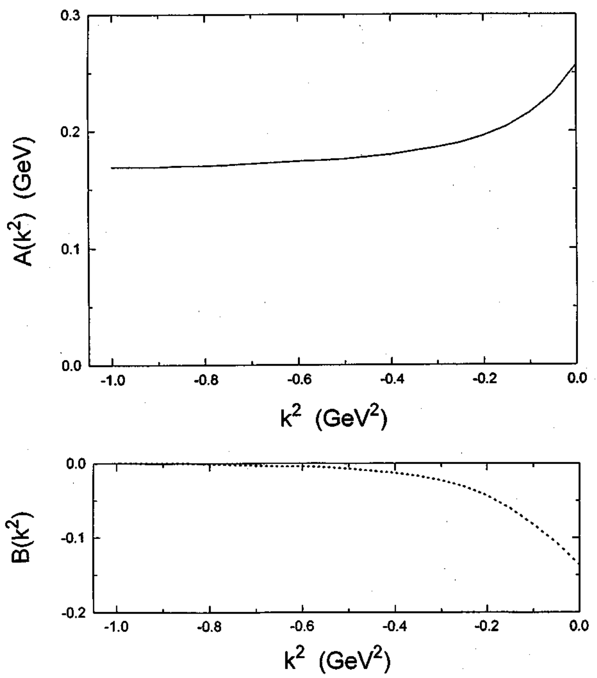

FIG. 6. Solutions of the coupled equations for $A(k^2)$ and $B(k^2)$, generalized to include a current mass, $m_q^0=5.5$ MeV, are shown. [See Eqs. (A1) and (A2).] Here we use vector confinement with $\kappa=-0.14/4$ GeV^2 and $G_S=8.65$ GeV^{-2}. (For this calculation we have used a Pauli-Villars regularization procedure. See caption of Fig. 2.)

$$k^2a_2(k^2)=-i\int \frac{d^4k'}{(2\pi)^4}\frac{2(k\cdot k')V^C(k-k')a_2(k'^2)}{k'^2[1-B(k'^2)]^2-A^2(k'^2)}. \tag{A7}$$

Now, let us ask if Eq. (A5) has a homogeneous solution. We write Eq. (A6), with the ''driving term'' removed, as

$$a_1(k^2)=i\int \frac{d^4k'}{(2\pi)^4}\frac{[4V^C(k-k')+4n_cn_fG_S]a_1(k'^2)}{k'^2[1-B(k'^2)]^2-A^2(k'^2)}. \tag{A8}$$

Now consider Eqs. (A7) and (A8). We see that comparison to Eq. (A1) shows that $a_1(k^2)\sim A(k^2)$. Further, we may put $a_2(k^2)=0$, since Eq. (A7) is homogeneous. Thus, in the chiral limit, the vertex for $q=0$ is $F_T^i(0,k)=\gamma_5\tau^iF_T(k^2)$, where $F_T(k^2)$ is proportional to $A(k^2)$. Since there is no driving term, $F_T(k^2)$ is related to the pion wave function

$$\psi_T(k^2)=\eta\frac{1}{k^2[1-B(k^2)]^2-A(k^2)}A(k^2), \tag{A9}$$

where η is a normalization constant. [Recall Eq. (4.7).] We remark that

$$\{k^2[1-B(k^2)]^2-A^2(k^2)\}\psi_T(k^2)=i\int\frac{d^4k'}{(2\pi)^4}[4V^C(k-k')+4n_cn_fG_S]\psi_T(k'^2) \tag{A10}$$

in the case of vector confinement.

In Fig. 6 we show values for $A(k^2)$ and $B(k^2)$ obtained when solving Eqs. (A1) and (A2). We have used $\kappa=-0.14/4$ GeV^2, $G_S=8.65\ \mathrm{GeV}^{-2}$ and have included a finite value of the current quark mass, $m_q^0=5.5$ MeV, in one case. [Note that $B(k^2)$ is dimensionless.]

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[1] For reviews of the NJL model and various applications see S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992); U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991).

[2] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Phys. Rev. D **51**, 3638 (1995); L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Ann. Phys. (N.Y.) **241**, 1 (1995).

[3] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Int. J. Mod. Phys. E **2**, 603 (1993); L. S. Celenza, C. M. Shakin, and J. Szweda, *ibid.* **2**, 437 (1993).

[4] F. Gross and J. Milana, Phys. Rev. D **43**, 2401 (1991); **45**, 969 (1992).

[5] C. R. Münz, Nucl. Phys. **A609**, 364 (1996).

[6] J. Parramore and J. Piekarewicz, Nucl. Phys. **A585**, 705 (1995).

[7] Eric Swanson, private communication.

[8] A. Szczepaniak and E. S. Swanson, LANL e-print archive: hep-ph/9611310 (unpublished).

[9] J. Resag and C. R. Münz, Nucl. Phys. **A590**, 735 (1995).