Two-body part of effective transition operators and higher-multipole transitions in the $1f_{7/2}$-shell nuclei

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$E4$, $E6$, and $M5$ transitions in the $1f_{7/2}$-shell nuclei, for which anomalous effective charges are required to reproduce experiment, are discussed by using the effective transition operators which are calculated within the framework of the first-order perturbation theory. The correlated operators thus obtained consist of one-body and two-body transition operators, the former of which is considered to be the origin of effective charges, while the latter cannot be incorporated into the idea of effective charges. The two-body contributions are very important in those higher-multipole transitions, to the extent that the concept of effective charge breaks down, and they are found indispensable in achieving reasonable agreement with experiment.
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I. INTRODUCTION

Electromagnetic transitions whose multipolarity is higher than ordinary $E2$ or $M1$ are often referred to as higher-multipole transitions. Those transitions are not as ubiquitous in the nuclear chart as $E2$ and $M1$ transitions, since there should be a certain spin gap between initial and final states. There is a small amount of experimental data for the higher-multipole transitions in the $1f_{7/2}$-shell nuclei [1–3], which has been obtained from decays of isomeric states. Theoretical interpretations of those data were carried out, but were found quite unsuccessful and have been abandoned for a long time. In the present paper, we will revisit these phenomena from a more sophisticated theoretical viewpoint based on effective transition operators.

Geesaman *et al.* [1] analyzed a number of $E4$ transitions in ^{44}Sc, ^{52}Mn, and ^{52,53}Fe by assuming the $f^n_{7/2}$ model with phenomenological effective charges. If the effective charges are defined as $e_p=1+\delta e$ for proton and $e_n=\delta e$ for neutron, they found that one has to assume a considerably large additive effective charge of $\delta e=1.1$ for ^{44}Sc in order to reproduce the enhancement of the experimental transition strength, whereas a large negative effective charge of $\delta e=-0.5$ should be assumed for ^{52}Mn and ^{52,53}Fe to explain the observed strong suppression of the transition strengths. Both $E6$ and $M5$ transitions observed in ^{53}Fe by Black *et al.* [2] are also strongly retarded and a large negative polarization charge $\delta e\sim-0.5$ is required for the $E6$ case, as long as the same approach as for $E4$ is adopted. The standard theoretical model requires by implication that effective charges be as less mass dependent and less state dependent as possible, because the origin of effective charges is considered to be a coupling to collective particle-hole excitations, or collective vibration, which should not cause serious mass and state dependence. Actually for the $E2$ transitions, $\delta e=0.9$ is assumed for the $f^n_{7/2}$ model analysis to fit the whole experimental data throughout the shell [4]. Thus, the question arises for the higher-multipole transitions as to why the effective charges are so large, are positive in the first half of the shell, and become negative in the second half of the shell.

It is widely known that valence particles carry an effective charge larger than their bare charge for electric transitions, i.e., δe is positive. The positive additive charge can be produced by the core-polarization mechanism under the assumption of the attractive nuclear interaction. In this respect the observed large negative δe for the higher-multipole transition has been a challenge for nuclear theorists to interpret. Bertsch [5] tried to interpret the negative δe for the $E6$ transition in ^{53}Fe by introducing the two-body spin-orbit interaction into the mixing interactions, but he concluded that the effect of core excitation via such interactions should be too small to account for the observation.

A sophisticated model with an $f^{13}_{7/2}+f^{12}_{7/2}f_{5/2}$ configuration was tried by Gloeckner and Lawson [6] for the $E6$ transition in ^{53}Fe, but it turned out that they also needed to assume the large negative additive charge of $\delta e=-0.4$. Later, more systematic calculations with $f^n_{7/2}+f^{n-1}_{7/2}(p_{3/2},p_{1/2},f_{5/2})^1$ configurations have been carried out [7], and the model was applied to interpret the higher-multipole transitions. A slight difference came out, but no significant improvement has been obtained. It was concluded that the improvement of the wave functions within the $0\hbar\omega$ model space did not help understand at all the large negative effective charges in ^{52}Mn and ^{52,53}Fe and the large positive charge in ^{44}Sc.

In the nuclear shell model, in order to make a calculation viable, one must introduce a model space that consists of a sufficiently small number of valence orbitals which we denote v hereafter. An orbit that is higher in energy than a valence orbit may be referred to as an empty orbit (e), and an orbit being lower than a valence orbit may be referred to as a filled orbit (f), which is usually assumed to be completely filled by nucleons to make an inert core. The general formalism has been developed so far by many authors [8–11], which defines the effective Hamiltonian by which every physical quantity can be calculated within the model space only. Effective transition operators to be used in the model space are calculated by taking into account all $f\rightarrow e$,

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$f \rightarrow v$, and $v \rightarrow e$ type excitations. Within the framework of the first-order perturbation theory, we get both one- and two-body transition operators [8,12]. The core polarization normally includes the $f \rightarrow e$ excitations only and is shown to be renormalized into the one-body operators, supporting the concept of effective charges. On the other hand, the two-body operators obtained from the $f \rightarrow v$ and the $v \rightarrow e$ excitations cannot be incorporated into the effective charges, since the one-body and two-body contributions differ in the dependence on the number of valence particles. The two-body contributions, if appreciably large, might influence the transition matrix elements. It is thus worthwhile making a systematic calculation in order to understand the general tendency of the higher-multipole transitions by constructing the effective transition operators.

In Sec. II, we describe the assumptions made on the wave functions and reformulate the effective transition operators suitable for single-j-shell many-particle configurations in isospin formalism. Results are discussed in Sec. III, and concluding remarks are presented in Sec. IV.

II. THEORY

The $f^n_{7/2}$ configuration is assumed for the sake of simplicity. The two-body effective interactions suitable in this model space are employed from those obtained with the least-squares-fitting calculations by Muto [13]. We use the set $A3$ which is the best-fit parameter set for Ca and Sc isotopes and the set $B5$ for the $N=27$ and 28 isotones. The $A3$ interaction is used in order to get the wave functions of ^{44}Sc, while $B5$ is used for those of ^{52}Mn and ^{52,53}Fe. These interaction matrix elements for the $f^n_{7/2}$ configurations are summarized in Table I together with the empirical matrix elements [14], which are also employed to check the dependence of the transition matrix elements on the wave functions.

TABLE I. Effective interactions $\langle f^2_{7/2}|V|f^2_{7/2}\rangle_J$ and single-particle energy ϵ in MeV.

J	$A3$	$B5$	^{42}Sc	^{54}Co
0	−2.718	−2.243	−3.174	−2.528
1	−2.453	−2.453	−2.563	−1.591
2	−0.924	−0.765	−1.588	−1.082
3	−1.139	−1.391	−1.683	−0.250
4	−0.161	+0.287	−0.357	+0.102
5	−1.038	−1.057	−1.663	−0.377
6	+0.316	+0.521	+0.063	+0.557
7	−2.268	−2.514	−2.556	−2.329
ϵ	−8.660	−9.433		

The correlated transition operator $\tilde{f}^{(\lambda)}$ with the tensorial rank λ is defined by the first-order perturbation in the mixing interaction V:

$$\tilde{f}^{(\lambda)} = f^{(\lambda)} + f^{(\lambda)} \frac{Q}{E-H_0} V + V \frac{Q}{E-H_0} f^{(\lambda)}. \tag{1}$$

We adopt the Rayleigh-Schrödinger perturbation expansion, so that the energy denominator is given only by the single-particle energies of the relevant states. It is shown [12] that the $f \rightarrow e$ excitations produce one-body matrix elements, the $f \rightarrow v$ excitations generate both one- and two-body parts, and the $v \rightarrow e$ excitations yield two-body parts. The perturbed matrix elements are therefore recombined as the sum of the one-body and the two-body matrix elements; $\delta f^{(\kappa\lambda)}_{\text{one body}} + \delta f^{(\kappa\lambda)}_{\text{two body}}$, where κ denotes the tensorial rank in isospin space. Letting $|n\alpha_f T_f J_f\rangle$ and $|n\alpha_i T_i J_i\rangle$ be the many-particle wave functions of the final and initial states, respectively, we have the specific expression for the matrix elements for the correlated operators. The one-body parts of the effective-operator matrix elements are given by

$$\delta f^{(\kappa\lambda)}_{\text{one body}} = \langle n\alpha_f T_f J_f \| u^{(\kappa\lambda)}(j,j) \| n\alpha_i T_i J_i \rangle \Bigg[\sum_{ph} \left(-\frac{1}{\epsilon_p - \epsilon_h} \right) \{ (h\|f^{(\kappa\lambda)}\|p)(hp|\overline{V}|jj)_{\kappa\lambda} + (jj|\overline{V}|ph)_{\kappa\lambda}(p\|f^{(\kappa\lambda)}\|h) \}$$
$$+ \sum_{h} \left(-\frac{1}{\epsilon_j - \epsilon_h} \right) \{ (h\|f^{(\kappa\lambda)}\|j)(hj|\overline{V}|jj)_{\kappa\lambda} + (jj|\overline{V}|jh)_{\kappa\lambda}(j\|f^{(\kappa\lambda)}\|h) \} \Bigg] \tag{2}$$

Here, ϵ_p, ϵ_h, and ϵ_j represent the single-particle energies for the particle, hole, and $j = f_{7/2}$ orbits, which are calculated by using the extrapolation formulas [15]. The term $(hp|\overline{V}|jj)_{\kappa\lambda}$ denotes the p-h interaction matrix element which is defined as

$$(hp|\overline{V}|h'p')_{TJ} = \sum_{T'J'} (-)^{1-T'+h+p'-J'} (2T'+1)(2J'+1) W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}; T'T) W(ph'hp'; J'J) \{ (ph'|V|hp')_{T'J'}$$
$$- (-)^{1-T'+h+p'-J'} (ph'|V|p'h)_{T'J'} \}, \tag{3}$$

where $(ab|V|cd)_{T'J'}$ is the matrix element between the nonantisymmetrized two-particle states. The two-body parts of the effective-operator matrix elements are given by

$$\delta f^{(\kappa\lambda)}_{\text{two body}} = \sum_{\alpha_1 T_1 J_1, \alpha_2 T_2 J_2} \langle n\alpha_f T_f J_f \| U^{(\kappa\lambda)}(\alpha_1 T_1 J_1, \alpha_2 T_2 J_2) \| n\alpha_i T_i J_i \rangle \Bigg[\sum_h \left(-\frac{1}{\epsilon_h - \epsilon_j} \right) \{ \langle 2\alpha_1 T_1 J_1 \| f^{(\kappa\lambda)} \| jh; T_2 J_2 \rangle \times \langle jh|V|2\alpha_2 \rangle_{T_2 J_2} + \langle 2\alpha_1 |V| jh \rangle_{T_1 J_1} \langle jh; T_1 J_1 \| f^{(\kappa\lambda)} \| 2\alpha_2 T_2 J_2 \rangle \} + \sum_p \left(-\frac{1}{\epsilon_p - \epsilon_j} \right) \{ \langle 2\alpha_1 T_1 J_1 \| f^{(\kappa\lambda)} \| jp; T_2 J_2 \rangle \times \langle jp|V|2\alpha_2 \rangle_{T_2 J_2} + \langle 2\alpha_1 |V| jp \rangle_{T_1 J_1} \langle jp; T_1 J_1 \| f^{(\kappa\lambda)} \| 2\alpha_2 T_2 J_2 \rangle \} \Bigg]. \tag{4}$$

The matrix element of the one-body unit-tensor operator between the basis functions $|n\alpha TJ\rangle$ and $|n\overline{\alpha TJ}\rangle$ is given explicitly as the sum of products of fractional parentage coefficients and Racah coefficients as follows:

$$\begin{aligned} \langle n\alpha TJ \| u^{(\kappa\lambda)}(j,j) \| n\overline{\alpha TJ} \rangle = n \sum_{\alpha' T' J'} & \langle n\alpha TJ \{ |n-1\alpha' T' J', j \rangle \langle n-1\alpha' T' J', j| \} n\overline{\alpha TJ} \rangle \\ & \times (-)^{T'-1/2-\overline{T}+\kappa} \sqrt{(2T+1)(2\overline{T}+1)}\, W(\tfrac{1}{2} T \tfrac{1}{2} \overline{T}; T' \kappa) \\ & \times (-)^{J'-j-\overline{J}+\lambda} \sqrt{(2J+1)(2\overline{J}+1)}\, W(jJj\overline{J}; J'\lambda). \end{aligned} \tag{5}$$

The matrix element of the two-body unit-tensor operator is given explicitly as the sum of products of double-fractional parentage coefficients and Racah coefficients by

$$\begin{aligned} \langle n\alpha TJ \| U^{(\kappa\lambda)}(2\alpha_1 T_1 J_1, 2\alpha_2 T_2 J_2) \| n\overline{\alpha TJ} \rangle = \frac{1}{2} n(n-1) \sum_{\alpha'' T'' J''} & \langle n\alpha TJ \{ |n-2\alpha'' T'' J'', 2\alpha_1 T_1 J_1 \rangle \\ & \times \langle n-2\alpha'' T'' J'', 2\alpha_2 T_2 J_2| \} n\overline{\alpha TJ} \rangle \\ & \times (-)^{T''-T_1-\overline{T}+\kappa} \sqrt{(2T+1)(2\overline{T}+1)}\, W(T_1 T T_2 \overline{T}; T'' \kappa) \\ & \times (-)^{J''-J_1-\overline{J}+\lambda} \sqrt{(2J+1)(2\overline{J}+1)}\, W(J_1 J J_2 \overline{J}; J''\lambda). \end{aligned} \tag{6}$$

The total correlated transition matrix element is therefore obtained by summing the zeroth-order matrix element that is calculated with the free-nucleon operators and the perturbed matrix elements. The transition strength is given as usual by

$$B(\lambda) = \frac{1}{(2T_f+1)(2J_i+1)} \left| \delta_{T_f, T_i} \langle n\alpha_f T_f J_f \| \tilde{f}^{(\kappa=0\lambda)} \| n\alpha_i T_i J_i \rangle + (T_i M_T 10 | T_f M_T) \langle n\alpha_f T_f J_f \| \tilde{f}^{(\kappa=1\lambda)} \| n\alpha_i T_i J_i \rangle \right|^2. \tag{7}$$

As the mixing interaction for the perturbational calculations, we use the following interactions. (1) $M3Y$ interaction (seven ranges with r^2 Yukawa-type tensor components) [16]. (2) Schiffer-True interaction, denoted by ST, the version of which includes two-range central, spin-orbit, and tensor-components [17]. (3) A δ-function interaction [15]: The radial shape is defined by $\delta(r)/r^2$ with the triplet-to-singlet ratio $V_t/V_s = 1.5$ and $V_s = -40$ MeV. (4) The Serber-Yukawa interaction [18] multiplied by 1.3, denoted by SY×1.3: The radial shape is defined by $\exp(-\mu r)/(\mu r)$. The strength parameters are $V_t = -67.6$ MeV with the radial range $1/\mu_t = 1.38$ fm and $V_s = -61.1$ MeV with $1/\mu_s = 1.17$ fm. (5) An ordinary (Wigner) interaction: The radial shape is defined by $\exp(-\mu r)/(\mu r)$. The strength parameters are $V_t = V_s = -50$ MeV with $1/\mu = 1.414$ fm. The harmonic-oscillator radial wave functions are assumed to have the oscillator constant $\nu = 0.96 \times A^{-1/3}$ fm^{-2}, where A denotes the mass number of a nucleus. Since we assume the long-wavelength approximation for the bare electromagnetic operators, nonvanishing perturbed matrix elements appear only up to $\lambda\hbar\omega$ excitations for the $E\lambda$ transitions and $(\lambda-1)\hbar\omega$ excitations for the $M\lambda$ transitions.

III. RESULTS AND DISCUSSIONS

Calculated transition strengths are summarized in Table II and are compared with the experimental ones [1–3]. Contributions from the unperturbed, one-body and two-body operators to each transition matrix element are presented diagrammatically in Fig. 1.

^{44}Sc: The contributions from the one-body operators are twice or three times as large as the unperturbed matrix element that is calculated with use of the bare charges; $e_p = 1$ for the proton and $e_n = 0$ for the neutron. Thus, the core polarization significantly enhances the matrix element and improves the anomalous situation, but the sum of these contri-

TABLE II. Comparison of calculated and experimental [1–3] transition strengths. $f^n_{7/2}$ denotes the calculation within the $f^n_{7/2}$ configurations and with the free-nucleon operators. M3Y, ST, δ function, SY×1.3 and ordinary denote the calculations within the same configurations but with the effective transition operators obtained from the first-order perturbation calculations.

	^{44}Sc $6^+\rightarrow 2^+$ $E4$ $\times 10^3\ e^2\ \mathrm{fm}^8$	^{52}Mn $2^+\rightarrow 6^+$ $E4$ $\times 10^2\ e^2\ \mathrm{fm}^8$	^{52}Fe $12^+\rightarrow 8^+$ $E4$ $\times 10^2\ e^2\ \mathrm{fm}^8$	^{53}Fe $19/2^-\rightarrow 11/2^-$ $E4$ $\times 10^2\ e^2\ \mathrm{fm}^8$	^{53}Fe $19/2^-\rightarrow 7/2^-$ $E6$ $\times 10^5\ e^2\ \mathrm{fm}^{12}$	^{53}Fe $19/2^-\rightarrow 9/2^-$ $M5$ $\times 10^5\ \mu_N^2\ \mathrm{fm}^8$
Experiment	1.9	3.3	$\leqslant 3.6\times 10^{-3}$	6.5	2.8	4.2
$f^n_{7/2}$	0.054	30	11	28	21	44
M3Y	1.1	10	26	16	0.42	9.2
ST	1.4	7.3	25	11	0.12	2.6
δ function	1.9	21	44	24	6.6	0.12
SY×1.3	1.5	17	37	21	3.6	1.1×10^{-3}
Ordinary	0.96	3.0×10^{-2}	8.6	1.6	5.2	4.7

butions is still small and is not sufficient to explain the experiment. The contributions from the two-body operators are roughly of the same size as the one-body contributions and are added with the same phase to the other two, giving a considerably large $E4$ matrix element. When the δ-function interaction is employed, the one-body contribution becomes significantly larger than the others and the total matrix element hits the experimental value. Figure 1 displays a diagrammatic representation of each contribution for the matrix elements.

^{52}Mn: This nucleus has three proton holes and one neutron hole, so far as the $f^n_{7/2}$ model is assumed, and is called the cross-conjugate nucleus of ^{44}Sc which has one proton particle and three neutron particles. The wave functions of the relevant levels with the same spin should be the same, as far as the same effective interactions are assumed throughout the shell. Therefore, the comparison with ^{44}Sc and ^{52}Mn is particularly interesting from a theoretical point of view: to see how cross-conjugate symmetry is broken. A slight difference in the effective interactions between these two nuclei, which causes deviation from the cross-conjugate symmetry, may be acceptable. However, the effective charge of $\delta e = +1.1$ for ^{44}Sc and that of $\delta e = -0.5$ for ^{52}Mn, which are required to fit the experiment, are far too different from each other, to such an extent that the fine-tuning for the effective interactions does not help at all. In ^{52}Mn, the one-body contributions, being smaller than the unperturbed contribution, are added in phase and thus enhance the $E4$ matrix elements, aggravating the disagreement with experiment. On the other hand, the two-body contributions are added out of phase, reducing the $E4$ matrix elements. When the δ function and the SY interactions are used, the two-body contributions are as large as the one-body contributions, whereas the former contributions become considerably larger when the M3Y, the ST, and the ordinary interactions are employed. The one-body contributions are completely wiped out in the latter cases, and furthermore when the ordinary force is used, the unperturbed contribution is reduced dramatically by the two-body contribution being added destructively. This cancellation mechanism explains the reason why the $E4$ matrix element in ^{52}Mn is so small.

^{52}Fe: The contributions from the one-body operators, which are not small, are added constructively to the unperturbed matrix element, producing the enhanced $E4$ matrix elements. The two-body contributions, on the other hand, are added destructively, but the cancellation of the matrix elements is not remarkable and still insufficient to explain the vanishingly small matrix element which is suggested by experiment.

^{53}Fe: For the $19/2^-\rightarrow 11/2^-$ $E4$ transition, the one-body contributions are added in phase to the unperturbed matrix element, giving enhanced $E4$ matrix elements. The two-body contributions are significantly larger than the one-body contributions. They are added destructively and the one-body contributions are completely wiped out. The two-body contributions further reduce the unperturbed contribution. When the ordinary force is employed, the total $E4$ matrix element becomes very close to the experimental value.

For the $19/2^-\rightarrow 7/2^-$ $E6$ transition, the one-body contributions, though not so large, are added in phase to the unperturbed matrix element, when the δ function and the SY forces are used as the mixing interaction, whereas they are added out of phase, when the M3Y, the ST, and the ordinary interactions are adopted. It is noted that the former group of interactions do not have odd components, while the latter group have strongly attractive odd components. This indicates that $E6$ matrix elements depend delicately on the mixing interaction and that the idea for positive δe due to the attractive mixing interaction might be too naive. The two-body contributions are, on the other hand, much larger than the one-body contributions, and contribute considerably to cancellation of the unperturbed contribution, giving nice agreement with experiment. It should be noticed that the experimental matrix element is defined by $M_{\mathrm{expt}} = \pm\sqrt{B(E6)_{\mathrm{expt}}}$.

For the $19/2^-\rightarrow 9/2^-$ $M5$ transition, the one-body contributions reduce the unperturbed matrix element appreciably, but there still remains a large discrepancy between theory and experiment. The two-body contributions are very large and further reduce the matrix elements. In the cases with the δ function and the SY forces, the cancellation is almost complete and the total $M5$ matrix elements diminish. When the ordinary force is used, the sum of the one-body and two-body contributions becomes larger than the unperturbed one, and the theoretical $M5$ matrix element, changing its sign, becomes very close to the experimental value.

In order to check the dependence on the wave functions,

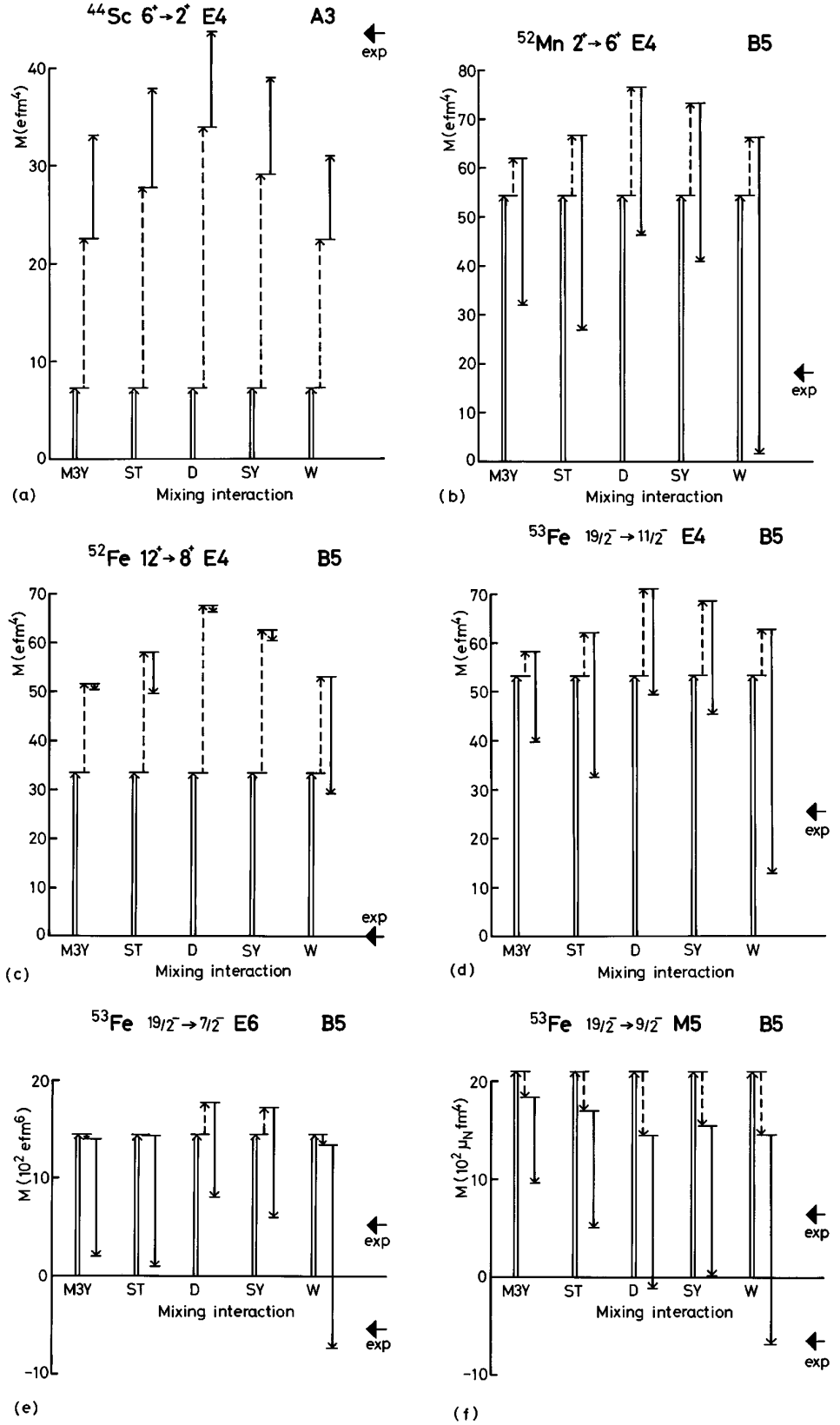

FIG. 1. Decomposition of the transition-matrix elements into the unperturbed, one-body, and two-body contributions, which are denoted by the double solid, the dashed, and the solid lines, respectively. M3Y, ST, D, SY and W represent the calculations with the effective operators generated from the M3Y, ST, δ function, SY$\times$1.3, and the ordinary (Wigner) interactions, respectively. $A3$ and $B5$ denoted in the upper-right position in the figure indicate the effective interactions within the $f^n_{7/2}$ configurations. The unperturbed contribution is taken to be positive in each figure. An experimental matrix element is defined by $M_{\rm expt}=\pm\sqrt{B(\lambda)}$ and is indicated by the horizontal arrow.

we make calculations replacing the $A3$ interaction by the ^{42}Sc interaction for ^{44}Sc and replacing the $B5$ by the ^{54}Co for ^{52}Mn and ^{52,53}Fe. For the $E4$ transition in ^{44}Sc, the calculated matrix elements become smaller typically by 10%. For the transitions in the latter half of the shell, the calculated matrix elements remain almost the same or become slightly smaller by less than 10%. When the great cancellation in the matrix elements has already been achieved, the change is significantly larger, but the matrix elements newly obtained still remain very small. Therefore, the discussions based on the $A3$ and $B5$ interactions hold for the calculations with the ^{42}Sc and ^{54}Co interactions.

IV. SUMMARY AND CONCLUSIONS

The effective transition operators are calculated within the framework of the first-order perturbation theory. The correlated operators thus obtained consist of the one-body and the two-body operators: The former can be incorporated into the effective charges, while the latter cannot. The $E4$, $E6$, and $M5$ transitions in the $1f_{7/2}$-shell nuclei, for which anomalous effective charges are required to reproduce experiment, are studied by using these effective transition operators.

The matrix elements obtained from the two-body operators are generally large and become even larger in most cases than the one-body matrix elements in the second half of the shell. The two-body contributions play a vital role and are indispensable for an understanding of the enhancement of the $E4$ matrix element in ^{44}Sc and the strong suppression of the $E4$, $E6$, and $M5$ matrix elements in the latter half of the shell. The changes in the wave functions, coming from the difference in the two-body effective interactions used in the $f^n_{7/2}$ model, do not cause any significant change in the transition matrix elements. We therefore conclude that reasonable agreement with experiment is obtained, only by properly taking into account the two-body contributions. Consequently the concept of effective charge, employed in previous analyses, has limitations in discussing these phenomena.

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[1] D. F. Geesaman, R. L. McGrath, J. E. Noé, and R. E. Malmin, Phys. Rev. C **19**, 1938 (1979).
[2] J. N. Black, Wm. C. McHarris, W. H. Kelly, and B. H. Wildenthal, Phys. Rev. C **11**, 939 (1975).
[3] H. K. Walter, A. Weitsch, and H. J. Welke, Z. Phys. **213**, 323 (1968).
[4] W. Kutchera, in *Proceedings of the Topical Conference on Physics of Medium-Light Nuclei*, Florence, 1977, edited by P. Blasi and R. A. Ricci (Editorice Compositori, Bologna, 1978), p. 120.
[5] G. F. Bertsch, in *Two-Body Forces in Nuclei*, edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972), p. 243.
[6] D. H. Gloeckner and R. D. Lawson, Phys. Rev. C **11**, 1832 (1975).
[7] A. Yokoyama and H. Horie, Phys. Rev. C **31**, 1021 (1985).
[8] F. Khanna, M. Harvey, D. W. L. Sprung, and A. Jopko, in *Two-body Forces in Nuclei* [5], p. 229; M. Harvey and F. C. Khanna, Nucl. Phys. **A152**, 588 (1970); **A155**, 337 (1970); F. C. Khanna, H. C. Lee, and M. Harvey, *ibid.* **A164**, 612 (1971).
[9] A. Rimini, in *Theory of Nuclear Structure: Trieste Lectures 1969* (International Atomic Energy Agency, Vienna, 1970), p. 713.
[10] P. J. Ellis and E. Osnes, Rev. Mod. Phys. **49**, 777 (1979).
[11] B. H. Brandow, Rev. Mod. Phys. **39**, 771 (1967).
[12] H. Horie, J. Phys. Soc. Jpn. Suppl. **34**, 461 (1973).
[13] K. Muto, Master thesis, Tokyo Institute of Technology, 1975.
[14] W. Kutchera, B. A. Brown, and K. Ogawa, Riv. Nuovo Cimento **1**, 1 (1978).
[15] H. Noya, A. Arima, and H. Horie, Prog. Theor. Phys. Suppl. **8**, 33 (1958).
[16] W. G. Love, in *The* (p,n) *Reaction and the Nucleon-Nucleon Force*, edited by C. D. Goodman, S. M. Austin, S. D. Bloom, J. Rapaport, and G. R. Satchler (Plenum, New York, 1980), p. 23.
[17] J. P. Schiffer and W. W. True, Rev. Mod. Phys. **38**, 191 (1976).
[18] A. de-Shalit and H. Feshbach, *Theoretical Nuclear Physics Vol. 1: Nuclear Structure* (Wiley, New York, 1974).