Nuclear temperature of the disassembling source in central heavy-ion collisions from isotope yields

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We modify the method of Albergo *et al.* for determining the temperature of an excited nucleus from double ratios of isotope yields and present a statistical model which accounts for the population and decay of excited states of the emitted fragments. Nuclear temperatures are extracted using experimental ratios of isotopic yields of fragments from helium through carbon for the reactions ^{40}Ca + ^{58}Ni, ^{40}Ar + ^{58}Ni, ^{40}Ca + ^{58}Fe, and ^{40}Ar + ^{58}Fe at 33 MeV/nucleon projectile energy. Using the model we obtain consistent values for the temperature from various isotope combinations within the experimental error when accounting for the population and decay of the excited fragments. [S0556-2813(96)50108-1]

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It is well established that heavy nuclei with excitation energies above 2–3 MeV/nucleon disintegrate into multiple fragments of different masses. This multifragmentation process has been the subject of intensive experimental and theoretical studies (see Refs. [1,2] and references therein) aimed to understand the underlying mechanism. Of particular interest is the question whether finite nuclei display a liquid-gas phase transition.

Recently [3], following the method proposed by Albergo *et al.*, [4] double ratios of the yields of He and Li isotopes were used to determine the nuclear temperature as a function of the excitation energy of the emitting source (the spectator) produced in Au+Au collisions at incident energy of 600 MeV/nucleon. An anomalous caloric curve was obtained, similar to that calculated in Ref. [5]. The extracted isotope temperature remained constant at about 5 MeV for excitation energies in the range of 3 to 10 MeV/nucleon, beyond which the temperature increased linearly with excitation energy possibly indicating the onset of a vapor regime. Further, in a recent work [6], the temperature extracted from double ratios of the yields of He and Li isotopes, emitted in central collisions of Ar+Au at 35 MeV/nucleon, was found to be similar to the one determined from relative populations of excited states of isotopes measured in the same heavy-ion reaction.

In the aforementioned results, the analysis was restricted to the yield ratios of He and Li isotopes. It would be informative to extract values of temperature from the yields of heavier isotopes, also taking into account the feeding of experimentally measured ground state populations of emitted fragments through particle- and γ-decay channels of fragments in excited states.

In this work, we modify the method of Albergo *et al.* [4] and present a statistical model which allows us to extract nuclear temperatures from fragment yield ratios taking into account the population of excited states of fragments and the post fragmentation γ and particle decay [7,8]. Using this model we investigate the impact of such feeding processes on the temperature extracted from ratios of fragment yields from helium through carbon produced in the reactions ^{40}Ca + ^{58}Ni, ^{40}Ar + ^{58}Ni, ^{40}Ca + ^{58}Fe, and ^{40}Ar + ^{58}Fe at 33 MeV/nucleon beam energy.

The measurements of isotope yields from He through C for these reactions were performed at the Cyclotron Institute at Texas A&M University (TAMU). Fragment yields were measured with isotopic resolution in telescopes consisting of a gas ionization chamber followed by a pair of ΔE-E silicon detectors and CsI(Tl) crystals. The telescopes covered laboratory angles ranging from 10° to 148° and were placed inside the Neutron Ball [9]. Further details of the experimental setup and analysis for these measurements can be found elsewhere [11]. The isotope yields were extracted from data collected in the ΔE-E silicon detectors of the telescope placed at 40° in the laboratory. The fragments detected at this angle originate predominantly from central events. The centrality was further assured by gating on the measured neutron multiplicity.

In order to obtain the total yields the experimentally observed yields $Y(A,Z)$ as a function of measured energy for each fragment of mass number A and proton number Z were transformed into the center of mass frame where the fragments are assumed to have a uniform angular distribution. A ratio of total yields $Y_{\rm tot}(A',Z')/Y_{\rm tot}(A,Z)$ can then be expressed in terms of the experimentally measured ratio $Y_{\rm exp}(A',Z')/Y_{\rm exp}(A,Z)$ by

$$Y_{\rm tot}(A',Z')/Y_{\rm tot}(A,Z)=Y_{\rm exp}(A',Z')/Y_{\rm exp}(A,Z)\left(\frac{A'}{A}\right)^{3/2}\times\frac{\int_{p_1(A,Z)}^{p_2(A,Z)}\int_{\theta_1(A,Z)}^{\theta_2(A,Z)}\int_{\phi_1(A,Z)}^{\phi_2(A,Z)}f(p,\theta,\phi,A)p^2\sin\theta d\phi d\theta dp}{\int_{p_1(A',Z')}^{p_2(A',Z')}\int_{\theta_1(A',Z')}^{\theta_2(A',Z')}\int_{\phi_1(A',Z')}^{\phi_2(A',Z')}f(p,\theta,\phi,A')p^2\sin\theta d\phi d\theta dp}, \quad (1)$$

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where ϕ,θ and p are the angles and the momentum of a fragment in the lab system, respectively. The limits of integration correspond to the experimental angular and energy ranges in the lab system, and

$$f(p,\theta,\phi,A)=\exp\left[-\frac{p^2+A^2m_n^2v^2-2pAm_nv\sin\theta\cos\phi}{2Am_nT_s}\right], \tag{2}$$

where m_n is the nucleon mass. The slope parameter T_s and the source velocity v were determined from a fit [10] to the experimental kinetic energy spectra of fragments assuming a single source. For 33 MeV/nucleon projectile energy we obtained a good description for the kinetic energy spectra for all fragments. The parameters T_s and $\beta=v/c$ were found to be 19.0 MeV and 0.046, respectively. For a given fragment, the lower and higher energy limits in the laboratory frame were determined by the threshold energy in the first silicon detector and the punch through energy in the second silicon detector of the telescope, respectively.

The procedure developed by Albergo *et al.* [4] was derived assuming both statistical and chemical equilibrium in the emitting source. In this derivation, the chemical equilibrium condition was written in the form

$$\mu(A,Z,T)=Z\mu_p(T)+(A-Z)\mu_n(T)+B(A,Z), \tag{3}$$

where $\mu(A,Z,T)$, $\mu_p(T)$ and $\mu_n(T)$ are the chemical potentials of the fragment (A,Z), the free proton and neutron at the temperature T, respectively, and $B(A,Z)>0$ is the binding (ground state) energy of the fragment (A,Z). If Boltzmann statistics is employed the temperature can be found from the double ratio:

$$R=\frac{Y(A_1',Z_1')/Y(A_1,Z_1)}{Y(A_2',Z_2')/Y(A_2,Z_2)}=F(A_1',Z_1',A_1,Z_1,A_2',Z_2',A_2,Z_2)\times\exp(\Delta B/T),$$

$$F(A_1',Z_1',A_1,Z_1,A_2',Z_2',A_2,Z_2)=\left(\frac{A_1'\cdot A_2}{A_1\cdot A_2'}\right)^{3/2}\frac{[2j(A_1',Z_1')+1][2j(A_2,Z_2)+1]}{[2j(A_1,Z_1)+1][2j(A_2',Z_2')+1]}. \tag{4}$$

Here $j(A,Z)$ and $Y(A,Z)$ are the total angular momentum of the ground state and ground state yield of the fragment (A,Z), respectively, and ΔB is given in terms of the binding energies of the fragments:

$$\Delta B=B(A_1',Z_1')-B(A_1,Z_1)+B(A_2,Z_2)-B(A_2',Z_2'). \tag{5}$$

The fragment yields in Eq. (4) must be selected in such a way that

$$(N_1'=N_1+n,N_2'=N_2+n)\ \text{ and }\ (Z_1'=Z_1+p,Z_2'=Z_2+p) \tag{6}$$

where $A=N+Z$ and n and p are integer numbers.

In Eq. (4) it is assumed that ground state populations are the experimentally observed fragment yields. A possible feeding of the ground state populations through particle and γ decay of excited fragments which takes place after fragments leave the source is, thus, ignored. In the following, we modify the method of Albergo *et al.* to include such a possibility.

In order to take into account the feeding due to γ decay, Eq. (4) was modified by adding the populations of γ-decaying states to the ground state populations for each fragment. One obtains the expression

$$\frac{Y(A_1',Z_1')/Y(A_1,Z_1)}{Y(A_2',Z_2')/Y(A_2,Z_2)}=\left(\frac{A_1'\cdot A_2}{A_1\cdot A_2'}\right)^{3/2}\frac{\omega(A_1',Z_1',T)\omega(A_2,Z_2,T)}{\omega(A_1,Z_1,T)\omega(A_2',Z_2',T)}\times\exp(\Delta B/T), \tag{7}$$

where

$$\omega(A,Z,T)=\sum_{i=1}^{N_\gamma}[2j_i(A,Z)+1]\exp(-\varepsilon_i(A,Z)/T), \tag{8}$$

and the sum extends over the ground state and all γ-decaying states with excitation energy $\varepsilon_i(A,Z)$ and angular momentum j_i of the fragment (A,Z) below the particle-decay threshold energy.

The feeding due to particle decay was taken into account in a similar fashion. In this case, populations of particle-decaying excited states calculated in the framework of the statistical model with chemical equilibrium were also added to the ground state population of the corresponding product fragment. We restricted ourselves only to nucleon and α decay and, at present, neglected multiple-step feeding. The yield $Y(A,Z)$ of the fragment (A,Z) for $A>4$ is then given by

$$Y(A,Z)=A^{3/2}\lambda_T^{-3}\exp\frac{Z\mu_p+(A-Z)\mu_n+B(A,Z)}{T}\times\left[\omega_\gamma(A,Z,T)+\left(\frac{A+1}{A}\right)^{3/2}\omega_p(A+1,Z+1,T)\right.$$
$$\times\exp\frac{\mu_p+B(A+1,Z+1)-B(A,Z)}{T}+\left(\frac{A+1}{A}\right)^{3/2}\omega_n(A+1,Z,T)\exp\frac{\mu_n+B(A+1,Z)-B(A,Z)}{T}$$
$$\left.+\left(\frac{A+4}{A}\right)^{3/2}\omega_\alpha(A+4,Z+2,T)\exp\frac{2\mu_n+2\mu_p+B(A+4,Z+2)-B(A,Z)}{T}\right], \tag{9}$$

$$\lambda_T=\frac{h}{\sqrt{2\pi Tm_n}},$$

TABLE I. Temperatures obtained from isotope yields in heavy-ion reactions at 33 MeV/nucleon projectile energy when the feeding is ignored (third column), when only γ-decay feeding is taken into account (fourth column), and when combined γ- and particle-decay feeding is considered (fifth column).

Isotope yields taken	Reaction	T_0 (MeV)	T_γ (MeV)	$T_{\gamma+p}$ (MeV)
^{3}He, ^{4}He, ^{6}Li, ^{7}Li	Ca+Ni	4.35 ± 0.28	3.89 ± 0.22	3.82 ± 0.38
	Ar+Ni	4.48 ± 0.30	3.99 ± 0.24	3.87 ± 0.38
	Ca+Fe	4.28 ± 0.28	3.83 ± 0.22	3.76 ± 0.36
	Ar+Fe	4.23 ± 0.27	3.79 ± 0.21	3.71 ± 0.35
^{3}He, ^{4}He, ^{9}Be, ^{10}Be	Ca+Ni	7.03 ± 0.72	3.71 ± 0.16	3.82 ± 0.45
	Ar+Ni	7.58 ± 0.84	3.83 ± 0.16	4.00 ± 0.53
	Ca+Fe	7.88 ± 0.90	3.89 ± 0.17	4.09 ± 0.58
	Ar+Fe	6.79 ± 0.67	3.66 ± 0.15	3.80 ± 0.46
^{3}He, ^{4}He, ^{10}B, ^{11}B	Ca+Ni	3.53 ± 0.27	3.20 ± 0.18	3.13 ± 0.35
	Ar+Ni	3.68 ± 0.30	3.30 ± 0.19	3.20 ± 0.35
	Ca+Fe	3.90 ± 0.33	3.43 ± 0.21	3.31 ± 0.37
	Ar+Fe	4.02 ± 0.36	3.51 ± 0.21	3.35 ± 0.36
^{3}He, ^{4}He, ^{12}C, ^{13}C	Ca+Ni	3.70 ± 0.18	3.49 ± 0.16	3.54 ± 0.38
	Ar+Ni	3.63 ± 0.17	3.43 ± 0.15	3.48 ± 0.37
	Ca+Fe	3.68 ± 0.17	3.47 ± 0.16	3.54 ± 0.38
	Ar+Fe	3.71 ± 0.18	3.50 ± 0.16	3.58 ± 0.38
^{6}Li, ^{7}Li, ^{11}C, ^{12}C	Ca+Ni	5.29 ± 0.49	4.27 ± 0.30	–
	Ar+Ni	5.68 ± 0.56	4.50 ± 0.33	–
	Ca+Fe	5.35 ± 0.50	4.31 ± 0.30	–
	Ar+Fe	5.21 ± 0.47	4.22 ± 0.29	–
^{11}C, ^{12}C, ^{12}C, ^{13}C	Ca+Ni	4.14 ± 0.25	3.72 ± 0.19	–
	Ar+Ni	4.12 ± 0.25	3.70 ± 0.19	–
	Ca+Fe	4.20 ± 0.26	3.77 ± 0.20	–
	Ar+Fe	4.22 ± 0.26	3.78 ± 0.20	–
^{6}Li, ^{7}Be, ^{11}B, ^{12}C	Ca+Ni	4.55 ± 0.40	3.74 ± 0.25	–
	Ar+Ni	4.80 ± 0.45	3.90 ± 0.27	–
	Ca+Fe	4.60 ± 0.41	3.78 ± 0.25	–
	Ar+Fe	4.71 ± 0.43	3.85 ± 0.26	–

where m_n is the nucleon mass.

Each term in Eq. (9) gives the contribution to the ground state population from a specific decay channel, where

$$\omega(A,Z,T)=\sum_i\,[2j_i(A,Z)+1]\exp[-\varepsilon_i(A,Z)/T], \tag{10}$$

and the subscripts by $\omega(A,Z,T)$ denote that corresponding excited states with excitation energies $\varepsilon_i(A,Z)$ decay by γ, proton, neutron, or α emission. Given a set of experimental yields of four fragments, one double ratio and two single ratios were constructed using Eq. (9), yielding a system of three independent equations. This system was solved by iteration for the temperature T and chemical potentials μ_p and μ_n.

In Table I the temperatures extracted from Eq. (4), where the feeding is ignored, are compared to the temperatures obtained from Eq. (7), where only the feeding through γ-decay channel is taken into account. According to Eq. (7), the error ΔT in the temperature is given by

$$\frac{\Delta T/T}{\Delta R/R}=\frac{T}{\Delta\overline{\varepsilon}-\Delta B}, \tag{11}$$

$$R=\frac{Y(A_1',Z_1')/Y(A_1,Z_1)}{Y(A_2',Z_2')/Y(A_2,Z_2)},$$

where ΔR is the experimental error in the double ratio R, and

$$\Delta\overline{\varepsilon}=\overline{\varepsilon}(A_1',Z_1',T)-\overline{\varepsilon}(A_1,Z_1,T)+\overline{\varepsilon}(A_2,Z_2,T)-\overline{\varepsilon}(A_2',Z_2',T). \tag{12}$$

Here $\overline{\varepsilon}(A,Z)$ is the average excitation energy at the temperature T for γ-decaying excited states of the fragment (A,Z):

$$\overline{\varepsilon}(A,Z)=\frac{\sum_{i=1}^{N_\gamma}\varepsilon_i(A,Z)[2j_i(A,Z)+1]\exp[-\varepsilon_i(A,Z)/T]}{\sum_{i=1}^{N_\gamma}[2j_i(A,Z)+1]\exp[-\varepsilon_i(A,Z)/T]}. \tag{13}$$

Since ΔT is inversely proportional to ΔB, the temperature found from Eq. (4) or Eq. (7) becomes very sensitive to the experimentally measured yields as ΔB gets small. For this reason, only the double ratios with sufficiently large values of ΔB were considered. In Eq. (7), experimentally measured excited states from Ref. [12] were used. The values of the binding energies were taken from Ref. [13]. Only states with the width $\Gamma<1$ MeV were included. The lifetime of other states (with $\Gamma>1$ MeV) is smaller than 200 fm/c, so we can expect that fragments in these states decay before they leave the source and therefore do not participate in the feeding.

As can be seen from Table I, taking into account γ-decay feeding alone gives quite consistent values of the temperature while ignoring the possibility of such feeding processes leads to large fluctuations in temperatures extracted from different combinations of fragment yield ratios.

When feeding due to both γ and particle decay is taken into account, individual yields of fragments for $A>4$ are given by Eq. (9). Due to the absence of comprehensive data on particle-decaying states of nuclei we made several drastic assumptions for the states selected to enter Eq. (9) via Eq. (10). First, particle-decaying states of fragments $(A+1,\ Z+1)$, $(A+1,\ Z)$, and $(A+4,\ Z+2)$ included in Eq. (10) were assumed to decay either to the ground state or a γ-decaying state of the fragment (A,Z) and have the width $\Gamma<1$ MeV. Furthermore, as in Refs. [7,8], only the dominant decay mode for all excited states was considered. For a specific fragment, the dominant decay mode was taken to be the one with the lowest Q value among proton-, neutron-, and α-decay modes. The lowest particle-decaying excited states that enter Eq. (10) were taken from Refs. [12] and [14] Contributions from states with higher excitation energies were found in terms of the effective level density $\rho^{\mathrm{eff}}(A,Z)$ using

$$\omega(A,Z,T)=\int d\varepsilon\,\rho^{\mathrm{eff}}(A,Z)\exp(-\varepsilon/T). \tag{14}$$

We adopted an empirical formula for the effective level densities of light nuclei obtained in Ref. [7]:

$$\rho^{\mathrm{eff}}(A,Z)=\frac{k_1}{A^{5/3}}\exp(2\sqrt{a\varepsilon})\exp\left(-\frac{(\varepsilon-B_{\min})^2}{2\varepsilon_{\mathrm{cut}}^2}\right),$$

$$\varepsilon>B_{\min}, \tag{15}$$

where

$$a=\frac{A}{8\ \mathrm{MeV}}\left(1-\frac{k_2}{A^{1/3}}\right),$$

$$\varepsilon_{\mathrm{cut}}(A)=\varepsilon_B[(A-3)^{1/2}-1],$$

$$A>4, \tag{16}$$

and $k_1=0.2\ \mathrm{MeV}^{-1}$, $k_2=0.8$, $\varepsilon_B=8$ MeV/nucleon. The second exponential factor in Eq. (15) is the probability that a state with the excitation energy ε above the dominant decay mode threshold energy $B_{\min}$ has the width $\Gamma<1$ MeV. Since the experimentally measured yields of ^{3}He and ^{4}He are much larger than those of heavier fragments, the feeding of ^{3}He and ^{4}He ground state populations from the decay of the excited states of heavier fragments was estimated to be relatively small. We thus neglected the particle feeding of ^{3}He and ^{4}He ground state populations.

The values of temperature for several combinations of fragment yields obtained with the combined γ- and particle-decay feeding are presented in Table I. They are compared to the temperatures obtained when the feeding is completely ignored as well as to those found when only γ-decay feeding is considered. In view of the mentioned assumptions, the temperatures obtained with the combined γ- and particle-decay feeding should be considered as estimates of the impact of such feeding on the calculated temperature. Within these assumptions, we were not able to achieve a convergence of the iteration procedure described above for the sets of yields (^{6}Li, ^{7}Be, ^{11}B, ^{12}C), (^{6}Li, ^{7}Li, ^{11}C, ^{12}C), and (^{11}C, ^{12}C, ^{12}C, ^{13}C). The double ratios of these yields, however, give reasonable values of temperatures when the particle-decay feeding is ignored (see Table I). This may be due to the fact that a double ratio becomes sensitive to the detailed structure of the particle-decaying excited states included in Eq. (9) when heavy fragments with small yields are considered. More detailed experimental information, as it becomes available, as well as development of a more comprehensive theoretical approach to the level density and decay of the particle unstable excited states of light nuclei, are believed to lead to an improvement of these results. However, for combinations of isotope yields listed in Table I the temperatures calculated with the combined γ- and particle-decay feeding do not significantly differ from those obtained with

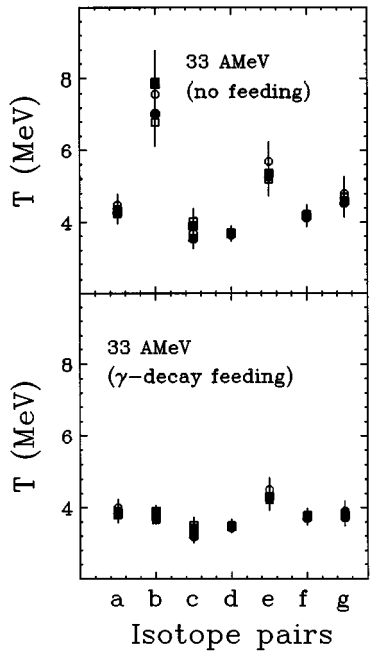

FIG. 1. Temperature extracted from double ratios of (a) ^{4}He/^{3}He, ^{7}Li/^{6}Li, (b) ^{4}He/^{3}He, ^{10}Be/^{9}Be, (c) ^{4}He/^{3}He, ^{11}B/^{10}B, (d) ^{4}He/^{3}He, ^{13}C/^{12}C, (e) ^{7}Li/^{6}Li, ^{12}C/^{11}C, (f) ^{12}C/^{11}C, ^{13}C/^{12}C, (g) ^{7}Be/^{6}Li, ^{12}C/^{11}B yield ratios for the reactions ^{40}Ca + ^{58}Ni (filled circles), ^{40}Ar + ^{58}Ni (open circles), ^{40}Ca + ^{58}Fe (filled squares), and ^{40}Ar + ^{58}Fe (open squares) at beam energies of 33 MeV/nucleon before (top) and after (bottom) accounting for populations of γ-decaying states.

γ-decay feeding only. So, within our model, for the listed yields, the γ-decay feeding has the dominant effect on the calculated temperature.

In summary, ratios of isotope yields of fragments from helium through carbon produced in near central collisions from the reactions ^{40}Ca + ^{58}Ni, ^{40}Ar + ^{58}Ni, ^{40}Ca + ^{58}Fe, and ^{40}Ar + ^{58}Fe at 33 MeV/nucleon projectile energy were used to extract nuclear temperatures for the emission zone. The statistical treatment of Albergo *et al.* was modified to include populations of excited states that γ decay to the ground state. The important effect of the γ-decay feeding is nicely demonstrated in Fig. 1. Inclusion of this correction results in temperature values that are consistent over various isotope/isotone pairs. The extracted temperature is consistent with values obtained from isotope yields and from relative yields of excited state populations in other measurements. The influence of sequential feeding on the extracted temperature was estimated for a few double ratios by considering particle unstable states in higher isotopes. Modifications to the statistical code to include a more comprehensive treatment of sequential feeding is currently under way.

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