Limits to the validity of the Glauber approximation for heavy-ion scattering, and a possible assessment of in-medium *NN* Pauli blocking

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A feature universally employed in the application of the Glauber approximation to the scattering of a projectile by a many-particle target is the assumption that the complex phase shift $\chi(b)/2=\delta(\ell)$ itself carries a phase which is independent of the impact parameter b. The most common assumption is that $\chi(b)=(\alpha+i)g(b)$, where $g(b)$ is real and $\alpha=\mathrm{Re}[f(0°)]/\mathrm{Im}[f(0°)]$ is the real-to-imaginary ratio of the forward amplitude for elastic scattering of the projectile (or one of its constituents) on the constituents of the target. Since $g(b)$ is proportional to an eikonal integral through an effective potential $V+iW$, this form also assumes that $V(r)=\alpha W(r)$, i.e., that the real and imaginary parts of this potential have the same radial shapes. In recent years this approximation has been applied to heavy-ion elastic scattering in the relatively low-energy range below 100 MeV/nucleon. Phenomenological optical potentials for these same nuclei and energies strongly violate the above condition, although they do approximate it in the surface region responsible for scattering to angles dominated by Fraunhofer oscillations. The Glauber approximation provides a qualitative fit to these oscillations, indicating that it is at least employing the correct nuclear radii. However, it grossly overestimates the absorption at larger angles in cases where these angles are sensitive to the interior of the potential. This failure may provide a sensitive means of assessing the effects of Pauli blocking on in-medium *NN* scattering. [S0556-2813(97)04903-0]

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I. INTRODUCTION

The scattering of a projectile nucleus P by a target T can, in principle, involve collisions between all A_PA_T possible pairs of nucleons, whatever the impact parameter b separating the centers of the two nuclei. The double-Glauber calculation [1] describing an elastic $P+T$ collision (which we shall simply call the Glauber calculation) recognizes this, and if done in detail for ''moderate'' energy collisions requires the summation of a very complex multiple-scattering series, as Franco and Tekou have described [2].

At sufficiently high energy, however, Czyz and Maximon [3] have pointed out that this series simplifies to the ''optical limit'' of the Glauber approximation. In this further approximation, the component nucleons move along straight-line paths during the collision, and the $P+T$ phase shift at impact parameter b is given by the elementary eikonal integral

$$\delta_{Gl}(b)=-\frac{k_{\mathrm{c.m.}}}{4E_{\mathrm{c.m.}}}\int_{-\infty}^{\infty}U_{\mathrm{DF}}(b,z)dz, \tag{1}$$

through the complex effective potential U_{DF}. Again, the simplest high-energy approximation to U_{DF} (*both* real and imaginary parts) is [1,3] just the zero-range double-folding potential

$$U_{\mathrm{DF}}(\vec{r})=-\frac{1}{2}\hbar v\sigma_T^{NN}(\alpha+i)\int\rho_T(\vec{r}_T)\rho_P(\vec{r}-\vec{r}_T)d^3r_T. \tag{2}$$

Here v is the relative velocity between the two nuclei, σ_T^{NN} is a spin-isospin-averaged NN total cross section at this relative velocity, $\alpha=\mathrm{Re}[f^{NN}]/\mathrm{Im}[f^{NN}]$, evaluated at 0° and the same velocity, and ρ_P and ρ_T are nuclear densities normalized to A_P and A_T. By the optical theorem,

$$\frac{1}{2}\hbar v\sigma_T^{NN}(\alpha+i)=t^{NN}(0°)=2\pi\frac{\hbar^2}{m}f^{NN}(0°), \tag{3}$$

where $f^{NN}(0°)$ is the forward NN scattering amplitude (actually assumed to be independent of angle in this zero-range approximation) and $t^{NN}(0°)$ is the corresponding t matrix. Consequently we recognize that the Glauber phase shift given by Eq. (1) is exactly that of the so-called ''$t_{\rho\rho}$'' approximation.

The last decade has seen the accumulation of a very substantial body of detailed elastic nucleus-nucleus angular distributions, to which it could be imagined that these calculations might be applied. These data are for certain light heavy-ion combinations like $^{12}C+^{12}C$, $^{16}O+^{16}O$, $\alpha+X$, etc., over the bombarding energy range below 100 MeV per nucleon. These are particularly appealing data because the occurrence of distinctive nuclear rainbows in many of the angular distributions permits the determination of nearly unique optical potential fits to them [4–8]. These fits in turn provide nearly unique phase shifts, so that the applicability of the above Glauber approximation to these data can be checked, either by a comparison of the folding potential of Eq. (2) with the empirical optical potentials ($V+iW$) or by a comparison of the Glauber and optical phase shifts. In Secs.

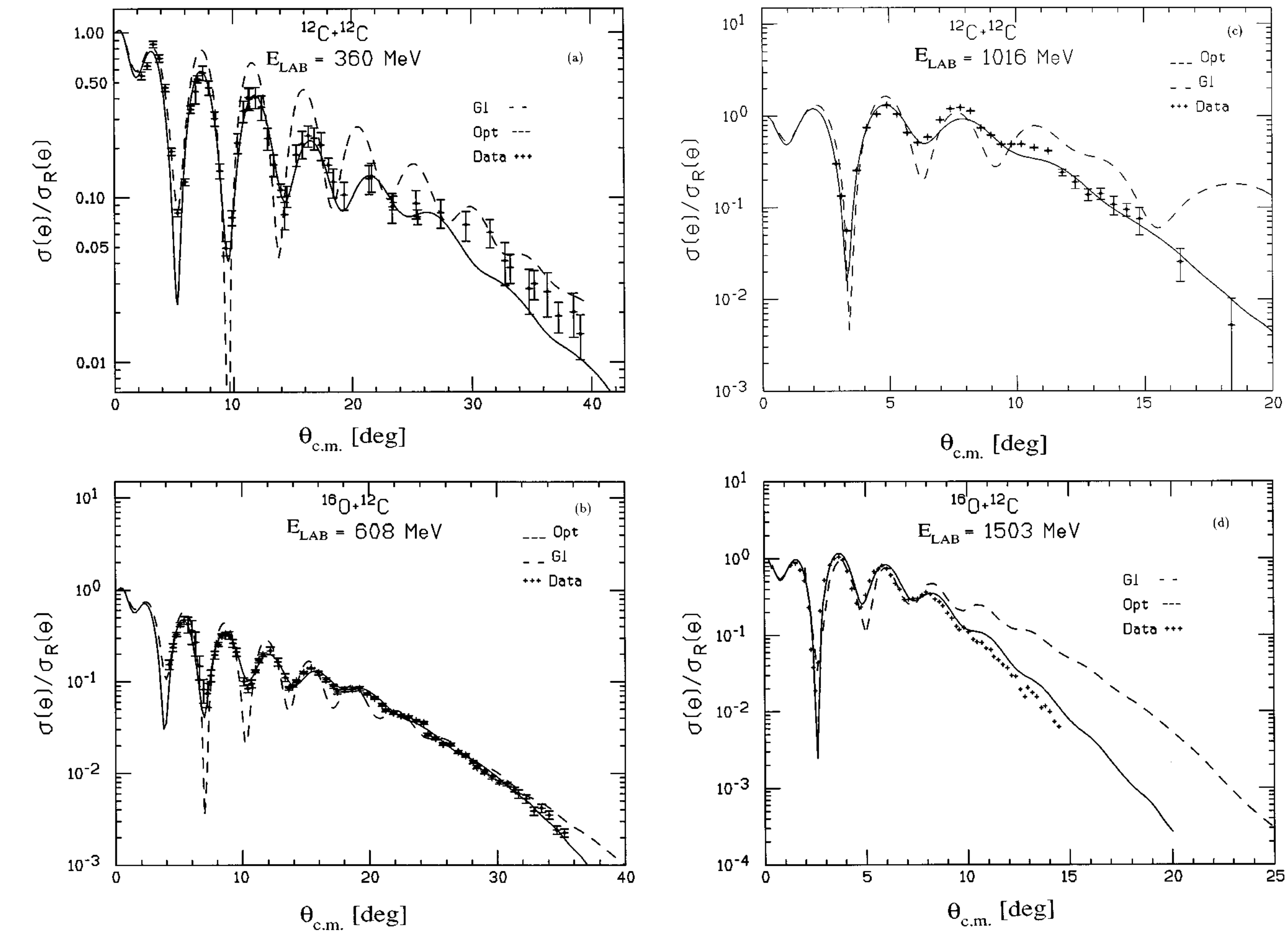

FIG. 1. Experimental angular distributions compared to optical-model fits [4] and Glauber calculations [11] for ^{12}C and ^{16}O projectiles, at four bombarding energies. Note that the Glauber calculations at the two higher energies (right-hand graphs) are *less* successful than those at the two lower energies.

II and III we provide comparisons of the phase shifts for a few representative cases, which show unambiguously that the Glauber approximation fails badly for these data, at angles sensitive to the interior of the potential, by grossly overestimating the absorption out of the entrance channel.

Disappointing as this might at first sight seem, we suggest in Sec. IV and in the following article [9] that this very failure may provide a means, via the ℓ dependence of the optical phase shifts [or equivalently via the r dependence of $W(r)/V(r)$], of investigating the way in which the Pauli blocking of in-medium NN scattering depends on the local density of that medium. Section V is a summary which points out that the rainbow data considered here appear to be unique within the entire body of strong-interaction collision data.

II. FEW REPRESENTATIVE EXAMPLES OF DOUBLE-GLAUBER FITS AND PHASE SHIFTS

Several years ago Chauvin *et al.* [10] and Lenzi *et al.* [11] published Glauber calculations of exactly this optical-limit, zero-range type and compared them with some of the above-mentioned elastic nuclear data. In Fig. 1 we show four of the results of Lenzi *et al.* and compare them both with the data [12–15] and with optical fits found in a generic study of light nuclei. Remarkably, the fits at the two lower energies give the appearance of being fairly acceptable, while those at the highest energy ($E_{c.m.} \approx 40-60$ MeV/nucleon), where the Glauber approximation would be expected to improve, are actually the worst.

Some insight into the reasons behind this puzzle is provided by Fig. 2, which shows the ratio $\mathrm{Re}[\delta(\ell)]/\mathrm{Im}[\delta(\ell)]$ for both the optical and Glauber phase shifts, as well as the corresponding ratio $V(b)/W(b)$ for the real and imaginary parts of the optical potentials; this latter ratio is evaluated at the Coulomb distance of closest approach, b, for each ℓ value, as described in the Appendix. The reason for displaying these ratios is in part because they are equal to a constant [the α of Eq. (3)] in the optical limit of the Glauber approximation and in part because we show in the succeeding article [9] that the inverse ratio W/V for the empirical optical potentials in the $E_{lab}/A < 100$ MeV energy range exhibits interesting systematics, independently of its relation to the Glauber approximation.

Figure 2 explains why we find the apparently acceptable Glauber fits of Fig. 1 at 360 and 608 MeV surprising: In contrast to these fits, the Glauber phases disagree strongly with the optical ones by being very much more absorptive. In

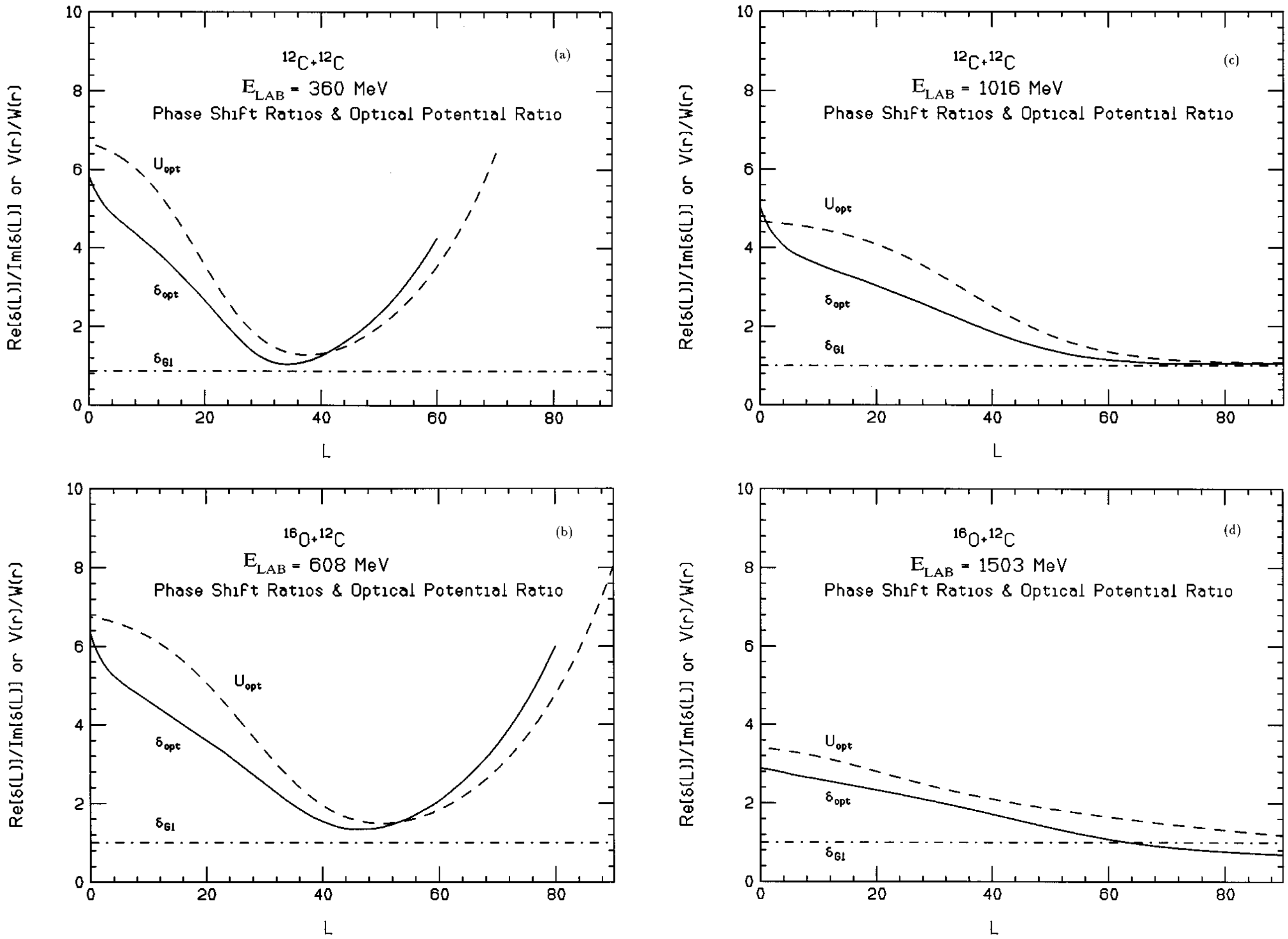

FIG. 2. Ratio of real to imaginary parts of (both Glauber and optical-model) phase shifts and of $U_{opt}(r)=V(r)+iW(r)$. The ratio for U_{opt} is plotted vs ℓ by using the approximation $r=b'(\ell)$, with $b'(\ell)$ given by Eq. (A2).

spite of the many well-documented cases of nuclear optical potential ambiguities, we have great confidence, as explained in the succeeding article, that the optical phases given here are the correct ones. Basically this is both because of the extensive quantities of elastic data available for these systems and because of the nuclear rainbows they exhibit at energies other than those which Lenzi *et al.* happened to study. These rainbows could never have existed if the potentials (and hence phase shifts) had been as absorptive as those of the Glauber calculation.

We recall from Eq. (1) that in the Glauber-eikonal approximation, the phase shift is *linear* in the potential U_{DF} and that in this limit the real and imaginary parts of this potential have the same radial shape, so that V_{DF}/W_{DF} is independent of r:

$$\frac{\mathrm{Re}[\delta_{Gl}(\ell)]}{\mathrm{Im}[\delta_{Gl}(\ell)]}=\frac{V_{DF}}{W_{DF}}=\alpha. \tag{4}$$

Over the nucleon bombarding energy range 30–100 MeV, α is close to unity [11], and so the Glauber approximation predicts the real and imaginary parts of the double-folding potential to be equal. This stands in flat contradiction to the occurrence of nuclear rainbows for the nucleus-nucleus scattering under consideration.

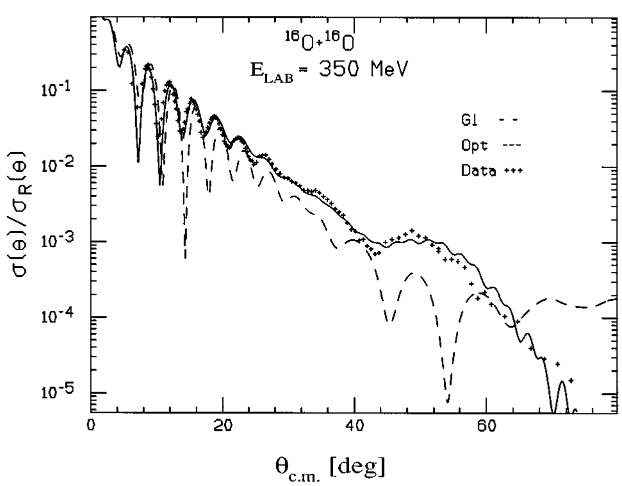

FIG. 3. Elastic angular distribution for $^{16}O+^{16}O$ at $E_{lab}=350$ MeV [21] compared with optical-model and Glauber calculations. The optical potential used is potential *A* of Kondō *et al.* [8]. The broad minimum at 44° in the optical calculation is due to a nuclear rainbow, which is entirely absent from the Glauber calculation. The Glauber minima at 45° and 55° are not rainbow minima, as can be verified by following their behavior when the imaginary part of U_{DF} from Eq. (2) is artificially decreased.

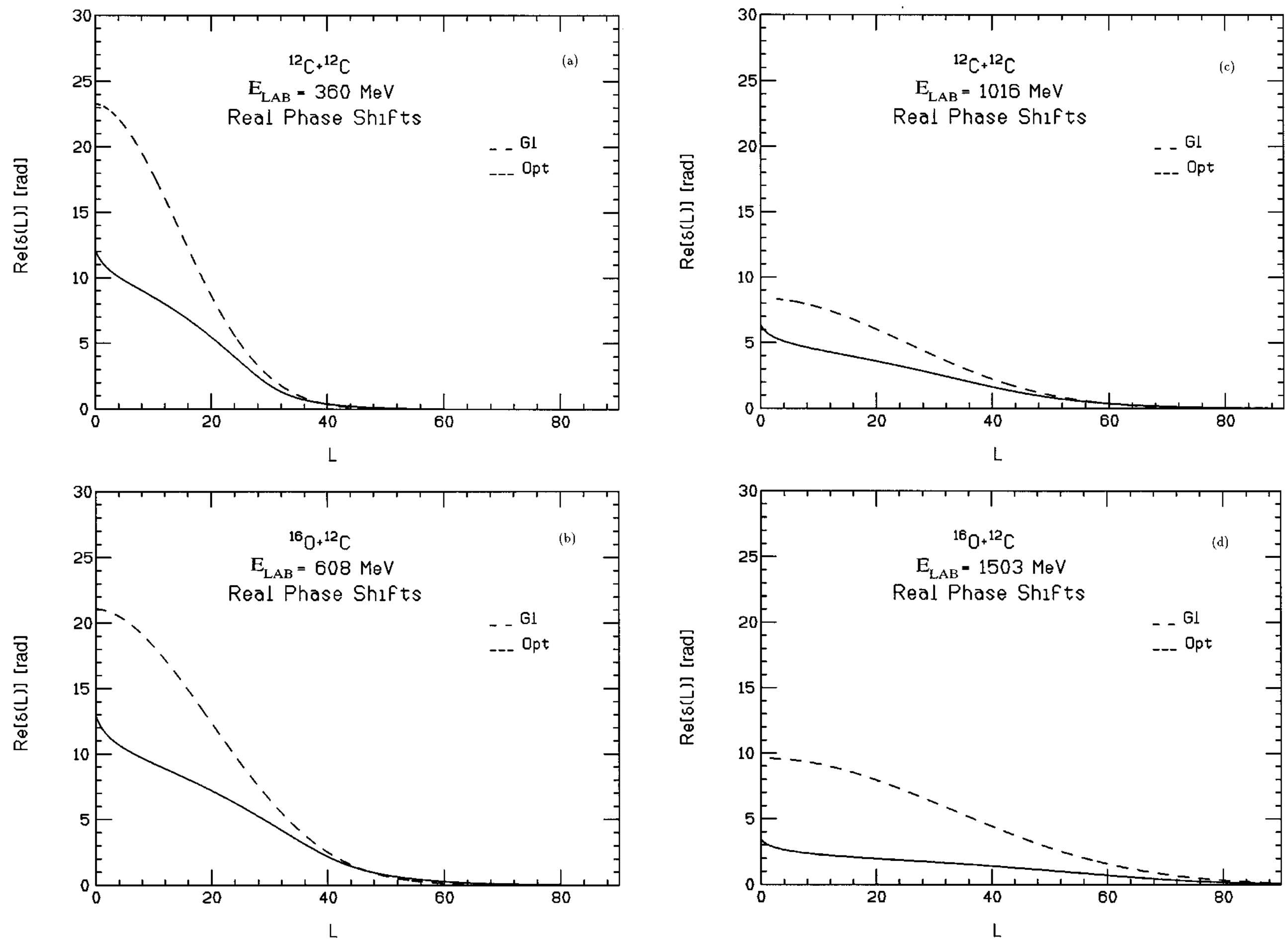

FIG. 4. Comparison of real parts of Glauber [11] and optical-model [4] phase shifts, in units of radians.

How, then, can even the rather poor approximation to the phase shifts provided by the Glauber expression give the appearance of ''reasonable'' fits to the data for the cases shown in Fig. 1? The answer seems to lie in the existence of well-known potential ambiguities at the lower two energies, but not at the higher ones. As discussed in [4,16,17], attempts to fit the 608 MeV data by searching on optical-model parameters have turned up two very different types of potential fits. One employs a unique, relatively ''transparent'' potential ($W \lesssim V/5$ at small r, $|S_0| \equiv |S(\ell=0)| \geq 10^{-2}$), and the other is actually a continuous *family* of highly absorptive potentials, with $|S_0| \leq 10^{-4}$: If little flux penetrates the interior of the potential, the scattering will be insensitive to the details of its interior region and so can be fit equally well by all members of an entire family. The optical model analysis of the 360 MeV data displays the same type of ambiguity. Fortunately, data are available for these systems at several energies, and taking all the measurements into account makes it clear that only the transparent potentials are acceptable over the entire energy range. The Glauber potentials, however, are clearly of the absorptive type. Although better absorptive fits than these can be found, the insensitivity to details within this family permits the Glauber potentials at 360 and 608 MeV to mimic the absorptive family well enough to produce the fits of Fig. 1. For $^{16}O+^{12}C$ at 1503 MeV, on the other hand, extensive optical model searches [18] have found two transparent-type potentials ($|S_0| \approx 10^{-2}-10^{-1}$), but never a continuously ambiguous absorptive family. For $^{12}C+^{12}C$ at 1016 MeV, a similar situation is encountered [19]: The only two ''acceptable'' potentials have $|S_0|=0.01$ and 0.07, corresponding to $W=43$ and 25 MeV at $r=0$, respectively. It is apparently the reduced absorption demanded by the data which explains the failure of the Glauber approximation at this energy. Even an ^{16}O projectile at 1503 MeV does not provide a large enough momentum for the Glauber approximation to be valid at small as well as large impact parameters.

This phenomenon of potential ambiguities shows clearly that achieving even quite a good fit to certain types of angular distributions does not guarantee that the fit is meaningful, i.e., that the correct phase shifts have been found. The ''problem'' with the angular distributions at 360 and 608 MeV, which permits these phase shift ambiguities to exist, is the lack of any structure in their smooth far-side components [20] in the angular range beyond the high-frequency Fraunhofer (''diffraction'') oscillations. In contrast, more recent data shown in Fig. 3 provides an $^{16}O+^{16}O$ angular distribution [21] which displays a prominent non-Fraunhofer dip in this angular region. It is due to a far-side or ''nuclear'' rainbow, i.e., a destructive interference between two far-side amplitudes, one peripheral and the other from a deeply penetrating trajectory. In order for this inner-trajectory amplitude to be large enough to beat significantly against the outer one, the corresponding potential must be relatively transparent—

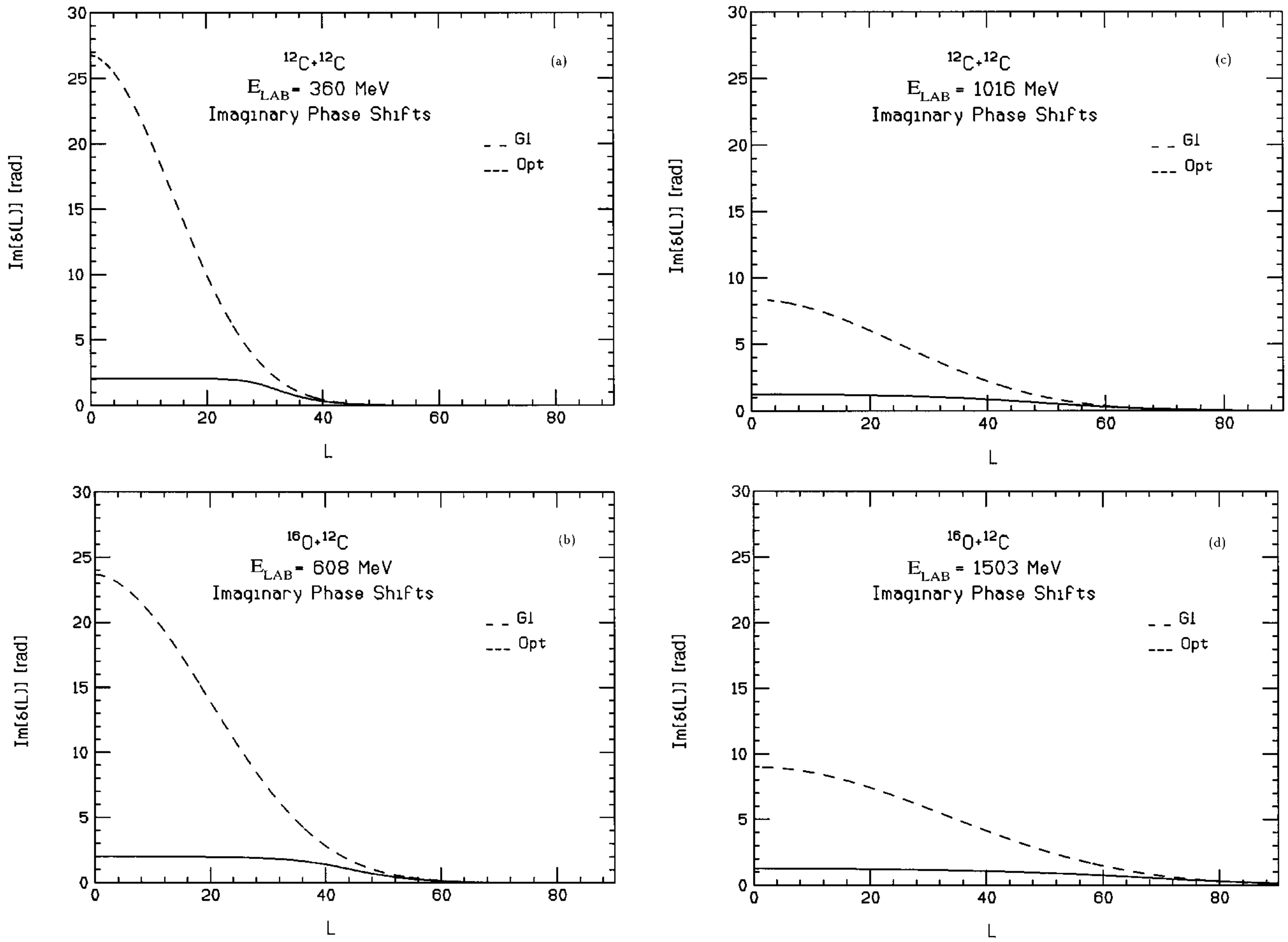

FIG. 5. Comparison of imaginary parts of Glauber [11] and optical-model [4] phase shifts, in units of radians.

certainly far more transparent than the Glauber potential U_{DF}. The Glauber calculation is guaranteed to fail in this case, and indeed Fig. 3 shows that it does. Had these data been available at the time of the above-cited Glauber studies, the failure of the Glauber approximation at these energies would have been much more apparent.

Before proceeding with our discussion of phase shifts and large-angle scattering, we should remark that Lenzi *et al.* have industriously published a substantial series of subsequent articles, on reactions as well as on elastic scattering [22], with quite remarkable success. In view of our findings in this article, we can only conjecture that this success must be heavily dependent on the fact that it is found predominantly at grazing angles, which sample only the surface region of the colliding nuclei—exactly the region where we find significant agreement between the Glauber and optical phases. Notch tests indicate, incidentally, that these angular distributions are in fact most sensitive to the potential in just this surface region. If the Glauber potential had not been reasonably accurate in this region, its inadequacy would have been obvious much sooner. If this interpretation is correct, our conjecture in the following article, that systematic deviations from the Glauber predictions will be found if more data are taken at angles *beyond* the grazing angle, becomes even more germane to tests of the Glauber prediction and to the possible interpretation of deviations in terms of Pauli blocking.

III. PHASE SHIFTS IN DETAIL

A direct comparison of both the real and imaginary parts of the Glauber phases with those of the optical potentials [4] is shown, for the curves of Fig. 1, in Figs. 4 and 5. The Glauber approximation to the real phases is quite acceptable at large ℓ values. This agrees with the well-known success of the folding procedure for the real part of the optical potential at large r values and emphasizes its need for a density-dependent modification at interior r values [16,23,24]. The imaginary part of the phase shifts, on the other hand, is overestimated at 360 MeV by as much as a factor of 13 [which gives $|S_{\mathrm{Gl}}(\ell=0)|\sim e^{-53}\sim 10^{-23}$, compared with $|S_{\mathrm{opt}}(\ell=0)|\sim e^{-4}\sim 10^{-2}$]. This suppression of the low-ℓ contributions removes all possible rainbow oscillations from the Glauber cross sections. It has been known for many years, in fact, that the simple folding potential unrealistically overestimates absorption [25]. For that reason it was abandoned long ago and has led to the use of a purely phenomenological imaginary potential in conjunction with a real folding potential for seeking empirical fits to heavy-ion data [23].

Figure 6 compares the Glauber and optical phase shifts for the cross sections shown in Fig. 3. Since the optical fit is necessarily of the transparent type, the difference between the two is very similar to those seen in Figs. 4 and 5, except that the optical potential used in Fig. 3 [8] is about twice as

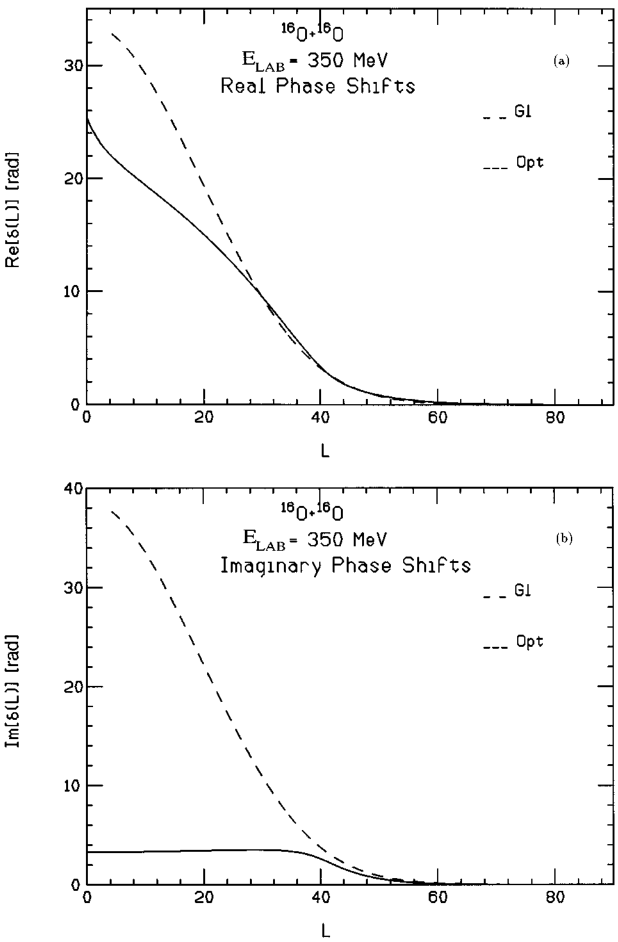

FIG. 6. Comparison of real and imaginary parts of Glauber and optical-model phase shifts, in units of radians, for the $^{16}O+{}^{16}O$ case of Fig. 3. The optical potential used is potential A of Kondo *et al.* [8].

deep as those used in Figs. 4 and 5, yielding a low-ℓ real phase shift which is about twice as large.

IV. PAULI BLOCKING AND THE FRIVOLOUS APPROXIMATION

The conclusion to be drawn from the above discussion is that the failure of the Glauber approximation for light heavy-ion scattering at $E_{\rm lab}/A<100$ MeV is due to a surprising ''transparency'' of these particular target and projectile combinations. It is exactly this transparency which manifests itself in the nuclear rainbows seen in the elastic scattering of these nuclei at these energies. Its most obvious interpretation is in terms of a suppression of nucleon-nucleon scattering in the nuclear medium, such as that due to Pauli blocking. The Glauber approximation entirely neglects the Pauli principle, both before the collision (by putting all nucleons of a given nucleus into the same single-particle state [2]) and after it (by allowing nucleons to recoil into all states, occupied or not).

To provide a bit of insight into this non-Pauli $U_{\rm DF}$, we recall a simple example (sometimes called the ''frivolous model'') in which it can be evaluated analytically: $A_P=A_T=A$, with identical ''square'' radial shapes of volume V for both target and projectile. From Eq. (2),

$$V_{\rm Gl}(\vec{r})+iW_{\rm Gl}(\vec{r})=-\frac{1}{2}\hbar v\,\sigma_T^{NN}(\alpha+i)\int \rho_T(\vec{r}_T)\times\rho_P(\vec{r}-\vec{r}_T)d^3r_T. \quad (5)$$

In this case $\rho=A/V$, and at complete overlap ($\vec{r}=0$), the above integral becomes $\int\rho^2 d^3\vec{s}=A^2/V=A\rho$; so the depth of the imaginary potential is given in this approximation by

$$\begin{aligned}W_{\rm Gl}^0&=-\frac{1}{2}\hbar v\,\sigma_T^{NN}(\alpha+i)A\rho\\&=-\frac{1}{2}\hbar[\hbar k_{NN}/(m/2)]\sigma_T^{NN}(\alpha+i)A\rho\\&=A[\hbar^2k_{NN}\sigma_{NN}/m]\rho\\&=\hbar^2k\sigma_{NN}\rho/m,\end{aligned} \quad (6)$$

using $k=Ak_{NN}$, where $\hbar k$ is the relative, or c.m., AA momentum, $\hbar k_{NN}$ is the same momentum for a nucleon pair, and m is the nucleon mass. Considering $^{12}C+{}^{12}C$ at $E_{\rm lab}=500$ MeV for definiteness and using the empirical value $V_0\approx200$ MeV, the internal c.m. kinetic energy of the two nuclei is $250+200=450$ MeV, giving $k_{\rm c.m.}=11.5$ fm^{-1}. Using $r_0=1.2$ fm for the nuclear radial parameter gives $\rho=0.138$ fm^{-3}, and using the empirical $\sigma_{NN}=7.3$ fm^2 at the corresponding $E_{\rm lab}^{\rm int}=75$ MeV/nucleon finally gives $W_{\rm Gl}^0=484$ MeV. This is some 19 times deeper than the typical $W_0=25$ MeV found empirically.

In other words, the $t_{\rho\rho}$ approximation to the $^{12}C+{}^{12}C$ optical potential at $E_{\rm lab}/A=40$ MeV grossly overestimates its imaginary part and predicts the scattering to be totally absorptive. This simple result is the essence of our message, and it is this which leads us to conclude, by comparison, that many (A_P+A_T) systems, with A_P or A_T or both below 25, are unusually transparent in this energy range. Although the factor of 19 is specific to the square-well potential with equal geometries for $V(r)$ and $W(r)$, numerical evaluation of the folding integral of Eq. (2) produces similar values for all light ions investigated.

It is useful to note that the estimate given by Eq. (6) can be obtained even more simply, by defining the real and imaginary parts of the relative momentum for propagation of the projectile through the ''infinite nuclear medium'' represented by the target at complete overlap:

$$\frac{\hbar^2(k+i\kappa)^2}{2\mu}=E_{\rm c.m.}+V_0+iW_0, \quad (7)$$

with $\mu=M_1M_2/(M_1+M_2)=Am/2$ for $(A+A)$ scattering. Thus

$$V_0+E_{\rm c.m.}=\hbar^2(k^2-\kappa^2)/2\mu\approx\hbar^2k^2/2\mu, \quad (8)$$

$$W_0=\hbar^2k\kappa/\mu, \quad (9)$$

using the empirical result $k\gg\kappa$. Since the system propagates through the medium with the wave function $\psi(x)=e^{i(k+\kappa)x}$, it decays according to $|\psi|^2=e^{-2\kappa x}$, from which

$$1/2\kappa = \lambda_{AA} = 1/\rho\sigma_{AA}, \tag{10}$$

with λ_{AA} the mean free path for propagation through the medium and σ_{AA} the $(A+A)$ total cross section. However, since $k = Ak_{NN}$, we must also have $\kappa = A\kappa_{NN} = A/2\lambda_{NN}$ or

$$\lambda_{AA} = \frac{\lambda_{NN}}{A}, \tag{11}$$

a result which is also given by the Glauber approximation. It is central to explaining the failure of this simple minded approach, for it says that the mean free path of a cluster of A nucleons is smaller than that of a single nucleon by the factor $1/A$. This is directly due to the $t_{\rho\rho}$ assumption that these A nucleons scatter independently. It also clearly requires that $\sigma_{AA} = A\sigma_{NN}$, empirically an overestimate by a factor of 2 or more.

Using $\kappa = \rho A \sigma_{NN}/2$ and $\mu = Am/2$, Eq. (9) reduces directly to the $t_{\rho\rho}$ result, Eq. (6). In their recent textbook, Siemens and Jensen [26] note that exactly this argument, applied to proton-nucleus scattering for $E_p < 100$ MeV, predicts a mean free path for protons in nuclei which is 10 times shorter than that observed—i.e., a W_0 that is 10 times larger than observed. They note further that Jeukenne, Lejeune, and Mahaux [27], using the Brueckner-Hartree-Fock approach to scattering in an infinite medium, together with a local-density approximation for finite nuclei, were successful in attributing this empirical suppression of NN scattering (long mean free path) to effects of the Pauli principle. (Crespo *et al.* [31] have, however, recently expressed certain reservations regarding the local-density approximation.) It thus seems highly likely that the corresponding suppressions in the heavy-ion case, at energies per nucleon not far above the Fermi energy, are likewise a manifestation of Pauli blocking, especially in view of recent indications [24,32] that densities in the overlap regions of these collisions approach twice their ground-state values. Although our interpretation of the observed transparency in terms of Pauli blocking seems the most obvious one, it must for the moment be recognized as conjecture. Indeed, the observation discussed in the following article, that even among these light nuclei some systems are substantially more absorptive than others, provides a warning that properties of individual nuclei (like low-lying level densities) can also play an important role, thus requiring something beyond the simple local-density approximation.

V. SUMMARY AND CONCLUSION

In the entire vast body of strong-interaction collision data, the elastic angular distributions for certain combinations of ''α-particle'' nuclei, like $^{12}C + ^{12}C$ and $^{16}O + ^{16}O$, stand out as being unique at laboratory energies below about 100 MeV/nucleon, in three related ways.

(1) The real part of their interaction is strong enough ($V_0 \sim 200$ MeV) to produce far-side or ''nuclear'' rainbow oscillations in their elastic amplitudes, at certain characteristic bombarding energies [28].

(2) The imaginary part of this same interaction is weak enough ($W_0 \lesssim 30$ MeV) that these non-Fraunhofer oscillations are not ''damped out.'' This requires that the inner or small-ℓ component of the far-side amplitude (which describes deep interpenetration of projectile and target during the collision) be large enough relative to the peripheral component that a detectable interference between the two can occur.

(3) The data available extend to angles well beyond the Fraunhofer crossover region, where the Airy or rainbow oscillations produced by this interference can be seen, unsullied by the higher-frequency Fraunhofer oscillations. This generally requires the measurement of very small cross sections.

It is the requirement that all three of these conditions be satisfied that makes these data so unusual in the strong-interaction world.

Furthermore, it is apparently only when all three are satisfied that the validity of the Glauber description in terms of component (NN) amplitudes can be checked. pp scattering, for instance, is apparently much too weak to exhibit far-side rainbows, at any energy. This may also be true for proton-nucleus scattering, which decreases so fast with increasing angle that a smooth far side at angles beyond the Fraunhofer crossover has never been measured [29]. Consequently we shall presumably never know whether the many fits of Glauber calculations to the insensitive Fraunhofer oscillations are in fact valid. The fact that most of them use the approximation that $\mathrm{Im}[\delta(b)]/\mathrm{Re}[\delta(b)]$ is constant, independent of b, makes them suspect, for this condition is violated by phenomenological optical fits.

However, given that the heavy-ion data are unique and remarkable, to what practical use can they be put? The following article provides a survey of the relevant light heavy-ion data and offers the conjecture that a study of the way in which the radial shape of the optical-potential ratio $W(r)/V(r)$ changes with bombarding energy may, by comparison with the NN parameter α^{-1} from Eq. (2), provide a means of studying the success or failure of the Glauber approximation as a function of collisional impact parameter. If, as we conjecture, the failure arises from neglect of Pauli blocking, this would also provide an assessment of Pauli blocking as a function of impact parameter.

We note in passing that the effect of Pauli blocking on nucleus-nucleus reaction cross sections σ_R was studied some years ago by DiGiacomo *et al.* [30]. They did not find nearly as large an effect as is suggested by the elastic angular distributions, but σ_R is a far less sensitive probe of blocking. It depends primarily on the radius of the imaginary part of the optical potential and is certainly not sensitive to the suppression of deep interior absorption that is responsible for the rainbow phenomena seen in $d\sigma/d\Omega$.

We also observe that the $V_{\mathrm{DF}}(r) = \alpha W_{\mathrm{DF}}(r)$ approximation, in which the real and imaginary parts of the Glauber potential have the same radial shape, is here a result of the zero-range approximation to the NN interaction, in which the angular dependence of the $f_{NN}(\vec{q})$ is neglected. This is reliable at sufficiently low energy, but even at energies where the $\vec{q}$ dependence cannot be neglected, customary Glauber calculations employ the approximation $f_{NN}(\vec{q}) = (\alpha + i)F(\vec{q})$, with $F(\vec{q})$ real, and this also leads to the $V_{\mathrm{DF}}(r) = \alpha W_{\mathrm{DF}}(r)$ result. One might justifiably ask whether it is this ''$(\alpha + i)$'' approximation, and not the more fundamental large-k approximations of the basic Glauber approach, which is the source of the trouble discussed here and which may only be noticeable in very good (i.e., large-q) data on unusually transparent nuclei. It seems unlikely to us, however, that the $(\alpha+i)$ approximation alone could explain the very large overestimate of absorption in the calculations described above or that, in the energy range considered here, the true $\mathrm{Im}[f_{NN}(\vec{q})]/\mathrm{Re}[f_{NN}(\vec{q})]$ could vary rapidly enough with q to produce the remarkable W/V curves displayed in the following article. Whatever $f_{NN}(\vec{q})$ is used, the Glauber approach certainly cannot produce the large W/V differences seen there between ''transparent'' and ''opaque'' nuclei, since in the optical limit the nuclei are identified only by their densities $\rho(r)$. Although $(\alpha+i)$ may be part of the problem, the energies considered here are too low for the Glauber approximation to be valid in any form, and a study of its improvement with increasing energy may provide useful insight into the importance of Pauli blocking in heavy-ion collisions.

Finally, we note that a curious aspect of all this is the remarkable surface transparency found in many empirical optical potentials for these systems: The Woods-Saxon surface-thickness parameters for their real parts are found to be as much as a factor of 2 larger than those of their imaginary parts. This is particularly surprising, since in the low-density matter of the nuclear surface, the Fermi energy (in the local-density approximation) should be correspondingly small, thus reducing Pauli blocking and permitting much stronger absorption, even at these low energies. This is currently a mystery, but one in which we have considerable confidence, for we have found that modifying the empirical potentials which fit the data, to make their real and imaginary tails approximately equal (as the Glauber approximation would predict at these energies), totally destroys the fit. Apparently much remains to be understood about the in-medium NN interaction.

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APPENDIX

The presence of the long-range Coulomb interaction between nuclei has a significant effect on the classical trajectory outside the range of the strong interaction. Lenzi *et al.* [11] included a rough approximation to this Coulomb trajectory effect by doing their straight-line eikonal integral at a ''local'' impact parameter b', which they included in the partial wave sum at the smaller asymptotic impact parameter b, by using ℓ conservation to require that

$$\ell = kb = k'b', \tag{A1}$$

with $k'(b')$ the Coulomb-reduced momentum at the separation b' between the two point charges. This gives

$$b' = [\eta + (\eta^2 + \ell^2)^{1/2}]/k, \tag{A2}$$

with η the usual Sommerfeld parameter. It is this $b' \leftrightarrow \ell$ correlation which was used in Fig. 2, to plot the potential ratio vs ℓ.

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