Skyrme-model πNN form factor and nucleon-nucleon interaction

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We apply the strong πNN form factor, which emerges from the Skyrme model, in the two-nucleon system using a one-boson-exchange (OBE) model for the nucleon-nucleon (NN) interaction. Deuteron properties and phase parameters of NN scattering are reproduced well. In contrast to the form factor of monopole shape that is traditionally used in OBE models, the Skyrme form factor leaves low-momentum transfers essentially unaffected while it suppresses the high-momentum region strongly. It turns out that this behavior is very appropriate for models of the NN interaction and makes it possible to use a soft pion form factor in the NN system. As a consequence, the πN and the NN systems can be described using the same πNN form factor, which is impossible with the monopole. [S0556-2813(97)00503-7]

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I. INTRODUCTION

It is well established that boson-exchange models are very successful in describing the low-energy nucleon-nucleon (NN) interaction [1]. Examples for such models are the Nijmegen [2], Paris [3], and Bonn [4] potentials [5]. Typically, these models take into account the non-strange mesons with masses below 1 GeV plus a 2π-exchange contribution. If the latter is approximated by a scalar-isoscalar boson (with mass 500–700 MeV), one speaks of the one-boson-exchange (OBE) model.

In meson-exchange models for the NN interaction, the meson-nucleon vertices are, in general, multiplied with so-called form factors, which are needed to avoid divergences in loop integrals. While the vertices are derived from effective meson-nucleon Langrangians which the models are based upon, the form factors are introduced essentially *ad hoc* and do not emerge from the underlying Lagrangians. Though the substructure of hadrons provides, in principal, a physical picture and justification for the form factors, in most OBE models no attempt is made to use form factors that have a theoretical basis in QCD or QCD-related models. Instead, a phenomenological ansatz is used for the form factor, like

$$F_\alpha(\mathbf{q}^2) = \left(\frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + \mathbf{q}^2} \right)^{n_\alpha}, \qquad (1)$$

where $\mathbf{q}$ is the three-momentum transfer, m_α the mass of the exchanged meson, and Λ_α the so-called cutoff mass; $n_\alpha = 1$ defines the monopole form factor and $n_\alpha = 2$ the dipole. In the contruction of OBE potentials, the cutoff parameters Λ_α are adjusted (together with the meson-nucleon coupling constants) such as to yield an optimal fit of the NN data. Typical values for Λ_α range between 1.3 and 2 GeV [4].

An example for a QCD-inspired form factor is the cloudy-bag form factor for the pion [6], which is given by

$$F_{\mathrm{CB}}(\mathbf{q}^2) = \frac{3 j_1(|\mathbf{q}|R)}{|\mathbf{q}|R}, \qquad (2)$$

where j_1 denotes the spherical Bessel function and R is the bag radius. The cutoff mass used in Eq. (1) with $n_\pi = 1$ and the R used in Eq. (2) are roughly related by $\Lambda_\pi = \sqrt{10}/R$, which implies $\Lambda_\pi \approx 780$ MeV for $R \approx 0.8$ fm. Unfortunately, pion form factors with these (seemingly very reasonable) parameters fail in the NN system, since they cut out too much of the tensor force provided by the pion: the deuteron quadrupole moment and asymptotic D/S state ratio and the ϵ_1 mixing parameter of NN scattering (which all depend crucially on the nuclear tensor force) come out too small [7]. A possible cure for this problem is the introduction of new short-range tensor-force generating mechanisms in the NN system, like the exchange of a heavy pion, $\pi'(1300)$ [8], which can also be viewed as a contribution from correlated π-ρ exchange [9]. However, this requires one to take the meson-exchange mechanism seriously at a very short distance between the interacting nucleons (namely, a distance equivalent to an exchanged mass of about 1300 MeV, that is ≈ 0.15 fm). This may be in conflict with the implications of a soft pion form factor ($R \approx 0.8$ fm), which leaves no room for the exchange of mesons or meson systems heavier than 1 GeV.

Another aspect of the problem is that models for πN scattering seem to require a soft πNN form factor ($\Lambda_\pi \approx 800$ MeV or $R \approx 0.8$ fm), if the analytic expressions Eqs. (1) or (2) are used for the πNN form factor [10]. Thus, with these types of form factors, it is impossible to describe the πN and NN systems consistently.

One reason for this problem may simply be that the shapes of the form factors conventionally used are not very appropriate. Note that simplicity and convenience is traditionally the main argument for Eq. (1).

Recently the strong πNN form factor has been extracted from Skyrme-type models which comprise the essential low-energy features of QCD in effective nonlinear meson dynamics and the description of nucleons as solitons in meson fields. It turned out that the shape of the resulting form factor is quite different from the conventional monopole form. This

suggests one take another look at the NN system to find out whether the implications of these models could be helpful for the form factor discussion.

The Skyrme model in its ''adiabatic'' approximation is able to give a quite convincing and unified description of the essential features of the πN system throughout and even beyond the resonance regions in all elastic scattering channels except for the S and P channels [11]. This qualitative statement should be seen in the light of the fact that the model contains only *one* free parameter, the strength of the Skyrme term. For the S and P waves the adiabatic approximation is not sufficient, due to the interplay between the collective zero modes and the continuum of soliton fluctuations. Although this makes it technically quite involved to analyze elastic πN scattering at low momentum transfers in the S and P channels, the Skyrme model has been shown to provide the right amount of isospin-independent background scattering and isospin splitting in the S channels [12], as well as in the $P13$ and $P31$ channels [13], and an accurate description of the $P33$ resonance [14]. Only in the $P11$ channel does the rise in the phase shift set in at too low energies due to the rather low-lying Roper resonance [13]. Again, this qualitative result is achieved with one parameter. Extensions of the Skyrme model (to chiral order 6, or inclusion of vector mesons [15]) can improve the agreement in some instances at the expense of additional parameters, but there has never been an attempt to find an optimal version which would quantitatively cover the experimental data in all scattering channels.

It should, perhaps, be noted that these results for S and P waves at low energies were obtained in a K-matrix unitarization which probably is not very sensitive to the high-energy cutoff of an underlying form factor. But it is at very low-momentum transfers where the Skyrme model form factor deviates crucially from the standard monopole type, and it is this difference which has been shown to significantly improve the agreement with the observed shape of the $P33$ resonance [14].

Altogether it is a fair statement to say that the Skyrme model and appropriate extensions work reasonably well in the πN system although this statement has not been analyzed in terms of underlying form factors (except for the case of the $P33$ resonance in [14]). It is therefore an interesting question to ask whether form factors extracted from the Skyrme model will work in the NN system. It is the purpose of this paper to investigate this question.

In Sec. II, we derive the strong πNN form factor in the Skyrme model, and in Sec. III we apply this form factor in the NN system. The paper is concluded in Sec. IV.

II. THE STRONG πNN FORM FACTOR

Analyzing the meson-baryon scattering S matrix in the soliton sectors of effective meson Lagrangians does not require one to separately consider meson-baryon form factors: the spatial structure of the interaction is determined by the self-consistently calculated soliton profiles which naturally enter in a consistent way into the scattering equations. This holds, of course, also for the analysis of the baryon-baryon interaction, or for the structure of the deuteron or other nuclei. Still, there have been attempts to extract meson-baryon

form factors from soliton solutions of mesonic actions, which would allow for a comparison with form factors typically used in conventional meson-exchange models of the baryon-baryon interaction.

In a fully consistent formulation in terms of soliton and soliton fluctuations the resulting S matrix will not depend on the choice of the field which interpolates between the asymptotic mesonic scattering states. Similarly, a form factor to be used for dressing conventional meson-baryon vertices should not depend on the choice of the interpolating field from which it is extracted. This raises the question whether it is possible at all to unambiguously extract form factors from effective meson theories. In the following we will argue that this is indeed possible if one takes due care of the local metric associated with a given choice of interpolating field.

These metrical factors have been disregarded in early attempts to relate the strong form factors to the soliton profiles [16,17]. The procedure suggested by Cohen [16] led to a shape of $G_{\pi NN}(t)$ which for small values of the momentum transfer q^2 was roughly compatible with the conventionally used monopole form, Eq. (1), but the resulting values of $\Lambda \approx 0.6$ GeV were less than half of the 1.3–1.7 GeV typically required in OBE potentials [1,4]. Later extensions including vector mesons explicitly in the effective action [17] led to some improvement ($\Lambda \approx 0.85$ GeV) without really resolving the problem.

In the following, we first give the general argument how the procedure in Refs. [16,17] to relate the strong form factors to the soliton profiles should be modified. We then calculate the πNN form factors for a purely pionic effective action (for the Skyrme model, and for its extension to chiral order six) and for the standard minimal action which includes ρ and ω mesons.

The procedure followed in Refs. [16,17] is based on the equation of motion (EOM) for a pion field π coupled to a (fermionic) axial source

$$(\Box + m_\pi^2)\pi^a(x) = J_5^a(x). \tag{3}$$

Taking matrix elements for nucleon states and using translational invariance leads to

$$(-q^2 + m_\pi^2)\langle N(p')|\pi^a(0)|N(p)\rangle = \langle N(p')|J_5^a(0)|N(p)\rangle \tag{4}$$

with $q = p' - p$. The matrix element on the right-hand side defines the form factor $G_{\pi NN}$ through

$$\langle N(p')|J_5^a(0)|N(p)\rangle = G_{\pi NN}(-q^2)\bar{u}(p')i\gamma_5\tau^a u(p) \tag{5}$$

while the matrix element on the left-hand side to lowest order in $\hbar/N_c$ is the Fourier transform of the classical meson field

$$\langle N(p')|\pi^a(0)|N(p)\rangle = \int e^{iqx}\pi_{cl}^a(x)dx. \tag{6}$$

Through Eqs. (4), (5), and (6) the πNN form factor thus is expressed in terms of the classical solution for the chiral field. It implies that in an EOM for the fluctuating pion field derived from any chiral effective action (conveniently formulated in terms of a unitary matrix field $U = \sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}$)

$$(\Box + m_\pi^2)\pi^a(x) = J_5^a[U(x)] \qquad (7)$$

the matrix elements of the functional $J_5^a[U(x)]$ in baryonic configurations may be identified with the corresponding fermionic matrix elements of $J_5^a(x)$.

It should be noted, however, that the EOM derived from some effective meson action is not immediately obtained in the form (7), because the kinetic part will generally contain a local metric. Only after a field redefinition to absorb this metric into the chiral field the correspondingly transformed source function can be compared with the fermionic matrix elements and the form factor. Evidently, this metric can only be identified from the time-derivative part of the action, because any deviation of the spatial part from the required structure $\nabla^2\pi^a$ could be absorbed into the source function $J_5^a[U(x)]$ without a redefinition of the field.

In terms of the Maurer-Cartan forms

$$L^\mu = U^\dagger \partial^\mu U = L_a^\mu \tau_a \qquad (8)$$

the kinetic part $\mathcal{T}$ of the Lagrangian which determines the dynamics of the field fluctuations generally is given by

$$\mathcal{T} = -\frac{f_\pi^2}{2}\int L_a^0 M_{ab} L_b^0 d^3x \qquad (9)$$

with

$$L_a^0 = i[-\dot{\sigma}\pi_a + \sigma\dot{\pi}_a + (\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})_a]. \qquad (10)$$

This also holds for effective theories which contain more than two time derivatives in their chiral action, because $\mathcal{T}$ is obtained by expanding the Lagrangian to second order in the fluctuations. In the Skyrme model and related models the classical field configuration $\pi_{\mathrm{cl}}^a(x)$ which characterizes the baryon is the hedgehog $U_0 = \exp[i\boldsymbol{\tau}\cdot\mathbf{x}F(r)]$ with chiral profile $F(r)$, rotating in isospace. For solitons of this type the only isovector which can appear in the metric tensor M_{ab} is the pion field itself, $(\boldsymbol{\pi} = |\boldsymbol{\pi}|\hat{\boldsymbol{\pi}})$, therefore M_{ab} has to be of the form

$$M_{ab} = M_L\hat{\pi}_a\hat{\pi}_b + M_T(\delta_{ab} - \hat{\pi}_a\hat{\pi}_b) \qquad (11)$$

with longitudinal and transverse metrical factors M_L and M_T depending on σ and $|\boldsymbol{\pi}|$. The metric in Eq. (9) can be removed from the kinetic energy by redefining

$$\tilde{L}_a^0 = M_{ab}^{1/2}L_b^0. \qquad (12)$$

For the hedgehog soliton $\boldsymbol{\pi}$ rotating in isospace with angular velocity $\boldsymbol{\Omega}$, the time derivative $\dot{\boldsymbol{\pi}}$ is purely transverse while the scalar part σ is static

$$\dot{\boldsymbol{\pi}} = \boldsymbol{\Omega}\times\boldsymbol{\pi}, \quad \dot{\sigma} = 0. \qquad (13)$$

This means that in this case $\tilde{L}_a^0$ absorbs only the transverse part of the metric

$$\tilde{L}_a^0 = i\sqrt{M_T}[\sigma\dot{\pi}_a + (\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})_a] \qquad (14)$$

and we have

$$-\tilde{L}_a^0\tilde{L}_a^0 = \dot{\tilde{\pi}}_a\dot{\tilde{\pi}}_a \qquad (15)$$

with redefined field $\tilde{\pi}_a = \sqrt{M_T}\pi_a$. This may seem a bit surprising because π is longitudinal (by definition), but it is clearly a consequence of the fact that the redefinition is determined through the time derivatives of the field.

Combining now Eqs. (4), (5), and (6) with $\pi_{\mathrm{cl}}^a(x)$ replaced by $\sqrt{M_T}\pi_{cl}^a$, the πNN form factor in the Breit frame then is obtained as

$$G_{\pi NN}(q^2) = \frac{8\pi}{3}\frac{M_N f_\pi}{q}(q^2 + m_\pi^2)$$

$$\times \int_0^\infty dr\, r^2 j_1(qr)\sqrt{M_T(r)}\sin F(r), \qquad (16)$$

where $M_T(r)$ is derived from the effective Lagrangian used to determine $F(r)$; M_N and m_π denote the nucleon and pion masses, respectively, and f_π is the pion decay constant. Notice that we have changed our notation in Eq. (16) defining now $q \equiv |\mathbf{q}|$ which will be used for the remainder of this paper.

As a typical example, we consider the standard Lagrangian for pseudoscalars with the dominant fourth- and sixth-order terms

$$\mathcal{L}_{\mathrm{PS}} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}, \qquad (17)$$

$$\mathcal{L}^{(2)} = \frac{f_\pi^2}{4}\int [-\mathrm{tr}L_\mu L^\mu + m_\pi^2\mathrm{tr}(U + U^\dagger - 2)]d^3x, \qquad (18)$$

$$\mathcal{L}^{(4)} = \frac{1}{32e^2}\int \mathrm{tr}[L_\mu, L_\nu]^2 d^3x,$$

$$\mathcal{L}^{(6)} = -\frac{1}{2}\left(\frac{3g_\omega}{m_\omega}\right)^2\int B_\mu B^\mu d^3x \qquad (19)$$

and baryon current $B_\mu = (1/24\pi^2)\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}L^\nu L^\rho L^\sigma$. It leads to the transverse metric to be used in Eq. (16):

$$M_T(r) = 1 + \frac{1}{e^2 f_\pi^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right) + \left(\frac{3g_\omega/m_\omega}{2f_\pi\pi^2}\right)^2\frac{\sin^2 F}{r^2}F'^2. \qquad (20)$$

In the original Skyrme model the term $\mathcal{L}^{(6)}$ is not present. The Skyrme term $\mathcal{L}^{(4)}$ therefore has to be supplied with sufficient strength $(3.5 < e < 4.5)$ to allow for reasonable soliton size. In the presence of a suitable sixth-order term comparable radii can be obtained with reduced fourth-order strength $(6 < e < 7)$. Both terms may be considered as local remnants of eliminated vector mesons. Therefore it may be of interest to extract the πNN form factor also from chiral models with explicit inclusion of vector mesons. Unfortunately, there are many ways to construct such models and for reasons of simplicity and definiteness we select a minimal model which comprises ρ and ω mesons together with the field U in a chiral-covariant way:

$$\mathcal{L}_{\mathrm{VM}} = \mathcal{L}^{(2)} + \mathcal{L}^{(\rho)} + \mathcal{L}^{(\omega)} \qquad (21)$$

with

$$\mathcal{L}^{(\rho)} = \int \left[-\frac{1}{8}\mathrm{tr}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{m_\rho^2}{4}\mathrm{tr}\left(\rho_\mu - \frac{i}{2g}(l_\mu - r_\mu) \right)^2 \right] d^3x, \tag{22}$$

$$\mathcal{L}^{(\omega)} = \int \left(-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu + 3g_\omega\omega_\mu B^\mu \right) d^3x, \tag{23}$$

and $l_\mu = \xi^\dagger \partial_\mu \xi$, $r_\mu = \partial_\mu \xi \xi^\dagger$. Here ξ denotes the square root of U

$$\xi = U^{1/2} = \Sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\Pi}. \tag{24}$$

In this case, to isolate the metric for the pseudoscalars we write the relevant kinetic parts of $\mathcal{L}_{\mathrm{VM}}$ as

$$\mathcal{T} = -\int \left(\frac{f_\pi^2}{4}\mathrm{tr}(l_0 + r_0)^2 + \frac{m_\rho^2}{16g^2}\mathrm{tr}(l_0 - r_0)^2 \right) d^3x. \tag{25}$$

Again, for the rotating classical hedgehog we have

$$\Sigma = 0, \quad \dot{\boldsymbol{\Pi}} = \boldsymbol{\Omega} \times \boldsymbol{\Pi} \tag{26}$$

and obtain

$$\mathcal{T} = \frac{f_\pi^2}{2} \int \left(4\Sigma^2 + \frac{m_\rho^2}{f_\pi^2 g^2}\Pi^2 \right) \dot{\boldsymbol{\Pi}}\cdot\dot{\boldsymbol{\Pi}} d^3x. \tag{27}$$

With the chiral profile $F(r)$ determined from the static minimization of Eq. (21) we have

$$\Sigma = \cos\frac{F}{2}, \quad \Pi = \sin\frac{F}{2}. \tag{28}$$

The transverse metric resulting from Eq. (27) then is

$$M_T(r) = \left(1 + \frac{m_\rho^2}{4f_\pi^2 g^2}\tan^2\frac{F}{2} \right). \tag{29}$$

Replacing the ω mesons by the baryon current in the lowest chiral-order local approximation ($\omega_\mu = -3g_\omega/m_\omega^2 B_\mu$) leads to the sixth-order contribution in Eq. (19). The elimination of the ρ mesons in lowest-order local approximation [$2g\rho_\mu = i(l_\mu - r_\mu)$] leads to the Skyrme term with $e = 2g$. If g is chosen to satisfy the KSRF relation $g^2 = m_\rho^2/(8f_\pi^2)$ with vector meson mass $m_\rho = 770$ MeV, i.e., $g = 2.925$, and $g_\omega \approx g$, both Lagrangians (17) and (21) stabilize solitons of reasonable size. However, it has been observed [18] that after renormalization of loop corrections the effective coupling constants in the soliton sector favor a stronger Skyrme term ($e \approx 4$) and, correspondingly, a weaker sixth-order term ($g_\omega < 1$). This is in accordance with ample past evidence, that the simple Skyrme model creates soliton profiles which are well suited for many applications.

In Fig. 1 we compare the form factors resulting from Eqs. (16) and (20) for the pure Skyrme model with strong Skyrme term and no sixth-order term ($e = 3.5$, $g_\omega = 0$; solid line in Fig. 1), and for the sixth-order extension with $e = 2g = 5.85$, and $g_\omega = 3.1$ (dashed line in Fig. 1). Both cases lead to the same values for the pion-nucleon coupling constant $G_{\pi NN}(0) = 0.99(2M_N/m_\pi) = 13.5$ and the axial cou-
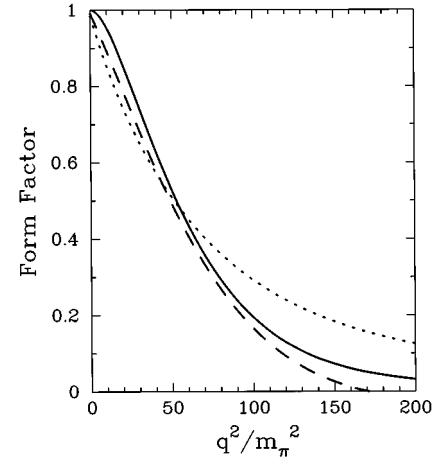

FIG. 1. Form factors emerging from Skyrme-type models. The solid line results from the ''simple'' Skyrme model [Eqs. (16) and (20) with $e = 3.5$ and $g_\omega = 0$], while the dashed line includes a sixth-order term ($e = 5.85$ and $g_\omega = 3.1$). The dotted line is based upon the vector meson model, Eq. (21). All form factors are normalized to unity at the pion pole.

pling constant $g_A = 1.30$. The same values for g and g_ω we use also in Eq. (29) for the form factor from the vector-meson model (dotted line in Fig. 1).

It is interesting to note that the πNN form factor which arises from the vector meson Lagrangian shows approximately a dipole form, Eq. (1) with $n_\alpha = 2$, with $\Lambda_\alpha \approx 1.5$ GeV. The ω mesons do not contribute at all to the pionic metric, because their coupling $\omega_\mu B^\mu$ to the baryon current contains at most one time derivative of the pion field. The term $2\tan^2(F/2)$ in Eq. (29) is due to the chiral invariant form of the ρ-π coupling in Eq. (22) and causes the deviation from the flat metric of $\mathcal{L}^{(2)}$. This results in the dipole form. The form factor derived from the corresponding local approximation (dashed line) shows an almost unchanged slope for small q^2 but it suppresses higher momenta more efficiently and displays small oscillations above $200m_\pi^2$ which may be traced directly to the nonvanishing sixth-order term.

Increasing the strength of the Skyrme term, however, produces a qualitative change in the low-q^2 behavior of the form factor: The soliton profile created through a strong Skyrme term causes the slope of the form factor near $q^2 = 0$ to become very small and, at the same time the curvature to become negative. This means that for small q^2 the effective πNN coupling strength stays much closer to its value at $q^2 = -m_\pi^2$ than for comparable monopole form factors. It is this feature of the Skyrme model which has been shown to improve the agreement of the calculated $P33$ phase shifts in π-N scattering with the data over the whole Δ-resonance region [14]. This very hard behavior of the form factor for small q^2 is compensated by a very soft behavior for $q^2 > 50$ m_π^2 which cuts off higher momenta much more efficiently than typical hard monopole form factors (cf. Fig. 2). Without the sixth-order term ($g_\omega = 0$) the form factor is monotonously decreasing without oscillations.

III. THE TWO-NUCLEON SYSTEM

In this section, we will apply the πNN form factor extracted from the ''simple'' Skyrme model in the NN system.
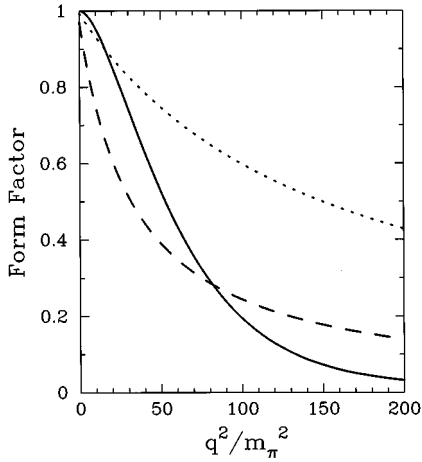
FIG. 2. Comparison of different πNN form factors. The solid line represents the form factor extracted from the ''simple'' Skyrme model (the same as the solid line in Fig. 1). The dashed and dotted lines are monopole form factors [Eq. (1) with $n_\alpha = 1$] with cutoff masses $\Lambda_\pi = 0.8$ and 1.7 GeV, respectively.

To facilitate the comparison with traditional work using monopole (or dipole) meson-nucleon form factors, we choose as our starting point the OBE model of Ref. [1], which has also become known as the Bonn-B potential [19].

A OBE potential is defined as the sum of one-particle-exchange amplitudes (V_α^{OBE}) of certain bosons α with given spin, parity, mass, coupling, etc. We use six bosons. Thus,

$$V(\mathbf{p}',\mathbf{p}) = \sum_{\alpha = \pi,\eta,\rho,\omega,\delta,\sigma} V_\alpha^{OBE}(\mathbf{p}',\mathbf{p})\{F_\alpha[(\mathbf{p}'-\mathbf{p})^2]\}^2 \tag{30}$$

with π and η pseudoscalar, σ and δ scalar, and ρ and ω vector bosons. Each vertex is multiplied with a form factor F_α (i.e., two factors per OBE diagram).

For the unitarizing scattering equation, we use the relativistic three-dimensional reduction of the Bethe-Salpeter equation suggested by Blankenbecler and Sugar [20]:

$$\hat{T}(\mathbf{p}',\mathbf{p}) = \hat{V}(\mathbf{p}',\mathbf{p}) + \int d^3k \hat{V}(\mathbf{p}',\mathbf{k}) \frac{M_N}{\mathbf{p}^2-\mathbf{k}^2+i\epsilon} \hat{T}(\mathbf{k},\mathbf{p}), \tag{31}$$

where $\hat{T}$ denotes the T matrix, and $\mathbf{p}, \mathbf{k}$, and $\mathbf{p}'$ are the initial, intermediate, and final relative three-momenta, respectively, of the two interacting nucleons. The relationship between $\hat{V}$ and V, the amplitude of Eq. (30), is

$$\hat{V}(\mathbf{p}',\mathbf{p}) = \sqrt{\frac{M_N}{E_{p'}}} V(\mathbf{p}',\mathbf{p}) \sqrt{\frac{M_N}{E_p}}, \tag{32}$$

with $E_p \equiv \sqrt{M_N^2 + \mathbf{p}^2}$ and $E_{p'}$ similarly. For further details see Appendix A of Ref. [1] and Ref. [21].

The meson parameters used in the original Bonn-B potential are listed in Table I, column Bonn-B. The phase-shift predictions by Bonn-B for neutron-proton (np) scattering below 300 MeV laboratory energy are shown in Fig. 3 by the dotted lines.

In the Bonn-B model, we replace now the monopole form factor applied to the πNN vertex by the ''simple'' Skyrme model πNN form factor, i.e., Eqs. (16) and (20) with $e = 3.5$ and $g_\omega = 0$ (solid curve in Fig. 2). The form factors of mesons other than the pion are not changed.

We make some minor adjustments of the coupling constants of the vector mesons to optimize the fit of the P-wave phase shifts, and we fine-tune the coupling constant of the sigma boson to accurately fit the S-wave effective range parameters and the deuteron binding energy. The new meson parameters are listed in Table I, column Skyrme FF. The phase-shift predictions for np scattering are plotted in Fig. 3 by the solid lines and deuteron properties are given in Table II. It is clearly seen that the model using the Skyrme form factor (FF) at the pion vertex reproduces the two-nucleon data as well as the original Bonn-B potential.

For comparison, we also show the results obtained when applying a soft monopole form factor (with $\Lambda_\pi = 0.8$ GeV) for the pion; see dashed line in Fig. 3. Note that, as customary in OBE models, the sigma-boson parameters are adjusted such as to fit the S waves. Obviously, a soft pion form factor of monopole shape yields disastrous results for several partial waves of NN scattering. In particular, the mixing parameters, ϵ_1 and ϵ_2, which depend entirely on the nuclear tensor force, are described badly. The same is true for the deuteron, see column ''$\Lambda_\pi = 0.8$'' of Table II. The common reason for

TABLE I. Meson parameters used in the OBE potential models considered in the present work.

Meson	J^P	I	m_α (MeV)	Bonn B[a] $g_\alpha^2/4\pi$ $[f_\alpha/g_\alpha]$	Λ_α (GeV)	n_α	Skyrme FF[b] $g_\alpha^2/4\pi$ $[f_\alpha/g_\alpha]$
π	0^-	1	138.03	14.4	1.7	1	13.5
η	0^-	0	548.8	3	1.5	1	3
ρ	1^-	1	769	0.9 [6.1]	1.85	2	0.9 [6.3]
ω	1^-	0	782.6	24.5	1.85	2	26
δ	0^+	1	983	2.488	2.0	1	2.488
σ [c]	0^+	0	550	8.9437	1.9	1	9.4369
			(720)	(18.3773)	(2.0)	(1)	(19.5806)

[a]Reference [19]; for definition of Λ_α and n_α see Eq. (1).
[b]OBE model that uses the ''simple'' Skyrme model form factor for the pion; see text for details.
[c]The σ parameters given in parenthesis apply to the $T=0$ NN potential, while the unparenthesized values are for $T=1$.
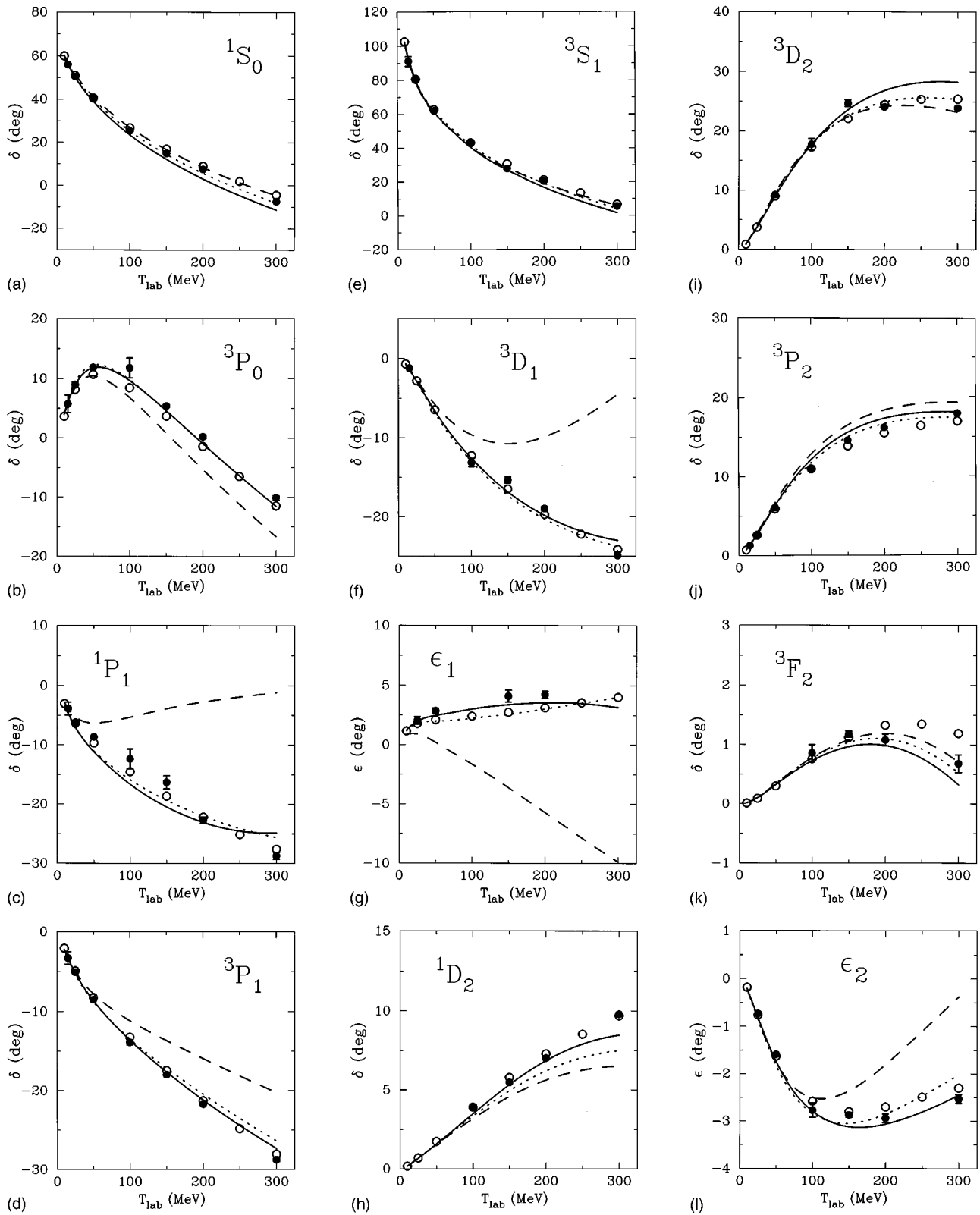
FIG. 3. Neutron-proton (np) phase shifts, δ, and mixing parameters, ϵ, for $J \le 2$ below 300 MeV laboratory energy, T_{lab}. The solid lines show the predictions by the present model using the Skyrme πNN form factor. The dotted lines are the predictions by the original Bonn-B model which applies a monopole form factor with $\Lambda_\pi = 1.7$ GeV at the πNN vertex, while the dashed lines are obtained by applying a monopole with $\Lambda_\pi = 0.8$ GeV for the pion. Open circles represent the Nijmegen multienergy np phase shift analysis [22], and solid dots are from the VPI single-energy analysis VS35 [23].

all these formidable predictions is that the soft monopole also cuts out part of the long-range tensor force created by the pion.

It is interesting to note that, at large momentum transfer ($q^2 \gtrsim 80 m_\pi^2$), the Skyrme FF (solid line in Fig. 2) is even softer than the soft monopole form factor (dashed line in Fig.

TABLE II. Deuteron properties as predicted by OBE potential models discussed in the text and from experiment.

	Bonn-B	Skyrme FF	$\Lambda_\pi = 0.8$[a]	Experiment
Binding energy (MeV)	2.2246	2.22454	2.2246	2.224575(9)[b]
D-state probability (%)	4.99	4.71	2.54	—
Quadrupole moment (fm^2)	0.278	0.274	0.242	0.276(3)[c]
Asymptotic D/S state	0.0264	0.0257	0.0236	0.0256(4)[d]

[a]OBE model that uses a monopole form factor with $\Lambda_\pi = 0.8$ GeV for the pion.
[b]Reference [24].
[c]Corrected for meson-exchange currents and relativity [25–27].
[d]Reference [28].

2). Thus, strong suppression at high-momentum transfer does not cause problems and is, in fact, the desired property of a form factor.

On the other hand, at low q^2, the Skyrme FF stays close to its value at the meson pole and at $q^2 \approx 0$. In contrast, the soft monopole falls off drastically already at low q^2. This causes problems in the NN system since it modifies the long-range part of the nuclear force. It also contradicts the idea of a form factor which is to regularize the short-range interaction.

For many years, it has been a great puzzle why NN models *seemingly* need a very hard πNN form factor. Based upon the above discussion, one can now explain this. Traditionally, OBE models use form factors of monopole shape which have the undesirable feature of cutting down also the low-q^2 region. The only way to avoid this within the monopole concept is to use a very large cutoff mass, like $\Lambda_\pi = 1.7$ GeV in the Bonn-B potential (cf. dotted curve in Fig. 2). This large cutoff mass then suggests that the required form factor is very hard. However, this is misleading. The large cutoff mass is needed to avoid an unreasonable suppression of the low-q^2 (equivalent to long-range) region. If this unwanted low-q^2 suppression can be avoided, a soft form factor is no problem in the NN system. The Skyrme FF proves the point.

There is one last item that deserves attention. The Bonn-B potential uses for the πNN coupling constant the large value $g_\pi^2/4\pi = 14.4$. In models that apply a monopole for the pion, a large value for the πNN coupling constant is needed to predict the deuteron quadrupole moment correctly. However, recent determinations of the πNN coupling constant have yielded the value $g_\pi^2/4\pi = 13.5 \pm 0.1$ [29] which is substantially smaller than the one above. As discussed in Refs. [30,31], the deuteron quadrupole moment is predicted far too small with $g_\pi^2/4\pi = 13.5$ in OBE models using monopole form factors.

An important by-product of our present investigation is the result that there is no such problem when the Skyrme FF is used. We use $g_\pi^2/4\pi = 13.5$ when applying the Skyrme FF for the pion, and the deuteron quadrupole moment, Q_d, is then predicted to be 0.274 fm^2 which is within the empirical range (cf. Table II). Note that, applying a monopole with $\Lambda_\pi = 1.7$ GeV, $Q_d = 0.266$ fm^2 is predicted when $g_\pi^2/4\pi = 13.5$ is used [31]. The deuteron quadrupole moment is a long-range property and, thus, sensitive to the low-q^2

behavior of the form factor. Again, the large values of the Skyrme FF at low q^2 are clearly preferred by the NN system.

IV. CONCLUSIONS

We have shown in this paper how to extract meson-baryon form factors from the soliton sector of effective meson theories which do not depend on the choice of the field that interpolates between the asymptotic meson states. The crucial ingredient is a redefinition of this field to absorb the local metric which characterizes the kinetic energy of the fluctuating field. The axial source in the resulting flat metric then can be used to extract the form factor in the usual way.

We have applied this procedure to two standard examples of effective meson theories: The minimal chiral model for $\pi, \rho,$ and ω mesons, and the Skyrme model (with or without sixth-order extension). Both models work qualitatively well in the πN system at least to the extent we could expect from one- or two-parameter models.

The resulting strong form factors are considerably affected by the respective local metric. Previous attempts [16,17] in which the metrical factors were omitted had led to very soft form factors of the conventional monopole type for low q^2. Our result for the chiral $\pi\rho\omega$ model is close to a dipole form with a cutoff mass of about 1.5 GeV. This difference, however (which originates in the chiral covariant $\rho\pi\pi$ coupling), is not sufficient for substantial improvement in the application of OBE potentials to the two-nucleon system.

On the other hand, the Skyrme term is responsible for a qualitative change in the form factor: It starts with almost vanishing slope and negative curvature for low q^2, and then falls off much faster than comparable monopole form factors. In OBE potentials this very hard behavior for low q^2 provides the necessary strength for the tensor force while at the same time the high momenta are still efficiently cut off. It is remarkable that in order to have the full advantage of this effect it is necessary to employ a Skyrme term with sufficient strength (Skyrme parameter $e \approx 4$, or less). The magnitude of e which is derived from the elimination of ρ mesons ($e = 2g \approx 6 - 7$) is not sufficient. The fact that $e \gtrsim 2g$ does not lead to a satisfactory soliton has been noticed in many instances and is supported by the recent discussion of loop corrections in the soliton sector.

We have applied the πNN form factor based upon the strong Skyrme term in the two-nucleon system using the

OBE model for the *NN* interaction. Deuteron properties and phase parameters of *NN* scattering are reproduced well.

Traditional OBE models use form factors of monopole shape and require a very hard pion form factor. This has been a long-standing puzzle. A comparison of the soft monopole with the Skyrme FF reveals that the latter leaves low-momentum transfers essentially unaffected while the former also suppresses the low-momentum region. To avoid the low-q^2 suppression, the monopole needs a large cutoff-mass parameter which results in an overall hard form factor.

Because of its strong suppression of large momenta, the Skrme FF can be termed as soft. On the other hand, since it does not suppress low momenta, it is compatible with the *NN* system. Deuteron properties can be reproduced with the small πNN coupling constant $g_\pi^2/4\pi = 13.5$, which does not work with the monopole.

In summary, the Skyrme FF is a soft pion form factor that is compatible with the πN and *NN* system. This is impossible to achieve with form factors of monopole shape.

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